Supplementary Materials: Definition of Multi–objective Benchmark Problems with Variable Number of Dimensions

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1. Dimension of Pareto Front Members

The dimension of a solution $\boldsymbol{u}$ residing on the Pareto front can be selected based on [1]:

$$D^{(\mathrm{opt})}(\boldsymbol{u}) = D_i^{(\mathrm{opt})}. \tag{S1}$$

where the optimal dimension for the Pareto front member is selected from the list of optimal dimensions $\left\{ D_{\min}^{\mathrm{opt}}, \ldots, D_{\max}^{\mathrm{opt}} \right\}$ having in total N_{opt} members. The index of the i-th member is selected accroding to:

$$i = 1 + \left\lfloor \left(1 - \frac{\theta(\boldsymbol{u})}{\theta_{\mathrm{M}}} \right) (N_{\mathrm{opt}} - 1) \right\rfloor \tag{S2}$$

for $\theta(\boldsymbol{u}) \leq \theta_{\mathrm{M}}$, and

$$i = 1 + \left\lfloor \left(\frac{\theta(\boldsymbol{u})}{\theta_{\mathrm{M}}} \right) (N_{\mathrm{opt}} - 1) \right\rfloor \tag{S3}$$

for $\theta(\boldsymbol{u}) > \theta_{\mathrm{M}}$. The symbol θ_{M} stands for the maximal angle (set to value $\pi/4$ in this study). The angle $\theta(\boldsymbol{u})$ reads:

$$\theta(\boldsymbol{u}) = \arccos \left(\frac{f_2}{\sqrt{f_1{}^2 + f_2{}^2}} \right) \tag{S4}$$

for the two–objective problems, and

$$\theta(\boldsymbol{u}) = \arccos \left(\max \left(\frac{f_1}{\sqrt{f_1{}^2 + f_2{}^2 + f_3{}^2}}, \frac{f_2}{\sqrt{f_1{}^2 + f_2{}^2 + f_3{}^2}}, \frac{f_3}{\sqrt{f_1{}^2 + f_2{}^2 + f_3{}^2}} \right) \right) \tag{S5}$$

for the three–objective problems. In (S.4), and (S.5), f_i, $i = 1, 2, 3$ denote the values of individual objective functions for the solution $\boldsymbol{u}$. By (S.2), and (S.3), the Pareto front is divided into two parts. Dimensions of the $\mathcal{PF}$s graudally decrease in the first part, while they start to gradually increase once the angle $\theta(\boldsymbol{u})$ overcomes the limit θ_{M}. The selection of optimal dimensions for individual members of Pareto front is visualized in Figure S.1.
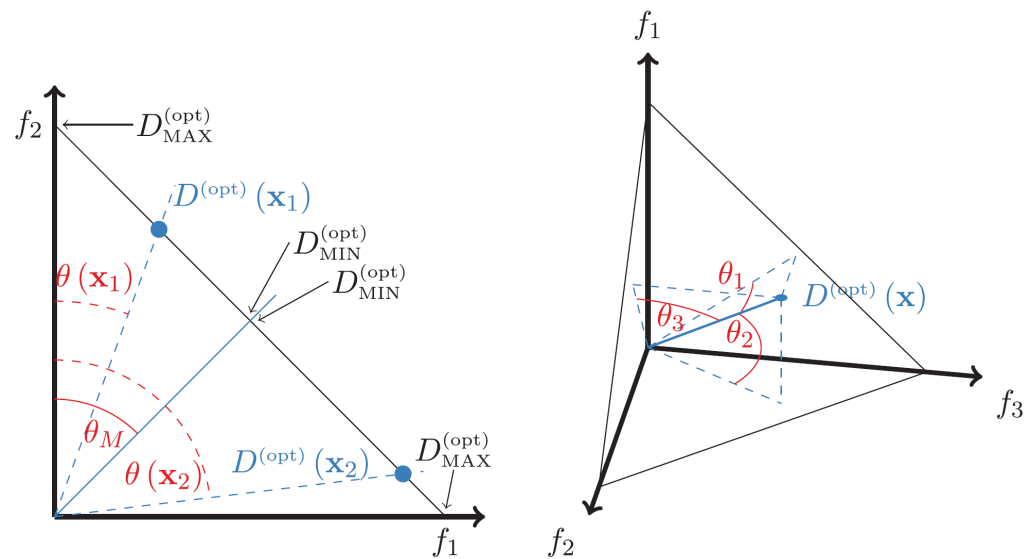
Figure S1. Selection of optimal dimension for two–objective and three–objective Pareto front members [3].

