

Article

A Novel Method That Is Based on Differential Evolution Suitable for Large-Scale Optimization Problems

Glykeria Kyrrou *, Vasileios Charilogis and Ioannis G. Tsoulos 

Department of Informatics and Telecommunications, University of Ioannina, 47150 Kostaki Artas, Greece; v.charilog@uoi.gr (V.C.); itsoulos@uoi.gr (I.G.T.)

* Correspondence: g.kyrrou@uoi.gr

Abstract

Global optimization represents a fundamental challenge in computer science and engineering, as it aims to identify high-quality solutions to problems spanning from moderate to extremely high dimensionality. The Differential Evolution (DE) algorithm is a population-based algorithm like Genetic Algorithms (GAs) and uses similar operators such as crossover, mutation and selection. The proposed method introduces a set of methodological enhancements designed to increase both the robustness and the computational efficiency of the classical DE framework. Specifically, an adaptive termination criterion is incorporated, enabling early stopping based on statistical measures of convergence and population stagnation. Furthermore, a population sampling strategy based on k-means clustering is employed to enhance exploration and improve the redistribution of individuals in high-dimensional search spaces. This mechanism enables structured population renewal and effectively mitigates premature convergence. The enhanced algorithm was evaluated on standard large-scale numerical optimization benchmarks and compared with established global optimization methods. The experimental results indicate substantial improvements in convergence speed, scalability and solution stability.

Keywords: optimization; differential evolution algorithm; evolutionary techniques; stochastic methods; large-scale problems

1. Introduction

The basic goal of global optimization is to find the global minimum of a continuous multidimensional function and is defined as

$$x^* = \arg \min_{x \in S} f(x) \quad (1)$$

with S :

$$S = [a_1, b_1] \times [a_2, b_2] \times \dots [a_n, b_n]$$

with a_i and b_i representing lower and upper bounds for each variable, x_i . In recent years many researchers have published important reviews on global optimization. Such methods find application in a wide range of scientific fields, such as mathematics [1], physics [2], chemistry [3], biology [4], medicine [5], agriculture [6] and economics [7]. A particular challenge is the large-scale global optimization (LSGO) problem, where the complexity increases significantly with increasing problem dimensions. Finding efficient and computationally feasible solutions has become particularly difficult, which has led the research



Academic Editor: Hsien-Chung Wu

Received: 4 December 2025

Revised: 15 January 2026

Accepted: 19 January 2026

Published: 23 January 2026

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community to focus on the development of innovative algorithms. LSGO problems are encountered in a wide range of applications, while their importance was also reflected in the organization of the first global large-scale optimization competition within the framework of the CEC in 2008. Other competitions followed in 2010 [8], 2013 [9] and 2015 [10], attracting the intense interest of the academic community.

To address these challenges, various heuristic and meta-heuristic approaches have been developed. Evolutionary algorithms (EAs) [11] are one of the most effective categories, as they mimic natural selection and genetic evolution to search for best solutions. Due to their adaptability and robustness, EAs can solve difficult optimization problems. Some of the most well-known EAs are DE [12], GAs [13], evolutionary strategies [14], evolutionary programming [15] and multimodal optimization algorithms [16]. Also, methods inspired by Swarm Intelligence [17] such as Particle Swarm Optimization (PSO) [18], Ant Colony Optimization (ACO) [19], Artificial Bee Colony (ABC) Optimization [20], the Firefly Algorithm (FA) [21] and the Bat Algorithm [22], Arithmetic Optimization Algorithm (AOA) [23] are strong alternatives.

DE is one of the most widely used optimization techniques, as it offers high robustness, simplicity and fast convergence. DE is a highly efficient evolutionary algorithm that has gained significant recognition since the late 1990s. DE, originally introduced in 1995 by Storn and Price [24], has proven to be a versatile optimization tool that can be applied in various scientific and engineering fields. It is particularly effective for symmetric optimization problems, as well as for dealing with discontinuous, noisy and dynamic challenges. In physics, it has been used in energy-related problems, including wind power optimization [25]. In chemistry, it has contributed to advances in atmospheric chemistry and the development of high-performance chemical reactors [26]. DE has also had significant implications in health-related areas, such as breast cancer research and medical diagnostics [27]. Despite its effectiveness, classical DE may face additional challenges when applied to high-dimensional and large-scale optimization problems. In such settings, the adaptation of control parameters becomes more demanding, while exploitation efficiency during later stages of the search process may be reduced. Moreover, as problem dimensionality increases, maintaining fast convergence and high solution accuracy can become more difficult. These observations have motivated extensive research efforts toward the development of enhanced DE variants, aiming to improve adaptability, scalability and the balance between exploration and exploitation in large-scale optimization problems.

Recent studies have proposed various strategies to address large-scale optimization challenges, including cooperative coevolution [28], Particle Swarm Optimization [29], a DE approach [30], distributed differential evolution with adaptive resource allocation [31], hybrid swarm-based algorithms combining multiple search mechanisms [32], a self-adaptive Fast Fireworks Algorithm [33], swarm-based methods with learning mechanisms [34] and advanced decomposition techniques such as dual Differential Grouping [35]. These approaches highlight the ongoing research interest in designing efficient and scalable optimization frameworks for large-scale problems.

In this work, a unified and modular DE framework is proposed to improve efficiency, robustness and convergence behavior in large-scale optimization problems. Instead of relying on a single evolutionary strategy or fixed parameter settings, the proposed approach integrates multiple enhancement mechanisms within a single DE framework. These mechanisms are designed to be independently configurable, allowing the algorithm to adapt more effectively to diverse problem characteristics while preserving the fundamental structure of classical DE.

