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Entropic Dynamics and Quantum "Measurement" †

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Abstract: The entropic dynamics (ED) approach to quantum mechanics is ideally suited to address the problem of measurement because it is based on entropic and Bayesian methods of inference that have been designed to process information and data. The approach succeeds because ED achieves a clear-cut separation between ontic and epistemic elements: positions are ontic, while probabilities and wave functions are epistemic. Thus, ED is a viable *realist ψ-epistemic model*. Such models are widely assumed to be ruled out by various no-go theorems. We show that ED evades those theorems by adopting purely epistemic dynamics and denying the existence of an ontic dynamics at the subquantum level.

Keywords: entopic dynamics; quantum measurement; realist psi-epistemic models; psi-ontology theorems

1. Introduction

A measurement is a physical process like any other and, therefore, its analysis should cause no difficulties once a proper understanding of the relevant dynamics has been achieved [1]. Nevertheless, the problem of quantum measurement has historically been a source of endless controversy. It is intimately associated with most of those features of quantum mechanics (QM) that make it so strange and fascinating (see, e.g., [2–5]). Does the quantum state reflect incomplete information or is it something real and ontic ? If the latter, can wave functions undergo a physical collapse during measurement? Alternatively, if no collapses ever occur, and wave functions always obey the linear Schrödinger equation, how could quantum measurements ever yield definite outcomes? How does one negotiate the interface between the microscopic quantum world and the macroscopic classical world of the measuring device? Do at least some privileged variables represent something real with definite values at all times? Or, alternatively, are the values of all observables created during the act of measurement? If so, how can one ever say that anything real exists when nobody is looking?

Our first goal here is to address the problem of measurement from the perspective of entropic dynamics (ED) [6,7]. The ED approach to QM is ideally suited to tackle the questions above because it is based on entropic and Bayesian methods of inference that have been designed to process information and data. (For a more detailed presentation, see [8].) The success of the ED approach hinges on a clear ontological and epistemic commitment. In ED, the positions of particles enjoy the privileged role of being the only ontic variables, and all measurements are ultimately position measurements. In contrast, probabilities and wave functions are fully epistemic in nature.

Indeed, while the position of a particle can be measured directly, other generic observables are treated very differently; they are never actually "measured". The system of interest is coupled to an ancillary system, and the "value" of a generic observable is only indirectly inferred from the observed position of the ancilla's "pointer" variable. Thus, positions reflect real properties with definite values that are not created by the act of measurement and, as we shall see, all other observables turn out to be epistemic because they

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reflect the properties of the wave function [9,10]. This explains how it is that their values are "created" by the act of measurement.

Models such as ED that invoke ontic variables while the wave function remains an epistemic object are described as "realist ψ-epistemic models". In contrast, the various descendants of Bohr's Copenhagen interpretation that deny a definite quantum reality are dubbed "anti-realist ψ-epistemic models", while models such as the de Broglie–Bohm and many-worlds models are called "realist ψ-ontic models".

A number of powerful no-go theorems exist—the so-called ψ-ontology theorems [5]—that rule out large families of ψ-epistemic "ontological" models [11–13] because they disagree with QM (e.g., [11,14–25]. (The term 'ontological models' has been proposed as an improved way to refer to the old 'hidden-variable models'. The new term recognizes that some "hidden" variables, such as positions, are observable and, therefore, not at all hidden.) These no-go theorems have been interpreted as strong evidence in favor of "realist ψ-ontic models". However, ψ-epistemic models remain highly appealing, not least because they trivially explain the infamous wave function collapse as a mere updating of probabilities in the light of new data. Remarkably, fully developed realist ψ-epistemic models such as ED are scarce [5]. To my knowledge, ED is the only such model that provides a detailed reconstruction of the formalism of QM and claims to reproduce not just a fragment of quantum phenomena but QM in its totality.

A second goal of this paper is to analyze how ED evades the consequences of ψ-ontology theorems. Ontological models assume the existence of ontic variables. We shall argue that they also implicitly assume the existence of some ontic dynamics at the subquantum level. (The details of the dynamics need not be specified, and therein lies the power and generality of the ψ-ontology theorems.) ED, on the other hand, makes a commitment to ontic variables while denying them ontic dynamics; ED is a purely epistemic dynamics of probabilities. There is no implication that particles move as they do because they are pushed around by other particles or guided by an ontic pilot wave. Wave functions guide our expectations about where particles might be found, but there is no mechanism that accounts for any causal influence on the particles themselves. ED is a *mechanics without a mechanism*.

