foundations

MDPI

Article

Dynamics of Information Quantifiers in the Damped Rabi Oscillator

Flavia Pennini [1,2] and Angelo Plastino [3,*]

[1] Departamento de Física, Universidad Católica del Norte, Av. Angamos 0610, Antofagasta 1270709, Chile; fpennini@ucn.cl
[2] Departamento de Física, Facultad de Ingeniería, Universidad Nacional de Mar del Plata (UNMDP), CONICET, Mar del Plata CP 7600, Argentina
[3] Instituto de Física La Plata–CCT-CONICET, Universidad Nacional de La Plata, C.C. 727, La Plata CP 1900, Argentina
* Correspondence: plastino@fisica.unlp.edu.ar

Abstract

This study examines the time evolution of structural and informational quantifiers in a damped Rabi oscillator, specifically focusing on fidelity, entropy, disequilibrium, and Fisher information. We observe that all four measures exhibit damped oscillatory behavior as the system approaches its steady state. However, the final asymptotic behavior is striking: while fidelity and disequilibrium indicate a residual, non-zero final state, and entropy quantifies the thermodynamic disorder, Fisher information uniquely vanishes. This vanishing implies a complete loss of dynamical information—the ability to infer the system's past evolution from its current state—even in the absence of complete thermodynamic disorder. Our findings introduce a new phenomenon where a system can be "informationally silent", meaning it becomes structurally ordered yet loses all inferential sensitivity to its own history, a detail that traditional entropy measures do not fully capture. This work highlights a critical distinction between thermodynamic disorder (entropy) and inferential sensitivity (Fisher information) in the context of open quantum systems.

Keywords: Fisher information; Rabi oscillator; decoherence; information degradation

1. Introduction

Simple and well-established models often play a decisive role in foundational physics. From the ideal gas and harmonic oscillator to the Ising and Rabi models, such systems are routinely employed not because they introduce new dynamics, but because they provide controlled settings in which conceptual distinctions can be isolated and analyzed with precision. The present work adopts this strategy in the context of open quantum systems, using the damped Rabi oscillator as a theoretical laboratory to investigate the relationship between entropy, information geometry, and inferential capability.

In recent years, information-theoretic quantities have assumed a prominent role in the analysis of quantum dynamics, decoherence, and thermalization. Measures such as Shannon or von Neumann entropy, fidelity, and statistical complexity are widely used to characterize loss of coherence, disorder, and structural change in open systems [1,2]. In parallel, Fisher information [3] has emerged as a central object in quantum metrology and information geometry, where it quantifies the sensitivity of a state to parameter changes and sets fundamental bounds on estimation precision [4,5].

Check for updates

Academic Editors: Ángel S. Sanz and Carlo Cafaro

Received: 12 December 2025
Revised: 22 January 2026
Accepted: 4 February 2026
Published: 13 February 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

Despite this growing body of work, the comparative dynamical behavior of entropy-based measures and information-geometric quantities remains insufficiently understood. In particular, it is often tacitly assumed that increasing entropy or decoherence necessarily implies a corresponding degradation of all informational capacities. Whether this assumption is justified is not obvious, especially in dissipative quantum systems [6] where different notions of information capture fundamentally different aspects of the state.

The central aim of this paper is to clarify this issue by performing a systematic, comparative analysis of several informational and structural quantifiers—entropy, disequilibrium, LMC statistical complexity, fidelity, and Fisher information—along the same open-system evolution. Our focus is not on introducing new measures or dynamics, but on identifying qualitative distinctions in how these quantities diagnose the system.

Using the damped Rabi oscillator governed by a Lindblad-type master equation as a paradigmatic example, we show that these quantifiers exhibit markedly different asymptotic behaviors. In particular, we identify a regime in which Fisher information vanishes at long times, signaling a complete loss of inferential sensitivity, while entropy, fidelity, and structural complexity remain finite. This result reveals a form of informational degradation that is not captured by thermodynamic or overlap-based measures.

This distinction has a clear conceptual significance. Entropy quantifies uncertainty or disorder in a state, whereas Fisher information measures the resolution with which physical parameters can be inferred from that state [3]. The vanishing of Fisher information therefore indicates not merely decoherence or mixing, but a collapse of statistical distinguishability. Our results demonstrate that these two notions of information loss need not coincide dynamically, even in simple and well-understood models.

From a foundational perspective, the significance of this finding lies precisely in its model independence. The Rabi oscillator is not introduced as a novel physical system, but as a minimal setting in which the distinction between thermodynamic disorder and inferential capability can be exposed without ambiguity. In this sense, the model plays a role analogous to that of the textbook systems frequently used to probe the conceptual foundations of quantum theory and statistical mechanics.

By highlighting a regime of inferential loss that is invisible to entropy-based diagnostics, this work contributes to a more nuanced understanding of decoherence and information loss in open quantum systems. It also underscores the importance of information geometry as a complementary tool to traditional entropic measures, particularly in contexts related to parameter estimation, metrology, and the interpretation of irreversibility.

Information Quantifier Used Here

Fisher information in a damped Rabi oscillator is crucial for understanding how a quantum system loses information due to dissipation and decoherence. Fidelity measures the overlap between the initial and time-evolved states, showing how the system's quantum information is lost. Entropy provides a measure of the system's uncertainty or disorder, and disequilibrium quantifies the deviation from a steady state. Fisher information is particularly important because it quantifies the precision of parameter estimation and can be used to detect quantum phenomena like synchronization, even when other measures fail [1,3,5–24]. Let us now delve on the importance of each of these quantifiers.

1. Fidelity: it (a) quantifies the loss of quantum coherence in the system over time; (b) helps determine the overall purity and structural integrity of the quantum state; and (c) a decline in fidelity indicates that the system is losing the ability to preserve quantum states accurately, which is a key indicator of how well information is being transmitted or stored.

