computer sciences and mathematics forum

MDPI

Proceeding Paper

Testing for the Asymmetric Optimal Hedge Ratios with an Application to Bitcoin †

Abdulnasser Hatemi-J

Department of Economics and Finance, College of Business and Economics, UAE University, Al Ain 15551, United Arab Emirates; ahatemi@uaeu.ac.ae

† Presented at the 12th International Conference on Time Series and Forecasting, Canaria, Spain, 15–17 July 2026.

Abstract

Reducing financial risk is of paramount importance to investors, financial institutions, and corporations. Since the pioneering contribution of Johnson, the optimal hedge ratio based on futures is regularly utilized. The current paper suggests an efficient method for testing the null hypothesis of a symmetric optimal hedge ratio against the asymmetric alternative one within a multivariate setting. If the null is rejected, the position-dependent optimal hedge ratios can be estimated via the suggested model. This approach is expected to enhance the accuracy of the implemented hedging strategies compared to the standard methods since it accounts for the fact that the source of risk depends on whether the investor is a buyer or a seller of the risky asset. An application is provided using spot and futures prices of Bitcoin. The results strongly support the view that the optimal hedge ratio for this cryptocurrency is position-dependent. The investor that is long in Bitcoin has a much higher conditional optimal hedge ratio compared to the one that is short in the asset. The difference between the two conditional optimal hedge ratios is statistically significant, which has important repercussions for implementing risk management strategies.

Keywords: hedging; asymmetry; hypothesis testing; Bitcoin; futures

1. Introduction

Hedging can increase profit stability for the seller as well as stability in the costs for the buyer, which is a fundamental factor for the success of long-run planning. One of the main functions of derivative securities is to achieve hedging, which means reducing (or neutralizing if possible) the price risk of an asset via the derivative security. Futures contracts are regularly used for this purpose according to [1]. Having a long position or a short position in a futures contract for hedging purposes depends on the need of the hedger. Assume that the relationship between the spot instrument and the futures as the hedging instrument is positive. If the hedger has a long position in the original asset, then the need is to sell in the future and therefore a short position in the futures contract is appropriate. On the other hand, if the hedger needs to buy the original asset in the future, then a long position in the futures is required for dealing with the underlying risk. Another important issue within this context that is usually neglected in the literature is the difference in the source of risk depending on whether the trader is a buyer or seller of the original asset. The source of risk for a trader that is short in a risky asset is increasing prices, while the source of risk for a trader with a long position is decreasing prices. For example, an oil-importing country as a buyer is exposed to the risk of increasing oil prices, whereas an oil-exporting country's risk exposure is determined by falling oil prices. Hence, a price

Check for updates

Academic Editors: Olga Valenzuela, Fernando Rojas, Luis Javier Herrera, Hector Pomares and Ignacio Rojas

Published: 17 November 2026

Copyright: © 2026 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

change is not part of the risk for every given trading position and there is a need for a position-dependent hedging strategy. Since there are potentially two different values for the optimal hedge ratio depending on whether the trader is short or long in the risky asset, it is crucial to assess not only if these estimated values are statistically significant individually, but also if their difference is statistically significant. Thus, this paper aims at introducing an efficient test of difference between the estimated values that can be implemented by using a system of equations for detecting whether the optimal hedge ratios are asymmetric or not. To the author's best knowledge, this is the first attempt to provide a method for testing the asymmetric optimal hedge ratios, which explicitly accounts for the position the trader is taking in the risky asset. An application to Bitcoin is provided, which is the cryptocurrency that has the largest market capitalization worldwide and also has a functional futures market.

The rest of this paper is structured as follows. Section 2 provides a brief review of the literature. The method for testing the null hypothesis of a symmetric optimal hedge ratio against the asymmetric alternative one is introduced in Section 3. A numerical application on hedging the price risk of Bitcoin is provided in Section 4. The conclusions are offered in the last section.

2. A Brief Literature Review

Two influential studies conducted by [2,3] exhibit empirically that a cryptocurrency can perform as a useful hedge for protecting the value of some other investment assets. Ref. [4] provides empirical results that show cryptocurrencies can be useful as a hedge again oil price fluctuations to some extent. Ref. [5] investigates the potential hedging performance of Bitcoin for protecting the equity markets of the BRIC countries via dynamic methods. They present empirical results showing that Bitcoin can perform as an efficient hedge for protecting the value of the financial markets in these countries. According to [6], Bitcoin can function as a hedge instrument for other currencies based on the empirical findings obtained using intraday data. Ref. [7] provides a complete literature review of hedge ratios.