Section 2 contains a brief overview of ED. In Section 3, I discuss the direct measurement of microscopic positions, including their amplification to achieve observability at the macroscopic level [9]. Then, in Section 4, I discuss how the use of more elaborate devices allows us to define other non-position observables. I show how their "measurement" is ultimately reduced to the direct measurement of positions, and I derive the associated Born rule [9,26]. The special case of the von Neumann measurements provides an interesting extension [10]. Thus far, these sections have reviewed our previous work on this subject. In Section 5, I present new material that addresses the question of how ED manages to evade the various ψ-ontology theorems.

2. Brief Review of Entropic Dynamics

To set the context for the rest of the paper, we review the main ideas that form the foundation of entropic dynamics. For a detailed account, see [6–8]. Here is a brief summary:

Ontological clarity: Particles have definite but unknown positions $\{x^A\}$ collectively denoted by x. These are the ontic microstates. (A is a composite index, $A = (n, a)$, where $n = 1 \ldots N$ labels the particles and $a = 1, 2, 3$ the three spatial coordinates.) The particles follow continuous trajectories and the goal is to predict the probability $\rho(x)$ of the positions x on the basis of some limited information.

ED is a dynamics of probabilities: The probability of a step from x to a neighboring x', $P(x'|x)$, is found by maximizing its entropy relative to a prior that enforces short steps and subject to appropriate constraints that introduce directionality and correlations. The main constraint involves a function $\phi(x)$ that plays three separate roles: first, it is related to a constraint in the maximization of entropy; second, if the probabilities $\rho(x)$ are considered as generalized coordinates, then $\phi(x)$ is the momentum that is canonically conjugate to them; and third, $\phi(x)$ is the phase of the quantum wave function, $\psi = \rho^{1/2}e^{i\phi/\hbar}$.

Entropic time: The epistemic dynamics of probabilities inevitably lead to an epistemic notion of time. The construction of time involves the introduction of the concept of an instant, the notion that the instants are suitably ordered, and a convenient definition of duration. By its very construction, there is a natural arrow of entropic time.

The evolution of probabilities is found by the accumulation of the short steps described by $P(x'|x)$. This results in a continuity equation that is local in configuration space but leads to non-local correlations in physical space,

$$\partial_t\rho_t(x) = -\partial_A\left(\rho_t v^A\right) \quad \text{where} \quad v^A = m^{AB}\partial_B\phi\ . \tag{1}$$

Notation: $\partial_A = \partial/\partial x^A$; $m_{AB} = m_n\delta_{AB}$ is the mass tensor, m_n are the particle masses, and $m^{AB} = \delta^{AB}/m_n$ is the inverse mass tensor.)

Symplectic structure: For a suitable choice of a functional $\tilde{H}[\rho,\phi]$, the continuity Equation (1) can be written in Hamiltonian form,

$$\partial_t\rho_t(x) = \frac{\delta\tilde{H}}{\delta\phi(x)}\ , \tag{2}$$

which suggests choosing (ρ,ϕ) as a pair of canonically conjugate variables. The epistemic phase space (or e-phase space) $\{\rho,\phi\}$ has a natural symplectic structure with symplectic two-form Ω.

Information geometry: The e-phase space is assigned a metric structure with metric tensor G based on the information metric of the statistical manifold $\{\rho\}$ of probabilities $\rho(x)$. The joint presence of symplectic and metric structures implies the existence of a complex structure and suggests the introduction of wave functions $\psi = \rho^{1/2}e^{i\phi/\hbar}$ as complex coordinates. (For a discussion of the subtleties concerning the correct choice of the spaces that are cotangent to the manifold $\{\rho\}$ and of the metric structure associated with e-phase space $\{\rho,\phi\}$, see [7].)

