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Technical Note

Cybertronics-Based Robust Control for Dynamic Supply Chains of AI Finished Products: A Theoretical Analysis

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Abstract

This technical note aims to present a theoretical analysis for cybertronics engineering (CE) for a class of dynamic supply chains for artificial intelligence (AI)-based products, services or hybrid solutions. The cybertronics-based analysis encompasses three classes of supply chains: (1) energy-based dynamic supply chains (DSC); (2) semiconductor manufacturing and quantum supply chains; and (3) retailing of the AI-based DSC solutions generated. Considering the nonlinear nature of DSC, to provide solutions for products and services based on AI-chained supply chains, novel robust control is addressed via sliding mode control (SMC) with proper stability analysis for the DSC.

Keywords: cybertronics; sliding mode control; dynamic supply chains; artificial intelligence (AI); Industry 6.0

1. Introduction

This technical note systematically presents an analysis on the use of cybertronics, which integrates solutions in automation and intelligent control, systems engineering, and theoretical physics to approximate a framework for products generated by dynamic supply chain artificial intelligence [1]. Industry 6.0 emerges as a contra-paradigm in industrial development, human-centric automation, and cognitive manufacturing, propelled by developments in artificial intelligence, quantum computing, and edge computing [2]. Specifically, it aims to generate sustainable supply chains based on energy supply chains (generation, production and distribution), semiconductor supply chains, and emerging technologies based on quantum computing. Finally, it addresses AI-based dynamic supply chain which can have three approaches: products, services, and a hybrid of the two. This is based on the high-volume generation and quality of electrical power, low-medium volume semiconductor products, and quantum computing. The final stage of the supply chain, linked to the marketing of artificial intelligence-based software, analyzes these finished goods as products, as services, and in a hybrid way. Cybertronics engineering (CE) represents an evolution in the field of systems design. It is the multidisciplinary convergence of industrial automation, systems engineering, and theoretical physics, to manage the complexities of

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dynamic supply chains. Unlike traditional engineering disciplines that focus on isolated components, CE bridges the gap between physical hardware and digital intelligence to create an environment capable of responding to real-world fluctuations in real-time towards the introduction of novel Industry 6.0.—Refs. [3–5]. In addition, a sustainable system is a balanced network that satisfies requirements without considering the resources. A sustainment process refers to the continuous cycle of supply chain coordination and engineering improvements required to ensure that a system stays functional. Moreover, an analysis for the basis of Industry 6.0—which emerges as a contra-paradigm of the proper Industries 4.0 and 5.0, but now with advent of quantum computing, human based automation, beyond the current state-of-the-art system mechatronics engineering, and powered by Internet of Things (IoT), control theory and theoretical physics—is presented looking at advantages of the application of CE in future industrial applications.

This technical note addresses the following analysis:

1. From the perspective of mathematical modeling, the dynamic supply chain of products generated by technologies based on artificial intelligence is analyzed.
2. An analysis of the energy contribution required to meet the supply chain demand for AI-based technologies is presented, using compartmental analysis.
3. The analysis of a fifth-order system that couples price dynamics and inventory levels with inputs in the demand and production rates to understand the dynamics of the final part of the retailing supply chain of AI-based technologies.
4. The classic sliding mode controller design, which does not incorporate disturbances in the system dynamics, and where it is established that there is chattering in the control signals, which are the demand and production rates.
5. A stability analysis is established using techniques based on Lyapunov stability, supporting SMC-based control strategies.

In Section 2, a methodology is presented for CE in sustainable DSC with energy-based DSC, semiconductor and quantum supply chains and retailing of AI-based DSC. In Section 3, a discussion of robust control via sliding mode control with a proper stability analysis and simulations is presented; in Section 4, the discussion is addressed; and finally, conclusions are presented in Section 5.

