computer sciences and mathematics forum

MDPI

Proceeding Paper

A Probabilistic Fuzzy Framework for Decision-Making Under Uncertainty in Complex Systems †

Chirag Dhankhar [1,*] and Shalini Yadav [2]

[1] Amity College of Commerce, Amity University Haryana, Gurugram 122413, India
[2] Department of Physical Science (Mathematics), Baba Mastnath University, Rohtak 124021, India; shaliniyadav147258@gmail.com
* Correspondence: cdhankhar@ggn.amity.edu
† Presented at the 2nd International Online Conference on Mathematics and Applications, 10–12 June 2026; Available online: https://sciforum.net/event/IOCMA2026.

Abstract

Fuzzy multi-criteria decision-making (MCDM) methods typically fix criterion weights as crisp numbers, ignoring the fact that decision makers are often uncertain about relative importance itself. We propose a Probabilistic Fuzzy Weighted Averaging (PFWA) operator that models this weight uncertainty explicitly, by placing a hesitancy-derived probability distribution over a finite space of q-rung orthopair fuzzy (q-ROFS) weight scenarios rather than embedding probability inside the ratings, as prior probabilistic q-ROFS operators do. We prove that PFWA is consistent, bounded, monotonic, and stable under small perturbations of the scenario probabilities, and illustrate the framework using an oil-supplier-selection case study, where it avoids the rank reversals produced by fixed-weight q-ROFWA aggregation.

Keywords: q-rung orthopair fuzzy sets; probabilistic aggregation operators; multi-criteria decision making; weight uncertainty

1. Introduction

Multi-criteria decision-making (MCDM) under uncertainty routinely combines two distinct sources of imprecision: vagueness in how alternatives are rated against each criterion, and disagreement, among decision makers or elicitation rounds, about how important each criterion is. Fuzzy MCDA methods, including q-rung orthopair fuzzy set (q-ROFS) extensions [1,2], address the first source by allowing membership and non-membership degrees rather than crisp scores. They almost universally address the second source by collapsing it into a single, deterministic weight vector, obtained by simple averaging, an eigenvector method, or a best–worst procedure. This discards precisely the information, disagreement across experts or elicitation rounds, that motivated a probabilistic treatment in the first place.

Existing probabilistic extensions of q-ROFS (e.g., probabilistic linguistic q-ROFS operators [3] and probabilistic hesitant orthopair fuzzy weight-optimisation models [4]) do incorporate probability, but they place it inside the ratings of the alternatives, not on the criterion weights themselves; the weight vector used in the final aggregation step remains a single crisp vector. Weight uncertainty is therefore never propagated into the ranking. Existing studies have advanced uncertainty modelling through q-rung neutrosophic fuzzy sets, fuzzy risk assessment, probabilistic system modelling, AI-assisted decision-making, optimisation, MCDM frameworks, sensor optimisation, entropy-based q-ROFS TOPSIS,

Check for updates

Academic Editor: Antonio Di Crescenzo

Published: 13 November 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

and other methods [5–15]. But these approaches generally address only specific aspects of uncertainty which lack a unified framework that simultaneously integrates probabilistic information with fuzzy representations for complex decision-making.

Outside the two closely related operators [3,4], the broader q-ROFS aggregation literature spans several distinct directions like Aczel–Alsina and Archimedean-based operators [16,17], dynamic and complex-valued extensions [18,19], and interval-valued or dual-hesitant variants [20,21]. Parallel probabilistic treatments have emerged under fuzzy environments like Fermatean, Pythagorean, and hesitant-linguistic frameworks [22–25]. Probability is again attached to ratings or attribute weights derived by optimisation rather than to an explicit weight-scenario distribution method. Soft-set and hypersoft extensions [26] and similarity-measure-based approaches [27–29] address some related but orthogonal sources of fuzziness. A recent systematic review that spans 119 q-ROFS-MCDM studies [30] confirms that deterministic weighting remains the important norm across this entire body of work, regardless of which aggregation operator is ultimately castoff.

This leaves a specific gap in that no existing q-ROFS aggregation operator treats the criterion weight vector itself as an uncertain, probabilistically distributed object. Once a weight vector is fixed—regardless of how it is derived—all downstream uncertainty about relative criterion importance is discarded before aggregation begins, even if the operators are explicitly designed for other kinds of fuzziness or hesitancy.

Motivated by this literature gap, this paper proposes a novel framework. Instead of treating the criterion weight vector as fixed, we treat them as uncertain. We build a finite set of reasonable q-ROFS-elicited weight scenarios, and give each one a specific probability, derived directly from the data itself rather than to be assumed from outside.

