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A Simple Proof of Chen–Ricci Inequality and Applications

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Abstract

In the present paper, we give a simple proof of the Chen–Ricci inequality for submanifolds in Riemannian and Lorentzian space forms, respectively. Moreover, we extend the Chen–Ricci inequality to submanifolds in Lorentzian manifolds with a semi-symmetric non-metric connection.

Keywords: Riemannian manifolds; Lorentzian manifolds; submanifolds; squared mean curvature; Ricci curvature; Chen–Ricci inequality; semi-symmetric non-metric connection

MSC: 53C20; 53C40

1. Introduction

General relativity is based on the Einstein field equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu},$$

where
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ is the Einstein tensor;
G is the Newton constant;
$T_{\mu\nu}$ is the energy-momentum tensor;
$g_{\mu\nu}$ is the metric;
$R_{\mu\nu}$ is the Ricci curvature tensor;
R is the scalar curvature, the contraction of Ricci tensor.

In differential geometry and geometric analysis, the Ricci flow, defined by Hamilton, is a certain partial differential equation for a Riemannian metric. It is called Ricci flow because of the presence of the Ricci curvature tensor. It can be considered an analogue of the diffusion of heat, because the equation of the Ricci flow is formally similar to the heat equation. By contrast, the equation of the Ricci flow is nonlinear and reveals many phenomena which are not present in the heat equation.

At any point p of a Riemannian manifold, g_p is a positive-definite inner product on the tangent space T_pM. The Ricci curvature tensor of a Riemannian manifold is a symmetric bilinear form on T_pM, for any $p \in M$. The Ricci tensor is the trace of the Riemannian curvature tensor. It can be viewed as an average of the sectional curvatures.

Let $(a, b) \subset \mathbb{R}$ be an open interval. A Ricci flow assigns a Riemannian metric g_t on M to any $t \in (a, b)$, satisfying

$$\frac{\partial}{\partial t} g_t = -2Ric_{g_t}.$$

In the theory of submanifolds, one of the basic problems is to establish simple relationships between intrinsic invariants and extrinsic invariants of a submanifold isometrically

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immersed in a space form. The classical intrinsic invariants are the sectional curvature, scalar curvature and Ricci curvature, and the main extrinsic invariant is the squared mean curvature.

B.-Y. Chen [1] estimated the squared mean curvature of an n-dimensional $(n \geq 2)$ submanifold M in a real space form of constant sectional curvature c, in terms of the Ricci curvature. More precisely, he has proven that

$$Ric(X) \leq (n-1)c + \frac{n^2}{4}\|H\|^2.$$

The above inequality is known as the Chen–Ricci inequality. He has also characterized the equality sign of the inequality.

The Chen–Ricci inequality for submanifolds in complex space forms was proven in [2].

A survey on recent developments on Chen–Ricci inequality is due to B.-Y. Chen and A. Blaga [3].

In the present paper, we give a very simple proof of the Chen–Ricci inequality.

In addition, we prove the Chen–Ricci inequality for submanifolds in Lorentzian space forms.

On the other hand, recently, in a joint paper with A. Mihai, we defined a sectional curvature for Riemannian manifolds admitting a semi-symmetric non-metric connection. We extend this definition to Lorentzian manifolds with a semi-symmetric non-metric connection.

For Lorentzian submanifolds in such manifolds, we establish the Chen-Ricci inequality.

2. New Proof of Chen–Ricci Inequality

Let (M, g) be an n-dimensional Riemannian manifold, p a point in M, X a unit vector tangent to M at p and π a 2-plane section in T_pM.

If X, Y is a basis of π, the sectional curvature $K(\pi)$ of π is given by

$$K(\pi) = \frac{g(R(X,Y)Y,X)}{g(X,X)g(Y,Y) - g^2(X,Y)}, \tag{1}$$

where R is the Riemannian curvature tensor with respect to the Levi-Civita connection associated with g.

We consider an orthonormal basis $\{e_1 = X, e_2, \cdots, e_n\}$ of T_pM. The Ricci curvature of X is given by

$$Ric\,(X) = \sum_{i=2}^{n} K(X \wedge e_i), \tag{2}$$

where $K(X \wedge Y)$ is the sectional curvature of the 2-plane section spanned by X and Y.

Let M be an n-dimensional submanifold in an m-dimensional Riemannian space form $\bar{M}(c)$ of constant sectional curvature c.

We denote by $\bar{\nabla}$ the Levi-Civita connection on $\bar{M}(c)$ and by $\bar{R}$ its Riemannian curvature tensor field.