The proposed method in this paper is used to test whether or not futures of Bitcoin can function as a hedge to protect the spot value of Bitcoin from the perspective of both a buyer and seller of this very risky asset. Bitcoin was introduced by [8], and has the highest market value currently compared to many alternative cryptocurrencies. Cryptocurrencies are characterized by illiquidity, structure breaks with alternations and extremely high volatility. Therefore, hedging is of paramount importance when dealing with cryptocurrencies. Currently, the only cryptocurrency that has a futures market is Bitcoin. Ref. [9] provides an interesting study of factors determining the adaptation of cryptocurrencies in 137 countries. Stimulating discussions on advantages and disadvantages of cryptocurrencies are offered by [10,11]. For a detailed review of the literature on cryptocurrencies, see [12]. Ref. [13] investigates the impact of cryptocurrencies on the natural environment. A stochastic difference equation for the pricing of cryptocurrencies is formulated in [14]. For a recent investigation of the dynamics in the cryptocurrency markets, see [15]. Ref. [16] provides new insights on the dynamic interaction between conventional financial markets and the markets for cryptocurrencies.

3. Methodology

In this section, symmetric and asymmetric optimal hedge ratios are defined. An efficient test is also introduced to test the null hypothesis of a symmetric optimal hedge ratio against the asymmetric alternative hypothesis. It is shown how the asymmetric optimal hedge ratios can be estimated when needed via a system of equations that is also flexible to

https://doi.org/10.3390/cmsf2026016032

include multivariate generalized autoregressive conditional heteroskedasticity (GARCH) effects with a potential multivariate t-distribution.

3.1. The Symmetric Optimal Hedge Ratio

The optimal hedge ratio (OHR), which was introduced by [1] originally, aims at making sure that the total value of the hedged portfolio remains unaltered (see also [17]) by having an offsetting position in the futures contract versus the original asset. The main question in this case is how many futures contracts are needed for each unit of the original asset to minimize the risk of the hedged portfolio? The answer to this question can be provided by estimating the OHR. Note that the hedged portfolio includes the quantities of the spot instrument (i.e., the original asset in this case) as well as the hedging instrument (i.e., the futures contract in this case) and it can be expressed mathematically as the following according to [1]:

$$V_h = Q_s S - Q_f F \tag{1}$$

where V_h represents the value of the hedged portfolio, Q_s and Q_f are the quantity of the spot and futures instrument respectively, and S and F are the prices of the spot and the futures, respectively. Equation (1) can be transformed into changes. Since the only source of uncertainty are the two prices, Equation (1) can be expressed as the following:

$$\Delta V_h = Q_s \Delta S - Q_f \Delta F \tag{2}$$

where Δ is the first difference operator. The ultimate goal of the hedging strategy is to achieve $\Delta V_h = 0$, which results in having $\frac{Q_f}{Q_s} = \frac{\Delta S}{\Delta F}$. Now, let us define the following:

$$h = \frac{Q_f}{Q_s} \tag{3}$$

which implies

$$h = \frac{\Delta S}{\Delta F} \tag{4}$$

Consequently, h represents the hedge ratio, which can be obtained as the slope parameter in a regression of the price change of the spot instrument (ΔS) on the price change of the futures (ΔF) as the hedging instrument. This point can be demonstrated mathematically. By substituting Equation (3) into Equation (2), the following equation can be obtained:

$$\Delta V_h = Q_s[\Delta S - h\Delta F] \tag{5}$$

The optimal hedge ratio (OHR) is the one that minimizes the risk of a change in the value of the portfolio that is hedged. This risk is measured by the variance of Equation (5), which is presented below:

$$V[\Delta V_h] = Q_s^2\left[\sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F\right] \tag{6}$$

Here σ_S^2 and σ_F^2 represent the variance of ΔS and ΔF, respectively. The correlation coefficient between ΔS and ΔF is denoted by ρ. In order to obtain the OHR, Equation (6) is minimized with regard to the parameter h. That is,

$$\frac{\partial[V[\Delta V_h]]}{\partial h} = Q_s^2\left[2h\sigma_F^2 - 2\rho\sigma_S\sigma_F\right] = 0, \tag{7}$$

which gives

$$h^* = \rho\frac{\sigma_S}{\sigma_F} \tag{8}$$

This OHR can also be attained by estimating the following regression model:

$$\Delta S_t = \alpha + h\Delta F_t + u_t, \tag{9}$$

https://doi.org/10.3390/cmsf2026016032

where $t = 1, \ldots, T$; α is an intercept; and u_t is an error term. The estimated slope in Equation (9), i.e., h, is the optimal hedge ratio that is exactly equal to the value obtained by Equation (8). It should be mentioned that the sign of the optimal hedge ratio matters. The strategies that are defined in Table 1 describe how the investor should create the hedged portfolio depending on whether the optimal hedge ratio is positive or negative.