2. Entropy: it (a) measures the degree of "disorder" or the spread of the system's probability distribution; (b) in the context of the damped Rabi oscillator, it shows how information spreads out as the system loses energy and coherence; and (c) by tracking entropy, researchers can identify how the system's uncertainty increases over time due to damping and decoherence.
3. Disequilibrium: it (a) quantifies the system's deviation from its final equilibrium state; (b) in the damped Rabi oscillator, it shows how far the system is from reaching a steady state and helps describe the dynamics leading to it; and (c) it is a useful measure for understanding how a system is evolving and how it approaches a stable, thermal state in the presence of dissipation.
4. Fisher Information: it (a) provides a measure of the precision with which a parameter can be estimated from the system's state; (b) it is a sensitive probe of quantum features, unlike entropy which can be insensitive to certain types of information loss; (c) in the damped Rabi oscillator, Fisher information can reveal phenomena like the "trapping" of information, where the system becomes "informationally silent" but still retains some structural order; and (d) it is a powerful tool for metrology and can be used to detect quantum synchronization, even when other measures fail.

The organization of the paper is as follows. Section 2 provides a brief contextual introduction to the Rabi model and to the dissipative mechanisms [6] relevant for the driven two-level system. Section 3 introduces the Lindblad master equation and presents the derivation of the Bloch equations that describe the damped dynamics. Section 4 develops the Fisher information formalism for the two-level dynamics and provides the unified expression valid in all dissipative regimes. Section 5 defines the Shannon entropy for the two-level system and gives the explicit time-dependent expression used in our analysis. Section 6 introduces disequilibrium and the LMC statistical complexity and applies them to the Rabi populations. Section 7 is devoted to the analysis of the fidelity, where we discuss its behavior under dissipation and its usefulness as a measure of coherence loss in the driven system. In Section 8, we define the additional information-based quantifiers used in this work—entropy, disequilibrium, and Fisher information—and comment on the specific dynamical features each one captures. Finally, Section 9 summarizes the main findings and outlines possible extensions of this framework to other driven open quantum systems.

We remark finally that relevant complementary studies include treatments of quantum Fisher information under dissipation [6], geometric formulations of decoherence and information contraction [21,22], and the thermodynamic length in quantum systems [23,24], which provide a natural conceptual framework for the informational degradation to be observed here.

2. Historical Context of the Rabi Model

The Rabi model [25], introduced in 1937 by Isidor Isaac Rabi [25], describes the simplest quantum interaction between a two-level system (spin-$\frac{1}{2}$ particle) and a classical oscillating field. Originally developed to explain the magnetic resonance of atomic nuclei in a rotating magnetic field, the Rabi model laid the groundwork for nuclear magnetic resonance (NMR), electron spin resonance (ESR), and numerous spectroscopic techniques.

The key insight of Rabi's work was the prediction of coherent oscillations between the two atomic states when subjected to a resonant field—what are now called Rabi oscillations. These oscillations reflect the time-dependent probability of population inversion in a driven two-level system and remain a cornerstone of coherent control in modern quantum technologies.

While the original Rabi model involves a classical driving field, it serves as the conceptual forerunner of fully quantum models such as the Jaynes–Cummings model, where the field itself is quantized. Nonetheless, the classical Rabi model retains fundamental importance due to its exact solvability and analytical accessibility under both resonant and detuned driving conditions.

With the advent of quantum computing and coherent control platforms (e.g., trapped ions, superconducting qubits, and NV centers in diamond), the Rabi model has gained renewed relevance. It underpins control protocols, gate dynamics, and the study of decoherence in driven quantum systems. Moreover, it has been generalized to include counter-rotating terms (yielding the quantum Rabi model), dissipative environments [6], and even multi-photon interactions.

In this work, we revisit the Rabi model from a quantum information-theoretic perspective. By coupling the two-level system to a classical drive and incorporating Lindblad-type dissipation, we expose a novel mechanism for informational degradation—the vanishing of Fisher information—even as the system retains low entropy and structural coherence. This highlights the Rabi model's continuing power as a theoretical laboratory for probing the boundaries of coherence, inference, and irreversibility in open quantum systems.

Let us insist that the novel contribution of this work is not the analysis of the Rabi model itself, but the identification of a dynamical regime in open quantum systems where Fisher information vanishes asymptotically despite finite entropy, fidelity, and structural order. Our results reveal a form of inferential loss not detectable by thermodynamic or overlap-based measures, highlighting a fundamental distinction between entropy-based disorder and information-geometric sensitivity.

Exact Theory of Damped Rabi Oscillations

One must have in mind, before proceeding, the importance and interesting findings on Ref. [25], that deal with a driven two-level system subject to dissipation exhibits Rabi oscillations whose amplitude decays due to population loss and dephasing. While the standard phenomenological model employs a single exponential envelope, an exact analysis shows that open-system Rabi dynamics generically contain three distinct decay channels. This structure was derived in full generality by Kosugi et al. [26], who extended Torrey's Laplace-transform method to include both internal relaxation and external loss mechanisms.

The starting point is the driven Hamiltonian matrix

$$\mathcal{H}(t) = \begin{pmatrix} E_0 & \frac{\Omega_0}{2} e^{i\omega t} \\ \frac{\Omega_0}{2} e^{-i\omega t} & E_1 \end{pmatrix}, \tag{1}$$

together with four dissipative rates: decay from the excited and ground states (Γ_1 and Γ_0), internal relaxation (Γ_{10}), and pure dephasing (Γ_ϕ). Introducing the usual Bloch variables u, v, w for coherence and population inversion, and $R = \rho_{11} + \rho_{00}$ for total population, the master equation reduces to a linear system $\dot{f} = Mf$ with $f = (u, v, w, R)^T$. Unlike the closed Bloch equations, $R(t)$ is no longer conserved because of population leakage.

The exact solution for each component has the universal form

$$f_i(t) = A_i e^{-at} + B_i e^{-bt} + C_i \cos(st)\, e^{-ct} + \frac{D_i}{s} \sin(st)\, e^{-ct}, \tag{2}$$

revealing two purely decaying exponentials (e^{-at}, e^{-bt}) and one damped oscillation with frequency s and decay constant c. For experimentally relevant near-resonant driving ($\Omega_0 \gg \Gamma$), the excited-state population is well approximated by

$$\rho_{11}(t) = \tfrac{1}{2}\big[A\, e^{-at} + C\, e^{-ct} \sin(\Omega_0 t + \phi)\big], \tag{3}$$

where $a = \frac{1}{2}(\Gamma_1 + \Gamma_0)$ and $c = \frac{1}{4}(2\Gamma_2 + \Gamma_1 + \Gamma_0 + 2\Gamma_{10})$ with $\Gamma_2 = \frac{1}{2}(\Gamma_1 + \Gamma_0 + \Gamma_{10}) + \Gamma_\phi$. This expression explains several features observed experimentally in driven Josephson phase qubits—including asymmetric upper and lower envelopes, multiple decay rates, and non-zero initial excited-state populations—that are not captured by the usual single exponential model.