We recall the Gauss and Weingarten formulae:

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y),$$

$$\bar{\nabla}_X \xi = -A_\xi X + \nabla^\perp_X \xi,$$

for all vector fields X, Y tangent to M and any vector field ξ normal to M.

In the above equations, h is the second fundamental form, A the shape operator and $\nabla^\perp$ the normal connection.

We denote by h^r_{ij}, $i,j = 1,\cdots,n, r = n+1,\cdots,m$, the components of the second fundamental form, i.e.,

$$h^r_{ij} = g(h(e_i,e_j),e_r),$$

where $\{e_{n+1},\cdots,e_m\}$ is an orthonormal basis of $T_p^\perp M$.

The mean curvature vector H is defined by $H = \frac{1}{n}\sum_{i=1}^n h(e_i,e_i)$.

The Gauss equation is

$$g(R(X,Y)Z,W) = g(\bar{R}(X,Y)Z,W) - g(h(X,Z),h(Y,W)) + g(h(X,W),h(Y,Z)),$$

for any vector fields X,Y,Z,W tangent to M.

B.-Y. Chen [1] established the Chen–Ricci inequality.

Theorem 1 ([1]). *Let M be an n-dimensional $(n \geq 2)$ submanifold isometrically immersed in a Riemannian space form $\bar{M}(c)$ of dimension m. Then, for each unit vector $X \in T_pM$, we have*

$$Ric\,(X) \leq (n-1)c + \frac{n^2}{4}\|H\|^2. \tag{3}$$

Moreover, one has:

(i) *If $H(p) = 0$, then a unit tangent vector X at p satisfies the equality case of (3) if and only if $X \in N_p$, where $N_p = \{X \in T_pM | h(X,Y) = 0, \forall Y \in T_pM\}$ is the kernel of the second fundamental form.*

(ii) *The equality case of (3) holds identically for all unit tangent vectors at p if and only if either p is a totally geodesic point or $n = 2$ and p is a totally umbilical point.*

We will give a simpler proof than the original proof.

Proof. Let $X \in T_pM$ be a unit tangent vector at p. We choose an orthonormal basis $\{e_1 = X, e_2,\cdots,e_n\}$ of T_pM and an orthonormal basis $\{e_{n+1},\cdots,e_m\}$ of $T_p^\perp M$. Then

$$Ric\,(X) = \sum_{i=2}^n K(X \wedge e_i) = \sum_{i=2}^n g(R(X,e_i)e_i,X).$$

By using the Gauss equation, we get

$$g(R(X,e_i)e_i,X) = c + g(h(X,X),h(e_i,e_i) - g(h(X,e_i),h(X,e_i)),$$

for any $i = 2,...,n$.

Then, the Ricci curvature of X is

$$Ric\,(X) = (n-1)c + \sum_{r=n+1}^m h^r_{11}\left(\sum_{i=2}^n h^r_{ii}\right) - \sum_{i=2}^r \|h(X,e_i)\|^2. \tag{4}$$

We will use the simple inequality $ab \leq \frac{(a+b)^2}{4}$, with equality if and only if $a = b$.

If we put $a = h^r_{11}$ and $b = \sum_{i=2}^n h^r_{ii}$, it follows that

$$Ric\,(X) \leq (n-1)c + \frac{1}{4}\sum_{r=n+1}^m (h^r_{11} + h^r_{22} + \cdots + h^r_{nn})^2, \tag{5}$$

or equivalently,

$$Ric\,(X) \leq (n-1)c + \frac{n^2}{4}\|H\|^2.$$

The equality holds if and only if

(a) $h(X, e_i) = 0, \forall i = 2, ..., n$,
(b) $h^r_{11} = h^r_{22} + ... + h^r_{nn}, \forall r = n+1, ..., m$.
(i) If $H(p) = 0$, then (b) implies $h(X, X) = 0$, which, together with (a), give $X \in N_p$.
(ii) Assume that the equality holds for all unit tangent vectors at p. Then $h(e_i, e_j) = 0$, for any $1 \leq i \neq j \leq n$, and

$$(n-2)\sum_{i=1}^{n} h^r_{ii} = 0, \forall r = n+1, ..., m.$$

We distinguish two cases:
1^0 $n \geq 3$. Then p is a totally geodesic point.
2^0 $n = 2$. Then p is a totally umbilical point. □

3. Ricci Curvature of Lorentzian Submanifolds

We will establish the Chen–Ricci curvature for Lorentzian submanifolds in Lorentzian space forms.