The main contributions of this paper are summarized as follows:

- A unified and modular DE framework is introduced, integrating multiple control mechanisms within a single optimization scheme, including mutation weighting, parent selection, local refinement activation and termination criteria.
- A k-means-based population sampling strategy is incorporated to preserve population structure and improve sampling efficiency in high-dimensional search spaces.
- Alternative mechanisms for computing the differential weight parameter are proposed, incorporating number-based, random and migrant strategies to enhance adaptability in large-scale optimization problems.
- An optional tournament-based parent selection strategy is employed to improve selection pressure while maintaining population diversity and robustness.
- A periodic local optimization refinement using deterministic local optimizers, such as BFGS, is integrated to enhance solution accuracy and accelerate convergence without compromising global exploration.
- A population-based termination criterion is introduced to enable early stopping when convergence stagnation is detected, significantly reducing unnecessary objective function evaluations.
- The proposed framework is specifically designed for large-scale global optimization problems and aims to achieve a more effective balance between exploration and exploitation compared to classical DE variants.

Although the individual enhancement mechanisms employed in the proposed framework, including adaptive termination strategies, clustering-based population initialization, adaptive differential weight schemes and local optimization techniques, have been previously studied in the literature, their combined integration within a single DE framework is not straightforward. Each mechanism operates in a distinct stage of the evolutionary process and affects different aspects of the search dynamics, such as diversity preservation, convergence behavior and computational efficiency.

The contribution of the present work lies in the systematic and modular coordination of these mechanisms within a unified optimization framework. By enabling controlled interaction between global exploration, adaptive parameter control, local exploitation and termination mechanisms, the proposed approach facilitates synergistic effects that cannot be readily achieved through the isolated application of individual enhancements. This structured integration provides a principled framework for improving robustness and scalability in large-scale global optimization problems.

Importantly, the novelty of the proposed approach lies not in the introduction of new standalone operators but in the principled way these mechanisms are coordinated within a single framework. Each component is explicitly designed to operate in synergy with the others, rather than being applied independently. In particular, structured sampling and adaptive differential weighting jointly influence population diversity and step-size control, while tournament selection ensures that informative population feedback is preserved for adaptive mechanisms to exploit. This coordinated interaction provides a theoretical justification for the combined framework and explains why its behavior cannot be reduced to a simple aggregation of existing techniques.

The remaining parts of this paper are divided as follows: In Section 2 the original DE algorithm, the proposed method and a flowchart with detailed description are presented. In Section 3 of the test functions used in the experiments as well as the related experiments are presented. In Section 4, there is a brief discussion of the results obtained from the experiments. In Section 5 some conclusions and directions for future improvements are discussed.

2. Differential Evolution Algorithm

2.1. The Original Differential Evolution Method

DE is a population-based evolutionary algorithm that has been widely used for continuous optimization problems. The method maintains a population of candidate solutions, which are iteratively evolved through the application of mutation, crossover and selection operators. In each iteration, new candidate solutions are generated by combining information from multiple population members, while selection is performed based on objective function comparisons. The DE procedure begins by defining the population size NP . In order to ensure the feasibility of the classical DE mutation operator, which requires three distinct population members in addition to the target vector, a minimum population size of $NP \geq 4$ is required. In practice, the population size is often related to the dimensionality of the optimization problem. A commonly adopted guideline in the DE literature is to set $NP = 10n$, where n denotes the problem dimension, as this choice has been shown to provide robust performance across a wide range of problems without extensive parameter tuning. We note that alternative formulations relating the population size to the problem dimension have also been proposed in the literature; however, such choices are problem-dependent and do not affect the general applicability of the DE framework. The initial population is generated randomly within the search space and evaluated using the objective function. During each iteration, for every target vector, $\mathbf{x}_i$, three distinct population members are randomly selected to construct a mutant vector through a differential mutation operation. This mutant vector is then combined with the target vector using a binomial crossover mechanism, producing a trial vector. If the trial vector achieves an objective function value that is not worse than that of the target vector, it replaces the target vector in the population. The evolutionary process continues until a termination criterion is satisfied, such as reaching a maximum number of iterations or meeting a convergence condition. The algorithm returns the best solution found during the search process. Regarding parameter settings, the crossover probability is set to $CR = 0.9$ and the differential weight to $F = 0.8$, following values commonly adopted in the DE literature [36]. These parameter values have been empirically shown to provide stable performance across a broad range of optimization problems without requiring problem-specific tuning. In this study, all parameters of the original DE algorithm are kept fixed throughout the experimental evaluation in order to ensure a fair and unbiased comparison with the proposed method. For clarity, the full steps of the original DE algorithm are summarized in Algorithm 1.

Algorithm 1 Original Differential Evolution Algorithm

INPUT

- f : objective function
- NP : population size
- CR : Crossover rate
- F : Differential weight
- n : Problem dimension

OUTPUT

- x_{best}

INITIALIZATION

-Generate an initial population of NP candidate solutions x_i , $i = 1, \dots, NP$, uniformly at random within the search bounds.
-Evaluate the objective function $f(x_i)$ for all individuals.
-Set x_{best} as the individual with the best objective value.

main pseudocode

```

01 while stopping criterion is not met do
02   for each individual  $i$ ,  $i \in \{1..NP\}$  do
03     Select randomly three agents  $a, b, c \in \{1..NP\}$ 
04     Generate mutant vector  $u = a_j + F \times (b_j - c_j)$ 
05     Select a random index  $R \in \{1..n\}$ 
06     for each dimension  $j = 1$  to  $n$  do
07       Generate a random number  $r_i \in [0, 1]$ 
08       if  $r_j < CR$  or  $j = R$  then
09         Set  $y_j = u_j$ .
10       else
11         Set  $y_j = x_{ij}$ .
12     endif
13   endfor
14   if  $f(y) \leq f(x_i)$  then
15     Replace  $x_i$  with  $y$ 
16   endif
17 endfor
18 endwhile
19 return  $x_{best}$ 