An important practical outcome of Ref. [26] is a method for extracting all relaxation rates from the measured envelopes of the damped oscillation. By forming the combinations $\rho_{11}^{(+)}(t) = \frac{1}{2}(Ae^{-at} + Ce^{-ct})$ and $\rho_{11}^{(-)}(t) = \frac{1}{2}(Ae^{-at} - Ce^{-ct})$, one obtains two single exponential curves with decay constants a and c. Together with the free-evolution decay rates in the absence of driving, these yield closed-form expressions for Γ_0, Γ_1, Γ_{10}, and Γ_ϕ, allowing for a complete reconstruction of decoherence channels in the two-level system.

3. Lindblad Formalism for the Damped Rabi Oscillator

Our references in this Section are [1,16]. To incorporate decoherence, we adopt the Lindblad master equation for Markovian open quantum systems [1,16]. The time evolution of the density matrix $\rho(t)$ is

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\mathcal{H}, \rho] + \gamma\left(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\right), \tag{4}$$

where γ is the spontaneous emission rate, $\sigma_- = |g\rangle\langle e|$, and $\sigma_+ = |e\rangle\langle g|$. The coherent part is governed by the driven two-level Hamiltonian

$$\mathcal{H} = \frac{\hbar\Delta}{2}\sigma_z + \frac{\hbar\Omega}{2}\sigma_x, \tag{5}$$

where σ_x and σ_z denote the Pauli matrices. Here, Ω is the (bare) Rabi frequency, proportional to the strength of the coherent driving field, while $\Delta = \omega_L - \omega_0$ is the detuning between the laser frequency ω_L and the internal transition frequency ω_0. The term proportional to σ_x induces coherent population oscillations between the ground and excited states, while the term proportional to σ_z reflects the energy splitting in the rotating frame.

In the basis $\{|e\rangle, |g\rangle\}$, the density operator is [1,16]

$$\rho(t) = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}, \qquad \rho_{gg} = 1 - \rho_{ee}.$$

From Equations (4) and (5) one obtains [1,16]

$$\frac{d}{dt}\begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix} = \begin{pmatrix} -\gamma\rho_{ee} + i\frac{\Omega}{2}(\rho_{ge} - \rho_{eg}) & -i\Delta\rho_{eg} + i\frac{\Omega}{2}(\rho_{gg} - \rho_{ee}) - \frac{\gamma}{2}\rho_{eg} \\ +i\Delta\rho_{ge} + i\frac{\Omega}{2}(\rho_{ee} - \rho_{gg}) - \frac{\gamma}{2}\rho_{ge} & +\gamma\rho_{ee} - i\frac{\Omega}{2}(\rho_{ge} - \rho_{eg}) \end{pmatrix} \tag{6}$$

The origin of each contribution is as follows [1,16]: the terms proportional to $i\Delta$ and $i\Omega$ arise from the coherent commutator $-\frac{i}{\hbar}[\mathcal{H}, \rho]$ and generate Rabi oscillations, whereas the terms proportional to γ originate from the Lindblad dissipator. The latter causes the irreversible decay of the excited-state population ($-\gamma\rho_{ee}$) and exponential damping of coherences ($-\frac{\gamma}{2}\rho_{eg}$ and $-\frac{\gamma}{2}\rho_{ge}$), driving the system toward the ground state. Units with $\hbar = 1$ are assumed throughout.

3.1. Bloch Equations, Analytic Solution and Dissipative Regimes

3.1.1. Bloch Equation Representation

Starting from the Lindblad Equation (4) with Hamiltonian (5), it is convenient to rewrite the dynamics in terms of the Bloch variables [1,16]

$$u(t) = \Re[\rho_{eg}(t)], \qquad v(t) = \Im[\rho_{eg}(t)], \qquad w(t) = \rho_{ee}(t) - \rho_{gg}(t).$$

One obtains the dissipative Bloch equations

$$\begin{aligned} \dot{u} &= -\frac{\gamma}{2}u + \Delta\, v, \\ \dot{v} &= -\frac{\gamma}{2}v - \Delta\, u - \Omega\, w, \\ \dot{w} &= -\gamma(w+1) + \Omega\, v. \end{aligned} \tag{7}$$

These three coupled linear equations encode the interplay between coherent driving (Ω, Δ) and spontaneous emission γ. The excited-state population is recovered as [1,16]

$$\rho_{ee}(t) = P_e(t) = \tfrac{1}{2}\big[1 + w(t)\big]. \tag{8}$$

3.1.2. Closed-Form Solution for $\rho(t)$ (Initial Ground State)

Let us define the convenient parameters [1,16]

$$\alpha = \frac{\gamma}{2}, \qquad \Omega_R = \sqrt{\Omega^2 + \Delta^2}, \qquad \Omega_R' = \sqrt{\Omega^2 + \Delta^2 - \alpha^2}, \tag{9}$$

and introduce also the amplitude factor

$$A = \frac{\Omega^2}{\Omega^2 + \Delta^2 - \alpha^2}. \tag{10}$$

We consider the initial condition $\rho(0) = |g\rangle\langle g|$, i.e., $u(0) = v(0) = 0$, $w(0) = -1$.

In the underdamped regime [1,16] $\Omega_{R'}^2 > 0$, the solution for the density matrix elements in the basis $\{|g\rangle, |e\rangle\}$ reads

$$\rho_{ee}(t) = P_e(t) = \frac{\Omega^2}{\Omega_R'^2}\, e^{-\alpha t}\, \sin^2\!\left(\frac{\Omega_R' t}{2}\right), \tag{11}$$

$$\rho_{gg}(t) = 1 - \rho_{ee}(t), \tag{12}$$

$$\rho_{eg}(t) = \frac{\Omega}{2\Omega_R'}\, e^{-\alpha t}\left[-i\sin(\Omega_R' t) \;+\; \frac{\Delta}{\Omega_R'}\big(1 - \cos(\Omega_R' t)\big)\right], \tag{13}$$

with $\rho_{ge}(t) = \rho_{eg}^*(t)$. These expressions are algebraically equivalent to the solution obtained by solving the linear system (7) and reconstructing $\rho_{eg} = u + iv$, $\rho_{ee} = (1+w)/2$.