This paper proposes a framework that instead treats the criterion weight vector as a random object, with a finite set of plausible q-ROFS-elicited weight scenarios and an explicit, endogenously derived probability distribution over them. Our contribution is threefold: (i) a formal definition of a probabilistic fuzzy weight space over q-ROFS weight scenarios; (ii) a hesitancy-derived rule for assigning probabilities to scenarios that requires no external calibration; and (iii) the Probabilistic Fuzzy Weighted Averaging (PFWA) operator, together with proofs of consistency, boundedness, monotonicity, and stability.

2. Preliminaries

We recall the q-ROFS foundation of orthopair fuzzy numbers, their score/accuracy ordering, and the standard q-ROFWA operator on which our probabilistic extension is built.

Definition 1 (q-Rung Orthopair Fuzzy Set [1]). *Let X be a fixed reference set. A q-rung orthopair fuzzy set (q-ROFS) A on X is $A = \{\langle x,\mu_A(x),\nu_A(x)\rangle | x \in \mathbf{X}\}$, where μ_A, ν_A:$X \to [0, 1]$ denote, respectively, the membership and non-membership degrees, subject to $\mu_A(x)^q + \nu_A(x)^q \leq 1$ for a rung $q \geq 1$. The hesitancy degree of x is $\pi_A(x) = \left(1 - \mu_A(x)^q - \nu_A(x)^q\right)^{1/q}$. A pair $\alpha = (\mu,\nu)$ satisfying the constraint is called a q-rung orthopair fuzzy number (q-ROFN).*

Definition 2 (Score and accuracy [2]). *For a q-ROFN $\alpha = (\mu,\nu)$, the score function is $S(\alpha) = \mu^q - \nu^q \in [-1, 1]$ and the accuracy function is $H(\alpha) = \mu^q + \nu^q$. For two q-ROFNs α_1,α_2: $\alpha_1 \succ \alpha_2$ if $S(\alpha_1) > S(\alpha_2)$, or if $S(\alpha_1) = S(\alpha_2)$ and $H(\alpha_1) > H(\alpha_2)$.*

Definition 3 (Basic operations [2]). *For q-ROFNs $\alpha_1 = (\mu_1,\nu_1)$, $\alpha_2 = (\mu_2,\nu_2)$, and $\lambda > 0$:*

$$\alpha_1 \oplus \alpha_2 = \left(\left(\mu_1^q + \mu_2^q - \mu_1^q\mu_2^q\right)^{1/q}, \nu_1\nu_2\right) \quad (1)$$

https://doi.org/10.3390/cmsf2026015020

$$\lambda\alpha_1 = \left(\left(1-\left(1-\mu_1^q\right)^{\lambda}\right)^{1/q}, \nu_1^{\lambda}\right) \tag{2}$$

Definition 4 (q-ROFWA operator [2]). *Given q-ROFNs* $\alpha_j = (\mu_j,\nu_j)$, $j = 1,\ldots,m$, *and a weight vector* $w = (w_1,\ldots,w_m)$, $w_j \geq 0$, $\sum_j w_j = 1$:

$$q\text{−ROFWA}_w(\alpha_1,\ldots,\alpha_m) = \bigoplus_{j=1}^{m} w_j\alpha_j = \left(\left(1-\prod_{j=1}^{m}\left(1-\mu_j^q\right)^{w_j}\right)^{1/q}, \prod_{j=1}^{m}\nu_j^{w_j}\right) \tag{3}$$

Liu and Wang [2] show that q-ROFWA is idempotent, bounded, and monotonic for any fixed crisp weight vector w. Our objective is to extend these guarantees to the case where w is itself uncertain.

3. Proposed Probabilistic Fuzzy Framework

In this section, we present the novel probabilistic fuzzy weight space and the PFWA operator itself, and establish the theoretical guarantees that justify its use in place of fixed-weight aggregation.

3.1. Weight-Scenario Space

Rather than eliciting a single crisp weight vector, we collect K independent weight elicitations (e.g., from K experts, or K elicitation rounds with the same panel), each expressed as q-ROFS linguistic judgments and converted, via Definitions 1–2, into a normalised crisp weight vector $w^{(k)} = \left(w_1^{(k)},\ldots,w_m^{(k)}\right)$, $\sum_j w_j^{(k)} = 1$, for $k = 1,\ldots,K$. The weight-scenario space is $\Omega = \{w^{(1)},w^{(2)},\ldots,w^{(K)}\}$. Each scenario retains the mean hesitancy degree $\bar{\pi}^{(k)} = \frac{1}{m}\sum_j \pi_j^{(k)}$ of the underlying q-ROFS judgments, which records how confident the corresponding expert or round was.