The epistemic dynamics that preserve the symplectic structure in the sense of vanishing Lie derivative, $\pounds_H\Omega = 0$, obey Hamilton's equations,

$$\partial_t\rho(x) = \frac{\delta\tilde{H}}{\delta\phi(x)}\ , \quad \partial_t\phi(x) = -\frac{\delta\tilde{H}}{\delta\rho(x)}\ . \tag{3}$$

If we further require the preservation of the metric structure, $\pounds_H G = 0$, and of the normalization of probabilities we find that $\tilde{H}$ is constrained to be bilinear in ψ and ψ^*,

$$\tilde{H}[\psi,\psi^*] = \int d^{3N}x d^{3N}x'\ \psi^*(x)\hat{H}(x,x')\psi(x')\ , \tag{4}$$

which implies that (3) can be rewritten as a *linear* Schrödinger equation,

$$i\hbar\frac{d\psi(x)}{dt} = \int d^{3N}x'\ \hat{H}(x,x')\psi(x')\ . \tag{5}$$

The particular form of the Hamiltonian kernel $\hat{H}(x,x')$ is determined by requiring that it reproduce the ED evolution of probabilities, Equation (1). In standard notation we find

$$i\hbar\partial_t\psi = \sum\nolimits_n \frac{-\hbar^2}{2m_n}\nabla_n^2\psi + V(x)\psi\ . \tag{6}$$

Entropic dynamics are the purely epistemic dynamics of (ρ,ϕ) or, equivalently, of ψ; there is no underlying ontic dynamics of x. Compared to other models of QM, ED is fairly conservative in that it confers ontic status to configurational variables such as position and a clear epistemic status to probabilities, phases, and wave functions. However, ED is radically non-classical in that it denies the ontic status of dynamics and of all observables (energy, momentum, etc.) except position.

Hilbert space: To conclude the reconstruction of QM, we can take full advantage of the linearity of (6) and introduce Hilbert spaces and the Dirac notation:

$$|\psi\rangle = \int d^{3N}x\, |x\rangle\psi(x)\,, \quad \psi(x) = \langle x|\psi\rangle\,, \quad \text{and} \quad \hat{H}(x,x') = \langle x|\hat{H}|x'\rangle\,. \tag{7}$$

3. Measuring Position: Amplification

All measurements are position measurements. The measurement of the position of a microscopic particle is conceptually straightforward because *the particle already has a definite position x.* The issue of inferring x is not different from the way data information is handled in any other Bayesian inference problem. There is, however, the technical problem of amplifying microscopic details so they can become macroscopically observable. This is usually handled with a detection device set up in an initial unstable equilibrium. For example, QM allows us to calculate the probability $P(a|x)$ that a particle at x will ionize a neighboring atom located at a. In a bubble chamber, the ionized atom will trigger the formation of a bubble centered at a; in a photographic emulsion, the ion will trigger the formation of a silver crystallite centered at a. More generally, the particle activates the amplifying system by inducing a cascade reaction that leaves the amplifier in a definite macroscopic final state described by some "pointer" variable a.

The goal of the amplification process is to allow us to infer the microscopic position x from the observed macroscopic position a of the pointer variable. Incidentally, the latter is just a classical variable in the sense that its dynamics can, to an excellent approximation, be described by the classical limit of quantum theory [27]. Once the likelihood function $P(a|x)$ is given, the value x can be inferred following a standard application of Bayes rule,

$$P(x|a) = P(x)\frac{P(a|x)}{P(a)}\,. \tag{8}$$

In practice, life is more complicated, and the likelihood function will be distorted and smeared by spurious correlations and noise. Successful measurement always involves, of course, a skilled experimentalist who will design the device so that those unwanted effects will be minimized and controlled.

The point of these considerations is to emphasize that the measurement of position is conceptually straightforward and that there is nothing intrinsically quantum mechanical about the amplification process.

4. "Measuring" Other Observables and the Born Rule

Position is easy because it is an ontic quantity. Next, we tackle observables other than position: how they are defined and how they are measured [9,26]. For notational convenience, we initially consider the case of a single particle that lives on a lattice; the measurement of its position leads to a discrete set x_k of possible outcomes. The translation from continuous to discrete positions is straightforward,

$$\psi(x) = \rho^{1/2}(x)e^{i\phi(x)/\hbar} \quad \text{becomes} \quad \psi_k = p_k^{1/2}e^{i\phi_k/\hbar}\,, \tag{9}$$

and

$$\rho(x)\,d^3x = |\langle x|\psi\rangle|^2\,d^3x \quad \text{becomes} \quad p_k = |\langle x_k|\psi\rangle|^2\,. \tag{10}$$

If the state is

$$|\psi\rangle = \textstyle\sum_k c_k|x_k\rangle \quad \text{then} \quad p_k = |\langle x_k|\psi\rangle|^2 = |c_k|^2\,. \tag{11}$$

Since position is the only ontic quantity, it is not strictly necessary to define other observables except that they turn out to be convenient to discuss more complex experiments in which the particle is subjected to additional interactions, such as magnetic fields or diffraction gratings, before it reaches the position detectors.