Theorem 2. *Let M be an n-dimensional Lorentzian submanifold isometrically immersed in a Lorentzian space form $\bar{M}(c)$ of dimension m. Then, for each unit time-like vector $X \in T_pM$, we have*

$$Ric\,(X) \geq (n-1)c - \frac{n^2}{4}\|H\|^2. \tag{6}$$

Moreover, one has:
If $H(p) = 0$, then a unit time-like vector $X \in T_pM$ satisfies the equality case of (6) if and only if $X \in N_p$, where $N_p = \{X \in T_pM | h(X, Y) = 0, \forall Y \in T_pM\}$.

Proof. Let $X \in T_pM$ be a unit time-like vector. We choose orthonormal bases $\{e_1 = X, e_2, \cdots, e_n\}$ of T_pM and $\{e_{n+1}, \cdots, e_m\}$ of $T_p^{\perp}M$. Then

$$Ric\,(X) = \sum_{i=2}^{n} K(X \wedge e_i) = -\sum_{i=2}^{n} g(R(X, e_i)e_i, X).$$

By using the Gauss equation, we get

$$g(R(X, e_i)e_i, X) = -c + g(h(X, X), h(e_i, e_i) - g(h(X, e_i), h(X, e_i)),$$

for any $i = 2, ..., n$.

Then, the Ricci curvature of X is

$$Ric\,(X) = (n-1)c - \sum_{r=n+1}^{m} h^r_{11}\left(\sum_{i=2}^{n} h^r_{ii}\right) + \sum_{i=2}^{r} \|h(X, e_i)\|^2. \tag{7}$$

We will use again the inequality $ab \leq \frac{(a+b)^2}{4}$, with equality if and only if $a = b$.
If we put $a = h^r_{11}$ and $b = \sum_{i=2}^{n} h^r_{ii}$, it follows that

$$Ric\,(X) \geq (n-1)c - \frac{1}{4}\sum_{r=n+1}^{m} (h^r_{11} + h^r_{22} + ... + h^r_{nn})^2, \tag{8}$$

or equivalently,

$$Ric\,(X) \geq (n-1)c - \frac{n^2}{4}\|H\|^2.$$

The equality holds if and only if

(a) $h(X, e_i) = 0, \forall i = 2, ..., n$,
(b) $2h(X, X) = nH(p)$.
If $H(p) = 0$, then (b) implies $h(X, X) = 0$, which, together with (a), give $X \in N_p$. □

4. Semi-Symmetric Non-Metric Connections

The notion of a semi-symmetric connection on differentiable manifolds was defined by Friedmann and Schouten in [4] and Hayden in [5], respectively. After that, Yano [6] investigated Riemannian manifolds with a semi-symmetric metric connection, and Agashe and Chafle [7,8] studied submanifolds in Riemannian manifolds endowed with semi-symmetric non-metric connections.

A linear connection $\overline{\nabla}$ on a Riemannian manifold $(\bar{M}, g)$ of dimension m is called a semi-symmetric connection if its torsion tensor

$$\overline{T}(\overline{X}, \overline{Y}) = \overline{\nabla}_{\overline{X}}\overline{Y} - \overline{\nabla}_{\overline{Y}}\overline{X} - [\overline{X}, \overline{Y}],\ \overline{X}, \overline{Y} \in \Gamma(T\bar{M}), \tag{9}$$

is given by

$$\overline{T}(\overline{X}, \overline{Y}) = \omega(\overline{Y})\overline{X} - \omega(\overline{X})\overline{Y}, \tag{10}$$

where ω is the 1-form associated with the vector field P on $\bar{M}$, i.e., $\omega(\overline{X}) = g(\overline{X}, P)$.

If $\overline{\nabla}g = 0$, then $\overline{\nabla}$ is said to be a semi-symmetric metric connection. Otherwise, if $\overline{\nabla}g \neq 0$, then $\overline{\nabla}$ is a semi-symmetric non-metric connection.

Let $\overset{\circ}{\bar{\nabla}}$ be the Levi-Civita connection with respect to the Riemannian metric g. A semi-symmetric non-metric connection $\overline{\nabla}$ on $\bar{M}$ is given by (see [7])

$$\overline{\nabla}_{\overline{X}}\overline{Y} = \overset{\circ}{\bar{\nabla}}_{\overline{X}}\overline{Y} + \omega(\overline{Y})\overline{X},$$

for all vector fields $\overline{X}$ and $\overline{Y}$ on $\bar{M}$.

In the following, we consider a pseudo-Riemannian manifold $(\bar{M}, g)$ equipped with a semi-symmetric non-metric connection $\overline{\nabla}$ and M an n-dimensional submanifold of $\bar{M}$.