3.2. Hesitancy-Derived Probability Assignment

We assign probability mass to each scenario inversely to its hesitancy, without any external calibration or auxiliary optimisation:

$$P^{(k)} = \frac{1-\bar{\pi}^{(k)}}{\sum_{l=1}^{K}\left(1-\bar{\pi}^{(l)}\right)}, \qquad k = 1,\ldots,K \tag{4}$$

Since $\bar{\pi}^{(k)} \in [0,1)$ for every k, each $P^{(k)} \in (0,1]$ and $\sum_k P^{(k)} = 1$, so $\{P^{(k)}\}$ is a valid discrete probability distribution over Ω. Lower-hesitancy (higher-confidence) scenarios are automatically weighted more heavily.

3.3. The Probabilistic Fuzzy Weighted Averaging (PFWA) Operator

Let $\alpha_1,\ldots,\alpha_m$ be the q-ROFN ratings of a given alternative under criteria $x_1,\ldots,x_m$. For each scenario k, define the conditional aggregate $\beta^{(k)} = (\mu_k,\nu_k)$ using the crisp weight vector $w^{(k)}$ of that scenario in the ordinary q-ROFWA operator (Definition 4):

$$\beta^{(k)} = q\text{−ROFWA}_{w^{(k)}}(\alpha_1,\ldots,\alpha_m) = \left(\left(1-\prod_{j=1}^{m}\left(1-\mu_j^q\right)^{w_j^{(k)}}\right)^{1/q}, \prod_{j=1}^{m}\nu_j^{w_j^{(k)}}\right)$$

Definition 5 (PFWA operator)**.** *The Probabilistic Fuzzy Weighted Averaging value of* $\alpha_1,\ldots,\alpha_m$ *over the weight-scenario space* Ω*, with hesitancy-derived probabilities* $\{P^{(k)}\}$*, is the probability-weighted q-ROFS combination of the conditional aggregates:*

$$\mathrm{PFWA}(\alpha_1,\ldots,\alpha_m) = \bigoplus_{k=1}^{K} P^{(k)}\beta^{(k)} = (\mu^*,\nu^*) \tag{5}$$

$$\mu^* = \left(1-\prod_{k=1}^{K}\left(1-\mu_k^q\right)^{P^{(k)}}\right)^{1/q}, \qquad \nu^* = \prod_{k=1}^{K}\nu_k^{P^{(k)}} \tag{6}$$

PFWA is thus a two-level, nested q-ROFWA aggregation: the inner layer combines criteria under each weight scenario (Equation (5)), and the outer layer combines the K conditional aggregates using the hesitancy-derived probabilities as its own weight vector (Equations (6) and (7)). Weight uncertainty is encoded in the aggregation operator itself, rather than folded into the input ratings as in [3,4].

Throughout, the closure of PFWA (i.e., that $\mu^{*q}+\nu^{*q}\le 1$, so $\mathrm{PFWA}(\alpha_1,\ldots,\alpha_m)$ is itself a valid q-ROFN) follows directly from the closure of q-ROFWA under Definition 4, applied twice: once to obtain each $\beta^{(k)}$, and once more to combine the $\beta^{(k)}$ using weights $P^{(k)}$.

Theorem 1 (Consistency)**.** *If* $\alpha_j=\alpha=(\mu,\nu)$ *for every criterion* $j=1,\ldots,m$ *(unanimous ratings across all criteria, for every scenario), then* $\mathrm{PFWA}(\alpha,\ldots,\alpha)=\alpha$.

Proof. Fix a scenario k. Since $\sum_j w_j^{(k)}=1$, Equation (5) gives

$$\mu_k = \left(1-\prod_j(1-\mu^q)^{w_j^{(k)}}\right)^{1/q} = \left(1-(1-\mu^q)^{\sum_j w_j^{(k)}}\right)^{1/q} = (1-(1-\mu^q))^{1/q} = \mu,$$

and analogously $\nu_k=\prod_j \nu^{w_j^{(k)}} = \nu^{\sum_j w_j^{(k)}} = \nu$. Hence, $\beta^{(k)}=(\mu,\nu)=\alpha$ for every $k=1,\ldots,K$. Repeating the same argument for the outer aggregation, since $\sum_k P^{(k)}=1$, Equation (7) gives $\mu^*=\mu$ and $\nu^*=\nu$. Therefore, $\mathrm{PFWA}(\alpha,\ldots,\alpha)=(\mu,\nu)=\alpha$. □

