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Vector Entropy Modeling as a Tool for Monitoring and Forecasting the State of Economic Systems †

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Abstract

Economic systems are complex, adaptive and interact with other systems. Their characteristics include stochastic and non-stationary behavior, complex interrelationships between elements, and small data samples. This makes it difficult to monitor and forecast their state. The purpose of this study is to describe and test a systematic approach to entropy monitoring and forecasting of the state of economic systems on real macroeconomic data. The article describes a vector entropy model that considers the main systemic patterns. The model is based on the decomposition of the differential entropy of a set of indicators into components—entropic measures of dynamism, randomness and self-organization. This model develops the extended version of the second law of thermodynamics proposed by Ilya Prigozhin for the entropy of open systems and allows it to be implemented in practice. An example of monitoring the German macroeconomic indicators system is considered. The results obtained generally correspond to the known estimates of the situation. Vector entropy modeling makes it possible to supplement well-known macro economical modeling and forecasting methods with a system-wide interpretation and assessment of the state of the analyzed processes and systems.

Keywords: economic system; monitoring; entropy; model; random vector; chaoticness; self-organization

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1. Introduction

Economic systems are adaptive and open, constantly interacting with their external environment. They can change their behavior and their structure to maintain or achieve an optimal state when external conditions change. Their features include stochastic and non-stationary behavior, the complex nature of the relationships between elements, and small data samples [1,2]. Monitoring and forecasting the state of economic systems in conditions of such limited information is a well-known problem, since it is necessary to assess not only the state of the system, but also analyze trends.

It appears that one way of overcoming this problem may be to adopt a systems approach [3,4]. An adequate model of an economic system should explain its behavior,

meet system-wide patterns, allow the system to be influenced using control variables and have an efficiency criterion. The purpose of the system approach is to detect system effects and understand the mechanisms of their occurrence based on modeling.

In systems modeling, one of the common approaches involves the use of entropy. This is because entropy is a fundamental property of any system exhibiting non-deterministic or probabilistic behavior [5,6]. In 1955, I. Prigozhin formulated an extended version of the second law of thermodynamics [7], according to which the total change in entropy of an open system should be represented in two parts. The first one is irreversible internal processes that are necessarily accompanied by the transfer of part of the energy of the ordered processes (kinetic energy of a moving body, electric current energy, etc.) into the energy of disordered processes, ultimately into heat; the second part is the exchange of energy and matter between the system and the environment

$$dS = dS_{out} + dS_{in}. \quad (1)$$

where dS is the total change in the entropy of an open system; dS_{out} is the change in entropy during the exchange processes with the environment; dS_{in} is the change in entropy during the processes occurring in the system. However, the issue of practical application of this theory to the study of real systems—for example, economic, organizational, living, technical, etc.—has not been disclosed yet.

The effect of entropy on the evolution of open systems has been studied by many scientists, for example, in publications [8–11] and others. Their publications note that changing open systems either leads to its degradation, or it is a process of self-organization, because of which more complex structures appear. The concept of entropy is often used in modern science to describe behavior, structural organization and disorganization, and the degree of destruction of connections between elements of various systems [12–15]. However, as a rule, the information entropy H_I Shannon is used [16]

$$H_I(D) = -\sum_{i=1}^{L} p_i \ln p_i, \quad (2)$$

where $p_1, \ldots, p_L$ are the probabilities that the system S accepts a finite number of corresponding states D_i, i.e., $p_i = P(S \in D_i)$.

Information entropy (2) is a function of the set of its states $D = \bigcup_{i=1}^{L} D_i$, and its use as a system model has several disadvantages:

i. Some states of D_i may not be known at all, and the principle of formation of system states is not clear.
ii. Estimating the probabilities of p_i requires appropriate samples, and it is almost impossible to obtain statistics for some states.
iii. Modeling interrelations between the elements of multidimensional systems is difficult.
iv. The issue of considering the main systemic patterns in (2) is not clear.
v. As a result, the use of information entropy for the study of complex systems is difficult, and adequate models have been developed only for specific tasks.

The purpose of the study is to propose and test on real data a systematic approach to entropy monitoring and forecasting of the state of economic systems that meets the main features of economic processes.