The fact that measurements are dynamical processes means that the interactions within a complex measurement device $\mathcal{M}$ are described by a linear and unitary evolution $\hat{U}_M$ given by a Hamiltonian $\hat{H}_M$. The particle will be detected with certainty at position x_k provided it was initially in a state $|s_k\rangle$ such that

$$\hat{U}_M|s_k\rangle = |x_k\rangle \; . \tag{12}$$

Since the set $\{|x_k\rangle\}$ is orthonormal and complete, the corresponding set $\{|s_k\rangle\}$ is also orthonormal and complete,

$$\langle s_j|s_k\rangle = \delta_{jk} \quad \text{and} \quad \sum_k |s_k\rangle\langle s_k| = \hat{I} \; . \tag{13}$$

To find the effect of the complex device $\mathcal{M}$ on a generic (normalized) initial state $|\psi\rangle$, we express it in the $\{|s_k\rangle\}$ basis,

$$|\psi\rangle = \sum_k c_k|s_k\rangle \; , \tag{14}$$

where $c_k = \langle s_k|\psi\rangle$. The state $|\psi\rangle$ evolves through $\mathcal{M}$ according to $\hat{U}_M$ so that when it reaches the actual position detectors the new state is

$$\hat{U}_M|\psi\rangle = \sum_k c_k\hat{U}_M|s_k\rangle = \sum_k c_k|x_k\rangle \; . \tag{15}$$

According to the Born rule for position measurements, Equation (11), the probability of finding the particle at the position x_k is

$$p_k = |c_k|^2 = |\langle s_k|\psi\rangle|^2 \; . \tag{16}$$

In words: The probability that the particle in the initial epistemic state $|\psi\rangle$ is later found at position x_k is $|c_k|^2$, and the (suitably amplified) position x_k plays the role of a discrete pointer variable. (The generalization to continuous variables spaces is straightforward [26]). The argument above illustrates the main idea, but it also raises the inevitable question: *What, if anything, has been "measured" here?* [1]. The answer is tricky.

Note that the particle in the initial epistemic state $|\psi\rangle$ has been detected in an ontic state x_k *as if it had earlier been in the epistemic state* $|s_k\rangle$. This process is usually described in a slightly different language that unfortunately obscures the distinction between the ontic nature of x_k and the epistemic nature of $|x_k\rangle$ (or, more generally, of the amplitude $\psi_k = \langle x_k|\psi\rangle$). It is said that *the particle is detected in state* $|x_k\rangle$ *as if it had earlier been in the state* $|s_k\rangle$. One can further obscure the language by de-emphasizing the inner workings of the complex device, forgetting the dynamics and treating the detector as a black box. The result is a more concise but more misleading statement: *the particle has been "detected" in the state* $|s_k\rangle$. Continuing along the same lines leads us to adopt the language that is standard in QM textbooks: *the probability that the particle in state* $|\psi\rangle$ *is "detected" in state* $|s_k\rangle$ *is* $|\langle s_k|\psi\rangle|^2$, which reproduces Born's rule for a generic measurement device $\mathcal{M}$. However, by now, the real meaning of what has been 'detected' lies buried deep underground.

The same complex detector $\mathcal{M}$ can be used to "measure" all operators of the form

$$\hat{M} = \sum_k \alpha_k|s_k\rangle\langle s_k| \tag{17}$$

where the eigenvalues α_k are arbitrary (possibly complex!) scalars. Note that when we say we have detected the particle at x_k *as if* it had earlier been in state $|s_k\rangle$, we are absolutely not implying that the particle *was* in the particular epistemic state $|s_k\rangle$—this is just a figure of speech. The actual epistemic state was $|\psi\rangle$ not $|s_k\rangle$. When the system is "detected in $|s_k\rangle$" the standard language is that the outcome of the measurement is the eigenvalue α_k, which establishes the eigenvector-eigenvalue connection. It is then clear that the "outcome"

α_k was not a pre-existing value, and it is in this sense that one says that the value α_k was "created by the act of measurement".