2. Mathematical Modeling

2.1. Cybertronics Based Sustainable DSC: A Framework

Cybertronics engineering (CE) refers to the application of industrial automation, system engineering and system dynamics towards the integration of solutions in the context of dynamic supply chains from operational, tactical and strategical decision-making [6–8]. Besides this, CE in DSC analyzes intelligent automation and control for cyber-physical supply chains, which allows them to achieve autonomous, resilient and sustainable operations. In an industrial context, CE presents solutions for supply chain analytics, applying automation and control theory (cybernetics), systems engineering and theoretical physics via artificial intelligence (AI), Internet of Things (IoT) and quantum computing, towards cutting-edge technologies related to Industry 6.0.

Furthermore, cybertronics engineering seeks to integrate solutions based on mathematical models through control theory, system dynamics, and theoretical physics towards the digitization of solutions in high-volume supply chains [9].

In Figure 1, CE is based on model-based control theory, systems engineering principles, and quantum computing to provide an AI-based supply chain for products and services within a sustainability approach. Therefore, Figure 1 illustrates that CE not only contributes to the design and optimization of the overall system but also incorporates information derived from control theory. This is based on the assumption that the energy supplied to the first state of the DSC is mainly nuclear.

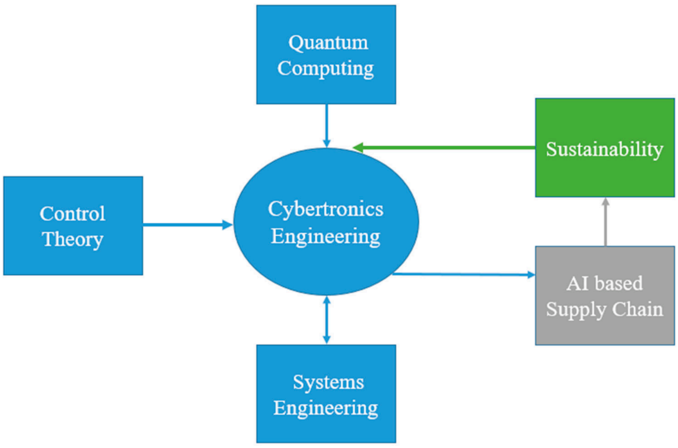

Figure 1. Cybertronics engineering-based sustainable DSC.

2.2. Chained Cybertronics DSC

2.2.1. Energy-Based DSC: The Creation of Value

In the first energy-related chain, the production, distribution, and generation of energy is analyzed [10,11], along with the elements necessary to produce energy based on renewable energies and nuclear energy. We consider sustainability factors in the high production volumes necessary to meet the needs of the chains in the stages of semiconductor generation, as well as technologies based on quantum computing, and finally the commercialization of the products, using the systemic approach based on cybertronics as a measure to understand the supply chain globally, starting from the local link for the purposes of the control, optimization, and stability of the electrical grid. At this stage, it is important to remember a sustainable system and sustainment process. Sustainability implies the ability to fulfill requirements independently of resource availability. To keep such a system operational, the sustainment process focuses on a perpetual cycle of supply chain alignment and engineering refinement, but not for sustainability purposes. The following analyses are based in compartmental analysis from an energy, $E(t)$, perspective:

$$\frac{dE}{dt} = (S_E(t) - D_E(t)) - \gamma E(t) \tag{1}$$

(a) $S_E(t) - D_E(t) = 0$

Where $S_E(t)$ refers to supplied energy into the system, $D_E(t)$ refers to demanded energy in the compartmental analysis, γ to the dissipation rate of energy in the system and K to an integration constant from condition $S_E(t) = D_E(t)$ in part (a).