2. Materials and Methods

Systems in economics are often described in the form of a set of indicators characterizing their functioning [17]. Therefore, we will further represent the economic system as a

random vector with correlated components $\mathbf{Y} = (Y_1, \ldots, Y_m)$ in order to understand the main socio-economic indicators characterizing the behavior of the analyzed system S.

A well-known universal characteristic based on the probability density $p_{\mathbf{Y}}(\mathbf{x})$ is the entropy of a continuous random vector (differential entropy) $H(\mathbf{Y})$ introduced in [16] as

$$H(\mathbf{Y}) = -\int_{-\infty}^{+\infty} \ldots \int_{-\infty}^{+\infty} p_{\mathbf{Y}}(\mathbf{x}) \ln p_{\mathbf{Y}}(\mathbf{x}) d\mathbf{x}, \tag{3}$$

for a one-dimensional random variable $H(Y) = -\int_{-\infty}^{+\infty} p_Y(x) \ln p_Y(x) dx$.

Formula (2) was proposed by Claude Shannon as a formal analog of information entropy for continuous messages [16]. In (3), it is assumed that the improper integral converges to $\mathbb{R}^m$. A special case is considered when $\mathbf{Y}$ has a multidimensional normal distribution with a covariance matrix $\Sigma_{\mathbf{Y}}$:

$$H(\mathbf{Y}) = 0.5 \ln \left[(2\pi e)^m |\Sigma_{\mathbf{Y}}| \right]. \tag{4}$$

However, in general formulation, the integral (3) was not considered. It was established in [18] that if everywhere on $G \subseteq \mathbb{R}^m$ the probability density $p_{\mathbf{Y}}(\mathbf{x})$ is absolutely continuous ($p_{\mathbf{Y}}(\mathbf{x}) > 0$) and all one-dimensional random variables have variances, then integral (3) converges and is equal to the difference between the entropy measure of chaoticness $H_V(\mathbf{Y})$ and the entropy measure of dependence (self-organization) $G_R(\mathbf{Y})$ [19]

$$H(\mathbf{Y}) = H_V(\mathbf{Y}) - G_R(\mathbf{Y}), \tag{5}$$

where $H_V(\mathbf{X}) = \sum_{j=1}^m \ln \sigma_{Y_j} + \sum_{j=1}^m \kappa_j$, $\kappa_j = H(Y_j^*) = H\left(\frac{X_j}{\sigma_{Y_j}}\right)$ is the entropy of the random variable Y_j^* with unit variance and the same type of distribution as Y_j (let us call κ_j the entropy of the type of distribution of the random variable Y_j), $G_R(\mathbf{Y}) = -0.5 \ln [\prod_{k=2}^m (1 - R^2_{Y_k|Y_1 \ldots Y_{k-1}})]$, $R^2_{Y_k|Y_1 \ldots Y_{k-1}}$ is the index of determination of the regression dependence of the random variable Y_k on $Y_1, \ldots, Y_{k-1}$.

In the special case for a Gaussian random vector $\mathbf{Y}$

$$H_V(\mathbf{Y}) = \sum_{j=1}^m \ln \sigma_{Y_j} + m \ln \sqrt{2\pi e}, \quad G_R(\mathbf{Y}) = -0.5 \ln |\mathbf{R}_{\mathbf{Y}}|, \tag{6}$$

where $|\mathbf{R}_{\mathbf{Y}}|$ is the determinant of the correlation matrix $\mathbf{R}_{\mathbf{Y}}$. Note that formulas (6) are easy to obtain using simple transformations of expression (4).

The contribution of any component Y_m to the change in entropy is equal to

$$\Delta H(Y_m) = H(\mathbf{Y}) - H(\mathbf{Y} \backslash Y_{m-1}) = \ln \sigma_{Y_m} + \kappa_m + 0.5 \ln \left(1 - R^2_{Y_m|Y_1 \ldots Y_{m-1}}\right), \tag{7}$$

for the Gaussian distribution $\Delta H(Y_m) = \ln \sigma_{Y_m} + \ln \sqrt{2\pi e} + 0.5 \ln \left(|\mathbf{R}_{\mathbf{Y}}| / \left| \mathbf{R}_{\mathbf{Y} \backslash Y_m} \right| \right)$.