As usual, we denote by $\overline{R}$ and $\overset{\circ}{\bar{R}}$ the curvature tensors of the Riemannian manifold $\overline{M}$ with respect to $\overline{\nabla}$ and $\overset{\circ}{\bar{\nabla}}$, respectively, and put

$$\overline{R}(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) = g(\overline{R}(\overline{X}, \overline{Y})\overline{W}, \overline{Z}).$$

It is known (see [7]) that $\overline{R}$ has the expression

$$\overline{R}(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) = \overset{\circ}{\bar{R}}(\overline{X}, \overline{Y}, \overline{Z}, \overline{W}) + s(\overline{X}, \overline{Z})g(\overline{Y}, \overline{W}) - s(\overline{Y}, \overline{Z})g(\overline{X}, \overline{W}), \tag{11}$$

for all vector fields $\overline{X}, \overline{Y}, \overline{Z}, \overline{W}$ on $\overline{M}$, with s a $(0,2)$-tensor defined by

$$s(\overline{X}, \overline{Y}) = (\overset{\circ}{\bar{\nabla}}_X \omega)\overline{Y} - \omega(\overline{X})\omega(\overline{Y}).$$

Let λ be the trace of s.

The Gauss formulae with respect to $\overline{\nabla}$ and $\overset{\circ}{\bar{\nabla}}$, respectively, are written as

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y),$$
$$\overset{\circ}{\bar{\nabla}}_X Y = \overset{\circ}{\nabla}_X Y + \overset{\circ}{h}(X, Y),$$

for all vector fields X, Y on the submanifold M, where $\overset{\circ}{h}$ is the second fundamental form of M and h is a $(0,2)$-tensor on M. In [8] it was proven that $\overset{\circ}{h} = h$.

We decompose the vector field P on M uniquely into its tangent and normal components $P^\top$ and $P^\perp$, respectively, i.e., $P = P^\top + P^\perp$.

The Gauss equation for the semi-symmetric non-metric connection is (see [8])

$$\begin{aligned}\overline{R}(X,Y,Z,W) = R(X,Y,Z,W) - g(h(X,Z),h(Y,W)) + g(h(X,W),h(Y,Z))\\ - g(P^\perp,h(Y,Z))g(X,W) + g(P^\perp,h(X,Z))g(Y,W),\end{aligned} \tag{12}$$

for all vector fields X, Y, Z and W on M.

To define the sectional curvature with respect to a non-metric connection, let $\pi \subset T_pM$ be a non-degenerate 2-plane section at a point $p \in M$ and $\{X, Y\}$ a basis of π. Since $R(X,Y,Z,W) \neq R(X,Y,W,Z)$, we cannot define the sectional curvature $K(\pi)$ by the standard definition. With respect to the induced connection ∇ the sectional curvature $K(\pi)$ is defined as follows (see [9]):

$$K(\pi) = \frac{1}{2}[g(R(X,Y)Y,X) + g(R(Y,X)X,Y)]/[g(X,X)g(Y,Y) - g^2(X,Y)]. \tag{13}$$

This definition is independent on the basis.

The scalar curvature τ is given by

$$\tau(p) = \sum_{1\leq i<j\leq n} K_{ij}, \tag{14}$$

where K_{ij} is the sectional curvature of the plane section spanned by e_i and e_j.

From the definition of the vector field P, we have

$$\omega(H) = \frac{1}{n}\sum_{i=1}^{n}\omega(h(e_i,e_i)) = g(P^\perp,H). \tag{15}$$

5. Chen–Ricci Inequality for Lorentzian Submanifolds in Lorentzian Space Forms Endowed with a Semi-Symmetric Non-Metric Connection

In this section, we establish the Chen–Ricci inequality for Lorentzian submanifolds in a Lorentzian space form admitting a semi-symmetric non-metric connection.

As we know, this is the first study of the Ricci curvature of submanifolds in a Lorentzian space form endowed with a semi-symmetric non-metric connection.

In contrast with metric connections on a Lorentzian manifold, the sectional curvature associated with a non-metric connection has a different formula (see (13)).

Therefore, the following result is more complicated than the corresponding result for metric connections.

