

Review

Analytical Modeling of Ancillary Items

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Abstract: Airlines profitability increasingly depends on the sale of ancillary items such as seat selection, baggage fees, etc. The modeling of ancillary items is becoming more important in the analytics literature. Much of the modeling is stylized and not immediately applicable. This paper contains a review of the approaches and modeling assumptions made in the literature. The focus is on the assumptions made so that models may be evaluated for how effective they are for applications and to highlight gaps in the literature.

Keywords: ancillary; revenue management; pricing

1. Introduction

Recently, there has been much interest in the modeling of ancillary add-on items in the analytics literature. The key concept is that a customer buys a core item (such as a hotel room or airline ticket) and may then purchase an extra item (such as parking in the hotel case or seat selection in the airline case). A major focus is on the pricing of ancillary items. Should they be priced into a bundle containing the primary item? Should they be priced separately? What should the price level be? If the primary retailer (e.g., hotel or airline) is selling through a platform (e.g., Expedia) what kind of contract should be utilized?

Ancillary items have become important in many industries but none so much as the airline industry. In the past airline tickets were sold inclusive of “ancillary” products such as seat assignments and baggage allowances. Cut price airlines and so called Ultra Low-Cost Carriers changed the landscape for the industry. Ryanair in Europe was at the forefront of the low-cost airline revolution. For the fiscal year up to March 2023, more than 30% of Ryanair’s revenue was from ancillary item sales [1]. Spirit Airlines ancillary revenue in 2022 was \$2.5 billion comprising over 50% of its revenue [2]. According to [3], projected airline ancillary revenue for 2024 will be \$148.4 billion worldwide.

Legacy airlines such as Delta and American in the US changed their models so that they can compete with low-cost airlines. They now have pricing strategies for basic economy fares that compete with those of low-cost airlines. Carriers other than specifically low-cost carriers have adapted their pricing models and now pose formidable competition. For 2023, United Airlines, Delta Airlines, American Airlines and Southwest Airlines had respective ancillary revenues of \$9.5 billion, \$9.4 billion, \$8.5 billion and \$6.8 billion [3]. Southwest Airlines was a well-known holdout in not charging for seat assignment. Even they have now changed their pricing model and will plan to impose charges for seat assignment.

Given the importance of ancillary items to revenue generation, it is remarkable how few analytics models have been developed that closely approximate the actual pricing and inventory issues facing airlines. Several valuable reviews provide a list of papers, authors and a brief description of the basic problem considered (e.g., [4,5]). The approach of this review is somewhat different. The major approaches to modeling and pricing ancillary items are summarized. One goal is to carefully review the methodology of



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analytical approaches. This will enable the analytics modeler or practitioner determine which approaches are relevant for particular situations. More importantly, it will help highlight where the current state of the art in modeling ancillary items needs improvement to model actual real-world applications.

The range of papers reviewed is meant to be representative of the current state of the art in modeling ancillary prices. Unlike other reviews, the focus is on the assumptions made in the models. This is important in evaluating how effective the models might be in practice. Would an airline planner want to use the models?

This paper starts with a general discussion of ethical and consumer reactions to ancillary pricing. Then, examples of how airlines market ancillary items are presented. Ancillary items may be priced individually or as part of a package (Section 4). Key to the optimization of ancillaries are assumptions about customers willingness to pay and cost considerations. This key element is summarized in Section 5. Dynamic approaches where optimization is performed at each time period or for each customer are summarized in Section 7. Choice models that describe the probability of a customer selecting a certain option were popular a few years ago and are summarized in Section 8. Artificial Intelligence is now starting to make inroads into airline optimization. AI and other approaches are summarized in Section 9.

2. Ethical and Political Considerations and Customer Reactions

Researchers are interested in consumer reaction to ancillary pricing. Ancillary items can engender customer controversy. Issues around customers perceptions of the fairness of ancillary pricing are considered in [6]: “ways to communicate ancillary fees in a way that reduces customers’ feeling that they are being unfairly treated” are discussed. Customers attitudes to ancillary approaches are empirically investigated in [7].