Under the equilibrium condition of energy produced equals energy consumed, this scenario presents the phase in which the market dynamics required to provide the energy conditions takes the form:

$$E(t) = Ke^{-\gamma t} \tag{2}$$

(b) $S_E(t) > D_E(t)$

Overproduction of energy via nuclear energy, is sustainment but not sustainable, this will require more AI-finished goods produced in a time horizon.

$$S_E(t) \gg D_E(t) \tag{3}$$

Equation (1) becomes:

$$\frac{dE}{dt} + \gamma E(t) = S_E(t) \tag{4}$$

Under the shortage conditions, if we assume

$$S_E(t) \ll D_E(t) \tag{5}$$

Equation (1) becomes:

$$\frac{dE}{dt} + \gamma E(t) = -D_E(t) \tag{6}$$

2.2.2. Semiconductor Manufacturing and Quantum Supply Chains

In the second stage, a chained supply chain for the production of semiconductors [12–14] and emerging components based on quantum computing [15,16], the analysis is conducted from the perspective of conventional optimization techniques for low and medium production volumes, such as mixed programming and combinatorial programming, which are approximated to meet the demand established in the next stage, which is the commercialization of products, services and hybrids of AI-based software. At this chained stage of the DSC, the dynamic pricing and inventory dynamics are as follows:

$$C\frac{d^2p}{dt^2} + I(t)\frac{dp}{dt} + k_1 p = -k_2 d(t) \tag{7}$$

$$\frac{d^3I}{dt^3} + \beta\frac{d^2I}{dt^2} + \rho\frac{dI}{dt} + kI = -S(t) \tag{8}$$

where C refers to capacity, *p(t)* is the dynamic pricing level, *I(t)* refers to the inventory level, and *S(t)* refers to the production rate. In addition to this, β, ρ and k refer to scaling factors which are not dimensional, for tuning purposes.

Equations (7) and (8) present the dynamics between inventory levels and dynamic pricing at the second state of chained DSC.

In Figure 2, a systemic framework for AI-based DSC within the context of sustainability and costs is presented, an input to the system is the energy supply required for production; nuclear energy and renewable energy are capable of providing the mandatory supply of energy that Semiconductor SCs and Quantum SCs need to develop the items that enable AI-based products and services to meet proper demand.

2.2.3. Retailing

In the final stage, the commercialization of products generated based on AI tools, such as the following, are presented: (1) price dynamics, (2) demand management, (3) dynamic marketing, and (4) hybrid game theory to market the products following the rules of the free market and with characteristics of the relationship to quantify the least amount of inventories. At the final stage of the supply chain are the products generated by AI, such

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as AI-based products (software) and/or services [17–20]. The final metric of the energy provided, and final amounts of products and services generated are:

$$\eta = \Delta E \cdot (D_p \alpha + (1-\alpha) D_s) \quad (9)$$

where D_p refers to demanded products, D_s are demanded services, and ΔE is the amount of energy provided to the entire chained supply chain, where α takes values between $0 \le \alpha \le 1$.

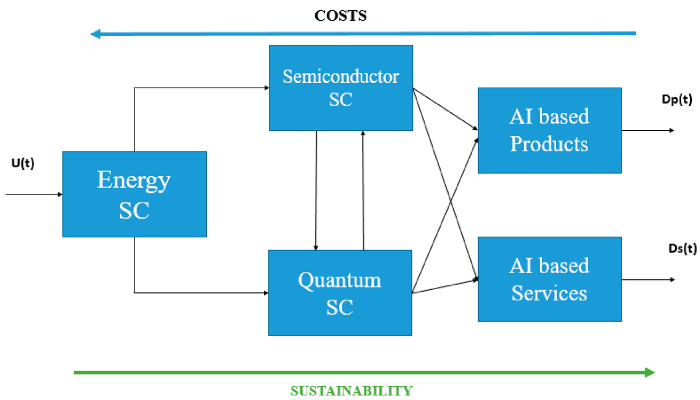

Figure 2. AI-based dynamic supply chain framework within sustainability context.

3. Robust Control in Supply Chains

Considering that supply chains are dynamical systems subject to variability and uncertainty. Several approaches have been addressed in the context of robust control, such as H∞ robust control and sliding mode control. For instance, [21] develops models for multi-echelon supply chain systems applying H∞ control methods, while [22] considers an inventory control technique for large-scale supply chains via adaptive control. The H∞ control technique addresses a class of supply chain model linearized about nominal operating conditions in [23].