Theorem 3. *Let M be an n-dimensional Lorentzian submanifold in a Lorentzian space form $\bar{M}(c)$ admitting a semi-symmetric non-metric connection $\overline{\nabla}$. Then, for any $p \in M$ and any unit time-like vector $X \in T_pM$, one has*

$$Ric\,(X) \geq (n-1)c - \frac{n^2}{4}\|H\|^2 + \frac{\lambda}{2} + \frac{n-2}{2}s(X,X) + \frac{1}{2}\omega(H) + \frac{n-2}{2}g(P^\perp,h(X,X)). \tag{16}$$

Moreover, if $H(p) = 0$, then the equality holds if and only if $X \in N(p)$.

Proof. Let $X \in T_pM$ be a unit time-like vector in T_pM. We choose an orthonormal basis $\{e_1, e_2, \cdots, e_n, e_{n+1}, e_{n+2}, \cdots, e_m\}$ in $T_p\overline{M}(c)$ such that $e_1, \cdots, e_n$ are tangent to M at p, with $e_1 = X$. Then

$$Ric\,(X) = \sum_{j=2}^{n} K(e_1 \wedge e_j). \tag{17}$$

From the Gauss equation, we have

$$g(R(e_1, e_j)e_j, e_1) = -c - s(e_j, e_j) + \sum_{r=n+1}^{m} [h_{11}^r h_{jj}^r - (h_{1j}^r)^2] - g(P^\perp, h(e_j, e_j)). \tag{18}$$

Similarly, from the Gauss equation, we have

$$g(R(e_j, e_1)e_1, e_j) = -c - s(e_1, e_1) + \sum_{r=n+1}^{m} [h_{11}^r h_{jj}^r - (h_{1j}^r)^2] - g(P^\perp, h(e_1, e_1)). \tag{19}$$

Since X is time-like, one has

$$K(e_1 \wedge e_j) = -\frac{1}{2}[g(R(e_1, e_j)e_j, e_1)) + g(R(e_j, e_1)e_1, e_j))]. \tag{20}$$

From the Equations (18)–(20), we get

$$\begin{aligned} K(e_1 \wedge e_j) = c &+ \frac{1}{2}[s(e_j, e_j) + s(e_1, e_1)] \\ &- \sum_{r=1}^{n} [h_{11}^r h_{jj}^r - (h_{1j}^r)^2] \\ &+ \frac{1}{2}[g(P^\perp, h(e_j, e_j)) + g(P^\perp, h(e_1, e_1))]. \end{aligned} \tag{21}$$

By substituting the Equation (21) in (17), we find

$$\begin{aligned} Ric\,(X) = (n-1)c &+ \frac{1}{2}[\lambda + (n-2)s(X, X)] \\ &+ \frac{1}{2}[n\omega(H) + (n-2)g(P^\perp, h(X, X))] \\ &- \sum_{j=2}^{n} \sum_{r=n+1}^{m} [h_{11}^r h_{jj}^r - (h_{1j}^r)^2]. \end{aligned}$$

From the last equation we have

$$\begin{aligned} &Ric\,(X) - (n-1)c - \frac{1}{2}[\lambda + (n-2)s(X, X)] \\ &- \frac{1}{2}[n\omega(H) + (n-2)g(P^\perp, h(X, X))] \\ &\geq - \sum_{r=n+1}^{m} \sum_{j=2}^{n} h_{11}^r h_{jj}^r \end{aligned} \tag{22}$$

As in Section 2,

$$\sum_{r=n+1}^{m} \sum_{j=2}^{n} h_{11}^r h_{jj}^r \leq \frac{n}{4}\|H\|^2, \tag{23}$$

which implies the inequality (16).

For each unit time-like vector X at p, the equality case of (16) holds if and only if

$$\begin{cases} h(X, e_i) = 0, \ \forall i = 2, ..., n, \\ 2h(X, X) = nH(p). \end{cases}$$

Therefore, if $H(p) = 0$, we have $h(X, X) = 0$, that is $X \in N_p$. □

6. Conclusions

Briefly, in the Introduction, we pointed out the crucial role of the Ricci curvature tensor in the general relativity. More precisely, it appears in the formula of the Einstein tensor.

Moreover, on the basis of the Ricci curvature, Hamilton defined the Ricci flow. Afterwards the Ricci soliton was introduced. The Ricci soliton and its generalizations is a current topic of research.

The present paper deals with some geometric properties of the Ricci curvature of submanifolds in Riemannian and Lorentzian space forms.

We simplified the proof of the well-known Chen–Ricci inequality. Regarding its applications, we established Chen–Ricci inequalities for Lorentzian submanifolds in Lorentzian space forms and Lorentzian space forms admitting semi-symmetric non-metric connections, respectively.

The same method can be used for obtaining Chen–Ricci inequalities for submanifolds in other space forms.

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