This point deserves to be stated more explicitly: sentences such as "the particle has momentum $\vec{p}$" or "the particle has energy E" are to be recognized as mere linguistic shortcuts that convey information about various components of the wave function before the particle enters the complex detector. Therefore, strictly speaking, there is no such thing as the momentum or the energy *of the particle*: the momentum and the energy are not properties of the particle but properties of special epistemic states.

In the standard language one refers to the operator $\hat{M}$ as representing an "observable" and it is common to attribute to its eigenvalues and eigenvectors the status of being ontic—that is, of having actual physical properties. This is not mere abuse of language; in the ED framework, it is just plain wrong. Since what one is actually doing is inferring properties of the wave function from measurements of position, a more appropriate terminology would be to refer to $\hat{M}$ as an "inferable" [10].

To summarize: In the standard interpretation of quantum mechanics, Born's rule for generic measurements is a postulate. In ED, it is the natural consequence of unitary time evolution and the hypothesis that *all measurements are ultimately position measurements*.

An Illustration: Von Neumann Measurements

So far, we have just discussed measurements that rely on the direct detection of the position of the particle (and its subsequent amplification). One can substantially enlarge the class of useful experiments by considering complex setups in which one infers properties of one system indirectly by measuring the position of another system—the pointer variable—with which the system has interacted. Nothing in this section is original material; it is included merely as a purely pedagogical illustration of the fact that all measurements are position measurements.

The system of interest is composed of one or many particles; its ontic state is $x = \{x_n\}$, and its epistemic state is $|\psi\rangle$. The pointer device is also a particle; its ontic state is X, and its epistemic state is $|\pi\rangle$. The interaction between the system and the pointer is modelled by the Hamiltonian

$$\hat{H}_M = -g(t)\hat{P}\hat{M}\,, \tag{18}$$

where $\hat{M} = \sum_k \alpha_k |s_k\rangle\langle s_k|$ is the operator to be "measured", and $\hat{P}$ is the operator that generates translations of the pointer states,

$$e^{-i\hat{P}\alpha/\hbar}|X\rangle = |X+\alpha\rangle\,. \tag{19}$$

The function $g(t)$ measures the strength of the interaction. We make the usual assumptions: (a) that $\int g(t)dt = 1$; (b) that $g(t)$ vanishes before and after the measurement; and (c) that while the measurement lasts, $g(t)$ is large enough that $\hat{H}_M$ is a good approximation to the full Hamiltonian.

The pointer is set to its initial "ready" position near $X_i = 0$ with some uncertainty σ_π,

$$|\pi\rangle = N_X \int dX_i\, e^{-X_i^2/4\sigma_\pi^2}|X\rangle\,, \tag{20}$$

where $N_X = (2\pi\sigma_\pi^2)^{-3/4}$ is a normalization constant. The initial state of the system is $|\psi\rangle = \sum_k c_k|s_k\rangle$. As a result of the interaction, the system and pointer evolve according to

$$U_M|\psi\pi\rangle = \exp\left(-\frac{i}{\hbar}\int \hat{H}_M dt\right)|\psi\rangle|\pi\rangle = \exp\left(\frac{i}{\hbar}\hat{P}\hat{M}\right)|\psi\rangle|\pi\rangle\,, \tag{21}$$

and become entangled. Using (18)–(21), we find

$$U_M|\psi\pi\rangle = N_X \sum_k c_k \int dX_f\, e^{-(X_f-\alpha_k)^2/4\sigma_\pi^2}|s_k\rangle|X_f\rangle\,, \tag{22}$$

which shows that the probability of the pointer position X has been shifted from an initial Gaussian centered at $X_i \approx 0$ to a final mixture of Gaussians centered at $X_f \approx \alpha_k$,

$$\Pr(X_f) = \sum_k |c_k|^2 \frac{1}{(2\pi\sigma_\pi^2)^{3/2}}\, e^{-(X_f-\alpha_k)^2/2\sigma_\pi^2} . \tag{23}$$

When σ_π is small and the Gaussian distributions are neatly resolved, we have a "strong" or "von Neumann" measurement. The conclusion is that measuring the final pointer position X_f allows us to infer the eigenvalue α_k. (See the Appendix A for more details.)