Theorem 2 (Boundedness)**.** *Let* $\alpha_j=(\mu_j,\nu_j)$, $j=1,\ldots,m$, *and define* $\alpha^-=(\min_j\mu_j,\max_j\nu_j)$ *and* $\alpha^+=(\max_j\mu_j,\min_j\nu_j)$*. Then,* $S(\alpha^-)\le S(\mathrm{PFWA}(\alpha_1,\ldots,\alpha_m))\le S(\alpha^+)$.

Proof. Fix scenario k and write $\mu^+=\max_j\mu_j$. Since $\mu_j\le\mu^+$ for all j, $1-\mu_j^q\ge 1-(\mu^+)^q$, and because $w_j^{(k)}\ge 0$ with $\sum_j w_j^{(k)}=1$, taking the weighted product preserves the inequality:

$$\prod_j\left(1-\mu_j^q\right)^{w_j^{(k)}} \ge \prod_j\left(1-(\mu^+)^q\right)^{w_j^{(k)}} = 1-(\mu^+)^q;$$

hence, $\mu_k\le\mu^+$. The symmetric argument with $\mu^-=\min_j\mu_j$ gives $\mu_k\ge\mu^-$; the same reasoning applied to ν_j gives $\nu^-\le\nu_k\le\nu^+$. Thus, $S(\alpha^-)\le S(\beta^{(k)})\le S(\alpha^+)$ for every $k=1,\ldots,K$. Applying the identical chain of inequalities to the outer aggregation (6)–(7), with $\mu_k,\nu_k,P^{(k)}$ in place of $\mu_j,\nu_j,w_j^{(k)}$, gives $\min_k\mu_k\le\mu^*\le\max_k\mu_k$ and $\min_k\nu_k\le\nu^*\le\max_k\nu_k$, and therefore, $S(\alpha^-)\le S(\mathrm{PFWA}(\alpha_1,\ldots,\alpha_m))\le S(\alpha^+)$. □

Theorem 3 (Monotonicity)**.** *Let* $\alpha_j=(\mu_j,\nu_j)$ *and* $\alpha_j'=(\mu_j',\nu_j')$, $j=1,\ldots,m$, *be two rating profiles for the same weight-scenario space* Ω *with* $\mu_j'\ge\mu_j$ *and* $\nu_j'\le\nu_j$ *for every j. Then,* $S(PFWA(\alpha_1',\ldots,\alpha_m'))\ge S(PFWA(\alpha_1,\ldots,\alpha_m))$.

Proof. Fix scenario k. Since $\mu_j'\ge\mu_j$ for all j, $1-\mu_j{}'^q\le 1-\mu_j^q$, and because $w_j^{(k)}\ge 0$,

$$\prod_j (1-\mu_j{}'^q)^{w_j^{(k)}} \le \prod_j \left(1-\mu_j^q\right)^{w_j^{(k)}};$$

hence, $\mu_k{}' \ge \mu_k$. Likewise, $\nu_j{}' \le \nu_j$ for all j gives $\nu_k{}' \le \nu_k$. So $\beta^{(k)'}$ dominates $\beta^{(k)}$ componentwise for every $k = 1,\ldots,K$. Since the hesitancy-derived probabilities $P^{(k)}$ depend only on the weight-scenario judgments and not on the α_j, they are unchanged between the two profiles; applying the same monotonicity argument to the outer aggregation (6)–(7) yields $\mu^{*'} \ge \mu^*$ and $\nu^{*'} \le \nu^*$, and therefore, $S(\mathrm{PFWA}(\alpha')) \ge S(\mathrm{PFWA}(\alpha))$. □

Theorem 4 (Stability under probability perturbation). *Let* $\{P^{(k)}\}$ *be the hesitancy-derived distribution used to compute* $\mathrm{PFWA}(\alpha) = (\mu^*,\nu^*)$, *and let* $\left\{\tilde{P}^{(k)}\right\}$ *be any perturbed distribution with* $\left|\tilde{P}^{(k)} - P^{(k)}\right| \le \delta$ *for every* k *and* $\sum_k \tilde{P}^{(k)} = 1$. *Let* $\widetilde{\mathrm{PFWA}}(\alpha) = \left(\tilde{\mu}^*,\tilde{\nu}^*\right)$ *denote the aggregate recomputed with* $\left\{\tilde{P}^{(k)}\right\}$. *Then,* $\left|S\left(\widetilde{\mathrm{PFWA}}(\alpha)\right) - S(\mathrm{PFWA}(\alpha))\right| \le C \cdot K \cdot \delta$ *for a constant* C *depending only on* q *and on the conditional aggregates* $\{\beta^{(k)}\}$.