2. Benchmark Problems Definition

This study considers following benchmark problems defined in [2,3]. All the benchmark problems are scalable, so that their true Pareto front can be made from decision space vectors u with arbitrary dimensions. The problems in the following subsections are defined by limimts for individual variables, a set of two, or three objective functions, and a list of feasible dimensions. The problems are defined based on well–known benchmark families DTLZ [4], ZDT [5], LI [1], and LZ [6]. In this study, the list of optimal dimensions is set as $D^{(\text{opt})}$.

2.1. VNDMODTLZ1

Based on DTLZ family of benchmark problems [4].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S6}
$$

Objective functions:

$$
\begin{aligned}
f_1(u) &= \left[1 + h\left(u_{M:N_{\mathrm{D}}}\right)\right] 0.5 u_1 \cdot u_2 + P, \\
f_2(u) &= \left[1 + h\left(u_{M:N_{\mathrm{D}}}\right)\right] 0.5 u_1 (1 - u_2) + P, \\
f_3(u) &= \left[1 + h\left(u_{M:N_{\mathrm{D}}}\right)\right] 0.5 (1 - u_1) + P, \\
h(z) &= 100 \left\{ k + \left[\sum_{i=1}^{k} (z_i - 0.5)^2 - 5\cos(20\pi(z_i - 0.5)) \right] \right\}, \\
P &= 0.50 \left(N_{\mathrm{D}} - D_{opt}\right)^2.
\end{aligned}
\tag{S7}
$$

Here, M stands for the number of objectives, and $k = N_{\mathrm{D}} - M + 1$.

2.2. VNDMODTLZ2

Based on DTLZ family of benchmark problems [4].

Limits:

$$
\begin{aligned}
u_{i,\text{min}} &= 0, \\
u_{i,\text{max}} &= 1.0, \\
i &= 1, 2, \ldots, N_{\text{max}}.
\end{aligned} \tag{S8}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_{\text{D}}}\right)\right] \cos(u_1 \pi/2) \cos(u_2 \pi/2) + P, \\
f_2(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_{\text{D}}}\right)\right] \cos(u_1 \pi/2) \sin(u_2 \pi/2) + P, \\
f_3(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_{\text{D}}}\right)\right] \sin(u_1 \pi/2) + P, \\
h(\boldsymbol{z}) &= \sum_{i=1}^{k} (z_i - 0.5)^2, \\
P &= 0.05\left(N_{\text{D}} - D_{opt}\right)^2.
\end{aligned} \tag{S9}
$$

Here, M stands for the number of objectives, and $k = N_{\text{D}} - M + 1$.

2.3. VNDMODTLZ3

Based on DTLZ family of benchmark problems [4].
Limits:

$$
\begin{aligned}
u_{i,\text{min}} &= 0, \\
u_{i,\text{max}} &= 1.0, \\
i &= 1, 2, \ldots, N_{\text{max}}.
\end{aligned} \tag{S10}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_{\text{D}}}\right)\right] \cos(u_1 \pi/2) \cos(u_2 \pi/2) + P, \\
f_2(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_{\text{D}}}\right)\right] \cos(u_1 \pi/2) \sin(u_2 \pi/2) + P, \\
f_3(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_{\text{D}}}\right)\right] \sin(u_1 \pi/2) + P, \\
h(\boldsymbol{z}) &= 100\left\{k + \left[\sum_{i=1}^{k} (z_i - 0.5)^2 - 5\cos(20\pi(z_i - 0.5))\right]\right\}, \\
P &= 0.80\left(N_{\text{D}} - D_{opt}\right)^2.
\end{aligned} \tag{S11}
$$

Here, M stands for the number of objectives, and $k = N_{\text{D}} - M + 1$.