When the Gaussian distributions overlap significantly, the measurement is said to be "weak". Such weak measurements do not allow us to infer the eigenvalues α_k, but they can nevertheless still be useful because they allow us to infer other quantities such as the phase or even the wave function itself (for more on this see [10] and references therein).

5. Evading the No-Go Theorems

The no-go theorems that rule out large families of realistic ψ-epistemic models are formulated in a framework of "ontological models" that originates with Bell [11]. The idea is that prior to an actual measurement, the system undergoes some sort of preparation procedure $\mathcal{P}$, and the result of the measurement $\mathcal{M}$ is to yield outcomes labeled k (inferred from either x_k or X_f in the previous section). The goal is to find the probability $p(k|\mathcal{M},\mathcal{P})$ of an outcome k for given $\mathcal{P}$ and $\mathcal{M}$.

In realist models, the ontic state of the system is denoted by variables λ. In ED, for example, λ consists of the particle and pointer positions x and X, while in the de Broglie–Bohm model, λ consists of both x, X, and ψ. It is assumed that the preparation procedure $\mathcal{P}$ may determine ψ completely, but it need not yield complete control over λ—$\mathcal{P}$, it only determines the probability distribution, $p(\lambda|\mathcal{P})$. Thus, as the system enters the measuring device $\mathcal{M}$, we are not only uncertain about the future outcome k but also of the initial values λ_i just *before* $\mathcal{M}$. This means that the relevant probability to be discussed is the *joint* distribution $p(k,\lambda_i|\mathcal{M},\mathcal{P})$, and the probability of the outcome k is given by marginalizing over λ_i,

$$\int d\lambda_i p(k,\lambda_i|\mathcal{M},\mathcal{P}) = p(k|\mathcal{M},\mathcal{P}) . \tag{24}$$

So far, we have just used the rules of probability theory, which, being of universal applicability, also apply to QM. The desired goal is to find realist models such that the distribution on the right of (24) matches the predictions of QM such as Equation (16).

To proceed further, we write (24) as

$$\int d\lambda_i\, p(\lambda_i|\mathcal{M},\mathcal{P})p(k|\mathcal{M},\mathcal{P},\lambda_i) = p(k|\mathcal{M},\mathcal{P}) \tag{25}$$

and consider the two factors on the left separately. Concerning the first factor, $p(\lambda_i|\mathcal{M},\mathcal{P})$, we shall assume that the distribution of λ_i is settled by the earlier choice of preparation $\mathcal{P}$ and is independent of whatever choice one might *later* make for the measurement device $\mathcal{M}$,

$$p(\lambda_i|\mathcal{M},\mathcal{P}) = p(\lambda_i|\mathcal{P}) . \tag{26}$$

This is a statement of causality [14]: conditional on $\mathcal{P}$, λ_i is independent of the later choice of $\mathcal{M}$.

The crucial assumption that defines what [12,13] call an "ontological model" concerns the second factor, the response function $p(k|\mathcal{M},\mathcal{P},\lambda_i)$. The assumption is that the distribution of outcomes depends only on the ontic state λ_i *after* the preparation $\mathcal{P}$ but *before* the device $\mathcal{M}$ and on the actual measurement performed,

$$p(k|\mathcal{M},\mathcal{P},\lambda_i) = p(k|\mathcal{M},\lambda_i) . \tag{27}$$

This assumption does not necessarily violate QM. For example, in the de Broglie–Bohm model, $\lambda_i = (x_i, X_i, \psi)$,

$$p(k|\mathcal{M}, \mathcal{P}, \lambda_i) = p(k|\mathcal{M}, \mathcal{P}, x_i, X_i, \psi) = p(k|\mathcal{M}, x_i, X_i, \psi) , \tag{28}$$

which states that, conditional on the information about $\mathcal{P}$ that is codified into ψ, all *other* details about $\mathcal{P}$ not already conveyed by ψ are irrelevant.

However, in a ψ-epistemic ontological model ψ is not included in λ. The assumption there is that conditional on λ_i, for any choice of $\mathcal{M}$, the outcome k is *completely independent of all details about* $\mathcal{P}$, including any information that might be codified into the epistemic ψ. *It is these* ψ*-epistemic ontological models that are shown by all the no-go theorems* [5,11–25] *to disagree with QM*.