Splitting the entropy $H(\mathbf{Y})$ into entropy measures of chaoticness and self-organization allows us to identify the main systemic patterns. The value $H_V(\mathbf{Y})$ is equal to the limiting entropy corresponding to the complete independence of macroeconomic indicators, as if all elements were functioning in a mutually random manner, i.e., when order is completely absent. It characterizes the consideration of the system as consisting of parts (additivity).

The value $G_R(\mathbf{Y})$ reflects the degree of interdependence between indicators, characterizing the properties of the system as a whole (integrity): when there is complete mutual independence between the elements of the system, $G_R(\mathbf{Y}) = 0$; when there is a strict functional dependence between at least two elements of the system, $G_R(\mathbf{Y}) \to +\infty$.

The larger $G_R(\mathbf{Y})$ is, the closer the links between the elements of the system and the more organized the system is, and the lower the system's entropy $H(\mathbf{Y})$ is. In fact, the value of $G_R(\mathbf{Y})$ is equal to the reduction in the entropy measure of chaoticness due to the orderliness between the elements of the system. Thus, it can be said that entropy $H(\mathbf{Y})$ characterizes the relationship between disorder and order in the system.

Since $H_V(\mathbf{Y})$ and $G_R(\mathbf{Y})$ characterize different aspects of the system's behavior, it was proposed in [18] to consider the entropy $H(\mathbf{Y})$ in vector form $(H_V(\mathbf{Y}); G_R(\mathbf{Y}))$.

If we consider the change in entropy (5) over time, its increment is expressed as

$$\Delta H(\mathbf{Y}) = \Delta H_V(\mathbf{Y}) - \Delta G_R(\mathbf{Y}). \tag{8}$$

Equation (8) is consistent with the extended version of the second law of thermodynamics (1). A change in the entropy measure of chaoticness (disorder) requires energy, which is obtained through exchange processes with the environment: $dS_{out} \sim \Delta H_V(\mathbf{Y})$. A change in the entropy measure of self-organization (order) occurs within the system itself because of self-organization processes taking place within the system, by increasing or decreasing the degree of interconnection between its elements: $dS_{in} \sim \Delta G_R(\mathbf{Y})$. The sign "–" in (8) before $\Delta G_R(\mathbf{Y})$ indicates that the change in $G_R(\mathbf{Y})$ is in the opposite direction: an increase in the system's degree of organization reduces its entropy, whereas a decrease in organization, conversely, increases it.

Another important feature (8) is that it becomes dimensionless. This simplifies the practical application of the vector entropy model.

It should be noted that the entropy measure $H_V(\mathbf{Y})$) characterizes changes in the system's behavior caused both by the trend dynamics of the system's elements and by the effect of randomness. Therefore, the behavior over time of each component Y_j of the system constitutes a random process consisting of the sum of the trend component $\widetilde{Y}_j$ and the random component (noise) ξ_j, which are mutually independent, i.e.,

$$Y_j = \widetilde{Y}_j + \xi_j, \;\; \sigma^2_{Y_j} = \sigma^2_{\widetilde{Y}_j} + \sigma^2_{\xi_j},$$

at the same time, the trend $\widetilde{Y}_j$ can be both deterministic (due to changes in the average level of the Y_j component) and stochastic (due to unaccounted-for factors), for example, random walk [20,21].

Given that $\ln \sigma_{Y_j} = \ln \sigma_{\widetilde{Y}_j} + \ln \frac{\sigma_{Y_j}}{\sigma_{\widetilde{Y}_j}}$, instead of the decomposition (5) have

$$H(\mathbf{Y}) = H_D(\mathbf{Y}) + H_S(\mathbf{Y}) - G_R(\mathbf{Y}), \tag{9}$$

where $H_D(\mathbf{Y}) = \sum_{j=1}^{m} \ln \sigma_{\widetilde{Y}_j} + \sum_{j=1}^{m} \kappa_j$ is an entropy measure of dynamism, and $H_S(\mathbf{Y}) = \sum_{j=1}^{m} \ln \frac{\sigma_{Y_j}}{\sigma_{\widetilde{Y}_j}}$ is an entropy measure of randomness.