On the other hand, if $\Omega_R'^2 < 0$ (so $\Omega^2 + \Delta^2 < \alpha^2$), the dynamics are [1,16] overdamped: write $\Omega_R' = i\,\kappa$ with $\kappa = \sqrt{\alpha^2 - \Omega^2 - \Delta^2} > 0$ and replace sines and cosines by hyperbolic functions:

$$\sin(\Omega_R' t) \mapsto i\sinh(\kappa t), \qquad \cos(\Omega_R' t) \mapsto \cosh(\kappa t),$$

so that $\rho_{ee}(t)$ becomes proportional to $e^{-\alpha t}\sinh^2(\kappa t/2)$.

The critically damped case corresponds to $\Omega_R' = 0$ (i.e., $\Omega^2 + \Delta^2 = \alpha^2$), where the trigonometric/hyperbolic expressions must be expanded to lowest nontrivial order (use series expansions) to obtain finite limits; in practice one expands $\sin(\Omega_R' t)/\Omega_R'$, etc., around $\Omega_R' = 0$ [1,16].

3.2. Unified Expression for All Dissipative Regimes [1,16]

For later use in Fisher information, Shannon entropy and LMC complexity, we adopt a single compact expression valid for underdamped, critically damped, and overdamped regimes. Introduce unified trigonometric/hyperbolic functions

$$S(t) = \begin{cases} \sin\left(\frac{\Omega_R' t}{2}\right), & \Omega_R'^2 > 0, \\ \sinh\left(\frac{|\Omega_R'| t}{2}\right), & \Omega_R'^2 < 0, \end{cases} \qquad C(t) = \begin{cases} \cos\left(\frac{\Omega_R' t}{2}\right), & \Omega_R'^2 > 0, \\ \cosh\left(\frac{|\Omega_R'| t}{2}\right), & \Omega_R'^2 < 0. \end{cases}$$

Then the excited-state population takes the compact form

$$P_e(t) = A\, e^{-\alpha t}\, S(t)^2. \tag{14}$$

3.3. Coherent and Dissipative Limits [1,16]

Before analyzing the behavior of information-theoretic quantifiers, it is useful to summarize the main limits and dynamical regimes implied by the Lindblad solution. Setting $\gamma = 0$ recovers the coherent Rabi evolution with generalized Rabi frequency $\Omega_R = \sqrt{\Omega^2 + \Delta^2}$,

$$P_e(t) = \frac{\Omega^2}{\Omega_R^2} \sin^2\left(\frac{\Omega_R t}{2}\right), \tag{15}$$

which will serve as a reference for the dissipative case. The complete density matrix solution also satisfies the expected conditions $\rho_{ee}(0) = 0$, $\rho_{gg}(0) = 1$, and $\rho_{eg}(0) = 0$, and in the long-time limit it relaxes to $\rho(\infty) = |g\rangle\langle g|$ for any $\gamma > 0$.

Dissipation modifies the coherent dynamics through the parameter

$$\Omega_R' = \sqrt{\Omega^2 + \Delta^2 - \gamma^2/4},$$

which determines the qualitative behavior of the system:

- Resonant case ($\Delta = 0$): the effective frequency reduces to $\Omega_R' = \sqrt{\Omega^2 - \gamma^2/4}$. Oscillations persist only in the underdamped regime $\gamma < 2\Omega$; otherwise the dynamics become non-oscillatory.
- Underdamped ($\Omega_R'^2 > 0$): damped oscillations at frequency Ω_R', modulated by $e^{-\gamma t/2}$, and
$$\Omega_R' = \sqrt{\Omega^2 - (\gamma/2)^2 - \Delta^2}$$
- Critical ($\Omega_R' = 0$): oscillatory and relaxational timescales coincide, yielding the slowest non-oscillatory response.
- Overdamped ($\Omega_R'^2 < 0$): no oscillations occur; the dynamics become purely relaxational with rates
$$\lambda_\pm = \alpha \pm \sqrt{\alpha^2 - \Omega^2 - \Delta^2}, \text{ where } \alpha = \gamma/2.$$

These limits provide the dynamical framework within which the behavior of entropy, disequilibrium, complexity, fidelity, and Fisher information will be analyzed in the following sections [1,16].

4. Fisher Information: Discrete Definition and Rabi Applications

4.1. Discrete Fisher Information

We consider a discrete distribution $\{P_i(\theta)\}$ depending on a parameter θ, with $i = 1, \ldots, N$. The Fisher information is defined as [3]

$$I(\theta) = \sum_i \frac{1}{P_i(\theta)} \left(\frac{\partial P_i(\theta)}{\partial \theta} \right)^2. \tag{16}$$

For a two-level system with $P_e(\theta)$ and $P_g(\theta) = 1 - P_e(\theta)$, choosing $\theta = t$ yields

$$I(t) = \frac{(\partial_t P_e(t))^2}{P_e(t)\,[1 - P_e(t)]}. \tag{17}$$

Using the unified expression (14) one finds

$$\partial_t P_e = A e^{-\alpha t} S(t) \big(-\alpha S(t) + \Omega_R' C(t) \big),$$

and therefore the Fisher information takes the compact unified form:

$$I(t) = A\, e^{-\alpha t}\, \frac{\big(\Omega_R' C(t) - \alpha S(t)\big)^2}{1 - A\, e^{-\alpha t} S(t)^2}. \tag{18}$$

This expression is valid in all dissipative regimes.

4.2. Coherent Limits

For $\gamma = 0$ and arbitrary detuning, Equation (18) reproduces

$$I(t) = \frac{\Omega^2 \Omega_R^2 \cos^2(\Omega_R t/2)}{\Omega_R^2 - \Omega^2 \sin^2(\Omega_R t/2)}.$$

For the resonant undamped case ($\Delta = \gamma = 0$), one recovers the constant result $I(t) = \Omega^2$.

5. Shannon Entropy and Rabi Applications

5.1. Definition

Shannon entropy was formally introduced by Shannon in 1948 with the purpose of mathematically quantify the statistical nature of lost information in communication systems and to define a measure of uncertainty [18].