Proof. Write $\mu^{*q} = 1 - \exp\left(\sum_k P^{(k)} a_k\right)$, where $a_k = \ln\left(1-\mu_k^q\right) \le 0$ is fixed once the ratings and Ω are fixed. The map $P \mapsto \sum_k P^{(k)} a_k$ is linear in P, so replacing P by $\tilde{P}$ with $\left|\tilde{P}^{(k)} - P^{(k)}\right| \le \delta$ changes this sum by at most $K\delta \max_k |a_k|$. Because μ_k stays bounded away from 1 for realistic ratings, $\max_k |a_k|$ is finite, and the compositions $t \mapsto 1 - e^t$ and $t \mapsto t^{1/q}$ are Lipschitz on the resulting compact domain; the same argument applies to ν^* via the analogous product form. Passing the resulting bound on $\left|\tilde{\mu}^* - \mu^*\right|$ and $\left|\tilde{\nu}^* - \nu^*\right|$ through the Lipschitz score map $S(\mu,\nu) = \mu^q - \nu^q$ gives $|\Delta S| \le C \cdot K \cdot \delta$ for a constant C absorbing the Lipschitz constants above. □

PFWA can be compared against ordinary q-ROFWA [2] and the two probabilistic q-ROFS operators most closely related to this work [3,4]. All three existing operators are idempotent, bounded, and monotonic under their respective conditions, and PFWA satisfies all properties of q-ROFWA (Theorems 1-3). The key difference lies elsewhere, as [3,4] place probability on the ratings of the alternatives, whereas PFWA places it on the weight vector itself. As a result, PFWA propagates disagreement about the importance of the criterion into the final ranking, rather than disagreement about how well an alternative performs. PFWA also offers a stability guarantee (Theorem 4) that neither [3] nor [4] provides, PFWA treats the weight vector as a random object subject to perturbation.

3.4. PFWA-Based MCDM Method

Input: Alternatives $A_1,\ldots,A_n$; criteria $x_1,\ldots,x_m$; ratings α_{ij} (q − *ROFNs*); K weight elicitations.

Step 1: For each elicitation $k = 1,\ldots,K$: convert linguistic weight judgments to q-ROFNs and normalise to a crisp weight vector $w^{(k)}$ (Section 3.1).

Step 2: Compute mean hesitancy $\overline{\pi}^{(k)}$ for each scenario.

Step 3: Compute scenario probabilities $P^{(k)}$ using Equation (4).

Step 4: For each alternative A_i and each scenario k, compute the conditional aggregate $\beta_i{}^{(k)}$ via Equation (5).

Step 5: For each alternative A_i, compute $PFWA(A_i) = (\mu_i{}^*, \nu_i{}^*)$ using Equations (6) and (7), using $P^{(k)}$ as outer weights.

Step 6: Compute score $S(PFWA(A_i))$ for each alternative.
Step 7: Rank alternatives by descending score (ties broken by accuracy H, Definition 2).
Output: Ranked list of alternatives

4. Case Study: Oil-Supplier Selection

We illustrate PFWA on an oil-procurement supplier-selection problem with $m = 6$ evaluation criteria (Table 1) and a set of $K = 4$ candidate alternatives. Linguistic ratings of each alternative under each criterion are converted to q-ROFNs (Definition 1, $q = 3$) using (Table 2), and scenario probabilities are computed from Equation (4). The ratings and weight scenarios below are presented as an illustrative numerical example rather than a field-collected dataset, in order to isolate and demonstrate PFWA's behaviour in weight disagreement without confounding it with measurement noise in the underlying ratings. We applied the algorithm outlined in Section 3.4 to this case study.

Table 1. Evaluation criteria for oil supplier.

Criteria	Evaluation Criterion	Description
C1	Price Stability	Consistency and predictability of crude oil pricing
C2	Supply Reliability	Ability to maintain uninterrupted supply under geopolitical uncertainty
C3	Transportation Cost	Logistics and shipping expenditure associated with import operations
C4	Environmental Compliance	Adherence to international environmental and sustainability standards
C5	Contract Flexibility	Adaptability of trade agreements and procurement contracts
C6	Strategic Alliance Strength	Long-term diplomatic and strategic relationship with importing nation

Table 2. Linguistic-to-q-ROFN conversion scale.