The entropy measure of randomness $H_S(\mathbf{Y})$ is dimensionless, and its minimum is bounded from below by zero (where all $\sigma_{Y_j} = \sigma_{\widetilde{Y}_j}$, or $\sigma_{\xi_j} = 0$). This allows us to standardize the assessment of the stability of the behavior of the system under analysis. In formula (9), the entropy measure of chaoticness is presented as the sum of the entropy measures of dynamism and randomness: $H_V(\mathbf{Y}) = H_D(\mathbf{Y}) + H_S(\mathbf{Y})$. We can therefore distinguish within $(H_D(\mathbf{Y}))$ and the influence of random factors on the system $(H_S(\mathbf{Y}))$. This refinement expands the capabilities of vector entropy modeling for the analysis of complex systems.

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3. Results

We shall monitor the state of the system as follows. The progress of the system's development can be described as the behavior of the vector $(H_V(\mathbf{Y}), G_R(\mathbf{Y}))$. This indicator allows us to consider both the overall trajectory of development and local fluctuations, in both $H_V(\mathbf{Y})$ and $G_R(\mathbf{Y})$. We shall interpret the stability of the system as the vector $(H_D(\mathbf{Y}), H_S(\mathbf{Y}))$, which characterizes the relative fluctuations of the entropy measure of dynamism. The smaller these fluctuations $H_S(\mathbf{Y})$ are relative to the measure of dynamism $H_D(\mathbf{Y})$, the more stable the system functions.

Let us examine the vector entropy approach using the example of a study of Germany's macroeconomic development based on annual statistical data from 1991 to 2024. We will conduct the study using data from the German Federal Statistical Office (https://www.statistikportal.de/de (accessed on 28 May 2026)) and the International Monetary Fund. This study utilized eight fairly common macroeconomic indicators, which are listed in Table 1. Indicators Y_1 and Y_2 relate to the population, Y_3, Y_4 and Y_5 relate to the economy, whilst Y_6, Y_7 and Y_8 are indicators of living standards. We shall analyze changes in entropy using the example of a Gaussian data representation: (1) the use of the normal distribution is based on the central limit theorem; (2) the samples under analysis are small, which does not allow us to identify the distribution of the random vector.

Table 1. Used macroeconomic indicators.

Designation	Indicator and Unit of Measure
Y_1	Natural increase rate per 1000 people
Y_2	Proportion of the workforce as a percentage of the total population
Y_3	Per capita investment (in constant 2015 US dollars)
Y_4	Total electricity generation per capita (MWh/capita)
Y_5	Balance of payments per capita (in constant 2015 US dollars)
Y_6	Infant mortality (under 5 years) per 1000 live births
Y_7	State budget expenditure per capita (in constant 2015 US dollars)
Y_8	Housing cost-to-income ratio (relative to 2015)

We will carry out the monitoring using samples covering 11-year periods: 1991–2001, . . ., 2014–2024. This period proved to be optimal in terms of statistical smoothing (the sample size must be greater than the number of indicators analyzed), on the one hand, and taking into account the dynamics of entropy change (the longer this period, the less sensitive the monitoring is to changes in the analyzed system), on the other. Data smoothing was performed using a sliding window of seven adjacent observations. Second-order polynomials were used as the smoothing filter. The results of the calculation of entropy indicators are presented in Table 2. The calculations were performed using Equations (6) to (9).