For a discrete probability distribution $\mathcal{P} = \{p_i\}_{i=1}^N$ with $p_i \geq 0$ and $\sum_{i=1}^N p_i = 1$, the Shannon entropy is defined as

$$H(\mathcal{P}) = -\sum_{i=1}^{N} p_i \ln p_i. \tag{19}$$

This time-dependent entropy provides a quantitative measure of the information content associated with the population dynamics. This quantity measures the statistical uncertainty (or information content) associated with the distribution. It satisfies $0 \leq H(\mathcal{P}) \leq \ln N$, with the lower bound reached for a fully certain distribution (one outcome with probability 1), and the upper bound reached for the uniform distribution $p_i = 1/N$.

5.2. Shannon Entropy for a Two-Level System

For a driven two-level system the relevant probability distribution is $\{P_e(t),\, P_g(t)\} = \{P_e(t),\, 1 - P_e(t)\}$, where $P_e(t)$ is the excited-state population obtained from the Lindblad dynamics. In this case the Shannon entropy reduces to the binary form

$$H(t) = -P_e(t)\, \ln P_e(t) - \big[1 - P_e(t)\big]\, \ln\big[1 - P_e(t)\big]. \tag{20}$$

This expression will be used below to construct the disequilibrium and the LMC complexity.

Inserting the general form of $P_e(t)$ gives

$$H(t) = -Ae^{-\alpha t}S(t)^2 \ln\left[Ae^{-\alpha t}S(t)^2\right] - \left[1 - Ae^{-\alpha t}S(t)^2\right] \ln\left[1 - Ae^{-\alpha t}S(t)^2\right], \tag{21}$$

which is an explicit closed expression valid in all regimes.

6. LMC Complexity

6.1. Definition

The LMC (López-Ruiz, Mancini and Calbet) statistical complexity is defined as [19]

$$C_{\mathrm{LMC}} = \overline{H}\, D, \tag{22}$$

where $\overline{H}$ is the normalized Shannon entropy and D the disequilibrium. For a discrete distribution of N states the normalized entropy is

$$\overline{H} = \frac{H(\mathcal{P})}{\ln N},$$

and the disequilibrium, that quantifies the departure of the system from the uniform distribution, is defined as

$$D = \sum_{i=1}^{N} \left(p_i - \frac{1}{N}\right)^2. \tag{23}$$

6.2. Rabi Applications

For the two-level system ($N = 2$), the disequilibrium, defined as the deviation from maximal mixedness, becomes

$$D(t) = \left(P_e(t) - \frac{1}{2}\right)^2 + \left(1 - P_e(t) - \frac{1}{2}\right)^2 = 2\left(P_e(t) - \frac{1}{2}\right)^2. \tag{24}$$

The disequilibrium vanishes at the fully mixed state $P_e(t) = \frac{1}{2}$, and increases as the distribution becomes more peaked.

In this case, the normalized Shannon entropy is

$$\overline{H}(t) = \frac{H(t)}{\ln 2},$$

with $H(t)$ given by Equation (20). Therefore, the LMC complexity is given by

$$C_{\mathrm{LMC}}(t) = \frac{H(t)}{\ln 2}\, 2\left(P_e(t) - \frac{1}{2}\right)^2. \tag{25}$$

This quantity is minimal both for perfectly ordered states ($P_e = 0, 1$) and for the maximally mixed state ($P_e = 1/2$), reaching its maximum at intermediate levels of disorder and disequilibrium.

7. Fidelity in Dissipative Dynamics

In the presence of dissipation, the time evolution of the two-level system is no longer unitary and the state of the system is described by a density matrix $\rho(t)$ satisfying the Lindblad master Equation (4). Since the state ceases to be pure for $\gamma \neq 0$, the usual expression $|\langle\psi(t)|\phi(t)\rangle|^2$ for pure states is no longer applicable. Instead, we employ the Uhlmann fidelity [20], which provides a consistent extension of state overlap to arbitrary mixed states.

Uhlmann fidelity.

For two density matrices ρ and σ, the fidelity is defined as [20]

$$F(\rho,\sigma) = \left[\mathrm{Tr}\sqrt{\sqrt{\rho}\,\sigma\,\sqrt{\rho}}\right]^2. \tag{26}$$

This expression reduces to the pure-state overlap whenever one (or both) of the states is pure.

Fidelity with respect to a reference pure state.

When the target state is pure, $\sigma = |\psi_{\mathrm{ref}}\rangle\langle\psi_{\mathrm{ref}}|$, Equation (26) simplifies to

$$F_{\psi_{\mathrm{ref}}}(t) = \langle\psi_{\mathrm{ref}}|\rho(t)|\psi_{\mathrm{ref}}\rangle, \tag{27}$$

which is often the most relevant quantity in dissipative quantum dynamics. For instance, choosing $|\psi_{\mathrm{ref}}\rangle = |e\rangle$ yields

$$F_e(t) = \rho_{ee}(t) = P_e(t), \tag{28}$$

so that the fidelity coincides with the excited-state population.

Fidelity between dissipative and coherent evolutions.

A useful quantifier of the effect of dissipation is the fidelity between the state evolving under Lindblad dynamics, $\rho_{\gamma\neq0}(t)$, and the state that would result from coherent evolution in the absence of decay, $|\psi_{\gamma=0}(t)\rangle$. Using Equation (27), this fidelity is

$$F_{\mathrm{coh}}(t) = \langle\psi_{\gamma=0}(t)\,|\,\rho_{\gamma\neq0}(t)\,|\psi_{\gamma=0}(t)\rangle. \tag{29}$$

This measure quantifies the loss of coherence and population induced by spontaneous emission. In particular, $F_{\mathrm{coh}}(t) \to 1$ as $\gamma \to 0$, while for $\gamma > 0$ it decays in time as the system relaxes toward the ground state.

The Uhlmann fidelity provides a natural extension of overlap-based quantifiers to dissipative systems and is therefore consistent with the use of the Shannon entropy, disequilibrium and statistical complexity introduced in the following sections. In the present context, the fidelity can be evaluated analytically once the matrix elements of $\rho(t)$ are known from the Bloch-equation solution.