2.4. VNDMODTLZ4

Based on DTLZ family of benchmark problems [4].
Limits:

$$
\begin{aligned}
u_{i,\text{min}} &= 0, \\
u_{i,\text{max}} &= 1.0, \\
i &= 1, 2, \ldots, N_{\text{max}}.
\end{aligned} \tag{S12}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_\mathrm{D}}\right)\right] \cos(u_1{}^{\alpha}\pi/2) \cos(u_2{}^{\alpha}\pi/2) + P, \\
f_2(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_\mathrm{D}}\right)\right] \cos(u_1{}^{\alpha}\pi/2) \sin(u_2{}^{\alpha}\pi/2) + P, \\
f_3(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_\mathrm{D}}\right)\right] \sin(u_1{}^{\alpha}\pi/2) + P, \\
h(\boldsymbol{z}) &= \sum_{i=1}^{k}(z_i - 0.5)^2, \\
P &= 0.05\left(N_\mathrm{D} - D_{opt}\right)^2, \\
\alpha &= 100.
\end{aligned}
\tag{S13}
$$

Here, M stands for the number of objectives, and $k = N_\mathrm{D} - M + 1$.

2.5. VNDMODTLZ5

Based on DTLZ family of benchmark problems [4].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S14}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_\mathrm{D}}\right)\right] \cos(\theta_1\pi/2) \cos(\theta_2\pi/2) + P, \\
f_2(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_\mathrm{D}}\right)\right] \cos(\theta_1\pi/2) \sin(\theta_2\pi/2) + P, \\
f_3(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_\mathrm{D}}\right)\right] \sin(\theta_1\pi/2) + P, \\
h(\boldsymbol{z}) &= \sum_{i=1}^{k}(z_i - 0.5)^2, \\
\theta_1 &= u_i, \\
\theta_i &= \frac{1 + 2hu_i}{2(1 + h)}, \\
i &= 2, 3, \ldots, M - 1, \\
P &= 0.05\left(N_\mathrm{D} - D_{opt}\right)^2.
\end{aligned}
\tag{S15}
$$

Here, M stands for the number of objectives, and $k = N_\mathrm{D} - M + 1$.

2.6. VNDMODTLZ6

Based on DTLZ family of benchmark problems [4].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S16}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_D}\right)\right] \cos(\theta_1 \pi/2) \cos(\theta_2 \pi/2) + P, \\
f_2(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_D}\right)\right] \cos(\theta_1 \pi/2) \sin(\theta_2 \pi/2) + P, \\
f_3(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_D}\right)\right] \sin(\theta_1 \pi/2) + P, \\
h(\boldsymbol{z}) &= \sum_{i=1}^{k} z_i^{0.1}, \\
\theta_1 &= u_1, \\
\theta_i &= \frac{1 + 2h u_i}{2(1+h)}, \\
i &= 2, 3, \dots, M-1, \\
P &= 0.08 \left(N_D - D_{opt}\right)^2.
\end{aligned}
\tag{S17}
$$

Here, M stands for the number of objectives, and $k = N_D - M + 1$.

2.7. VNDMODTLZ7

Based on DTLZ family of benchmark problems [4].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \dots, N_{\max}.
\end{aligned}
\tag{S18}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + P, \\
f_2(\boldsymbol{u}) &= u_2 + P, \\
f_3(\boldsymbol{u}) &= \left[1 + h\left(u_{M:N_D}\right)\right] \left\{ M - \sum_{i=1}^{M-1} \left[\frac{f_i}{1 + h\left(u_{M:N_D}\right)} \left(1 + \sin(3\pi f_i)\right) \right] \right\} + P, \\
h(\boldsymbol{z}) &= 1 + \frac{9}{k} \sum_{i=1}^{k} z_i, \\
P &= 0.30 \left(N_D - D_{opt}\right)^2.
\end{aligned}
\tag{S19}
$$

Here, M stands for the number of objectives, and $k = N_D - M + 1$.