Considering that closed-loop supply chains (CLSC) present novel topologies, in [24] robust H∞ control strategies constrain all uncertainties for this CLSC. Another robust control technique is sliding mode control (SMC); in [25] SMC for inventory policy is proposed, which assures that the demand is satisfied. Considering that SC systems are modeled as networked systems in [26], a H∞ control is present for a discrete SC.

3.1. Sliding Mode Control (SMC)

Variable control structures, such as sliding mode control, are robust control approaches that have been applied to electromechanical systems with fast actuator dynamics. (Refer Appendix B). For the dynamical system present in Equations (7) and (8), as in [27,28], fifth-order state-space minimum realization is presented in the form:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{-1}{C} x_2 x_3 - \frac{k_1}{C} x_1 - u_1(t) \\ \dot{x_3} &= x_4 \\ \dot{x_4} &= x_5 \\ \dot{x_5} &= -\beta x_5 - \rho x_4 - \gamma x_3 - u_2(t) \end{aligned} \quad (10)$$

We define the sliding surface $s_1 \rightarrow (x_1, x_2)$.
And the error as $e_1 = x_1, \dot{e_1} = x_2$. Therefore:

$$s_1 = \dot{e_1} + \lambda_1 e_1 = x_2 + \lambda_1 x_1 \tag{11}$$

For the sliding mode: $s_1 = 0$

$$\dot{x_1} = -\lambda_1 x_1 \tag{12}$$

We define the sliding surface $s_2 \rightarrow (x_3, x_4, x_5)$.
The error definition is:

$$e_3 = x_3, \dot{e_3} = x_4, \dddot{e_3} = x_5, \tag{13}$$

Developing the sliding surface:

$$s_2 = \ddot{e_1} + \lambda_2 \dot{e_1} + \lambda_3 e_3 = x_5 + \lambda_2 x_4 + \lambda_3 x_3 \tag{14}$$

Assuming the conditions $\lambda_2 > 0$, $\lambda_3 > 0$.
For the sliding mode $s_2 = 0$, we achieve:

$$\ddot{x_3} + \lambda_2 \dot{x_3} + \lambda_3 x_3 = 0 \tag{15}$$

Applying the derivatives for the surfaces:

$$\dot{s_1} = \frac{-1}{C} x_2 x_3 - \frac{k_1}{C} x_1 - u_1(t) + \lambda_1 x_2 \tag{16}$$

$$\dot{s_2} = -\beta x_5 - \rho x_4 - \gamma x_3 - u_2(t) + \lambda_2 x_5 + \lambda_3 x_4 \tag{17}$$

After some algebra the control laws are:

$$u_1(t) = \frac{-1}{C} x_2 x_3 - \frac{k_1}{C} x_1 + \lambda_1 x_2 + k_{s1} s_1 + \eta_1 sgn(s_1) \tag{18}$$

where $k_{s1} > 0$ and $\eta_1 > 0$ are terms to compensate for uncertainty. For the control law $u_2(t)$

$$u_2(t) = -\beta x_5 - \rho x_4 - \gamma x_3 + \lambda_2 x_5 + \lambda_3 x_4 + k_{s2} s_2 + \eta_2 sgn(s_2) \tag{19}$$

where $k_{s2} > 0$ and $\eta_2 > 0$ represent terms to compensate for uncertainty. (Refer to Appendix A).

A literature review of sliding mode control for supply chain is present in Table 1. In Appendix B, relevant literature on sliding mode control with applications is addressed in Table A1. Also, for comparison of listed methods in Table A2 with classical SMC, refer to Appendix C.