```

2.2. The Proposed Differential Evolution Method

The main contribution of the proposed approach is the formulation of a unified and modular DE framework that systematically integrates multiple control mechanisms within a single optimization scheme. Unlike most existing DE variants, which typically modify a single algorithmic component (e.g., mutation strategy or parameter adaptation), the proposed method allows the independent configuration and combined use of several algorithmic mechanisms while preserving the fundamental evolutionary structure of the original DE algorithm. The proposed method extends the classical DE framework by introducing a modular design that incorporates additional control components related to differential weight computation, parent selection, local refinement and termination criteria. This design enables both the independent analysis of each component and their joint exploitation within a single, coherent optimization process, facilitating a systematic investigation of robustness and parameter sensitivity. The main methodological contributions of the proposed framework can be summarized as follows:

- Alternative mechanisms for differential weight computation.
- A k-means-based population sampling strategy is incorporated to preserve population structure and improve sampling efficiency in high-dimensional search spaces.
- Optional tournament-based parent selection strategies.
- Periodic local refinement using a deterministic local optimizer.
- A population-based termination criterion.

The algorithm starts with an initialization phase in which a population of NP agents is randomly generated and evaluated using the objective function. Additional control parameters are also initialized in this stage, including the local search rate p_l , which determines the frequency of local refinement, the tournament size N_t , which controls selection pressure, the maximum number of generations N_g , the termination criteria N_l and the iteration counter k . During the evolutionary process, different strategies for computing the differential weight F can be employed. These include a constant value and a random mechanism defined as $F = -0.5 + 2r$, where $r \in [0, 1]$, as well as a migrant-based strategy [37]. For each agent, candidate solutions are generated through mutation and crossover opera-

tions, followed by a selection step based on objective function comparisons. In addition to the evolutionary operators, a deterministic local search procedure based on the BFGS method [38] may be periodically applied to refine promising solutions. The optimization process proceeds iteratively until a termination condition is satisfied, which may be defined either by a maximum number of generations or by a population-based convergence criterion. By jointly integrating stochastic variation, adaptive differential weight mechanisms and deterministic local refinement within a unified framework, the proposed method aims to improve search efficiency and to achieve a more effective balance between exploration and exploitation. Moreover, the modular structure of the algorithm allows the systematic evaluation of the effect of individual algorithmic components, as demonstrated in the sensitivity analysis presented in Section 3. For clarity, the complete steps of the proposed DE algorithm are summarized in Algorithm 2.

The main steps of the proposed DE algorithm are illustrated in the flowchart presented in Figure 1.

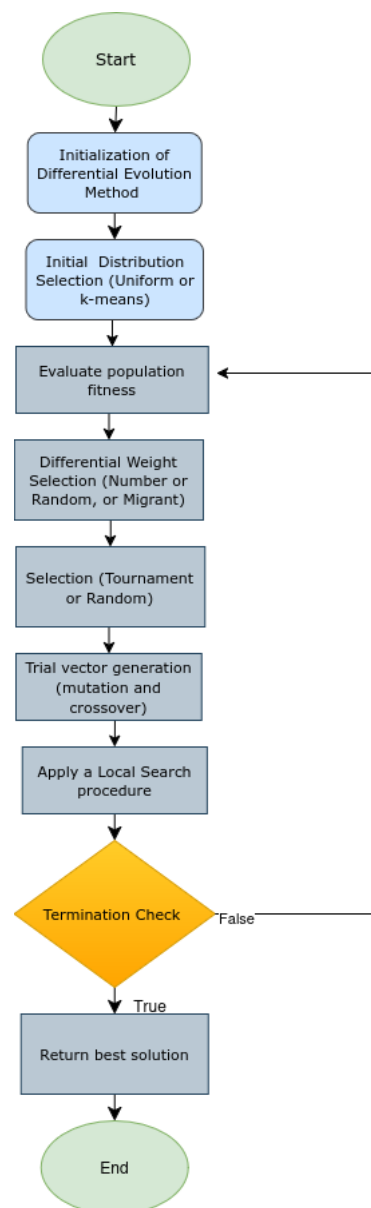


Figure 1. The steps of the proposed DE algorithm.

Algorithm 2 Proposed Algorithm

INPUT

- f : objective function
- NP : population size
- N_t : tournament size
- N_g : maximum number of iterations
- N_I : termination criteria
- CR : Crossover rate
- F : Differential weight
- n : Problem dimension
- k : iteration counter

OUTPUT
 $-x_{best}$
INITIALIZATION

- Set as NP the population size (number of agents)
- Create randomly NP agents $x_i, i = 1, \dots, NP$
- Compute the fitness value $f_i = f(x_i)$ for each agent
- Set as p_l the local search rate
- Set as N_g maximum number of iterations
- Set as N_I termination criteria
- Set as N_t tournament size
- Set $k \leftarrow 0$ as the iteration counter
- Set the parameter CR , with $CR \leq 1$
- Select the differential weight method F :
 - (a) Number : F is constant value.
 - (b) Random : $F = -0.5 + 2r, r \in [0, 1]$.
 - (c) Migrant : migrant-based mechanism [39].