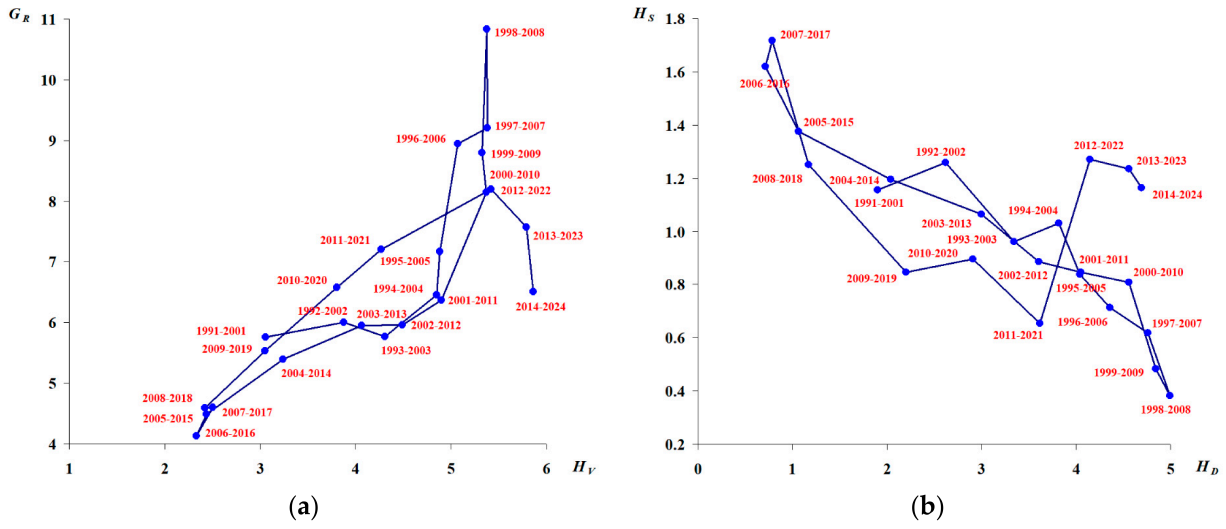
Table 2. Entropy indicators calculation results.

Period	H_V	H_D	H_S	G_R	H
1991–2001	3.056	1.900	1.156	5.756	−2.700
1992–2002	3.878	2.619	1.259	6.000	−2.123
1993–2003	4.306	3.344	0.962	5.764	−1.458
1994–2004	4.851	3.820	1.031	6.459	−1.608
1995–2005	4.881	4.043	0.838	7.168	−2.287

Table 2. *Cont.*

Period	H_V	H_D	H_S	G_R	H
1996–2006	5.069	4.356	0.713	8.949	−3.880
1997–2007	5.379	4.760	0.619	9.206	−3.826
1998–2008	5.374	4.992	0.382	10.839	−5.464
1999–2009	5.324	4.841	0.483	8.796	−3.472
2000–2010	5.369	4.560	0.810	8.148	−2.779
2001–2011	4.901	4.054	0.847	6.371	−1.470
2002–2012	4.491	3.605	0.886	5.960	−1.468
2003–2013	4.066	3.000	1.065	5.954	−1.888
2004–2014	3.237	2.042	1.196	5.391	−2.153
2005–2015	2.439	1.063	1.376	4.490	−2.051
2006–2016	2.332	0.712	1.620	4.130	−1.798
2007–2017	2.505	0.787	1.718	4.598	−2.093
2008–2018	2.421	1.170	1.252	4.596	−2.174
2009–2019	3.051	2.204	0.847	5.535	−2.484
2010–2020	3.806	2.911	0.895	6.575	−2.769
2011–2021	4.271	3.617	0.654	7.201	−2.930
2012–2022	5.418	4.147	1.272	8.200	−2.782
2013–2023	5.792	4.558	1.234	7.566	−1.774
2014–2024	5.860	4.696	1.164	6.509	−0.649

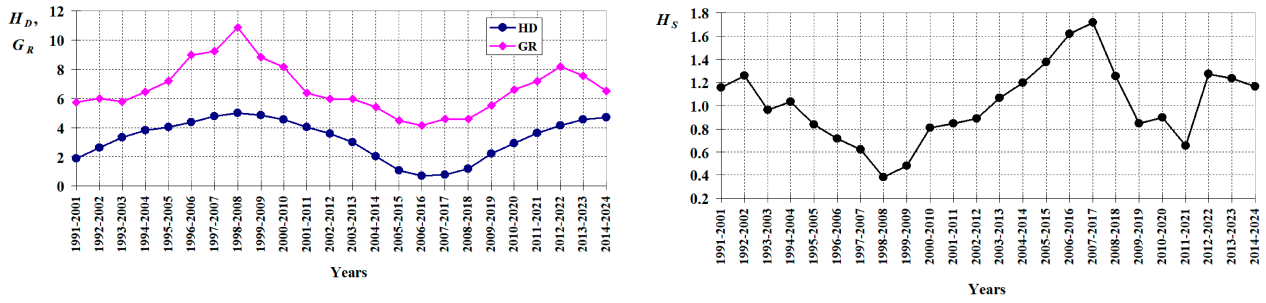
Figure 1 shows the hodographs of entropy measures of the vector of German macroeconomic indicators for 1991–2024: (a) (H_V(**Y**), G_R(**Y**)); (b) (H_D(**Y**), H_S(**Y**)).