3.2. Stability Analysis

In order to achieve a stability analysis,

Theorem 1. *By Lyapunov's Theorem, the origin of the system in terms of S1 and S2 is asymptotically stable, which guarantees that they tend to zero, if they satisfy:*

$$\dot{V} = -\left(k_{s1}\, s_1^2 + \eta_1 |s_1| + k_{s2} s_2^2 + \eta_2 |s_2| \right)$$

Table 1. Sliding mode control for supply chains (literature review).

Author	Contribution	Control Approach
Johansyah et al., 2025, [29]	This paper introduces a five-dimensional Chaotic Supply Chain Mode by incorporating a transport lag variable into a previously established four-dimensional model.	Integral Sliding Mode Control
Johansyah et al., 2024, [30]	This paper extensively explores dynamical analysis and multistability analysis to understand nonlinear behaviors via SMC.	Sliding Mode Control
Zhang et al., 2021, [31]	A three-level single-chain nonlinear supply chain system is studied for a production change optimization model, which conducts stability analysis for the designed controller via SMC.	Adaptive Sliding Mode Control
Wang et al., 2021, [32]	This work presents the control of supply chain systems subject to control input limitations. A new control scheme is proposed to tackle this issue; the proposed controller is applied to the symmetric supply chain system.	Constrained Control Input Super-Twisting Sliding Mode
Liu et al., 2021, [33]	The chaotic attractors of a supply chain network are shown, and its corresponding multi-agent system is presented.	Super-Twisting Sliding Mode
Cuong et al., 2021 [34]	This paper presents a system dynamics approach to analyze dynamical behavior and synthesize control in a three-stage production–distribution supply chain model.	Fractional Order Sliding Mode Control
Bartoszewicz, and Latosiński 2019, [35]	This paper addresses variable structure control of discrete-time systems and introduces a reaching-law-based sliding mode control approach.	Discrete Time SMC
Askari et al., 2024 [36]	This paper analyzes the dynamics of a chaotic three-echelon supply chain and designs a finite-time global nonlinear Super-Twisting Sliding Mode Controller.	Nonlinear Super-Twisting Sliding Mode Controller

Proof of Theorem 1. We define a candidate Lyapunov function:

$$V = \frac{1}{2}s_1^2 + \frac{1}{2}s_2^2 \tag{20}$$

Applying the derivative of Equation (20):

$$\dot{V} = s_1\dot{s}_1 + s_2\dot{s}_2 \tag{21}$$

By substitution of $\boldsymbol{u_1}(\boldsymbol{t})$ in Equation (16), after some algebra:

$$\dot{s_1} = \boldsymbol{k_{s1}s_1} + \boldsymbol{\eta_1 sgn}(\boldsymbol{s_1}) \tag{22}$$

Therefore: $s_1\dot{s}_1 = \boldsymbol{k_{s1}s_1^2} + \boldsymbol{\eta_1}|s_1|$
Similar procedure, for the control law $\boldsymbol{u_2}(\boldsymbol{t})$, we have:

$$\dot{s_2} = \boldsymbol{k_{s2}s_2} + \boldsymbol{\eta_2 sgn}(\boldsymbol{s_2}) \tag{23}$$

Substituting Equations (22) and (23), in Equation (21) we have:

$$\dot{V} = \boldsymbol{s_1}(-\boldsymbol{k_{s1}s_1} - \boldsymbol{\eta_1 sgn}(\boldsymbol{s_1})) + \boldsymbol{s_2}(-\boldsymbol{k_{s2}s_2} - \boldsymbol{\eta_2 sgn}(\boldsymbol{s_2})) \tag{24}$$

using the property: $s_i \cdot sgn(s_i) = |s_i|$. Finally:

$$\dot{V} = -\left(\boldsymbol{k_{s1}}\,\boldsymbol{s_1^2} + \boldsymbol{\eta_1}|s_1| + \boldsymbol{k_{s2}s_2^2} + \boldsymbol{\eta_2}|s_2|\right) \tag{25}$$

Since V > 0 and $\dot{V} < 0$—the sliding surfaces $s_1 \to 0$ and $s_2 \to 0$ as t →∞—by Lyapunov's Theorem the origin of the system in terms of S_1 and S_2 is asymptotically stable, which guarantees that trajectories will converge to the sliding surfaces. □

3.3. Simulations

This section presents the simulations of the design of the classic sliding mode controller, which does not incorporate disturbances in its design given that the fifth-order dynamics of the system, which incorporate a second-order system coupled to one with jerk dynamics, present a fast dynamic.