main pseudocode

```

01 while stopping criterion is not met do
02   for each individual  $i, i \in \{1..NP\}$  do
03     Select the agent  $x_i$ 
04     Select randomly three distinct agents  $x_a, x_b, x_c$ 
05     Choose a random integer  $R \in [1, n]$ 
06     Create a trial point  $x_t$ 
07     for  $j \in \{1..n\}$  do
08       Select a random number  $r \in [0, 1]$ 
09       if  $r < CR$  or  $j = R$  then
10         Set  $x_{t,j} = x_{a,j} + F \times (x_{b,j} - x_{c,j})$ 
11       else
12         Set  $x_{t,j} = x_{ij}$ 
13       endif
14     endfor
15     Set  $y_t = f(x_t)$ 
16     if  $(y_t) \preceq f(x_i)$  then
17       Replace  $x_i$  with  $x_t$ 
18     endif
19     Select a random number  $r \in [0, 1]$ 
20     if  $r \leq p_l$  then
21       Apply local search  $x_i = LS(x_i)$  [38]
22     endif
23   endfor
24   Set  $k \leftarrow k + 1$ 
25   if  $k \geq N_g$  then terminate
26   Compute  $\delta^{(k)} = \left| \sum_{i=1}^{NP} f_i^{(k)} - \sum_{i=1}^{NP} f_i^{(k-1)} \right|$ 
27   if  $\delta^{(k)} \leq \varepsilon$  for  $N_I$  iterations then terminate.
28 endwhile
29 return  $x_{best}$ 

```

3. Experiments

This section begins with a description of the functions that were used in the experiments and then presents in detail the experiments that were performed, in which the parameters available in the proposed algorithm were studied, in order to study their reliability and adequacy.

3.1. Test Functions

A variety of benchmark test functions were used in the conducted experiments. These functions have been widely adopted in previous studies [40–43]. In the present work, the test functions were evaluated using dimensionalities ranging from 25 to 150, where the constant n denotes the dimension of the objective function. The benchmark test functions used in the experimental study are summarized in Table 1, including their mathematical formulation, dimensionality and global optimum values.

Table 1. Benchmark test functions used in experimental study.

NAME	FORMULA	DIM	G_{min}
ATTRACTIVE SECTOR	$f(x) = (\sum_{i=1}^n (s_i x_i)^2)^{0.9}$	2	0
BUCHE RASTRIGIN	$f(x) = \sum_{i=1}^n [z_i \cdot (1 + 0.1 \cdot \sin(10\pi z_i))]$	n	0
DIFFERENT POWERS	$f(x) = \sqrt{\sum_{i=1}^n x_i ^{2+4\frac{i-1}{n-1}}}$	n	0
DISCUS	$f(x) = 10^6 x_1^2 + \sum_{i=2}^n x_i^2$	n	0
ELLIPSOIDAL	$f(x) = \sum_{i=1}^n (10^6)^{\frac{i-1}{n-1}} x_i^2$	n	0
LLAGHER101	$f(x) = \max_{i=1}^{101} [h_i - w_i \sqrt{\sum_{j=1}^n (x_j - c_{ij})^2}] \min : 100 + 1$	n	0
LLAGHER21	$f(x) = \max_{i=1}^{21} [h_i - w_i \sqrt{\sum_{j=1}^n (x_j - c_{ij})^2}] \min : 10 + 10 + 1$	n	0
GRIEWANK ROSENBROCK	$f(x) = \underbrace{\left(\frac{\ x\ ^2}{4000} - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \right)}_{\text{Griewank}} \cdot \underbrace{\left(\frac{1}{10} \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2] \right)}_{\text{Rosenbrock}}$	n	0
GRIEWANK	$f(x) = 1 + \frac{1}{200} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \frac{\cos(x_i)}{\sqrt{i}}$	n	0
RARSTIGIN	$f(x) = An + \sum_{i=1}^n [x_i^2 - A \cos(2\pi x_i)] \quad A = 10$	n	0
ROSENBROCK	$f(x) = \sum_{i=1}^{n-1} (100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2), \quad -30 \leq x_i \leq 30$	n	0
SHARP RIDGE	$f(x) = x_1^2 + \alpha \sum_{i=2}^n x_i^2, \quad \alpha > 1$	n	0
SPHERE	$f(x) = \sum_{i=1}^n x_i^2$	n	0
STEP ELLIPSOIDAL	$f(x) = \sum_{i=1}^n [x_i + 0.5]^2 + \alpha \sum_{i=1}^n \left(10^6 \cdot \frac{i-1}{n-1} \right) x_i^2, \quad \alpha = 1$	n	0
ZAKHAROV	$f(x) = \sum_{i=1}^n x_i^2 + \left(\sum_{i=1}^n \frac{i}{2} x_i \right)^2 + \left(\sum_{i=1}^n \frac{i}{2} x_i \right)^4$	n	0

3.2. Experimental Results

A series of experiments were carried out for the previously mentioned functions and these experiments were executed on an AMD RYZEN 5950X with 128 GB of RAM. The operating system of the running machine was Debian Linux. Each experiment was conducted 30 times, with different random numbers each time, and the averages were recorded. The software used in the experiments was coded in C++ using the freely available optimization environment of OPTIMUS [44], which can be downloaded from <https://github.com/itsoulos/OPTIMUS> (accessed on 18 January 2026). In addition to the proposed DE framework, a GA was employed as a baseline evolutionary method for

comparative evaluation. The inclusion of the GA provided a well-established reference approach in global optimization, allowing a clearer assessment of the performance of the proposed method. All comparative methods were independently implemented and evaluated within the same experimental framework and were not directly adopted from the corresponding literature. All algorithms were implemented in C++ and executed under identical hardware and software conditions to ensure a fair and consistent comparison. All algorithms were run using the same termination criterion, in order to ensure a fair and reproducible comparison. To guarantee comparability across all methods, common control parameters were fixed, as summarized in Table 2. Specifically, the number of agents (population size) was set to 200 for all algorithms, the maximum number of iterations was fixed at 200 and the local search rate was uniformly set to 0.05. These shared settings ensured that performance differences arose from algorithmic design rather than differences in experimental configuration. Algorithm-specific parameters were selected according to standard practices reported in the literature and were kept constant throughout all experiments. No problem-specific parameter tuning was applied in order to avoid bias and to maintain methodological consistency. The parameter settings for both the proposed method and the GA are summarized in the Table 2.