2.8. VNDMOLI1

Based on LI family of benchmark problems [1].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \dots, N_{\max}.
\end{aligned}
\tag{S20}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + h\left(u_{2:N_D}\right), \\
f_2(\boldsymbol{u}) &= 1 - u_1 + h\left(u_{2:N_D}\right) + P, \\
h(\boldsymbol{z}) &= \sum_{i=1}^{k} \left[z_i - \sin\left(\frac{D_{\text{opt}}}{2N_D} \pi \right) \right]^2, \\
P &= 0.01 \left(N_D - D_{opt}\right)^2.
\end{aligned}
\tag{S21}
$$

Here, $k = N_D - 1$.

2.9. VNDMOLZ1

Based on LZ family of benchmark problems [6].
Limits:

$$
\begin{aligned}
&u_{i,\min} = 0, \\
&u_{i,\max} = 1.0, \\
&\qquad i = 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S22}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left[u_j - h(j, u_1) \right]^2, \\
f_2(\boldsymbol{u}) &= 1 - \sqrt{u_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left[u_j - h(j, u_1) \right]^2 + P, \\
h(j) &= u_1^{0.5\left(1 + \frac{3j-6}{N_D - 2}\right)}, \\
J_1 &= 3, 5, \ldots, N_D, \\
J_2 &= 2, 4, \ldots, N_D, \\
P &= 0.05\left(N_D - D_{opt}\right)^2.
\end{aligned}
\tag{S23}
$$

2.10. VNDMOLZ2

Based on LZ family of benchmark problems [6].
Limits:

$$
\begin{aligned}
&u_{1,\min} = 0, \\
&u_{1,\max} = 1.0, \\
&u_{i,\min} = -1.0, \\
&u_{i,\max} = 1.0, \\
&\qquad i = 2, 3, \ldots, N_{\max}.
\end{aligned}
\tag{S24}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left[u_j - h(j, u_1) \right]^2, \\
f_2(\boldsymbol{u}) &= 1 - \sqrt{u_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left[u_j - h(j, u_1) \right]^2 + P, \\
h(j) &= \sin\left(6\pi u_1 + \frac{j\pi}{N_D} \right), \\
j &= 1, 2, \ldots, N_D, \\
J_1 &= 3, 5, \ldots, N_D, \\
J_2 &= 2, 4, \ldots, N_D, \\
P &= 0.06\left(N_D - D_{opt}\right)^2.
\end{aligned}
\tag{S25}
$$

2.11. VNDMOLZ3

Based on LZ family of benchmark problems [6].

Limits:

$$
\begin{aligned}
&u_{1,\min} = 0, \\
&u_{1,\max} = 1.0, \\
&u_{i,\min} = -1.0, \\
&u_{i,\max} = 1.0, \\
&\quad i = 2, 3, \ldots, N_{\max}.
\end{aligned}
\tag{S26}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left[u_j - h_1(j, u_1) \right]^2, \\
f_2(\boldsymbol{u}) &= 1 - \sqrt{u_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left[u_j - h_2(j, u_1) \right]^2 + P, \\
h_1(j) &= 0.8 u_1 \cos\left(6\pi u_1 + \frac{j\pi}{N_{\mathrm{D}}} \right), \\
h_2(j) &= 0.8 u_1 \sin\left(6\pi u_1 + \frac{j\pi}{N_{\mathrm{D}}} \right), \\
j &= 1, 2, \ldots, N_{\mathrm{D}}, \\
J_1 &= 3, 5, \ldots, N_{\mathrm{D}}, \\
J_2 &= 2, 4, \ldots, N_{\mathrm{D}}, \\
P &= 0.06\left(N_{\mathrm{D}} - D_{opt} \right)^2.
\end{aligned}
\tag{S27}
$$