For the system under classical SMC, Figure 3 presents the evolution of the five system states, which are set to zero at the end of the analysis time horizon. For controller tuning, it was considered necessary to scale the controller gains to magnitudes comparable to the system parameters. Figure 4 shows the demand rate for the system control law 1.

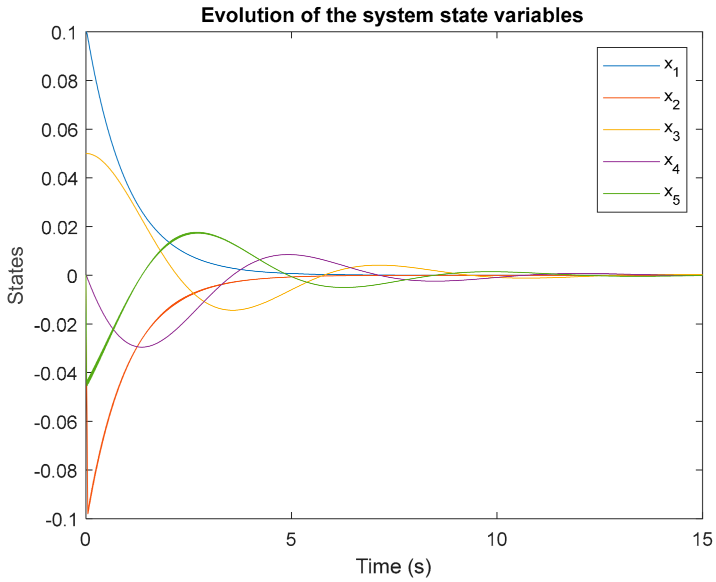

Figure 3. Evolution of the system state variables.

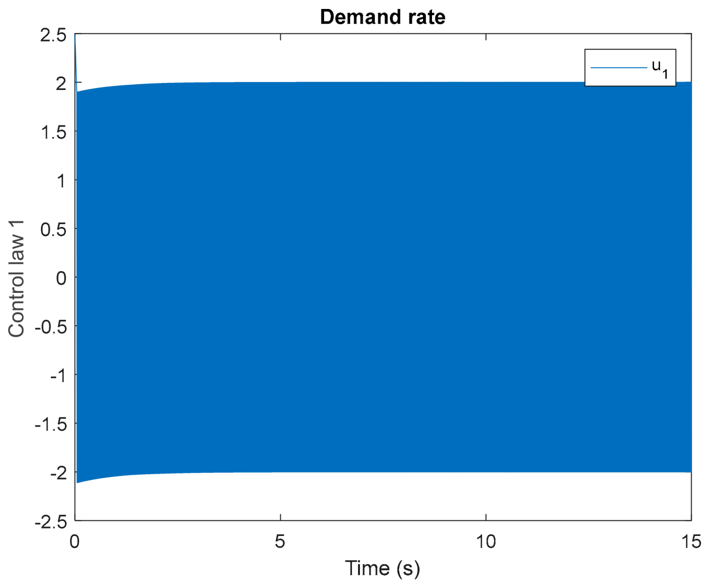

Figure 4. Evolution of demand rate control law 1.

Figure 5 represents the production rate for control law 2, where the chattering level can be determined because the system's fifth-order dynamics exhibit this phenomenon. To reduce this chatter, a saturation function can be incorporated instead of the sign signal, as explained in Appendix A.