Linguistic Term	μ	ν
Very Poor	0.10	0.85
Poor	0.30	0.65
Fair	0.50	0.45
Good	0.70	0.25
Very Good	0.90	0.05

Under PFWA, *Supplier* 2 obtains the highest score (≈0.68), reflecting the most favourable balance of membership and non-membership degrees once all K weight scenarios are aggregated. *Supplier* 1 follows closely with a score of ≈0.61, indicating strong but slightly less consistent performance across the criteria. *Supplier* 3 attains a moderate score of ≈0.54, while Supplier 4 records the lowest score at ≈0.49, suggesting comparatively weaker performance across the evaluation criteria once weight uncertainty is accounted for. This yields the overall ranking *Supplier* 2 $\succ$ *Supplier* 1 $\succ$ *Supplier* 3 $\succ$ *Supplier* 4, with *Supplier* 2 emerging as the most preferred alternative under the proposed framework.

5. Comparative Analysis

This section compares PFWA against fixed-weight q-ROFWA aggregation directly, using the case study of Section 4, and examines the stability of that comparison under perturbation of the scenario probabilities. Applying Equations (5)–(7) to the alternative ratings yields the PFWA scores and the induced ranking, compared against the ranking produced by ordinary q-ROFWA aggregation under a single, averaged crisp weight vector (Figure 1).

https://doi.org/10.3390/cmsf2026015020

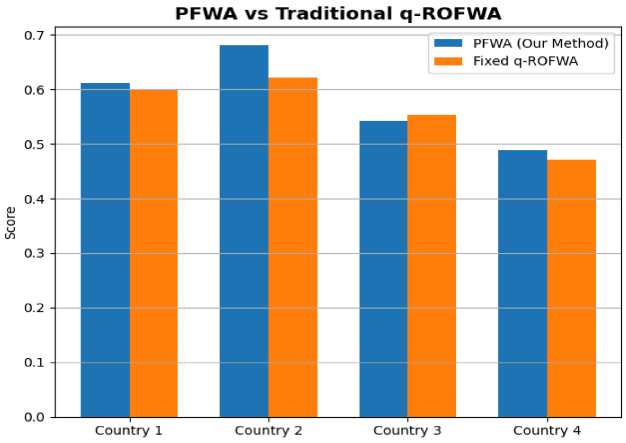
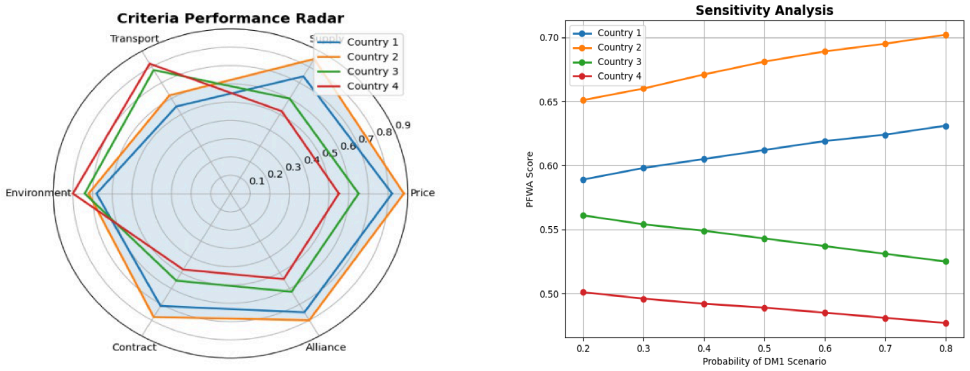


Figure 1. Comparative analysis of PFWA and traditional q-ROFWA rankings.

The fixed-weight q-ROFWA ranking reverses the order of two alternatives whose scores are close under one plausible weight scenario but not under others; PFWA, by averaging across the full scenario space with hesitancy-derived probabilities, produces a ranking that is not sensitive to which single scenario is treated as authoritative.

Figure 2b perturbs each $P^{(k)}$ by up to $\pm 10\%$ (renormalised) and recomputes the ranking; consistent with Theorem 4, the ranking order of the alternatives is unchanged throughout this range.



(**a**) (**b**)

Figure 2. This figure displays the following: (**a**) Radar analysis of criterion-wise performance of supplier alternatives. (**b**) Sensitivity analysis of PFWA scores under probability perturbation.