Table 1. Strategies for hedging via futures.

When $h > 0$	When $h < 0$
Long original asset + short futures	Long original asset + long futures
Short original asset + long futures	Short original asset + short futures

Note that if the strategies defined in Table 1 are not exactly implemented and an opposite position is undertaken in any case, it will lead to increasing risk, which would imply speculation. Thus, it is very crucial for the investor to make sure that the goal and the strategy are consistent with each other before considering any strategy.

3.2. The Asymmetric Optimal Hedge Ratios

There have been previous attempts to allow some form of asymmetry in the volatility for the optimal hedge ratio without explicitly providing position-dependent optimal hedge ratios, such as [18], among others. However, the explicit asymmetric optimal hedge ratios were introduced in the literature by [19]. Allowing for asymmetry accords well with reality because investors tend to react stronger to negative changes compared to the positive ones. In addition, the asymmetric hedge ratios provide position-dependent hedging values. For the stochastic optimal hedge ratios, see [20]. These ratios are denoted by h^+ and h^-, which can be calculated via the following equations:

$$h^+ = \rho^+ \frac{\sigma_S^+}{\sigma_F^+} \tag{10}$$

or

$$h^- = \rho^- \frac{\sigma_S^-}{\sigma_F^-} \tag{11}$$

Here σ_S^+ signifies the standard deviation of ΔS^+, σ_F^+ is the standard deviation of ΔF^+, and the correlation coefficient between ΔS^+ and ΔF^+ is denoted by ρ^+. Similar values are defined for the negative component. The following equations are used to transform the data into positive and negative components:

$$\Delta S_t^+ = MAX(\Delta S_t, 0\ ) \tag{12}$$

$$\Delta F_t^+ = MAX(\Delta F_t, 0\ ) \tag{13}$$

$$\Delta S_t^- = MIN(\Delta S_t, 0\ ) \tag{14}$$

and

$$\Delta F_t^- = MIN(\Delta F_t, 0\ ) \tag{15}$$

In the next section, an explicit test for the null hypothesis of no asymmetry in the optimal hedge ratio is presented.

4. The Asymmetric Optimal Hedge Ratio Test

In order to detect whether or not the hedge ratio is symmetric, we suggest estimating the following system of regression equations:

$$S_t^+ = \alpha^+ + h^+ \Delta F_t^+ + u_t^+ \tag{16a}$$

$$\Delta S_t^- = \alpha^- + h^- \Delta F_t^- + u_t^- \tag{16b}$$

$$\sigma_t^{2+} = \gamma^+ + \sum_{i=1}^{K^+} \phi_j^+ u_{t-i}^{2+} + \sum_{j=1}^{Q^+} \lambda_j^+ \sigma_{t-j}^{2+} \tag{16c}$$

$$\sigma_t^{2-} = \gamma^- + \sum_{i=1}^{K^-} \phi_j^- u_{t-i}^{2-} + \sum_{j=1}^{Q^-} \lambda_j^- \sigma_{t-j}^{2-} \tag{16d}$$

$$\sigma_t^+ \sigma_t^- = \gamma^\mp + \sum_{i=1}^{K^\mp} \phi_j^\mp u_{t-i}^+ u_{t-i}^- + \sum_{j=1}^{Q^\mp} \lambda_j^\mp \sigma_{t-j}^+ \sigma_{t-j}^- \tag{16e}$$

