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Favourable Conditions for Majorana Phase Appearance in Neutrino Oscillation Probabilities †

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Abstract: The Majorana phases of neutrino mixing matrix do not appear either in vacuum or in matter modified oscillation probabilities. It was previously shown that for some particular forms of decoherence, the neutrino oscillations do depend on Majorana phases. Here, we show that such dependence also occurs for neutrino decay scenarios where mass eigenstates are not the decay eigenstates. We calculate two flavour survival/oscillation probabilities in such a scenario and discuss their CP and CPT properties.

Keywords: neutrino oscillations; Majorana phase; neutrino decay

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1. Introduction

Neutrinos are the only known elementary neutral fermions in nature, which makes them interesting because their chargelessness permits them to be their own antiparticles, i.e., they can be Majorana fermions. In the SM, the neutrinos are massless. The discovery of neutrino oscillations proves the existence of nonzero neutrino mass. Whether this mass is a Dirac mass or a Majorana mass is still an open question. The Majorana mass of neutrino violates lepton number by two units and leads to an interesting signal of neutrinoless double beta decay. Also, in this case the mixing matrix connecting the flavour eigenstates to mass eigensates has extra phases, which are called Majorana phases. However, neutrino oscillation probabilities, that depend on the mixing matrix elements and mass squared differences, do not depend on the Majorana phase [1–4] both in case of vacuum and matter neutrino oscillations.

It was shown in Ref. [5] that for a new form of neutrino decoherence, with an off-diagonal term in the decoherence matrix, the neutrino oscillation probabilities depend on Majorana phases. Also, these probabilities are CP-violating [6]. In this paper, which is based on Ref. [7], we discuss another possibility that leads to the appearance of Majorana phase in two flavour neutrino oscillation probabilities by considering the most general neutrino evolution Hamiltonian.

We discuss the dynamics of two-flavour neutrino oscillations in the next section. We conclude with a discussion of the CP and CPT properties of the survival and oscillation probabilities.

2. Vacuum Neutrino Oscillations

In general, the two flavour eigenstates $\nu_\alpha = (\nu_e \quad \nu_\mu)^T$ are connected to the two mass eigenstates $\nu_i = (\nu_1 \quad \nu_2)^T$ via a unitary matrix as

$$\nu_\alpha = U\,\nu_i = O\,U_{ph}\,\nu_i,$$

with

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \text{and} \quad U_{ph} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}. \tag{1}$$

The mixing angle θ and the Majorana phase ϕ parameterize the mixing matrix U. The phase ϕ in Equation (1) becomes absorbed in the neutrino mass eigenstates in the case of Dirac neutrinos through rephasing and we are left with the orthogonal mixing matrix. However, such rephasing of mass eigenstates is not valid for Majorana neutrinos. (Here, we assume that two other phases are pulled out on the left and are absorbed in the flavour states).

The evolution of neutrino flavour states can be given as

$$i\frac{d}{dt}\nu_\alpha(t) = -\left[\frac{(a_2 - a_1)}{2} O U_{ph} \sigma_z U_{ph}^\dagger O^T\right] \nu_\alpha(t), \tag{2}$$

where σ_z is the diagonal Pauli matrix, and $a_1 = m_1^2/2E$ and $a_2 = m_2^2/2E$. Here m_i are the mass eigenvalues of the neutrinos and E is the neutrino energy. Since $[U_{ph}, \sigma_z] = 0$, we have $OU_{ph}\sigma_z U_{ph}^\dagger O^T = O\sigma_z O^T$. This leads to flavour transition probability being independent of ϕ and we get

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{(a_2 - a_1)t}{2}\right) \equiv P_{e\mu}^{\text{vac}}, \tag{3}$$

where t is the time of travel by neutrino. Here, the different probabilities follow trivial relations such as, $P_{ee}^{\text{vac}} = 1 - P_{e\mu}^{\text{vac}} = P_{\mu\mu}^{\text{vac}}$ and $P_{\mu e}^{\text{vac}} = P_{e\mu}^{\text{vac}}$.

3. Oscillations with General Decay-Hamiltonian

In this section, we consider the most general neutrino evolution Hamiltonian, including decay terms

$$\mathcal{H} = M - i\Gamma/2, \tag{4}$$

where M is the mass matrix and Γ is the decay matrix with the following form

$$\mathcal{M} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \quad \Gamma/2 = \begin{pmatrix} b_1 & \frac{1}{2}\eta e^{i\xi} \\ \frac{1}{2}\eta e^{-i\xi} & b_2 \end{pmatrix}. \tag{5}$$

The above form of the decay matrix is allowed for a system of two particles which can oscillate into each other, e.g., neutral meson system [8–10] as well as neutrinos. In Equation (5), b_1, b_2, η and ξ are real decay parameters. The matrix Γ needs to be positive semi-definite which leads to the following constraints: $b_1, b_2 \geq 0$ and $\eta^2 \leq 4b_1 b_2$. In this case, the equation of motion has the form

$$i\frac{d}{dt}\nu_\alpha(t) = -\left[\frac{(a_2 - a_1)}{2} O\sigma_z O^T + \frac{i}{2}(b_1 + b_2)\sigma_0 + \frac{i}{2} O U_{ph}(\vec{\sigma}.\vec{\Gamma}) U_{ph}^\dagger O^T\right] \nu_\alpha(t), \tag{6}$$

where $\vec{\Gamma} = [\eta\cos\xi, -\eta\sin\xi, -(b_2 - b_1)]$. Since σ_x and σ_y do not commute with U_{ph} matrix, the phase ϕ remains in the evolution equation.