Considering consumer reactions can be an important factor in a retailer’s strategy. General add-on fees, often referred to as “junk fees” drew the attention of regulators in the U.S. In April 2023, Congress introduced the *Junk Fee Prevention Act*. Airline ancillary item charges are not hidden fees and are not often referred to as “junk fees”. However, family seating fees have been referred to as junk fees [8]. A Senate Committee held hearings on “junk fees” and pricing strategies such as dynamic pricing. A five-year report on ancillary fees from the majority staff of the Senate Permanent Subcommittee on Investigations concluded that US Airlines ancillary fees “have led to higher costs and worse experiences for consumers”.

Frontier has been reported as paying commissions to gate agents who collect bag fees at the gate. Police were called to an Air Canada flight due to reportedly aggressive gate agents falsely sizing bags [9]. “Frontier and Spirit paid \$26 million to gate agents and other personnel between 2022 and 2023 to catch passengers allegedly not following airline bag policies, often forcing those passengers to pay a bag fee or miss their flight” [10]. American Airlines, perhaps because of consumer perception, have removed the “carry-on bag restriction that is currently part of its domestic and short-haul international Basic Economy fare rules”. the [11].

Some ([12]) actually propose that airlines purposely split up the seats of known groups of customers to incentivize them to pay for seat selection. The “importance of effective communication” of pricing schemes is emphasized in [6], where they state “it is more effective for the sellers to be transparent and upfront about ancillary pricing”. One wonders how customers perception of the provider would be impacted if knowledge of a strategy of purposefully splitting up groups become well known! “Ancillary revenue can damage brand equity if not correctly applied” [13]. Clearly, issues outside of formal analytical

modeling need to be considered when analyzing ancillary items ((For more on ethical issues in algorithmic pricing, see [14]).

3. Presentation of Offers

Figures 1 and 2 are typical of the traditional way airlines communicate their ancillary offerings.

☒ Included (\$) Additional Charge	Basic Fare	Economy Bundle	Premium Bundle	Business Bundle
Personal Item	☒	☒	☒	☒
Carry-on Bag	(S)	☒	☒	☒
Choose Your Seat	(S)	☒ Standard	☒ Premium Extra Legroom	☒ UpFront Plus Empty Middle Seat Extra Legroom
No Change or Cancel Fees	—	☒	☒	☒
Board First SM	(S)	(S)	☒	☒
Checked Bags	(S)	(S)	(S)	☒ 2 Bags (50lbs ea)

Figure 1. Frontier Airline’s Ancillary Bundles.

COMPARE OPTIONS Find the best fit for you.	Go Purchase bags, seats and other options separately	Go Savvy Standard seat selection and a carry-on or checked bag	Go Comfy Middle seat blocked, Group 2 boarding, bags, a drink and snack	Go Big Big Front Seat®, Group 1 boarding, bags, drinks, snacks and Wi-Fi
1 Personal Item Smaller than 18"x14"x8"	☒	☒	☒	☒
1 Carry-On Bag Maximum of 22"x18"x10"	Not Allowed	☒ Carry-On Bag or ☐ Checked Bag	☒	☒
1 Checked Bag Up to 50 lbs. Maximum of 62 linear inches	(S)	☐ Checked Bag	☒	☒
Seat Selection	(S)	☒ Standard Seat	☒ Comfy Seat Small snack & drink included	☒ Big Front Seat® Premium snacks & drinks included
Snacks & Drinks	(S)	(S)	☒	☒
Priority Boarding	—	—	☒	☒
Wi-Fi* Streaming (Subject to availability)	(S)	(S)	(S)	☒
Shortcut Security* Save time in line (Subject to availability)	(S)	(S)	(S)	☒
Priority Check-In* At participating airports	—	—	—	☒

Figure 2. Spirit Airline’s Ancillary Bundles.

A new approach, particularly for selling class upgrades, is the presentation of a slider. For instance, customers who buy an economy class ticket will receive an email from the carrier asking if they want to upgrade to first class (An example from Air Canada is shown in Figure 3). The slider consists of a low dollar amount at the lower end of the slider and a high dollar amount at the upper end. The starting point of the slider may be at any point between. The customer chooses an offer price. This is an example of a Name-Your-Own-Price auction (see [15] for a review).