Table 2. The values of the parameters of the proposed method.

PARAMETER	MEANING	VALUE
NP	Number of agents for all methods	200
n	Maximum number of allowed iterations for all methods	200
p_l	Local search rate	0.05
F	Differential weight for classic DE	$F \in [0, 1.0]$
F	Differential weight for proposed method	0.8
CR	Crossover probability	0.9
N_f	Number of iterations used in the termination rule	8
-	Mutation rate	0.05 (5%)
-	Selection rate	0.05 (5%)
-	Selection method	Roulette

The parameter values selected for the GA were chosen based on standard practices in evolutionary computation and preliminary empirical evaluation. All GA parameters were kept fixed throughout the experimental study to avoid problem-specific tuning and to ensure a fair and consistent comparison with the proposed method.

3.3. The Effect of Differential Weight Mechanism

Table 3 presents the impact of the three differential weight strategies, NUMBER(T), RANDOM(T) and MIGRANT(T), on the performance of the algorithm across a broad set of benchmark functions, where (T) denotes tournament-based selection. The results clearly show that MIGRANT(T) is by far the most efficient method. The MIGRANT(T) strategy consistently achieves the best outcomes, requiring the fewest objective function evaluations overall (387,335) compared with NUMBER(T) (527,444) and RANDOM(T) (543,201). This improvement is particularly noteworthy given that all three strategies achieve the same overall success rate (0.85), indicating that the performance advantage arises purely from efficiency rather than reliability differences. This trend is visible across multiple test functions. In the Attractive Sector family (25–150 dimensions), MIGRANT(T) demonstrates uniformly superior performance. For example, in Attractive Sector_25, it requires 1697 calls,

compared with 1743 for NUMBER(T) and 1756 for RANDOM(T). As dimensionality increases, this advantage becomes even more pronounced. The performance gap becomes even more substantial in multimodal landscapes such as Buche Rastrigin. In Buche Rastrigin_25, MIGRANT(T) needs 5893 function calls (success rate: 0.90), significantly fewer than NUMBER(T) (12,243) and RANDOM(T) (12,035). The difference becomes overwhelming in the highest-dimensional case (Buche Rastrigin_150): MIGRANT(T) completes the optimization with 23,466 calls, while NUMBER(T) requires 40,240, and RANDOM(T) needs 39,263. These results underline MIGRANT(T)'s superior adaptability in sharply multimodal and high-variance landscapes. For example, Ellipsoidal_150 is solved in 16,930 calls by MIGRANT(T), compared with 19,311 for NUMBER(T) and 19,940 for RANDOM(T). This difference becomes particularly important for large-scale smooth problems, where maintaining efficiency is critical.

Table 3. Experiments using different weight selection for the proposed method.

FUNCTION	MIGRANT(T)	NUMBER(T)	RANDOM(T)
ATTRACTIVE SECTOR_25	1697	1743	1756
ATTRACTIVE SECTOR_50	1761	1823	1828
ATTRACTIVE SECTOR_100	1832	1879	1880
ATTRACTIVE SECTOR_150	1867	1900	1920
BUCHERASTRIGIN_25	5893 (0.90)	12,243 (0.90)	12,035 (0.90)
BUCHERASTRIGIN_50	12,585 (0.50)	19,529 (0.50)	20,457 (0.50)
BUCHERASTRIGIN_100	16,490 (0.53)	30,055 (0.53)	31,465 (0.53)
BUCHERASTRIGIN_150	23,466 (0.27)	40,240 (0.27)	39,263 (0.27)
DISCUS_25	1992	1857	1896
DISCUS_50	2060	1926	1971
DISCUS_100	2104	1978	1989
DISCUS_150	2144	2006	2040
DIFFERENTPOWERS_25	6478	11,422	11,629
DIFFERENTPOWERS_50	11,183	15,258	15,179
DIFFERENTPOWERS_100	16,225	21,451	20,659
DIFFERENTPOWERS_150	21,495	24,429	24,670
ELLIPSOIDAL_25	3590	3751	3958
ELLIPSOIDAL_50	6424	6864	7184
ELLIPSOIDAL_100	11,549	13,756	13,890
ELLIPSOIDAL_150	16,930	19,311	19,940
LLAGHER21_25	2261 (0.90)	3815 (0.90)	6364 (0.90)
LLAGHER21_50	4503 (0.50)	5277 (0.50)	9643 (0.50)

Table 3. Cont.