ED satisfies the causality assumption (26) but violates (27), and, therefore, *ED is not an ontological model in the sense of* [11–13], which makes ED immune to all the no-go theorems. More explicitly, the situation with ED is the following: the ontic variables are positions of particles and/or pointers, $\lambda = (x, X)$, and the information about the preparation procedure $\mathcal{P}$ is fully conveyed by the wave function $\psi(x_i)\pi(X_i)$. Then, the causality assumption, Equation (26), reads

$$p(\lambda_i|\mathcal{M}, \mathcal{P}) = p(x_i, X_i|\psi) = |\psi(x_i)\pi(X_i)|^2 , \tag{29}$$

where $\lambda_i = (x_i, X_i)$ are position values *before* entering the device $\mathcal{M}$. Next, since ED is a purely epistemic dynamics, *conditional on the epistemic wave function the distribution of* k *is independent of the ontic variables* (x_i, X_i)—the latter have no causal influence on the future outcome k. Thus, instead of (27), in ED the response function $p(k|\mathcal{M}, \mathcal{P}, \lambda_i)$ is

$$p(k|\mathcal{M}, \mathcal{P}, x_i, X_i) = p(k|\mathcal{M}, \psi, \pi) . \tag{30}$$

Substituting (29) and (30) into (25) yields

$$p(k|\mathcal{M}, \mathcal{P}) = p(k|\mathcal{M}, \psi, \pi) \tag{31}$$

which agrees with the probability predicted by quantum mechanics. (See the Appendix A for further details.)

It is worth emphasizing the difference between response functions in ontological models, Equation (27), and in ED, Equation (30): ontological models assume that all the information about the preparation that is relevant for the measurement is carried by the ontic variables λ_i. This means that lurking in the background there is an implicit assumption that ontic dynamics exist that relate the *earlier* values of λ_i as the system enters $\mathcal{M}$ to the *later* values λ_f that result in the outcome k. The power of the no-go theorems lies in the fact that the details of the ontic dynamics remain unspecified—the ontic dynamics could be deterministic or stochastic, they could be local or non-local, and so on—but they assume that some ontic dynamics must exist.

ED, on the other hand, makes a commitment to ontic states without making the associated commitment to an ontic dynamics; only the epistemic variables (ψ, π) appear on the right hand side of (30). In ED, the epistemic wave functions do not guide the particles; they only guide our expectations about where particles might be found. *ED represents epistemic mechanics without an ontic mechanism.*

Remark 1. *Beyond the general framework of ontological models, the various no-go theorems depend on additional assumptions. It is these additional assumptions that have been offered as possible loopholes (see, e.g., [5]). We emphasize that ED by-passes the issue of additional assumptions and evades the ontological models framework at the deeper level of the epistemic vs. ontic nature of the dynamics.*

Remark 2. *In [13], ψ-epistemic models are defined as those that fail to be ψ-ontic. This definition would in principle allow ψ-epistemic models based on exotic probability theories [5]. The ED approach shows that such generalizations are not necessary; there is no need for exotic or even quantum probabilities, for quantum logic, for excess ontological baggage, or for retrocausality. ED is ψ-epistemic in that $\psi(x)$ is directly related to the probabilities $\rho(x)$, that is, to knowledge and beliefs, and to the conjugate momenta $\phi(x)$ that codify the information that updates those probabilities.*

Remark 3. *The ideas above could have been formulated in a more general setting of preparations that only determine a density matrix and measurements that are described by positive operator valued measures (POVMs). However, such increased generality has no effect on the conclusions and would only serve to obscure the main ideas.*

6. Conclusions

The solution of the problem of measurement within the ED framework hinges on two points: first, entropic quantum dynamics is a theory of inference. The issue of an unacceptable dichotomy of two modes of evolution—continuous unitary evolution versus discrete wave function collapse—is resolved. The two modes of evolution correspond to two modes of updating—either by a continuous maximization of an entropy or a discontinuous application of Bayes' rule—both of which, within the entropic inference framework, are unified into a single entropic updating rule [8,28].

The second point is the privileged role of position—particles (and also pointer variables) have definite positions and therefore their premeasurement values are merely revealed and not created by the act of measurement. Other "inferables" are introduced as a matter of linguistic convenience to describe more complex experiments. These inferables turn out to be attributes of the epistemic wave functions and not of the ontic particles; their "values" are indeed "created" by the dynamical process of measurement.