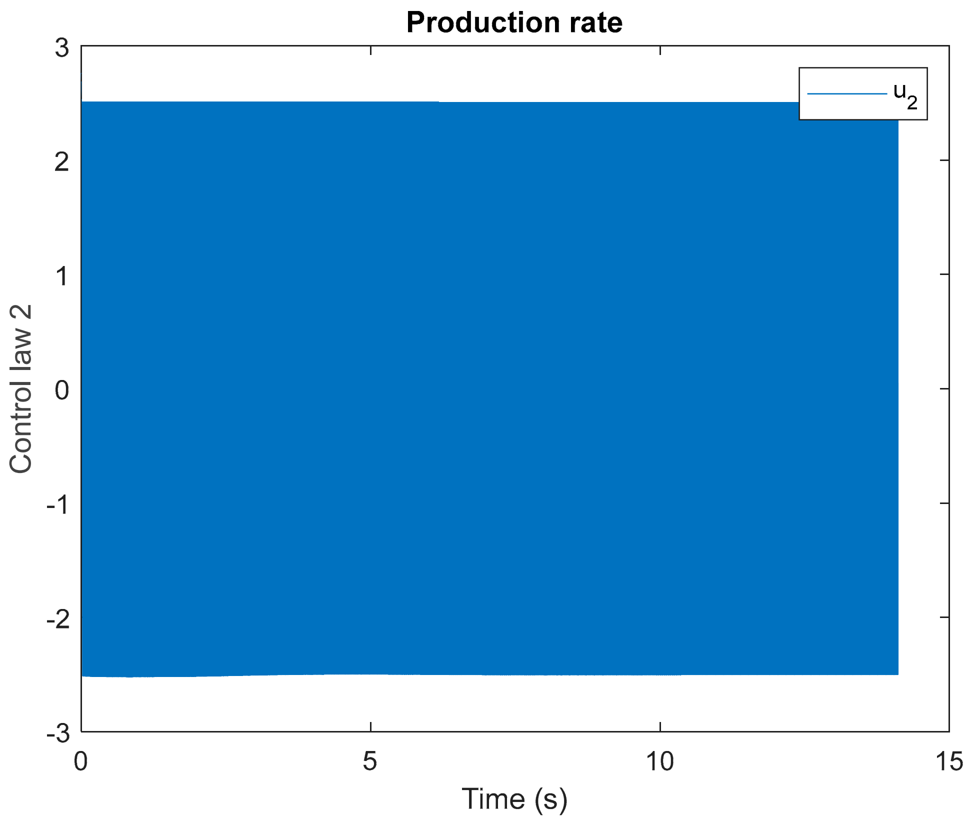

Figure 5. Evolution of production rate control law 2.

4. Discussion

CE represents a transformation in the evolution of systems theory and design, moving beyond traditional systems mechatronics into multi-dimensional and interdisciplinary frameworks; it is the convergence of industrial automation, systems engineering, and theoretical physics, synthesized to model and optimize the dynamic supply chains (linear or nonlinear). By integrating the power of theoretical physics with the structural discipline of systems engineering, CE presents the creation of resilient and sustainable DSC networks. These DSCs utilize real-time data, ensuring that global logistics remains resilient and responsive to the volatile fluctuations of dynamic market. Therefore, CE architecture begins at the operational level, where it serves as a critical measure for stability and energy optimization. By treating each DSC as a dynamic node within the global electrical grid, CE ensures that the massive power required for high-volume semiconductor supply chain generation does not compromise the energy generated via nuclear and renewable energy sources. This systemic approach follows the products and services through to the commercialization stage, where sustainability factors, such as externalities and circular economy, are present and quantified.

Finally, by applying the retailing tools through a CE arena, the supply chain achieves observability and controllability. The retailing stage becomes the sensor that tells the actuator (the DSC) exactly how much to produce, as an analogy to a control system. This inventory minimization tends to near-zero levels, satisfying the sustainable system requirement for proper losses and wasting resources.