7.1. Fidelity for the Dissipative Rabi Oscillator

To quantify how much the system remains in its initial state under Lindblad dissipation, we consider the fidelity with respect to the initial pure state. Since the system starts in the ground state $\rho(0) = |g\rangle\langle g|$, the fidelity at time t reduces to

$$F(t) = \langle g|\rho(t)|g\rangle = 1 - \rho_{ee}(t), \tag{30}$$

where $\rho_{ee}(t)$ is the excited-state population. This expression holds for any Markovian decay model where population leaks from $|e\rangle$ to $|g\rangle$, and it automatically satisfies the physical requirement $F(0) = 1$.

7.2. Closed Expression for the Fidelity

The Lindblad master equation with spontaneous emission rate γ and Rabi Hamiltonian (5) yields, in the underdamped regime, the well-known analytical solution for the excited-state population $\rho_{ee}(t)$ given by Equation (11).

Substituting Equation (11) into Equation (30), one obtains the closed expression for the fidelity:

$$F(t) = 1 - \frac{\Omega^2}{\Omega_R'^2} e^{-\gamma t/2} \sin^2\left(\frac{\Omega_R' t}{2}\right). \tag{31}$$

We have the following consistency checks:

1. Initial condition. Setting $t = 0$ in Equation (31) gives $F(0) = 1$.
2. Undamped limit $\gamma \to 0$. Since $\Omega_R' \to \Omega_R$, Equation (31) reduces to

$$F(t) \to 1 - \frac{\Omega^2}{\Omega_R^2} \sin^2\left(\frac{\Omega_R t}{2}\right),$$

   recovering the coherent Rabi result.
3. Long-time limit. Because $\rho_{ee}(t) \to 0$ as $t \to \infty$, the fidelity satisfies $F(t) \to 1$, meaning the system fully relaxes back to the ground state.

The fidelity therefore provides a concise and physically transparent measure of the dissipative evolution, fully determined by the same parameters (Ω, Δ, γ) that govern the Fisher information and Shannon entropy analyzed in previous sections.

8. Quantifiers Interpretation

Figure 1 illustrates the dynamical behavior of the information quantifiers of Shannon entropy $H(t)$, disequilibrium $D(t)$, complexity $C_{\mathrm{LMC}}(t)$, fidelity $F(t)$, and Fisher information $I(t)$ in the strongly underdamped regime, obtained for $\Delta = 5$, $\Omega = 1$, and a small dissipation rate $\gamma = 0.1$. In this parameter range the generalized Rabi frequency is dominated by the detuning, $\Omega_R \approx \sqrt{\Delta^2 + \Omega^2}$, and satisfies $\gamma \ll \Omega_R$, so the system remains close to the coherent limit. Accordingly, all quantities exhibit long-lived oscillations with only a slow exponential decay in amplitude. The coherence $I(t)$ displays large-amplitude revivals that follow the unitary oscillation frequency, while the Shannon entropy $H(t)$ and the disequilibrium $D(t)$ oscillate with comparatively small amplitude, reflecting the fact that the state remains nearly pure during the evolution. The LMC statistical complexity $C_{\mathrm{LMC}}(t)$ reproduces the same oscillatory pattern, showing intermittent peaks that progressively decrease in magnitude as dissipation accumulates. In contrast to the overdamped regime, the fidelity $F(t)$ stays close to unity throughout the dynamics, with only small modulations, indicating that the system does not significantly depart from its initial state. Overall, the figure captures the characteristic signature of the weak-damping regime: persistent Rabi oscillations that imprint their structure simultaneously on entropy, disequilibrium, complexity, coherence, and fidelity.

All five quantifiers display damped oscillations due to coherent Rabi dynamics suppressed by spontaneous emission. The vanishing of Fisher information indicates that, although the system remains structurally ordered, it becomes dynamically silent. That is, infinitesimal changes in time produce negligible statistical differences—an effect we term informational degradation without entropy increase. Our graph illustrates how decoherence, induced by $\gamma = 0.1$, affects the Rabi-coherent dynamics: the system evolves towards a mixed state. Fisher informational dynamics degrades and one observes the transition from a coherent quantum regime to a more "classical" and stationary one. Such a transition can be said to be the "heart" of the decoherence phenomenon.

Figure 2 illustrates the dynamical behavior of the same quantifiers as in Figure 1 in the overdamped regime, using the parameters $\Omega = 1$, $\Delta = 2$, and $\gamma = 5$. Since the dissipation rate greatly exceeds the coherent Rabi drive ($\gamma \gg \Omega$), the dynamics are fully non-oscillatory. All quantities relax monotonically toward their stationary values as the system is rapidly driven into the ground state by spontaneous emission. The coherence-based quantifier $I(t)$

decays on the fastest timescale, reflecting the strong suppression of off-diagonal density matrix elements by the Lindblad damping. The entropy-like measures $H(t)$ and $D(t)$ display smooth transient behavior without oscillations, reaching steady values once the system approaches its asymptotic state. In contrast, the complexity measure $C_{\mathrm{LMC}}(t)$ exhibits only a small and short-lived maximum, characteristic of transient mixedness, before vanishing at long times as the system becomes effectively pure. Finally, the fidelity $F(t)$ monotonically approaches unity, consistent with irreversible relaxation toward the ground state. Overall, the figure captures the defining features of the overdamped regime, where coherent Rabi oscillations are fully suppressed by strong dissipation.

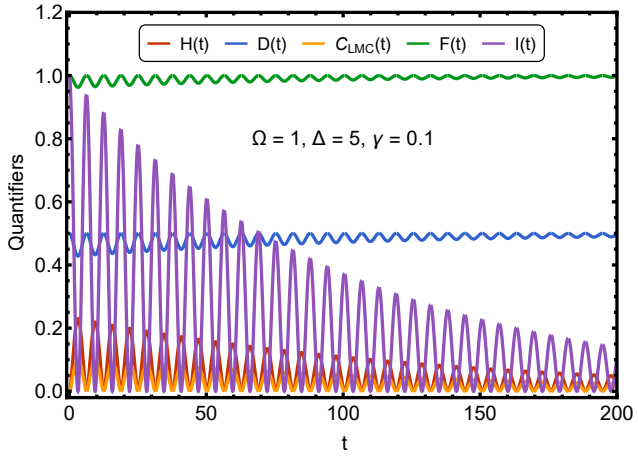


Figure 1. Underdamped case: Time evolution of Shannon entropy $H(t)$ (red), disequilibrium $D(t)$ (blue), complexity $C_{\mathrm{LMC}}(t)$ (Orange), fidelity $F(t)$ (green), and quantum Fisher information $I(t)$ (purple) for a underdamped Rabi oscillator. The parameters are normalized by setting $\Omega = 1$; the detuning is $\Delta = 5$, and the damping rate is $\gamma = 0.1$. The time variable is expressed in units of $1/\Omega'_R$, so that t is dimensionless.