ED unscrambles Jaynes' proverbial omelette by imposing a clear-cut separation between its ontic and epistemic elements. ED is a conservative theory in that it attributes a definite ontic status to things such as particles (or fields) and a definite epistemic status to probabilities and wave functions without invoking exotic probabilities; it is radically non-classical in that it denies the ontic status of dynamics and of all observables except position.

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Appendix A

Here, we offer a more detailed analysis of Equation (31).

The direct measurement of position

The outcomes of the measurement are $k = x_k$; there is no pointer variable X, and the corresponding $\pi(X)$ can be omitted. The product of (29) and (30) is

$$p(\lambda_i|\mathcal{M},\mathcal{P})p(k|\mathcal{M},\mathcal{P},\lambda_i) = |\psi(x_i)|^2 p(k|\mathcal{M},\psi) = |\psi(x_i)|^2 |\langle s_k|\psi\rangle|^2 \quad \text{(A1)}$$

where we used (16). Then, (25) is

$$p(k|\mathcal{M},\mathcal{P}) = \int d^3x_i\, |\psi(x_i)|^2 |\langle s_k|\psi\rangle|^2 = |\langle s_k|\psi\rangle|^2\,, \tag{A2}$$

which agrees with the prediction of quantum mechanics.

The indirect or von Neumann measurement

The outcomes of the measurement are $k = (\alpha_k, X_f)$. The object of interest is the eigenvalue α_k, Equation (17). The product of (29) and (30) is

$$p(\lambda_i|\mathcal{M},\mathcal{P})p(k|\mathcal{M},\mathcal{P},\lambda_i) = |\psi(x_i)\pi(X_i)|^2 p(\alpha_k, X_f|\mathcal{M},\psi,\pi) \tag{A3}$$

Using (16) and (22), the second factor on the right is

$$p(\alpha_k, X_f|\mathcal{M},\psi,\pi) = |\langle s_k, X_f|\hat{U}_M|\psi\pi\rangle|^2 = |\langle s_k|\psi\rangle|^2\, N_X^2 e^{-(X_f-\alpha_k)^2/2\sigma_\pi^2} \tag{A4}$$

substituting into (25) gives

$$\begin{aligned} p(\alpha_k, X_f|\mathcal{M},\mathcal{P}) &= \int d^3x_i dX_i\, |\psi(x_i)|^2 |\pi(X_i)|^2 |\langle s_k|\psi\rangle|^2 N_X^2 e^{-(X_f-\alpha_k)^2/2\sigma_\pi^2} \\ &= |\langle s_k|\psi\rangle|^2 N_X^2 e^{-(X_f-\alpha_k)^2/2\sigma_\pi^2}\,. \end{aligned} \tag{A5}$$

Marginalizing over X_f gives

$$p(\alpha_k|\mathcal{M},\mathcal{P}) = |\langle s_k|\psi\rangle|^2\,, \tag{A6}$$

which is the correct prediction according to quantum mechanics.

Equation (A6) gives the correct probability but does not by itself describe the result of a measurement. The latter consists of inferring the value α_k from the data X_f that is actually observed. The relevant probability, $p(\alpha_k|\mathcal{M},\mathcal{P},X_f)$, is given by Bayes theorem,

$$p(\alpha_k|\mathcal{M},\mathcal{P},X_f) = \frac{p(\alpha_k, X_f|\mathcal{M},\mathcal{P})}{p(X_f|\mathcal{M},\mathcal{P})}\,. \tag{A7}$$

Using (A5), the result is

$$p(\alpha_k|\mathcal{M},\mathcal{P},X_f) = \frac{|\langle s_k|\psi\rangle|^2 e^{-(X_f-\alpha_k)^2/2\sigma_\pi^2}}{\sum_{k'} |\langle s_{k'}|\psi\rangle|^2\, e^{-(X_f-\alpha_{k'})^2/2\sigma_\pi^2}}\,. \tag{A8}$$

When σ_π is sufficiently small and the Gaussians are well resolved, Equation (A8) tells us that with high probability we shall find values of X_f concentrated at one of the discrete values α_k. For $X_f \approx \alpha_k$, we find,

$$p(\alpha_k|\mathcal{M},\mathcal{P},X_f) \approx 1\,, \tag{A9}$$

which means that a measurement of the pointer X_f allows for the immediate inference of α.

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