FUNCTION	MIGRANT(T)	NUMBER(T)	RANDOM(T)
LLAGHER21_100	1756 (0.53)	1523 (0.53)	1521 (0.53)
LLAGHER21_150	1662 (0.27)	1521 (0.27)	1526 (0.27)
LLAGHER101_25	2769 (0.90)	3472 (0.90)	5657 (0.90)
LLAGHER101_50	4890 (0.50)	6950 (0.50)	7454 (0.50)
LLAGHER101_100	5886 (0.53)	6846 (0.53)	9505 (0.53)
LLAGHER101_150	8646 (0.27)	7701 (0.27)	12,352 (0.27)
GRIEWANK_25	4084	5276	5145
GRIEWANK_50	5039	5138	5729
GRIEWANK_100	6460	5726	6002
GRIEWANK_150	6542	5870	6164
GRIEWANK_ROSENBROCK_25	4466	7458	6939
GRIEWANK_ROSENBROCK_50	5325	9766	9255
GRIEWANK_ROSENBROCK_100	6465	11,776	11,001
GRIEWANK_ROSENBROCK_150	7272	13,482	12,543
ROSENBROCK_25	5950	7824	7955
ROSENBROCK_50	8963	13,970	13,057
ROSENBROCK_100	15,930	23,402	22,348
ROSENBROCK_150	22,135	32,850	31,562
RARSTIGIN_25	4577 (0.90)	9691 (0.90)	10,242 (0.90)
RARSTIGIN_50	7746 (0.50)	13,134 (0.50)	12,740 (0.50)
RARSTIGIN_100	9147 (0.53)	13,128 (0.53)	13,184 (0.53)
RARSTIGIN_150	11,620 (0.27)	15,105 (0.27)	15,602 (0.27)
SPHERE_25	1481	1507	1512
SPHERE_50	1509	1534	1539
SPHERE_100	1524	1555	1556
SPHERE_150	1535	1568	1567
STEP ELLIPSOIDAL_25	1625 (0.90)	1642 (0.90)	2090 (0.90)
STEP ELLIPSOIDAL_50	2300 (0.50)	2774 (0.50)	4021 (0.50)
STEP ELLIPSOIDAL_100	2465 (0.53)	1598 (0.53)	1571 (0.53)
STEP ELLIPSOIDAL_150	3143 (0.27)	1531 (0.27)	1521 (0.27)
SHARP RIDGE_25	5104	6215	6026
SHARP RIDGE_50	5226	6850	7123
SHARP RIDGE_100	5995	7782	7649
SHARP RIDGE_150	6481	8112	8237
ZAKHAROV_25	2185	2752	2639
ZAKHAROV_50	3027	4063	3864
ZAKHAROV_100	5572	6265	5634
ZAKHAROV_150	6304	7574	7553
	387,335 (0.85)	527,444 (0.85)	543,201 (0.85)

Overall, the evidence strongly indicates that MIGRANT(T) is the most effective differential weight mechanism among the tested variants. It consistently reduces the number of function evaluations across a wide variety of functions both unimodal and multimodal while preserving identical success rates. This combination of efficiency, robustness, and stability makes MIGRANT(T) a particularly advantageous choice for enhancing the performance of DE in high-dimensional and challenging optimization scenarios.

Figure 2 presents a pairwise statistical comparison of the three migration strategies, MIGRANT(T), NUMBER(T) and RANDOM(T), with respect to the required number of function evaluations. The Kruskal–Wallis test ($p = 0.3$) does not indicate the presence of statistically significant overall differences among the three strategies, suggesting comparable distributions of computational cost. This outcome is consistent with the results of the pairwise t -tests, for which none of the comparisons reach statistical significance ($p > 0.05$). Accordingly, the observed differences in medians and dispersion across the strategies are not supported as statistically significant. The MIGRANT(T) strategy exhibits a slightly lower average number of function evaluations; however, this difference is not accompanied by statistical significance and remains within the range of stochastic variability. Overall, the results indicate similar behavior of the three migration strategies in terms of computational cost.

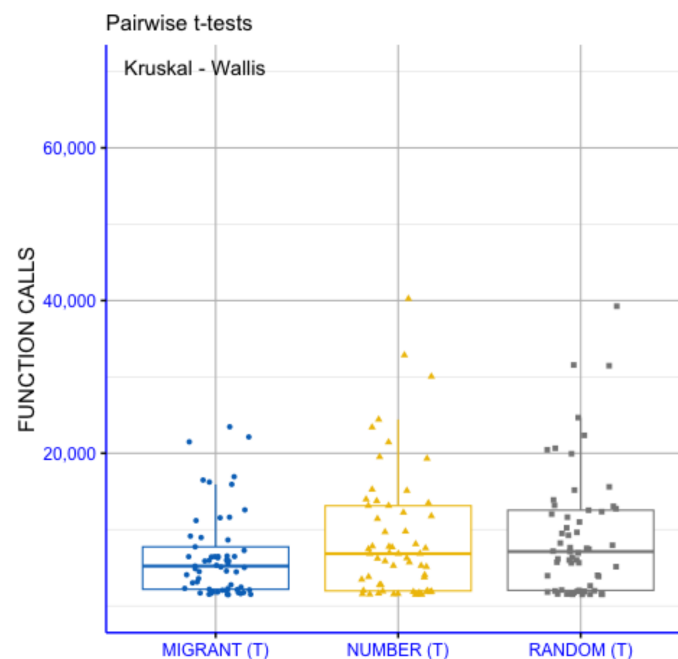


Figure 2. A statistical comparison of the proposed with different weight selection.

3.4. The Effect of Selection Mechanism

Table 4 compares the four selection strategies, RANDOM(R), RANDOM(T), MIGRANT(R) and MIGRANT(T), where (R) denotes purely random selection and (T) denotes tournament-based selection. The results clearly show that MIGRANT(T) is by far the most efficient method. It achieves the lowest total number of objective function evaluations (387,335), significantly outperforming MIGRANT(R) (962,599), RANDOM(T) (543,201) and RANDOM(R) (767,225). Since all methods achieve the same success rate (0.85), the performance differences are due solely to efficiency, demonstrating the importance of the selection mechanism. This advantage becomes evident across nearly all tested functions. For the Attractive Sector family (25–150 dimensions), MIGRANT(T) consistently requires the fewest evaluations. For example, in Attractive Sector_25, it needs only 1697 calls, com-

pared to 2174 MIGRANT(R), 1756 for RANDOM(T) and as many as 2162 for RANDOM(R). The differences are even more striking for multimodal benchmarks such as Buche Rastrigin. In the 25-dimensional case, MIGRANT(T) achieves 5893 calls (0.90 success), while MIGRANT(R) needs 15,894, RANDOM(T) needs 12,035 and RANDOM(R) 11,921. At the 150-dimensional level, the gap widens dramatically: MIGRANT(T) requires 23,466 calls, whereas MIGRANT(R) rises to 77,590, RANDOM(T) 39,263 and RANDOM(R) to 54,663. These results highlight the strong stabilizing effect that tournament selection has on the MIGRANT mechanism. The superiority of MIGRANT(T) is even more pronounced in the Sharp Ridge functions. In Sharp Ridge_150, MIGRANT(T) completes the optimization with 6481 calls, while MIGRANT(R) requires 12,053, RANDOM(T) 8237 and RANDOM(R) 12,395. Finally, in the Zakharov functions, MIGRANT(T) again shows consistently superior performance. In Zakharov_150, MIGRANT(T) needs just 6304 calls, compared to 25,370 for MIGRANT(R), 7553 RANDOM(T) and 16,240 for RANDOM(R). Even in the easier 25-dimensional case, MIGRANT(T) requires 2185 calls, whereas RANDOM(R) requires over twice as many (4605).