Making use of a system of equations is required for efficiently estimating the parameters in the model since positive and negative components are not independent of each other. The variables σ_t^{2+} and σ_t^{2-} represent a measure of time-dependent conditional variance for u_t^+ and u_t^-, respectively. The symbols γ, Φ and λ represent volatility parameters to be estimated for each case. The optimal lag orders K^+, Q^+, K^- and Q^- can be determined by minimizing an information criterion. It is also possible to include the cross-product $\sigma_t^+ \times \sigma_t^-$ term. Model (16) can be estimated by using the multivariate GARCH method, which can capture the clustering property of volatility that usually characterizes financial assets. For a survey on multivariate GARCH models, see [20]. The issue of whether there are multivariate GARCH effects or not can also be settled empirically by testing the null hypothesis of constant variance against time-varying conditional variance by conducting a multivariate ARCH test [21]. If the null is not rejected, then Equation 16(a–e) can be estimated via the seemingly unrelated regression equations (SURE) method. The model can also be estimated by making use of the multivariate t-distribution in order to remedy the potential fat-tails effect to some extent. Since our proposed approach emphasizes two hedge ratio parameters, i.e., h^+ and h^-, it is crucial to assess not only if these coefficients are statistically significant individually, but also if their difference $(h^+ - h^-)$ is statistically significant. Thus, the following hypothesis can be tested by the test in [22]: H_0: $h^+ - h^- = 0$ against the alternative H_1: $h^+ - h^- \neq 0$. If the underlying null hypothesis is not supported empirically, it is to be concluded that the optimal hedge ratios are asymmetric. In that case, the investor with a short position in the asset needs to make use of h^+ for hedging purposes and the investor that is long should rely on h^-.

5. An Application

The suggested asymmetric test is applied to estimate the optimal hedge ratio for Bitcoin. The sample period covers the last week of December in 2017 until the first week of March in 2024. The spot and futures exchange rates of Bitcoin against the US dollar are used. The choice of the start of the sample period is restricted by data availability since Bitcoin futures were launched by the CBOE (the Chicago Board Options Exchange) and CME (the Chicago Mercantile Exchange) in December 2017 as an instrument for hedging the price risk of Bitcoin. The source of the data is Yahoo Finance. The estimation results based on the system of Equation (14), which is estimated via a multivariate GARCH (1, 1) model assuming a multivariate t-distribution, are presented in Table 2. Here the focus is on the estimated values for the optimal hedge ratios only. Thus, the rest of the estimated volatility parameters are not presented.

As is evident from the estimation results, each individual optimal hedge ratio is statistically strongly significant. The null hypothesis that the two hedge ratios are equal to each other is also rejected strongly. Thus, the optimal hedge ratio for Bitcoin is indeed asymmetric. This in turn implies that the trader who is short in Bitcoin needs to have a long position of 0.399432 units in the futures for each unit of spot for optimal hedging. On the other hand, the trader that is long in Bitcoin should have a short position of 0.713761 units

in the futures for each unit of spot for ideal hedging. Surprisingly, the optimal hedge ratio for a long position in Bitcoin is 78.69% higher than the optimal hedge ratio for a trader with a short position in the underlying asset.

Table 2. The test results for arch effects (estimation results).

Parameters	Parameter Values
h^+	0.399432
h^-	0.713761
Null Hypothesis	***p*-Value**
$H_0 : h^+ = 0$	<0.0001
$H_0 : h^- = 0$	<0.0001
$H_0 : h^+ - h^- = 0$	<0.0001

6. Conclusions

Hedging is a crucial strategy utilized by investors and enterprises for reducing risk. This paper proposes an efficient methodological framework for testing whether or not the optimal hedge ratio is asymmetric. In case the null hypothesis of symmetry is rejected, each position-dependent hedge ratio can be obtained via the suggested model. An application is provided using the spot and futures prices of Bitcoin. The results show that the null hypothesis of a symmetric optimal hedge ratio can be strongly rejected. The estimation results also reveal that the optimal hedge ratio for a long position in Bitcoin is almost 78% higher compared to the optimal hedge ratio for a short position in this very risky asset. Thus, using the same hedge ratio for both positions, as is common practice in the existing literature (see [23–25] among others), can be an inefficient strategy for reducing risk. However, in order to establish the robustness of the obtained empirical evidence, further empirical investigations with regard to other assets and different time horizons are needed in the future using the suggested approach. Ref. [26] provides an extension and application of the method introduced in this article within a multivariate cointegration framework.

Funding: This project is funded by the college of business and economics, the UAE university, via the CARP 2026 grant, which is greatly appreciated by the author.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data is available on request.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Johnson, L.L. The Theory of Hedging and Speculation in Commodity Futures. *Rev. Econ. Stud.* **1960**, *27*, 139–151. [CrossRef]
2. Dyhrberg, A.H. Hedging capabilities of bitcoin. Is it the virtual gold? *Financ. Res. Lett.* **2016**, *16*, 139–144. [CrossRef]
3. Dyhrberg, A.H. Bitcoin, gold and the dollar—A GARCH volatility analysis. *Financ. Res. Lett.* **2016**, *16*, 85–92. [CrossRef]
4. Będowska-Sójka, B.; Kliber, A. Can cryptocurrencies hedge oil price fluctuations? A pandemic perspective. *Energy Econ.* **2022**, *115*, 106360. [CrossRef]
5. Shahzad, S.J.H.; Bouri, E.; Rehman, M.U.; Roubaud, D. The hedge asset for BRICS stock markets: Bitcoin, gold or VIX. *World Econ.* **2022**, *45*, 292–316. [CrossRef]
6. Urquhart, A.; Zhang, H. Is bitcoin a hedge or safe haven for currencies? An intraday analysis. *Int. Rev. Financ. Anal.* **2019**, *63*, 49–57. [CrossRef]
7. Chen, S.S.; Lee, C.F.; Lin, F.L.; Shrestha, K. Hedge Ratios: Theory and Applications. In *Handbook of Investment Analysis, Portfolio Management, and Financial Derivatives*; World Scientific: Singapore, 2024; pp. 1277–1328.