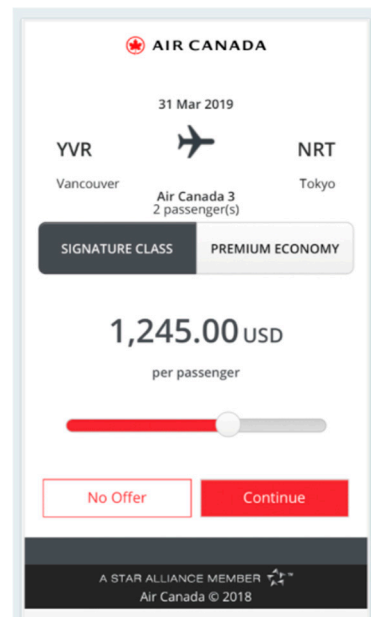


Figure 3. Air Canada’s Slider.

4. Bundles vs. A La Carte

Ancillaries may be priced in different ways. One approach is to have them as part of a bundle. Ancillary items are also sometimes priced “a la carte”. Of course, with only one ancillary item, the distinction between bundle and add-on pricing becomes mathematically irrelevant. As demonstrated below, it starts to make a difference when there are at least two ancillary items.

Consider the case where the primary product is an airline seat and there are two ancillary products: seat assignment and a checked bag. Then, there are four possible combinations: Combination 1—seat only; Combination 2—seat and seat assignment; Combination 3—seat, seat assignment and checked bag; Combination 4—seat and checked bag. The respective costs of the four combinations may be denoted by q_1, q_2, q_3 and q_4 . For bundle pricing, there are no restrictions on the values for the q_i other than common sense ones such as $q_4 \geq q_1$. For a la carte pricing, $q_1 = p_1$, $q_2 = p_1 + p_2$, $q_3 = p_1 + p_2 + p_3$, $q_4 = p_1 + p_3$, where p_1, p_2, p_3 are the individual prices for the seat, seat assignment and checked bag, respectively.

Pricing bundles is an old problem in the economics literature [16,17]. The general question of optimal bundling in various industries is considered in [17]. Bundles in the revenue management literature are different in that each one must contain the primary item... the basic air fare.

Spirit Airlines initially basically adopted a la carte pricing. The form of pricing can be important to profitability. For instance, Spirit Airlines would offer low fares (some as low as \$9 one-way) and hoped customers would purchase ancillary items. As pointed out in [18], this could lead to suboptimal results if bargain hunters rush to purchase most of the available seats and then do not purchase ancillary items. They assume a probability distribution over the arrival time of customers: the probability that given customer i will select product i during a period t equals to r_{it} . Then, the number of products of type i is modeled as a hypergeometric random variable using r_{it} . For a \$9 fare, there are passengers who would be more than willing to forego seat selection and carry on or checked bags for a low total cost. Perhaps because of this Spirit Airlines has now changed its model from a la carte pricing to bundle pricing. As seen in Figure 2, the options are now “Go Big”, “Go Comfy”, “Go Savvy” and “Go”. Ryanair also offers bundle options.

The case of bundling versus not for one ancillary item but two types of customer (e.g., business and leisure) is analyzed in [19]. There are λ_h “high” valuation customers all of whom value the primary item at v_h . Similarly, there are λ_l “low” valuation customers who value the primary item at v_l . Not all customers want the one ancillary service. The proportions of each group that value the ancillary service are β_h and β_l with respective valuations of u_h and u_l . All of the quantities are assumed fixed and known. Thus, pricing strategies for bundling or not bundling the main service and one ancillary item can be compared algebraically.

The numbers in each customer class are known as are their respective fixed valuations for the primary product. Fixed proportions of both types of customer value the ancillary item with known and fixed valuations. They show that, under these restrictive assumptions, unbundling can sometimes be profitable.

5. Customer Demand

Every model needs to assume something about the amount customers will pay. The most general is to assume a probability distribution of values. For tractability, this may be further restricted by assuming a specific distribution such as the uniform. In the most restrictive cases, fixed values are assumed. Demand functions at a given price might also be assumed. In this case, linearity of demand in price is the most common approach.