Table 4. Effect of random and tournament selection strategies on optimization performance.

FUNCTION	MIGRANT(T)	MIGRANT(R)	RANDOM(T)	RANDOM(R)
ATTRACTIVE SECTOR_25	1697	2174	1756	2162
ATTRACTIVE SECTOR_50	1761	2212	1828	2162
ATTRACTIVE SECTOR_100	1832	2177	1880	2192
ATTRACTIVE SECTOR_150	1867	2206	1920	2174
BUCHER RASTRIGIN_25	5893 (0.90)	15,894 (0.90)	12,035 (0.90)	11,921 (0.90)
BUCHER RASTRIGIN_50	12,585 (0.50)	50,438 (0.50)	20,457 (0.50)	20,542 (0.50)
BUCHER RASTRIGIN_100	16,490 (0.53)	59,214 (0.53)	31,465 (0.53)	34,570 (0.53)
BUCHER RASTRIGIN_150	23,466 (0.27)	77,590 (0.27)	39,263 (0.27)	54,663 (0.27)
DISCUS_25	1992	2588	1896	2542
DISCUS_50	2060	2601	1971	2552
DISCUS_100	2104	2553	1989	2617
DISCUS_150	2144	2608	2040	2591
DIFFERENTPOWERS_25	6478	13,918	11,629	14,477
DIFFERENTPOWERS_50	11,183	20,100	15,179	20,064
DIFFERENTPOWERS_100	16,225	27,396	20,659	29,408
DIFFERENTPOWERS_150	21,495	35,710	24,670	35,070
ELLIPSOIDAL_25	3590	6424	3958	5932
ELLIPSOIDAL_50	6424	11,704	7184	10,585
ELLIPSOIDAL_100	11,549	20,736	13,890	20,887
ELLIPSOIDAL_150	16,930	29,835	19,940	28,265
LLAGHER21_25	2261 (0.90)	5412 (0.90)	6364 (0.90)	2891 (0.90)
LLAGHER21_50	4503 (0.50)	11,988 (0.50)	9643 (0.50)	3311 (0.50)

Table 4. *Cont.*

FUNCTION	MIGRANT(T)	MIGRANT(R)	RANDOM(T)	RANDOM(R)
LLAGHER21_100	1756 (0.53)	1565 (0.53)	1521 (0.53)	1524 (0.53)
LLAGHER21_150	1662 (0.27)	1490 (0.27)	1526 (0.27)	1520 (0.27)
LLAGHER101_25	2769 (0.90)	5180 (0.90)	5657 (0.90)	3016 (0.90)
LLAGHER101_50	4890 (0.50)	21,179 (0.50)	7454 (0.50)	4878 (0.50)
LLAGHER101_100	5886 (0.53)	26,739 (0.53)	9505 (0.53)	4507 (0.53)
LLAGHER101_150	8646 (0.27)	46,866 (0.27)	12,352 (0.27)	3400 (0.27)
GRIEWANK_25	4084	8148	5145	9902
GRIEWANK_50	5039	7894	5729	5203
GRIEWANK_100	6460	9083	6002	4145
GRIEWANK_150	6542	9154	6164	4075
GRIEWANK_ROSENBROCK_25	4466	11,510	6939	17,429
GRIEWANK_ROSENBROCK_50	5325	14,658	9255	24,666
GRIEWANK_ROSENBROCK_100	6465	15,890	11,001	34,019
GRIEWANK_ROSENBROCK_150	7272	17,910	12,543	39,208
ROSENBROCK_25	5950	13,718	7955	15,591
ROSENBROCK_50	8963	21,827	13,057	23,980
ROSENBROCK_100	15,930	34,948	22,348	40,245
ROSENBROCK_150	22,135	49,061	31,562	53,073
RARSTIGIN_25	4577 (0.90)	11,276 (0.90)	10,242 (0.90)	9910 (0.90)
RARSTIGIN_50	7746 (0.50)	26,967 (0.50)	12,740 (0.50)	14,234 (0.50)
RARSTIGIN_100	9147 (0.53)	27,639 (0.53)	13,184 (0.53)	16,666 (0.53)
RARSTIGIN_150	11,620 (0.27)	34,865 (0.27)	15,602 (0.27)	19,135 (0.27)
SPHERE_25	1481	1620	1512	1627
SPHERE_50	1509	1641	1539	1634
SPHERE_100	1524	1635	1556	1644
SPHERE_150	1535	1644	1567	1639
STEP ELLIPSOIDAL_25	1625 (0.90)	2073 (0.90)	2090 (0.90)	1750 (0.90)
STEP ELLIPSOIDAL_50	2300 (0.50)	5937 (0.50)	4021 (0.50)	1664 (0.50)
STEP ELLIPSOIDAL_100	2465 (0.53)	6546 (0.53)	1571 (0.53)	1523 (0.53)
STEP ELLIPSOIDAL_150	3143 (0.27)	11487 (0.27)	1521 (0.27)	1520 (0.27)
SHARP RIDGE_25	5104	10,153	6026	11,776
SHARP RIDGE_50	5226	11,108	7123	12,123
SHARP RIDGE_100	5995	11,592	7649	12,704
SHARP RIDGE_150	6481	12,053	8237	12,395
ZAKHAROV_25	2185	3941	2639	4605
ZAKHAROV_50	3027	8972	3864	7963
ZAKHAROV_100	5572	23,782	5634	14,514
ZAKHAROV_150	6304	25,370	7553	16,240
	387,335 (0.85)	962,599 (0.85)	543,201 (0.85)	767,225 (0.85)

Overall, these results highlight the strong interaction between the MIGRANT mechanism and tournament selection. Tournament selection dramatically enhances the perfor-

mance of MIGRANT, reducing the computational cost by large margins across all functions while preserving identical success rates. As a result, MIGRANT(T) emerges as the most balanced, stable, and efficient strategy, making it highly suitable for optimization scenarios where minimizing objective function evaluations is essential.