8. Nakamoto, S. Bitcoin: A Peer-to-Peer Electronic Cash System. Technical Report. 2008. Available online: https://bitcoin.org/bitcoin.pdf (accessed on 17 February 2022).
9. Bhimani, A.; Hausken, K.; Arif, S. Do national development factors affect cryptocurrency adoption? *Technol. Forecast. Soc. Change* **2022**, *181*, 121739. [CrossRef]
10. Taleb, N.N. Bitcoin, currencies, and fragility. *Quant. Financ.* **2021**, *21*, 1249–1255. [CrossRef]
11. Lipton, A. Cryptocurrencies change everything. *Quant. Financ.* **2021**, *21*, 1257–1262. [CrossRef]
12. Bariviera, A.F.; Merediz-Solà, I. Where do we stand in cryptocurrencies economic research? A survey based on hybrid analysis. *J. Econ. Surv.* **2021**, *35*, 377–407. [CrossRef]
13. Erdogan, S.; Ahmed, M.Y.; Sarkodie, S.A. Analyzing asymmetric effects of cryptocurrency demand on environmental sustainability. *Environ. Sci. Pollut. Res.* **2022**, *29*, 31723–31733. [CrossRef] [PubMed]
14. El-Khatib, Y.; Hatemi-J, A. On a regime switching illiquid high volatile prediction model for cryptocurrencies. *J. Econ. Stud.* **2024**, *51*, 485–498. [CrossRef]
15. Yan, H. Understanding Cryptocurrency Market Dynamics: An Efficiency Perspective. 2024. Available online: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4872474 (accessed on 2 December 2024).
16. Huang, L. The relationship between cryptocurrencies and convention financial market: Dynamic causality test and time-varying influence. *Int. Rev. Econ. Financ.* **2024**, *91*, 811–826. [CrossRef]
17. Hull, J.C. *Options, Futures, and Other Derivatives*; Pearson Education: New York, NY, USA, 2012.
18. Brooks, C.; Henry, O.T.; Persand, G. The effect of asymmetries on optimal hedge ratios. *J. Bus.* **2002**, *75*, 333–352. [CrossRef] [PubMed]
19. El-Khatib, Y.; Hatemi-J, A. Asymmetric Optimal Hedge Ratio with an Application. *IAENG Int. J. Appl. Math.* **2011**, *41*, 330–333.
20. Hatemi-J, A.; El-Khatib, Y. Stochastic optimal hedge ratio: Theory and evidence. *Appl. Econ. Lett.* **2012**, *19*, 699–703. [CrossRef]
21. Hacker, S.; Hatemi-J, A. A Test for Multivariate ARCH Effects. *Appl. Econ. Lett.* **2005**, *12*, 411–417. [CrossRef]
22. Wald, A. Statistical decision functions. *Ann. Math. Stat.* **1949**, *20*, 165–205. [CrossRef]
23. Corbet, S.; Lucey, B.; Peat, M.; Vigne, S. Bitcoin Futures—What use are they? *Econ. Lett.* **2018**, *172*, 23–27. [CrossRef]
24. Sebastião, H.; Godinho, P. Bitcoin futures: An effective tool for hedging cryptocurrencies. *Financ. Res. Lett.* **2020**, *33*, 101230. [CrossRef]
25. Matsui, T.; Knottenbelt, W.J. Optimal Hedge Ratio Estimation for Bitcoin Futures using Kalman Filter. In Proceedings of the 2023 IEEE International Conference on Blockchain and Cryptocurrency (ICBC), Dubai, United Arab Emirates, 1–5 May 2023; pp. 1–5.
26. Gatarek, L.; Johansen, S. Optimal Portfolio Hedging with Cointegration. *Oxf. Bull. Econ. Stat.* **2026**, 1–15. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.