Supplementary Material

Table S1. Climatic conditions recorded inside the barn of each herd (hourly loggers), January 2023 to October 2025.

Herd	Days with valid records	Coverage (%)	Mean T (°C)	Mean daily max T (°C)	Mean daily min T (°C)	Mean RH (%)	Mean THI
A	991	98.3	15.0	18.5	12.2	80.4	58.8
B	958	95.1	16.8	19.6	14.3	77.4	61.7
C	976	96.7	15.7	18.4	13.4	77.3	60.0
D	947	93.9	14.3	18.5	11.1	84.1	57.7

T = air temperature; RH = relative humidity; THI = temperature-humidity index. Coverage = days with ≥ 20 hourly readings / days between the first and last record. Precipitation was not recorded, because the loggers were placed inside the barns, where the cows were permanently housed.

Table S2. Sensitivity of the main estimates to model specification.

(a) Severity-band model: within-cow change in milk yield (kg per hour in each THI band, 7-d window).

Specification	68-71: β (SE)	P	72-79: β (SE)	P	≥ 80: β (SE)	P
Cow random effect only (original)	+0.015 (0.004)	<0.001	-0.040 (0.008)	<0.001	-0.180 (0.035)	<0.001
+ herd-test-day	+0.019 (0.011)	0.093	-0.048 (0.025)	0.055	-0.168 (0.110)	0.131
+ herd-test-day + year	+0.016 (0.011)	0.144	-0.048 (0.024)	0.053	-0.192 (0.109)	0.082
+ herd-test-day + year + season (main model)	+0.005 (0.013)	0.687	-0.048 (0.024)	0.047	-0.187 (0.108)	0.089
+ herd-test-day + year × herd + month	-0.015 (0.013)	0.237	-0.021 (0.020)	0.279	-0.170 (0.089)	0.062
Main model excluding $\|z\| > 4$	+0.004 (0.013)	0.758	-0.046 (0.024)	0.056	-0.195 (0.108)	0.076

(b) Distributed-lag non-linear model of daily maximum THI (centred at 64).

Max lag (d)	df var	df lag	Adjustment	Cumulative at THI 80, kg (95% CI)	Peak lag	Dissipation lag
21	4	3	Cow only (original)	-4.96 (-6.07 to -3.85)	1	9
21	4	3	+ herd-test-day	-5.16 (-8.27 to -2.05)	1	7

Max lag (d)	df var	df lag	Adjustment	Cumulative at THI 80, kg (95% CI)	Peak lag	Dissipation lag
21	4	3	+ herd-test-day + year + season (main)	-6.05 (-9.30 to -2.81)	1	7
7	4	3	main	-5.78 (-9.60 to -1.96)	7	7
14	4	3	main	-6.88 (-9.89 to -3.86)	6	10
21	4	4	main	-6.33 (-9.52 to -3.14)	6	9
21	4	5	main	-7.19 (-10.73 to -3.66)	6	9
21	3	3	main	-5.05 (-8.19 to -1.91)	1	8
21	5	3	main	-5.28 (-8.75 to -1.81)	1	7

df = degrees of freedom of the natural cubic splines in the exposure (var) and lag dimensions. Peak lag = lag with the most negative estimate at a daily maximum THI of 80; dissipation lag = first lag after the peak whose 95% CI includes zero. With a maximum lag of 7 d, the minimum falls on the last lag (boundary). Mean correlation between daily maximum THI on consecutive days: r = 0.89.