(**a**) (**b**)

Figure 1. Hodographs of entropy measures of the vector of German macroeconomic indicators for 1991–2024: (**a**) entropy measures of chaoticness and self-organization (H_V(**Y**), G_R(**Y**)); (**b**) entropy measures of dynamism and randomness (H_V(**Y**), G_R(**Y**)).

Figure 2 shows the values of entropic measures of dynamism, self-organization, and randomness of the vector of German macroeconomic indicators for 1991–2024.



(**a**) (**b**)

Figure 2. Entropic measures of the vector of German macroeconomic indicators for 1991–2024: (**a**) measures of dynamism and self-organization; (**b**) randomness.

4. Discussion

Four characteristic periods can be distinguished from the graphs.

1991–2008. The increase in entropy measures of chaoticness self-organization and dynamism, and the decrease in the entropy measure of randomness, characterize the progressive development of the system of macroeconomic indicators. During this period, Y_6 and Y_8 made the greatest contribution to the growth in the interrelationship between indicators, whilst Y_7 and Y_8 had the greatest influence on the growth in chaos. The sharp spike in entropy measures in 2008 was caused by the global financial crisis.

2009–2016. A significant decrease in the entropy measures of chaoticness, self-organization and dynamism by 2.99, 4.67 and 4.13, respectively, and an increase in the entropy measure of randomness by 1.24 indicate unstable behavior in the system of macroeconomic indicators. During this period, the indicator Y_2 made the greatest contribution to the entropy measure of self-organization, whilst Y_4 made the smallest. The indicators Y_7 and Y_8 had the greatest influence on the decrease in the entropy measures of chaoticness and dynamism.

2017–2021. Once again, a period of steady development in the system of macroeconomic indicators was observed: an increase in entropy measures of chaoticness, self-organization and dynamism, and a decrease in the entropy measure of randomness. Here, the increase in chaos was most strongly influenced by Y_4, Y_7 and Y_8. The main contribution to the increase in entropy measures of chaoticness and dynamism came from indicators Y_1 and Y_7. It should also be noted that there was a sharp decrease in the influence of indicator Y_6 on chaoticness and dynamism; this was caused by the stabilization of the indicator at a level of 3.5.

2022–2024. A controversial period: entropy measures of chaoticness, dynamism and randomness show a slight increase, whilst the entropy measure of self-organization decreases (due to indicators Y_3, Y_7 and Y_8).

Overall, two periods of instability can be observed. The first period was triggered by the global financial crisis. Once it had ended, the system of indicators began to gradually return to its 2007 state. However, the COVID-19 pandemic and the subsequent conflict in Ukraine halted this stabilization. Each period of instability is accompanied by a decrease in the entropy measure of self-organization and the entropy measure of randomness. The increase in the entropy measure of dynamism during the most recent period may also represent a stochastic trend caused by international instability and the influence of a number of random factors.

5. Conclusions

The vector entropy model is based on the system-wide principles of additivity and integrity. It enables known macroeconomic modeling and forecasting methods to be sup-

plemented with a system-wide interpretation and assessment of the state of the processes and systems under analysis.

The approach described here builds on the extended version of the second law of thermodynamics for the entropy of open systems proposed by Ilya Prigogine and enables its practical application.

This paper examines an example of monitoring Germany's macroeconomic indicators. The results obtained are generally consistent with established assessments of the situation.

The vector entropy model uses measurable macroeconomic indicators for which forecasting methods have been developed. Therefore, the proposed model can be used in forecasting multidimensional systems in the economy.

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