6. Discussion

Existing probabilistic extensions of q-ROFS probabilistic linguistic q-ROFS, Archimedean operators [3] and weight-optimisation in (p,q)-rung probabilistic hesitant orthopair fuzzy environments [4] place the probabilistic component inside the ratings of the alternatives, while the criterion weight vector used at the aggregation step remains a single crisp vector obtained by some deterministic weighting method. To the best of our knowledge, PFWA is the first operator to place an explicit, formally defined probability distribution directly over q-ROFS criterion-weight scenarios, with the distribution itself derived endogenously from the hesitancy of the weight elicitation rather than fitted or assumed. This distinction matters in practice precisely where fixed-weight methods are least reliable: when two or more alternatives are closely ranked and small changes in the weight vector would reverse their order.

7. Conclusions and Future Work

We introduced a probabilistic fuzzy weight space over q-ROFS criterion-weight scenarios and the associated PFWA aggregation operator, proved that it is consistent, bounded, and monotonic under the same conditions as ordinary q-ROFWA, and established a stability bound governing its behaviour under perturbation of the hesitancy-derived scenario probabilities. The oil-supplier case study shows that PFWA avoids the rank reversals produced by fixed-weight aggregation when plausible weight scenarios disagree. Future work will extend the framework to interval-valued q-ROFS settings, apply it to supply-chain and healthcare MCDA problems, and develop a software toolkit implementing this method for practitioners.

Author Contributions: Conceptualization, C.D.; methodology, C.D. and S.Y.; formal analysis, C.D.; writing—original draft preparation, C.D.; writing—review and editing, S.Y.; visualisation, C.D. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Yager, R.R. Generalized orthopair fuzzy sets. *IEEE Trans. Fuzzy Syst.* **2017**, *25*, 1222–1230. [CrossRef]
2. Liu, P.; Wang, P. Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *Int. J. Intell. Syst.* **2018**, *33*, 259–280. [CrossRef]
3. Ranjan, M.J.; Kumar, B.P.; Bhavani, T.D.; Padmavathi, A.V.; Bakka, V. Probabilistic linguistic q-rung orthopair fuzzy Archimedean aggregation operators for group decision-making. *Decis. Mak. Appl. Manag. Eng.* **2023**, *6*, 473–496. [CrossRef]
4. Bao, J.; Kong, X. Weight optimization decision algorithm in (p,q)-rung probabilistic hesitant orthopair fuzzy environments. *Symmetry* **2023**, *15*, 898. [CrossRef]
5. Alqurashi, O.S.; Alsager, K.M. Enhancing complex decision-making under uncertainty: Theory and applications of q-rung neutrosophic fuzzy sets. *Symmetry* **2025**, *17*, 1224. [CrossRef]
6. Božić, V. Fuzzy approach to risk management: Enhancing decision-making under uncertainty. *Preprint* **2023**. [CrossRef]
7. Meng, X.; Zhu, J.; Chen, G.; Shi, J.; Li, T.; Song, G. Dynamic and quantitative risk assessment under uncertainty during deepwater managed pressure drilling. *J. Clean. Prod.* **2022**, *334*, 130249. [CrossRef]
8. Luther, B.; Gunawan, I.; Nguyen, N. Identifying effective risk management frameworks for complex socio-technical systems. *Saf. Sci.* **2023**, *158*, 105989. [CrossRef]
9. Pouya, P.; Madni, A.M. Probabilistic system modeling for complex systems operating in uncertain environments. In *Current Trends and Advances in Model Based Systems Engineering*; Springer: Berlin/Heidelberg, Germany, 2022.
10. Zarei, E.; Khan, F.; Abbassi, R. How to Account Artificial Intelligence in Human Factor Analysis of Complex Systems? *Process Saf. Environ. Prot.* **2023**, *171*, 736–750. [CrossRef]
11. Meng, D.; Zhu, S.P. *Multidisciplinary Design Optimization of Complex Structures under Uncertainty*; Taylor & Francis: Abingdon, UK, 2024.
12. Sharma, M.K.; Dhiman, N.; Kumar, S.; Rathour, L.; Mishra, V.N. Neutrosophic Monte Carlo simulation approach for decision making in medical diagnostic process under uncertain environment. *Int. J. Neutrosophic Sci.* **2023**, *22*, 8–16. [CrossRef]
13. Ferdous, J.; Bensebaa, F.; Milani, A.S.; Hewage, K.; Bhowmik, P.; Pelletier, N. Development of a generic decision tree for the integration of multi-criteria decision-making (MCDM) and multi-objective optimization (MOO) methods under uncertainty. *Sustainability* **2024**, *16*, 2684. [CrossRef]
14. Suslu, B.; Ali, F.; Jennions, I.K. Understanding the role of sensor optimisation in complex systems. *Sensors* **2023**, *23*, 7819. [CrossRef] [PubMed]
15. Arora, R.; Dhankhar, C.; Yadav, A.K.; Kumar, K. A TOPSIS Method Based on Entropy Measure for q-Rung Orthopair Fuzzy Sets and Its Application in MADM. In *Soft Computing for Problem Solving: Proceedings of the SocProS 2022*; Springer Nature: Singapore, 2023; pp. 709–718.