5.1. General Willingness-to-Pay

It makes sense to assume that different customers have a different willingness to pay. This could also loosely be described as a customer’s “valuation” or the “reserve price”, the maximum a customer is willing to pay for the product (e.g., [5]). Marketers have pursued various approaches to estimate how much customers are willing to pay for an item. Expert judgements may be utilized. Customer surveys may be used. Sometimes a direct question such as “Above which price would you definitely not buy the product, because you can’t afford it or because you didn’t think it was worth the money” [20]. Data from test markets where differing prices are used may be analyzed. A review on methods for estimating willingness to pay can be found in [20]. The modeler assumes a distribution over the possible values that customers are willing to pay. The maximum price a customer is willing to pay is unknown. A customer drawn from a population where the willingness to pay across members of that population is described by the distribution function $F(\cdot)$ is modeled as willing buy at a price p with a probability equal to $1 - F(p)$. The expected revenue from such a customer is given by $p(1 - F(p))$.

One can also envision a difference between valuation and willingness-to pay: a customer may believe a product is worth a value of \$500, say, but, for various reasons, be only willing to pay, say, \$400. For this situation, the modeling needs to carefully distinguish between the possibilities. The distribution of customers willingness-to-pay distributions may differ for different product bundles. The distribution may depend on a set of independent covariates. (See, e.g., [21,22] for examples of this approach to willingness-to-pay). Understanding and estimating customer willingness-to-pay remains a highly relevant task. This is something that has been neglected in the analytics literature [23]. In [23], there is a discussion of dynamic personalized pricing recommendation models for ancillaries that are specific to each customer.

A general framework can be found in [23], where the willingness-to-pay for a flight is a random variable θ , while the willingness to pay for ancillary item k is assumed to be a random variable γ_k . A customer will purchase a flight and ancillaries in a particular bundle if that bundle has the maximum value of $\{\theta + \sum_{k \text{ in bundle}} \gamma_k\} - p$, where p is the price of the fare and bundle of ancillaries. The willingness to pay in [24] for a set of ancillaries

is the sum of the willingness to pay for each item and these individual willingness to pay amounts are the values of normal random variables. It is also assumed that each ancillary item is chosen according to a Bernoulli distribution with ancillary and segment specific probabilities.

For the case of one ancillary product, ref. [25] allow three groups of customers. Group 1 customers will purchase the ancillary item in addition to the base item if the price is attractive. Group 2 customers are only interested in the base item, while Group 3 customers are only interested in purchasing both the primary and ancillary items together. The willingness to pay for each group is modeled by the general distribution functions $F_i(\cdot)$, $i = 1, 2, 3$.

5.2. Demand Functions

Another approach is to model demand as a function of price. If $F(\cdot)$ is the distribution function of willingness-to-pay and there are N possible customers, then the expected demand at the price p is $E[N](1 - F(p))$. This is similar to ref. [26] who assume a fixed market size of M customers with a willingness to pay drawn from a known distribution. For tractability, it is then assumed that this distribution has an increasing generalized failure rate.

5.3. Linearity Assumptions

Little has been done to estimate $F(\cdot)$. Generally, data are not available for a wide enough range of price values. One approach to overcome these difficulties is to assume that the expected demand is linear in price. In [27], it is assumed that the utilities to customer i of choices $j \in \{1, 2\}$, are given by:

$$u_{i,j} = \beta_0 - \beta_1 p_j + \epsilon_{i,j}$$

where p_j is the price of choice j and $\epsilon_{i,j}$ are error terms that are of type-1 extreme value. How reasonable is it to suppose an extreme value distribution for the distribution of error terms in customers utilities? Linearity is assumed in [28]: given fixed values for all other variables it is assumed that a unit increase in price has a constant effect on the number of bookings. Multiple bundles are considered in [29] but demand is a linear function of price.

Demand being linear in price is a common assumption but how realistic? How does one model the reserve price? Ideally, the retailer will have some data from which a distribution may be derived. One of the issues with this is the censoring problem: observations are generally only available for those who actually purchased. The case where customers would bid on a product thus enabling a more robust description of customers' reserve price distribution was considered on [30]. Along the way, they demonstrated that modeling demand as a linear function of price often does not do a good job of reflecting reality.