The time evolution operator for neutrinos in the mass eigenbasis can be expanded in the basis spanned by σ_0 and Pauli matrices [11,12] as

$$\mathcal{U} = e^{n_0}\left[\cosh n\, \sigma_0 + \frac{\vec{n}.\vec{\sigma}}{n} \sinh n\right]. \tag{7}$$

This expansion is parameterized by a complex four-vector $n_\mu \equiv (n_0, \vec{n})$, where $n = (n_x^2 + n_y^2 + n_z^2)^{1/2}$ and $n_\mu = Tr[(-i\mathcal{H}t).\sigma_\mu]/2$. The evolution matrix in flavour basis can be obtained as $\mathcal{U}_f = U\mathcal{U}U^{-1}$, where U is defined in Equation (1). The survival and oscillation probabilities are given by

$$P_{\alpha\beta} = \left| \left(\mathcal{U}_f \right)_{\alpha\beta} \right|^2. \tag{8}$$

In the approximation $b_1 = b_2 = b$ and $\eta \ll |a_2 - a_1|$, we obtain the survival probabilities as

$$\begin{aligned} P_{ee} &= e^{-2bt}\left(P_{ee}^{\text{vac}} - \eta\cos(\xi - \phi)\frac{\sin(2\theta)\sin[(a_2 - a_1)t]}{(a_2 - a_1)} \right) \\ P_{\mu\mu} &= e^{-2bt}\left(P_{\mu\mu}^{\text{vac}} + \eta\cos(\xi - \phi)\frac{\sin(2\theta)\sin[(a_2 - a_1)t]}{(a_2 - a_1)} \right) \end{aligned} \tag{9}$$

and the oscillation probabilities as

$$\begin{aligned} P_{e\mu} &= e^{-2bt}\left(P_{e\mu}^{\text{vac}} + 2\eta\sin(\xi - \phi)\frac{\sin(2\theta)\sin^2\left[\frac{1}{2}t(a_2 - a_1)\right]}{(a_2 - a_1)} \right) \\ P_{\mu e} &= e^{-2bt}\left(P_{\mu e}^{\text{vac}} - 2\eta\sin(\xi - \phi)\frac{\sin(2\theta)\sin^2\left[\frac{1}{2}t(a_2 - a_1)\right]}{(a_2 - a_1)} \right), \end{aligned} \tag{10}$$

where terms of order η^2 are neglected.

4. Results

In obtaining Equations (9) and (10), we made the approximation $b_1 = b = b_2$ and $\eta \ll |a_2 - a_1|$ so that the appearance of ϕ in neutrino oscillation probabilities has a simple algebraic form. It can be seen that the Majorana phase ϕ appears in the probability expressions only if the neutrino evolution equation contains the off-diagonal term of the decay matrix $\Gamma_{12} \propto \eta$. The presence of this term also violates the equalities $P_{\mu\mu} = P_{ee}$ and $P_{\mu e} = P_{e\mu}$ unlike the case of two flavour vacuum oscillations. Further, we also note that the second terms in the oscillation probabilities have opposite signs for the two cases $a_2 > a_1$ and $a_2 < a_1$, i.e., the oscillation probabilities are sensitive to the neutrino mass ordering.

In case of antineutrinos the relations $\bar{M} = M$ and $\bar{\Gamma} = \Gamma^*$ hold for the mass and decay matrices [13]. Here we have assumed that CPT is conserved. Therefore, antineutrino probabilities expressions can be obtained by making the substitutions $\phi \to -\phi$ and $\xi \to -\xi$ in the neutrino probability expressions. We find $P_{\bar{e}\bar{e}} = P_{ee}$, $P_{\bar{\mu}\bar{\mu}} = P_{\mu\mu}$ and $P_{\bar{\mu}\bar{e}} = P_{e\mu}$. However, $P_{\bar{e}\bar{\mu}} \neq P_{e\mu}$ and $P_{\mu e} \neq P_{e\mu}$, i.e., there is CP and T-violation.

The CP violating term in the oscillation probabilities in Equation (10) is proportional to $\eta\sin(\xi - \phi)$. Hence, based on the values of the parameters η, ξ and ϕ, we can classify different forms of CP-violation. For $\eta \neq 0$ and $\xi = 0$ the decay matrix Γ is real and is CP-conserving. The CP-violation occurs due to the nonzero value of ϕ, which we call CP-violation in mass because the phase ϕ arises from the neutrino mass terms. For $\phi = 0$, CP-violation is possible if $\eta \neq 0$ and $\xi \neq 0$. This can be termed CP-violation in decay because the CP-violating phase comes from the decay matrix. The most general possibility is CP-violation due to both mass and decay if $\eta \neq 0$, $\xi \neq 0$ and $\phi \neq 0$, but $\phi \neq \xi$. In all these cases $\eta \neq 0$. However, for the two special cases, (a) $\phi = 0 = \xi$ and (b) $\phi = \xi$, there is no CP-violation even if $\eta \neq 0$. This implies that a non-zero value of η is a necessary condition for CP-violation but not a sufficient condition. In these two special cases the CP violating terms vanish and the flavour conversion probabilities are the same as the vacuum probabilities multiplied by the decay term. However, the presence of a non-zero value of η is visible in the survival probabilities.

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Abbreviations

The following abbreviations are used in this manuscript:

SM	Standard Model
CP	Charge-Parity
CPT	Charge-Parity-Time reversal

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