2.12. VNDMOLZ4

Based on LZ family of benchmark problems [6].
Limits:

$$
\begin{aligned}
&u_{1,\min} = 0, \\
&u_{1,\max} = 1.0, \\
&u_{i,\min} = -1.0, \\
&u_{i,\max} = 1.0, \\
&\quad i = 2, 3, \ldots, N_{\max}.
\end{aligned}
\tag{S28}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left[u_j - h_1(j, u_1) \right]^2, \\
f_2(\boldsymbol{u}) &= 1 - \sqrt{u_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left[u_j - h_2(j, u_1) \right]^2 + P, \\
h_1(j) &= 0.8 u_1 \cos\left(\frac{6\pi u_1 + \frac{j\pi}{N_{\mathrm{D}}}}{3} \right), \\
h_2(j) &= 0.8 u_1 \sin\left(6\pi u_1 + \frac{j\pi}{N_{\mathrm{D}}} \right), \\
j &= 1, 2, \ldots, N_{\mathrm{D}}, \\
J_1 &= 3, 5, \ldots, N_{\mathrm{D}}, \\
J_2 &= 2, 4, \ldots, N_{\mathrm{D}}, \\
P &= 0.05\left(N_{\mathrm{D}} - D_{opt} \right)^2.
\end{aligned}
\tag{S29}
$$

2.13. VNDMOLZ5

Based on LZ family of benchmark problems [6].
Limits:

$$
\begin{aligned}
u_{1,\min} &= 0, \\
u_{1,\max} &= 1.0, \\
u_{i,\min} &= -1.0, \\
u_{i,\max} &= 1.0, \\
i &= 2, 3, \ldots, N_{\max}.
\end{aligned}
\tag{S30}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1 + \frac{2}{|J_1|} \sum_{j \in J_1} \left[u_j - h_1(j, u_1) \right]^2, \\
f_2(\boldsymbol{u}) &= 1 - \sqrt{u_1} + \frac{2}{|J_2|} \sum_{j \in J_2} \left[u_j - h_2(j, u_1) \right]^2 + P, \\
h_1(j) &= \left[0.3u_1{}^2 \cos\left(24\pi u_1 + \frac{4j\pi}{N_D} \right) + 0.6u_1 \right] \cos\left(6\pi u_1 + \frac{j\pi}{N_D} \right), \\
h_2(j) &= \left[0.3u_1{}^2 \cos\left(24\pi u_1 + \frac{4j\pi}{N_D} \right) + 0.6u_1 \right] \sin\left(6\pi u_1 + \frac{j\pi}{N_D} \right), \\
j &= 1, 2, \ldots, N_D, \\
J_1 &= 3, 5, \ldots, N_D, \\
J_2 &= 2, 4, \ldots, N_D, \\
P &= 0.05 \left(N_D - D_{opt} \right)^2.
\end{aligned}
\tag{S31}
$$

2.14. VNDMOLZ6

Based on LZ family of benchmark problems [6].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \\
u_{j,\min} &= -2.0, \\
u_{j,\max} &= 2.0, \\
j &= 3, 4, \ldots, N_{\max}.
\end{aligned}
\tag{S32}
$$

Objective functions:

$$f_1(\boldsymbol{u}) = \cos(0.5u_1\pi)\cos(0.5u_2\pi) + \frac{2}{|J_1|}\sum_{j\in J_1}\left[u_j - h(j, u_1, u_2)\right]^2 + P,$$

$$f_2(\boldsymbol{u}) = \cos(0.5u_1\pi)\sin(0.5u_2\pi) + \frac{2}{|J_2|}\sum_{j\in J_2}\left[u_j - h(j, u_1, u_2)\right]^2 + P,$$

$$f_3(\boldsymbol{u}) = \sin(0.5u_1\pi) + \frac{2}{|J_3|}\sum_{j\in J_3}\left[u_j - h(j, u_1, u_2)\right]^2 + P,$$

$$h(j) = 2u_2\sin\left(2\pi u_1 + \frac{j\pi}{N_\mathrm{D}}\right),$$

$$j = 1, 2, \dots, N_\mathrm{D},$$

$$J_1 = 4, 7, \dots, N_\mathrm{D},$$

$$J_2 = 5, 8, \dots, N_\mathrm{D},$$

$$J_3 = 3, 6, \dots, N_\mathrm{D},$$

$$P = 0.12(N_\mathrm{D} - D_{opt})^2. \tag{S33}$$

2.15. VNDMOLZ7

Based on LZ family of benchmark problems [6].
Limits:

$$u_{i,\min} = 0,$$
$$u_{i,\max} = 1.0, \tag{S34}$$
$$i = 1, 2, \dots, N_{\max}.$$