5.4. Distributional Assumptions

Often, authors will arbitrarily make assumptions about the distribution of the reserve price. The most common distribution to assume is the uniform. This allows for analytical solutions. However, this assumption might not reflect reality and may result in suboptimal pricing. Competition between two firms is considered in [31]. There are two possible retailers with known and fixed prices for the base and ancillary items. Each customer has the same (fixed and known) valuations for the base good and only ancillary item. An extra cost of $t\theta$ where θ is uniform over the interval $(0, 1)$ is incurred if the purchase is from retailer number 1, and $t(1 - \theta)$ if from retailer 2. In [32], the valuation of the core product, such as an airline ticket, is assumed to be uniform over $(0, 1)$.

In [33], ancillary item k is relevant to a consumer with probability ϕ_k and a customer's willingness to pay is the sum of the random variables for willingness to pay for the primary item and the ancillary items. They consider the case of normally distributed willingness to pay. A customer's valuation for the primary product is uniformly distributed over $(0, 1)$ in [34]. A fixed proportion α have a fixed utility for the single ancillary item and other customers have zero utility for it.

For the case of one ancillary item [19], assume two customer types. Each type has a fixed and known valuations for the primary products and the ancillary items.

For the slider case (see, e.g., Figure 1) [35], do not restrict the values or distribution for a customer's willingness-to-pay. If, however, this distribution is uniform and the customers' perceptions of the threshold beyond which an airline will accept an offer is distributed as uniform or triangle random variable, they obtain Nash equilibria for the optimal points to present on the slider.

The valuation of the main service (ticket) is uniform over a range in [36]. A known fraction α want the ancillary item and value it at a value equal to the fraction β (fixed and known) of the valuation for the primary item. The assumption is that "customers do not value the add-on as much as the main service" (this assumption would not apply for situations such as *Spirit Airlines* original pricing model where ancillaries were often priced much higher than the airfare).

Considering seat choice as the ancillary possibility [12], model the cost of seat number i as $base_i + b(rank_i)$, where $base_i$ is the price charged for the seat, $rank_i$ is the historical preference ranking among purchasers of that seat and b is a parameter. So, a seat of rank 1, would be modeled as worth b more than the price.

Fixed valuations for the primary and ancillary products are assumed in [19]. For one ancillary item, where and how a company might subcontract sales to an online platform is considered in [34]. The major assumption is that a fixed fraction of customers value the ancillary item at a common fixed value.

6. Customer Decisions

There are different ways to consider customer decisions on purchase. A customer who has a willingness-to-pay or a utility of w_i experiences a "surplus" of $w_i - p_i$ for product i when it costs p_i . A customer then chooses the product that has the highest surplus (e.g., [24]). In the case of one offering, the customer purchases if the surplus is positive (e.g., [25]) In discrete choice modeling, a probability that a customer will choose product i is calculated. In [34], a customer who purchases the (only) ancillary item receives a utility of $\theta + \theta_a - p$, where θ is uniform over $(0, 1)$ and represents the customer's valuation of the primary item, θ_a is the fixed utility to the customer of the ancillary item and p is the price of the primary and ancillary items.

7. Dynamic Approaches

When ancillary fees were first introduced, they tended to stay unchanged in the periods before the flight. Now, however, dynamic approaches where ancillary fees may change throughout the periods before the flight as a function of current and projected sales are considered (see, e.g., [37,38]). An overview of dynamic pricing definitions in airline revenue management is provided in [39]. An overview of various dynamic pricing approaches can be found in [40].

It has been recognized in the literature that, to date, the reality of dynamic pricing for ancillary items has not been particularly successful. There are numerous issues. An issue with ancillary pricing is "its static nature, where prices are typically predetermined and known to customers" [41]. A big issue with dynamic approaches is a lack of data. As

pointed out by [42], “most airlines employ static pricing for ancillary upsells; however, they have begun to experiment with dynamic ancillary pricing due to static ancillary pricing, many airlines lack data on how customers respond to dynamic ancillary prices”.