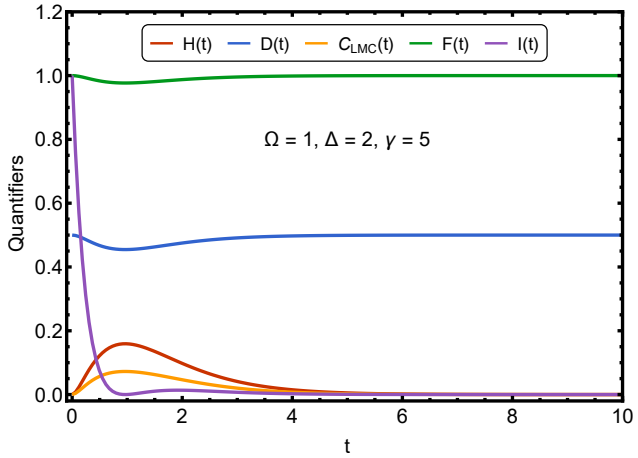


Figure 2. Overdamped case: Time evolution of Shannon entropy $H(t)$ (red), disequilibrium $D(t)$ (blue), Complexity $C_{\mathrm{LMC}}(t)$ (Orange), fidelity $F(t)$ (green), and quantum Fisher information $I(t)$ (purple) for a overdamped Rabi oscillator. The parameters are normalized by setting $\Omega = 1$; the detuning is $\Delta = 2$, and the damping rate is $\gamma = 5$. The time variable is expressed in units of $1/\Omega'_R$, so that t is dimensionless.

Figure 3 displays the time evolution of the informational quantifiers $H(t)$, $D(t)$, $C_{\mathrm{LMC}}(t)$, $F(t)$, and $I(t)$ for the resonant Rabi regime, using the parameters $\Omega = 1$, $\Delta = 2$, and $\gamma = 0$. In the absence of dissipation all quantifiers exhibit strictly periodic oscillations governed by the Rabi frequency. Despite sharing the same underlying period, each measure displays a distinct amplitude and dynamical pattern—$F(t)$ oscillates around a value

close to unity, reflecting the preservation of quantum coherence; $D(t)$ fluctuates around a mid-range value with reduced amplitude; and $C_{\mathrm{LMC}}(t)$ remains confined to very small values—capturing the weak interplay between disorder and structure in this regime. The quantifiers $H(t)$ and $I(t)$ show faster and higher-contrast oscillations, originating from their nonlinear dependence on the state populations. Altogether, the figure highlights how different informational measures encode complementary aspects of the same coherent two-level dynamics.

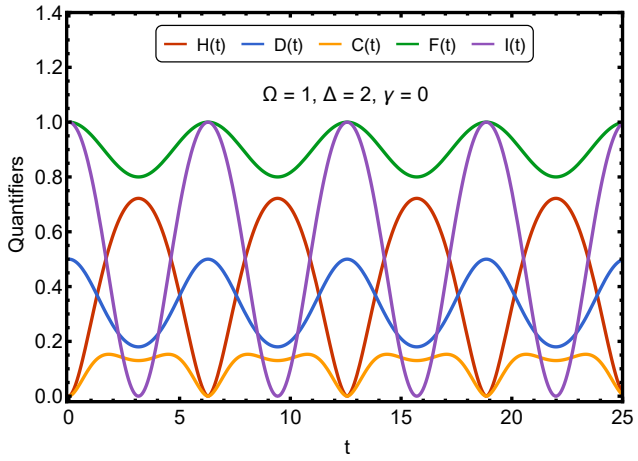


Figure 3. Resonant case: Time evolution of Shannon entropy $H(t)$ (red), disequilibrium $D(t)$ (blue), Complexity $C_{\mathrm{LMC}}(t)$ (Orange), fidelity $F(t)$ (green), and quantum Fisher information $I(t)$ (purple) for a resonant Rabi regime. The parameters are normalized by setting $\Omega = 1$; the detuning is $\Delta = 2$, and the damping rate is $\gamma = 0$. The time variable is expressed in units of $1/\Omega'_R$, so that t is dimensionless.

9. Conclusions

In this work we have examined the dynamical behavior of several information-theoretic and structural quantifiers—entropy, disequilibrium, LMC statistical complexity, fidelity, and Fisher information—along the dissipative evolution of a damped Rabi oscillator. The purpose of this analysis was not to introduce a new physical model or dynamical mechanism, but to use a well-controlled and widely understood open quantum system as a conceptual laboratory for clarifying the distinct roles played by different notions of information.

Our central result is the identification of a dynamical regime in which Fisher information vanishes asymptotically, even though entropy, fidelity, and statistical complexity remain finite. This behavior reveals a form of informational degradation that is not captured by entropic or overlap-based measures. While entropy tracks uncertainty or disorder in the state, Fisher information quantifies inferential sensitivity and statistical distinguishability. The decoupling of these quantities demonstrates that loss of inferential capability is not equivalent to thermodynamic mixing or decoherence.

From a foundational perspective, this distinction is significant precisely because it emerges in a simple and well-established model. The use of the Rabi oscillator is not a limitation but a strength: by eliminating model-specific complications, it allows for the conceptual content of the diagnostic measures themselves to be isolated and compared. In this respect, the present analysis follows a long tradition in theoretical physics in which elementary systems are used to probe general principles.

The results also have implications beyond conceptual clarification. Since Fisher information sets fundamental bounds on parameter estimation, its asymptotic decay identifies a regime in which no further information about system parameters can be extracted, even though the system retains structural order. This observation is directly relevant to quantum

metrology and to the interpretation of decoherence as a loss of statistical resolution rather than merely a loss of coherence.