Objective functions:

$$f_1(\boldsymbol{u}) = u_1 + \frac{2}{|J_1|}\sum_{j\in J_1}\left[4y_j^2 - \cos(8y_j\pi) + 1\right],$$

$$f_2(\boldsymbol{u}) = 1 - \sqrt{u_1} + \frac{2}{|J_2|}\sum_{j\in J_2}\left[4y_j^2 - \cos(8y_j\pi) + 1\right] + P,$$

$$y_j = u_j - h(j),$$

$$h(j) = u_1^{0.5\left(1 + \frac{3j-6}{N_\mathrm{D} - 2}\right)},$$

$$j = 1, 2, \dots, N_\mathrm{D},$$

$$J_1 = 3, 5, \dots, N_\mathrm{D},$$

$$J_2 = 2, 4, \dots, N_\mathrm{D},$$

$$P = 0.10(N_\mathrm{D} - D_{opt})^2. \tag{S35}$$

2.16. VNDMOLZ8

Based on LZ family of benchmark problems [6].
Limits:

$$u_{i,\min} = 0,$$
$$u_{i,\max} = 1.0, \tag{S36}$$
$$i = 1, 2, \dots, N_{\max}.$$

Objective functions:

$$f_1(\boldsymbol{u}) = u_1 + \frac{2}{|J_1|}\left[4\sum_{j\in J_1}y_j{}^2 - 2\prod_{j\in J_1}\cos\left(\frac{20y_j\pi}{\sqrt{j}}\right) + 2\right],$$

$$f_2(\boldsymbol{u}) = 1 - \sqrt{u_1} + \frac{2}{|J_2|}\left[4\sum_{j\in J_2}y_j{}^2 - 2\prod_{j\in J_2}\cos\left(\frac{20y_j\pi}{\sqrt{j}}\right) + 2\right] + P,$$

$$y_j = u_j - h(j),$$

$$h(j) = u_1^{0.5\left(1+\frac{3j-6}{N_D-2}\right)}, \tag{S37}$$

$$j = 1, 2, \ldots, N_D,$$

$$J_1 = 3, 5, \ldots, N_D,$$

$$J_2 = 2, 4, \ldots, N_D,$$

$$P = 0.35\left(N_D - D_{opt}\right)^2.$$

2.17. VNDMOLZ9

Based on LZ family of benchmark problems [6].
Limits:

$$u_{1,\min} = 0,$$
$$u_{1,\max} = 1.0,$$
$$u_{i,\min} = -1.0, \tag{S38}$$
$$u_{i,\max} = 1.0,$$
$$i = 2, 3, \ldots, N_{\max}.$$

Objective functions:

$$f_1(\boldsymbol{u}) = u_1 + \frac{2}{|J_1|}\sum_{j\in J_1}\left[u_j - h(j, u_1)\right]^2,$$

$$f_2(\boldsymbol{u}) = 1 - u_1{}^2 + \frac{2}{|J_2|}\sum_{j\in J_2}\left[u_j - h(j, u_1)\right]^2 + P,$$

$$h(j) = \sin\left(6\pi u_1 + \frac{j\pi}{N_D}\right), \tag{S39}$$

$$j = 1, 2, \ldots, N_D,$$

$$J_1 = 3, 5, \ldots, N_D,$$

$$J_2 = 2, 4, \ldots, N_D,$$

$$P = 0.06\left(N_D - D_{opt}\right)^2.$$

2.18. VNDMOZDT1

Based on ZDT family of benchmark problems [5].
Limits:

$$u_{i,\min} = 0,$$
$$u_{i,\max} = 1.0, \tag{S40}$$
$$i = 1, 2, \ldots, N_{\max}.$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1, \\
f_2(\boldsymbol{u}) &= h(\boldsymbol{u})\left[1 - \sqrt{\frac{u_1}{h(\boldsymbol{u})}}\right] + P, \\
h(\boldsymbol{u}) &= 1 + 9\frac{\sum_{i=2}^{N_{\mathrm{D}}} u_i}{N_{\mathrm{D}} - 1}, \\
P &= 0.10\left(N_{\mathrm{D}} - D_{opt}\right)^2.
\end{aligned}
\tag{S41}
$$

2.19. VNDMOZDT2

Based on ZDT family of benchmark problems [5].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S42}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1, \\
f_2(\boldsymbol{u}) &= h(\boldsymbol{u})\left[1 - \left(\frac{u_1}{h(\boldsymbol{u})}\right)^2\right] + P, \\
h(\boldsymbol{u}) &= 1 + 9\frac{\sum_{i=2}^{N_{\mathrm{D}}} u_i}{N_{\mathrm{D}} - 1}, \\
P &= 0.10\left(N_{\mathrm{D}} - D_{opt}\right)^2.
\end{aligned}
\tag{S43}
$$

2.20. VNDMOZDT3

Based on ZDT family of benchmark problems [5].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S44}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1, \\
f_2(\boldsymbol{u}) &= h(\boldsymbol{u})\left[1 - \sqrt{\frac{u_1}{h(\boldsymbol{u})}} - \frac{u_1}{h(\boldsymbol{u})}\sin(10\pi u_1)\right] + P, \\
h(\boldsymbol{u}) &= 1 + 9\frac{\sum_{i=2}^{N_{\mathrm{D}}} u_i}{N_{\mathrm{D}} - 1}, \\
P &= 0.10\left(N_{\mathrm{D}} - D_{opt}\right)^2.
\end{aligned}
\tag{S45}
$$

2.21. VNDMOZDT4

Based on ZDT family of benchmark problems [5].

Limits:

$$
\begin{aligned}
u_{1,\min} &= 0, \\
u_{1,\max} &= 1.0, \\
u_{i,\min} &= -5.0, \\
u_{i,\max} &= 5.0, \\
i &= 2, 3, \ldots, N_{\max}.
\end{aligned}
\tag{S46}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= u_1, \\
f_2(\boldsymbol{u}) &= h(\boldsymbol{u}) \left[1 - \sqrt{\frac{u_1}{h(\boldsymbol{u})}} \right] + P, \\
h(\boldsymbol{u}) &= 1 + 10(N_{\mathrm{D}} - 1) + \sum_{i=2}^{N_{\mathrm{D}}} u_i^2 - 10 \cos(4\pi u_i), \\
P &= 0.20 \left(N_{\mathrm{D}} - D_{opt} \right)^2.
\end{aligned}
\tag{S47}
$$

2.22. VNDMOZDT6

Based on ZDT family of benchmark problems [5].
Limits:

$$
\begin{aligned}
u_{i,\min} &= 0, \\
u_{i,\max} &= 1.0, \\
i &= 1, 2, \ldots, N_{\max}.
\end{aligned}
\tag{S48}
$$

Objective functions:

$$
\begin{aligned}
f_1(\boldsymbol{u}) &= 1 - \exp(-4u_1) \sin^6(6\pi u_1), \\
f_2(\boldsymbol{u}) &= h(\boldsymbol{u}) \left[1 - \left(\frac{f_1(\boldsymbol{u})}{h(\boldsymbol{u})} \right)^2 \right] + P, \\
h(\boldsymbol{u}) &= 1 + 9 \left[\frac{\sum_{i=2}^{N_{\mathrm{D}}} u_i}{N_{\mathrm{D}} - 1} \right]^{0.25}, \\
P &= 0.10 \left(N_{\mathrm{D}} - D_{opt} \right)^2.
\end{aligned}
\tag{S49}
$$

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