Dynamic pricing is appealing in principle since it allows for time and customer dependency when optimizing pricing. Many sequential decision problems are easily formulated using the Bellman equations. However, finding solutions when realistic assumptions are used is notoriously difficult. An indication of the general complexity of the dynamic problem can be found [25] who formulated and solved a dynamic pricing problem with realistic assumptions on willingness to pay but their methods only worked for one ancillary item and do not appear to be easily extendable to more items. As pointed out in [41], “while some studies have explored dynamic pricing of ancillary services in bundled offerings ([25,43,44]) this approach has not been widely implemented in the real market”.

For the slider approach of Figure 3, [35] dynamically consider the case where seats might sell out due to customer demand. The airline sends the offer to upgrade to a better seat to a limited number of passengers. The decision to accept or reject a customer’s offer is made dynamically as offers randomly arrive. The state information when an offer arrives is (n, c) , where c = the number of seats still left n = the number left to bid. The state space then transitions to $(n - 1, c - 1)$ if the offer is accepted or $(n - 1, c)$ if it is not. The decision on whether to accept the bid is also influenced by the probability distribution for the number of customers who might arrive and be willing to pay full price. No restrictions on willingness to pay are imposed.

Dynamic approaches often assume a Poisson arrival rate. When a customer arrives an optimization can, theoretically, be performed. For instance, ref. [33] formulate a model where customers consider ticket/bundle number i at a rate of λ_i . This is common in airline revenue management. Dynamic pricing of the primary product with this approach is proposed in [45]. For a single leg, exactly one customer can arrive with a probability equal to λ_i during period i . An item is purchased with a probability depending on the price. The authors point out that the model as formulated is “intractable” and investigate pooling and heuristics. Of course, the complexity will increase when the rate is allowed to be a function of time and customer choice behavior is factored in. As is pointed out in [35] “Dynamic offer creation for ancillary products is still in its infancy due to “inadequate...scientific modeling”. While dynamic pricing “has the potential to increase airline revenues, in practice it is difficult to obtain the true optimal pricing solution” [5].

The promise of full dynamic optimization has not been fulfilled. It is a little like the old aphorism “to a person with a hammer, everything looks like a nail”. Discussing a sequential decision-making problem in terms of the Bellman equations is often straightforward and mathematically appealing. However, finding optimal solutions when reasonable assumptions that realistically approximate customer behavior are made is often fraught with difficulty. For instance, ref. [35] considered dynamic pricing with reasonable assumptions but only for one ancillary product. The full extension of the dynamic pricing approach to more than one ancillary item has not been accomplished either by them or others.

8. Choice Models

The multinomial logit model, well known previously in the statistics and economics literature, was introduced for revenue management in [46]. Customer i ’s utility for option j is modeled as:

$$u_{i,j} = u_j + \epsilon_{i,j},$$

where u_j is constant and $\epsilon_{i,j}$ is the value of the Gumbel extreme value distribution. The component u_j is often represented as:

$$u_j = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k,$$

where $x_1, \dots, x_k$ is a vector of attributes associated with option j and $\beta_0, \dots, \beta_k$ are constants to be estimated. Under these assumptions, the probability that option j maximizes utility and is thus chosen is given by:

$$\frac{e^{u_j/u}}{\sum e^{u_i/u} + e^{u_0/u}},$$

where u is a scaling parameter and the sum is over all options (indexed by i). A review of choice models in revenue management can be found in [47]. There are many issues with using choice models in practice. As commented in [33] “Many popular discrete choice models used for price optimization, such as the multinomial logit (MNL) and probit, either do not produce logical prices or are mathematically intractable for joint optimization in general” and “customer choice models commonly used in the airline literature do not possess the properties needed to solve the dynamic offer creation problem logically and at scale”.

Use of popular choice models can lead to impracticable pricing behavior where one ancillary item could be priced identical to a bundle containing multiple ancillary items (see, e.g., [48,49]).