More broadly, our findings underscore the need for caution when entropy is used as a stand-alone diagnostic of information loss in open quantum systems. Entropy-based measures provide valuable insight into disorder and mixing, but they do not exhaust the informational content of a quantum state. Information geometry, and Fisher information in particular, offers a complementary perspective that captures aspects of dynamical degradation invisible to thermodynamic indicators.

In summary, this work contributes to a more nuanced understanding of information loss in open quantum systems by distinguishing thermodynamic disorder from inferential collapse. By demonstrating this distinction in a paradigmatic model, we aim to clarify the conceptual scope and limitations of commonly used informational measures and to motivate their careful and context-dependent application in both foundational studies and practical settings. Our results reveal a form of inferential loss not detectable by thermodynamic or overlap-based measures, highlighting a fundamental distinction between entropy-based disorder and information-geometric sensitivity.

Author Contributions: Conceptualization, F.P. and A.P.; methodology, F.P. and A.P.; formal analysis, F.P. and A.P.; investigation, F.P. and A.P.; writing—original draft preparation, F.P. and A.P.; writing—review and editing, F.P. and A.P.; funding acquisition, F.P. All authors have read and agreed to the published version of the manuscript.

Funding: This research was partially supported by ANID/FONDECYT, Grant No. 1251928.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Breuer, H.-P.; Petruccione, F. *The Theory of Open Quantum Systems*; Oxford University Press: Oxford, UK, 2002.
2. Deffner, S.; Lutz, E. Nonequilibrium entropy production for open quantum systems. *Phys. Rev. Lett.* **2011**, *107*, 140404. [CrossRef]
3. Frieden, B.R. *Physics from Fisher Information*; Cambridge University Press: Cambridge, UK, 1998.
4. Braunstein, S.L.; Caves, C.M. Statistical distance and the geometry of quantum states. *Phys. Rev. Lett.* **1994**, *72*, 3439–3443. [CrossRef] [PubMed]
5. Paris, M.G.A. Quantum estimation for quantum technology. *Int. J. Quantum Inf.* **2009**, *7*, 125–137. [CrossRef]
6. Alipour, S.; Mehboudi, M.; Rezakhani, A.T. Quantum Metrology in Open Systems: Dissipative Cramér–Rao Bound. *Phys. Rev. Lett.* **2014**, *112*, 120405. [CrossRef] [PubMed]
7. Petz, D.; Ghinea, C. Introduction to Quantum Fisher Information. In *Quantum Probability and Related Topics*; Rebolledo, R., Orszag, M., Eds.; World Scientific: Singapore, 2011; pp. 261–281. [CrossRef]
8. Girolami, D.; Tufarelli, T.; Adesso, G. Characterizing nonclassical correlations via local quantum uncertainty. *Phys. Rev. Lett.* **2013**, *110*, 240402. [CrossRef]
9. Flego, S.P.; Plastino, A.; Plastino, A.R. Physical implications of Fisher-information's scaling symmetry. *Cent. Eur. J. Phys.* **2012**, *10*, 390–397. [CrossRef]
10. Venkatesan, R.; Plastino, A. Fisher Information Framework for Time Series Modeling. *Physica A* **2017**, *480*, 22–38. [CrossRef]
11. Pennini, F.; Plastino, A. The Fisher thermodynamics of quasi-probabilities. *Entropy* **2015**, *17*, 7848–7858. [CrossRef]
12. Venkatesan, R.; Plastino, A. Hellmann-Feynman connection for the relative Fisher information. *Ann. Phys.* **2015**, *359*, 300–316. [CrossRef]
13. Rosso, O.A.; Olivares, F.; Plastino, A. Noise versus Chaos in a causal Fisher-Shannon plane. *Pap. Phys.* **2015**, *7*, 070006. [CrossRef]
14. Shi, Y.-S.; Sun, G.-H.; Jing, J.; Dong, S.-H. Shannon and Fisher entropy measures for a parity-restricted harmonic oscillator. *Laser Phys.* **2017**, *27*, 125201. [CrossRef]
15. Nielsen, M.A.; Chuang, I.L. *Quantum Computation and Quantum Information*; Cambridge University Press: Cambridge, UK, 2000. [CrossRef]
16. Gorini, V.; Kossakowski, A.; Sudarshan, E.C.G. Completely positive dynamical semigroups of *N*-level systems. *J. Math. Phys.* **1976**, *17*, 821–825. [CrossRef]

17. Potts, P.P.; Brask, J.B.; Brunner, N. Fundamental limits on low-temperature quantum thermometry with finite resolution. *Quantum* **2019**, *3*, 161. [CrossRef]
18. Shannon, C.E. A Mathematical Theory of Communication. *Bell Syst. Tech. J.* **1948**, *27*, 623–656. [CrossRef]
19. López-Ruiz, R.; Mancini, H.L.; Calbet, X. A statistical measure of complexity. *Phys. Lett. A* **1995**, *209*, 321–326. [CrossRef]
20. Uhlmann, A. The "transition probability" in the state space of a $*$-algebra. *Rep. Math. Phys.* **1976**, *9*, 273–279. [CrossRef]
21. Alsing, P.M.; Milburn, G.J. Information Geometry, Entanglement, and Decoherence. *Quantum Inf. Comput.* **2002**, *2*, 487–512.
22. Braun, D.; Adesso, G.; Datta, A.; Benatti, F.; Floreanini, R.; Marzolino, U.; Mitchell, M.W.; Pirandola, S. Quantum-Enhanced Measurements Without Entanglement. *Rev. Mod. Phys.* **2018**, *90*, 035006. [CrossRef]
23. Deffner, S.; Lutz, E. Generalized Clausius Inequality for Nonequilibrium Quantum Processes. *Phys. Rev. Lett.* **2010**, *105*, 170402. [CrossRef] [PubMed]
24. Cafaro, C.; Mancini, S. Quantum stabilizer codes for correlated and asymmetric depolarizing errors. *Phys. Rev. A* **2010**, *82*, 012306. [CrossRef]
25. Rabi, I.I. Space Quantization in a Gyrating Magnetic Field. *Phys. Rev.* **1937**, *51*, 652. [CrossRef]
26. Kosugi, N.; Matsuo, S.; Konno, K.; Hatakenaka, N. Theory of damped Rabi oscillations. *Phys. Rev. B* **2005**, *72*, 172509. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.