5. Conclusions

The application of robust control theory via sliding mode control yields interesting results for nonlinear systems with rapid dynamics. Dynamic supply chains, when mathematically modeled as systems of ordinary differential equations with couplings, offer promising avenues for future development through bi-level optimal control and model-predictive control.

Cybertronics engineering, unlike mechatronic systems, offers the advantage of separating mechanical aspects from mechatronic design and approximating solutions using theoretical physics, the Internet of Things, system dynamics, and control engineering. In this case, the results presented by sliding mode control are of interest, especially considering the nonlinearity and fast dynamics involved in inventory management within the class of dynamic supply chains.

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Abbreviations

The following abbreviations are used in this manuscript:

CE	Cybertronics engineering
DSC	Dynamic supply chain
SMC	Sliding Mode Control
AI	Artificial intelligence

Appendix A

Chattering Mitigation

To avoid oscillations, the sign function is replaced by a smooth saturation function, Refs. [37–39]:

$$\boldsymbol{sat}(s_i, \boldsymbol{\Delta}) = \begin{cases} \boldsymbol{sgn}(s_i)\ \boldsymbol{if}\ \lceil s_i \rceil \geq \boldsymbol{\Delta} \\ \frac{s_i}{\boldsymbol{\Delta}}\ \boldsymbol{if}\ \lceil s_i \rceil < \boldsymbol{\Delta} \end{cases}$$

Appendix B

Table A1. Relevant literature on sliding mode control with applications.

Author	Paper Contribution	Application Field
Deng et al., 2026 [40]	This paper presents an automated microinjection system for visual positioning with adaptive neural network sliding-mode motion control.	Motion control
Wang et al., 2026 [41] Yaşkıran et al., 2026 [42]	This study introduces a fast-reaching law in SMC that ensures a fixed-time convergence to the sliding surface. This study provides a comprehensive review of rotary-wing UAVs, with an emphasis on quadcopter and helicopter systems via SMC.	Robotics UAV control
Borja-Jaimes et al., 2026 [43]	This paper incorporates explicit disturbance compensation into a backstepping–sliding framework using a super-twisting observer (STO).	Quadrotor UAV control
Arsalan et al., 2026 [44]	This paper proposes a multi-input smooth sliding mode control strategy that integrates radiotherapy and chemotherapy for a nonlinear tumor–immune system modeled by ordinary differential equations.	Medicine
Zhang and Zhu, 2026 [45]	This work proposes a fully model-free terminal sliding mode control approach to reduce sensitivity in multilink robotic manipulators.	Robotics
Zhou et al., 2025 [46]	This study proposes a fractional-order adaptive Super-Twisting Sliding Mode control (strategy to handle nonlinearities, in hydraulic excavator path-following.	Hydraulic excavator

Appendix C

Compared with classical SMC:

(a) Integral SMC improves robustness from the initial instant.
(b) Adaptive SMC improves adaptability and reduces excessive control effort.
(c) Fractional order SMC improves modeling capability and dynamic precision.
(d) Super-twisting algorithm provides one of the best reductions in chattering while preserving the robustness.

Table A2. Comparative Sliding Mode Control methods tradeoffs.

Method/Main Performance	Relative Chattering Level	Chattering Criteria	Major Application
Classical SMC (Basic robust control)	High	Discontinuous sign switching causes high-frequency oscillations [47,48]	General nonlinear systems
Integral SMC (Replace reaching phase)	Medium	Reduces reaching-phase oscillations, but discontinuous control remains [49]	Robotics
Adaptive SMC (Adaptive gains)	Low-Medium	Reduce switching gains, decreasing chattering amplitude [50]	Unknown uncertainties
Fractional order SMC (Fractional dynamics)	Low	Fractional dynamics smooth system response and reduce switching aggressiveness [51]	Complex systems
Super-Twisting SMC (Continuous higher-order control)	Very low	Continuous second-order SMC suppresses chattering [52]	High performance control

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