16. Ali, G.; Alsager, K.; Ali, A. Novel linguistic q-rung orthopair fuzzy Aczel-Alsina aggregation operators for group decision-making with applications. *AIMS Math.* **2024**, *9*, 32328–32365. [CrossRef]
17. Senapati, T.; Chen, G.; Ullah, I.; Khan, M.S.A.; Hussain, F. A novel approach towards multiattribute decision making using q-rung orthopair fuzzy Dombi–Archimedean aggregation operators. *Heliyon* **2024**, *10*, e27969. [CrossRef] [PubMed]
18. Farid, H.M.A.; Riaz, M.; Simic, V.; Peng, X. q-Rung orthopair fuzzy dynamic aggregation operators with time sequence preference for dynamic decision-making. *PeerJ Comput. Sci.* **2024**, *10*, e1742. [CrossRef] [PubMed]
19. Liu, P.; Mahmood, T.; Ali, Z. Complex q-rung orthopair fuzzy aggregation operators and their applications in multi-attribute group decision making. *Information* **2020**, *11*, 5. [CrossRef]
20. Liu, P.; Cheng, S. Interval-valued probabilistic dual hesitant fuzzy sets for multi-criteria group decision-making. *Int. J. Comput. Intell. Syst.* **2019**, *12*, 1393–1411. [CrossRef]
21. Naz, S.; Akram, M.; Alsulami, S.; Ziaa, F. Decision-making analysis under interval-valued q-rung orthopair dual hesitant fuzzy environment. *Int. J. Comput. Intell. Syst.* **2021**, *14*, 332–357. [CrossRef]
22. Ruan, C.; Chen, X.; Yan, L. Fermatean hesitant fuzzy multi-attribute decision-making method with probabilistic information and its application. *Axioms* **2024**, *13*, 456. [CrossRef]
23. Joshi, D.K.; Beg, I.; Kumar, S. Hesitant probabilistic fuzzy linguistic sets with applications in multi-criteria group decision making problems. *Mathematics* **2018**, *6*, 47. [CrossRef]
24. Saqlain, M.; Kumam, P.; Kumam, W.; Phiangsungnoen, S. Proportional distribution based Pythagorean fuzzy fairly aggregation operators with multi-criteria decision-making. *IEEE Access* **2023**, *11*, 72209–72226. [CrossRef]
25. Rahman, K.; Abdullah, S.; Ali, A.; Amin, F. Pythagorean fuzzy Einstein hybrid averaging aggregation operator and its application to multiple-attribute group decision making. *J. Intell. Syst.* **2018**, *29*, 736–752. [CrossRef]
26. Zulqarnain, R.M.; Siddique, I.; Iampan, A.; Baleanu, D. Aggregation operators for interval-valued Pythagorean fuzzy soft set with their application to solve multi-attribute group decision making problem. *Comput. Model. Eng. Sci.* **2022**, *131*, 1717–1750. [CrossRef]
27. Xing, X.; Luo, C.; Zhou, J.; Yan, M.; Liu, C.; Xu, T. Combining regional energy and intuitionistic fuzzy sets for infrared and visible image fusion. *Sensors* **2021**, *21*, 7813. [CrossRef] [PubMed]
28. Xia, M.; Xu, Z. Some new similarity measures for intuitionistic fuzzy values and their application in group decision making. *J. Syst. Sci. Syst. Eng.* **2010**, *19*, 430–452. [CrossRef]
29. Baccour, L.; Alimi, A.M.; John, R.I. Intuitionistic fuzzy similarity measures and their role in classification. *J. Intell. Syst.* **2015**, *25*, 221–227. [CrossRef]
30. Öztaş, S. Exploring q-rung orthopair fuzzy sets based on multi criteria decision making: A systematic literature review. *Egypt. Inform. J.* **2025**, *31*, 100755. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.