9. AI and Other Approaches

AI and machine learning is becoming prevalent in all sorts of applications (see, e.g., [50]). Use of AI to price ancillary items is relatively new. At this stage it is unclear the value AI approaches will bring to ancillary pricing. The advantage of model-based approaches is a greater understanding of underlying mechanisms. AI is, in many ways, sophisticated “pattern matching”. If there is enough data in training data sets, good predictions can be made. However, if the training data does not capture certain scenarios, it is not clear that an AI solution will be valuable. PROS [37] suggest the use of “reinforcement learning models”.

Machine learning and a dynamic approach is considered in [43]. One base item and one ancillary item is assumed in [51] (although they remark the algorithm can be “easily extended”). Dynamic pricing is via a Markov Decision Process and an optimization algorithm. “Indeed, the pricing of the primary product (and similarly, the ancillary item) can influence the sales of the other”. No modeling assumptions are made about customer behavior. Instead, the historical empirical distribution of purchase for each price pair action is estimated. This presupposes that there are enough price/purchase decisions available for meaningful estimation.

Several machine learning approaches are considered in [43]. They consider a naïve Bayes model and Deep Neural Networks to estimate ancillary purchase probabilities.

10. Discussion and Conclusions

This paper has focused on the modeling of ancillary items. This can be thought of as part of yield/revenue management. A summary of yield management approaches in the airline industry may be found in [52].

Assuming that customers are heterogeneous and that willingness to pay is drawn from a probability distribution with covariates is the most general way to model customers reactions to ancillary prices. However, estimating this distribution is not an easy task and has not received much attention in the literature. Instead, assumptions that may or may

not reflect reality are often used (e.g. fixed and known values and uniform in [32], uniform in [35], increasing generalized failure rate in [24], fixed values in [19]).

In general, limited data are available. Consequently, finding optimal price points for actual flights is not generally possible. “Due to limitations in airline distribution technology, RM has typically not focused on determining the set of possible price points itself” [39].

Ancillary pricing is of increasing economic importance to airlines. However, the academic modeling and statistical estimation associated with ancillaries is lacking and still in its infancy. Strong assumptions about customers willingness to pay and reaction to price changes are made that do not necessarily reflect reality. The papers involving more realistic assumptions look at scaled down versions of the real problem—e.g., allowing for only one ancillary item.

Estimating customers responses to different prices has not been widely considered. Linearity assumptions are often used. This remains an area in need of new statistical methodology.

Discrete choice modeling of customers’ probability of purchase comes with its own set of problems. Many popular discrete choice models used for price optimization, such as the multinomial logit (MNL) and probit, either do not produce logical prices or are mathematically intractable for joint optimization in general” [33].

Dynamic pricing and allocation is clearly the preferred mathematical method of optimization. However, while often easy to formulate Bellman equations, finding solutions is not necessarily easy. So far, the promise is more than the reality when it comes to actual applications of pricing using dynamic approaches and realistic assumptions.

The assumption of fixed customer valuations for a population of customers does not reflect reality. Moving toward distributional assumptions for willingness-to pay-distributions is a move towards recognizing customer diversity. However, many of the distributional assumptions made for mathematical tractability can be restrictive. The number of ancillary items considered in the analyses is often restricted to no more than one or two, which is far too few to be useful to airlines in deciding on pricing strategies.

Joint pricing of ancillaries and the primary item has not received much consideration and can be important. Some movement has been made but the literature “does not offer specific models for the pricing and optimization of bundled offers” [24].

Statistical estimation is in its infancy when it comes to ancillary pricing. A common assumption is that demand is linear in prices. This might be somewhat realistic over restricted ranges but is unrealistic in general. Dynamic pricing is mathematically superior. However, there is a dearth of models that allow for realistic scenarios with multiple ancillary items and realistic assumptions about customer behavior. AI and machine learning are the new trendy methods but, at this stage, offer only black box solutions where underlying market forces are masked. It remains to be seen if these will evolve into truly useful approaches.

The bottom line and practical implications for airline revenue managers is that the assumptions made in most research on ancillary items make unrealistic assumptions. For ease of mathematical analysis, models often assume just a few ancillary items. Modeling of demand functions often requires assumptions such as linearity in price or uniform willingness to pay distributions which makes their use in applications problematic.

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