Figure 3 presents a pairwise statistical comparison of the four strategies, MIGRANT(T), MIGRANT(R), RANDOM(T) and RANDOM(R), based on their distributions of function calls. The Kruskal–Wallis test indicates a statistically significant overall difference among the groups ($p = 0.0019$), suggesting that at least one strategy differs from the others in terms of computational cost. To further examine these differences, pairwise t -tests were conducted, with p -values annotated using standard significance notation (ns: $p > 0.05$; *: $p < 0.05$; **: $p < 0.01$; ***: $p < 0.001$; ****: $p < 0.0001$). The pairwise analysis shows that MIGRANT(T) differs significantly from MIGRANT(R) and RANDOM(R), with the corresponding comparisons reaching higher levels of statistical significance. The strategies MIGRANT(R) and RANDOM(T) exhibit intermediate behavior, with several pairwise comparisons indicating statistically significant differences at moderate significance levels. Comparisons labeled as “ns” indicate pairs for which no statistically significant differences are detected. Overall, the distributions of function calls indicate lower evaluation counts for the MIGRANT-based strategies, particularly MIGRANT(T), relative to the RANDOM-based variants. These results describe differences in computational cost among the examined strategies under the considered experimental conditions.

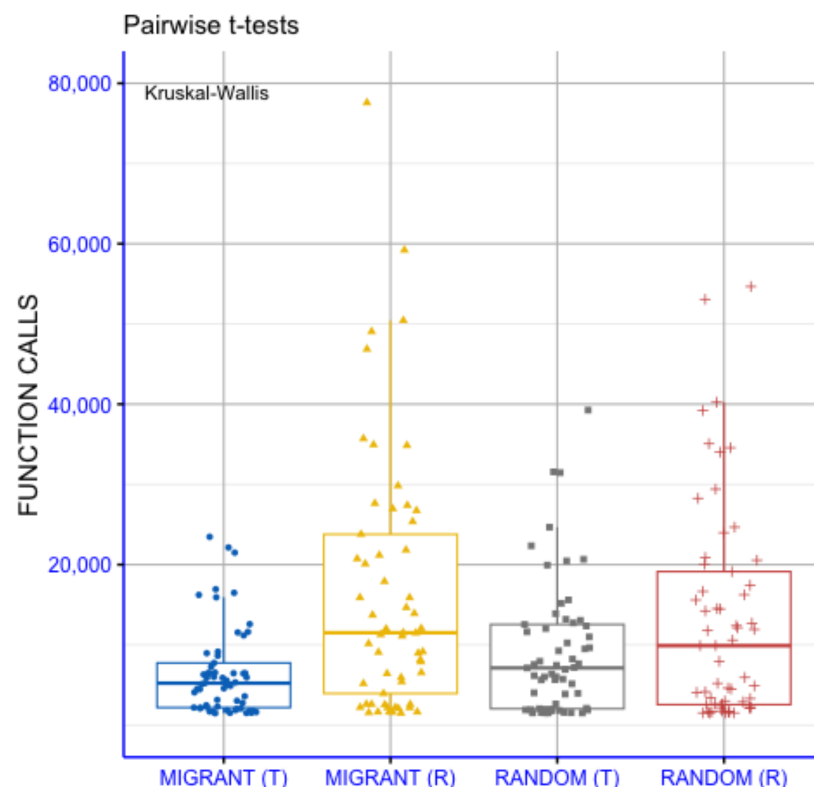


Figure 3. Statistical comparison of random and tournament selection strategies for optimization performance.

3.5. The Effect of Sampling Method

In Table 5 tournament selection is used to choose the samples that participate in the core operator of DE. Four strategies for computing the differential weight are evaluated: random weight with uniform sampling (RANDOM(U)), random weight with k-means

sampling [45,46] (RANDOM(K)), migrant weight with uniform sampling (MIGRANT(U)) and migrant weight with k-means sampling (MIGRANT(K)). The k-means method, originally proposed by MacQueen [47] is employed not only to determine cluster centers but also as a structured sampling mechanism. Across all test functions, MIGRANT(K) consistently achieves the lowest total number of function calls (387,335) with a success rate of 0.85, outperforming all other sampling strategies. This advantage becomes clear when examining individual benchmarks. For the Attractive Sector family (dimensions 25–150), MIGRANT(K) systematically requires fewer evaluations than MIGRANT(U) and both RANDOM methods. For example, in Attractive Sector_25, MIGRANT(K) uses only 1697 evaluations compared to 1738 for MIGRANT(U), while RANDOM(K) and RANDOM(U) require 1756 and 1792, respectively. This pattern holds across all dimensionalities, showing the benefit of structured sampling in unimodal landscapes. The effect becomes far more pronounced in multimodal functions such as Buche Rastrigin. In Buche Rastrigin_25, MIGRANT(K) needs 5893 function calls with a success rate of 0.90, in contrast to MIGRANT(U)'s 12,818 calls (0.03). RANDOM(K) performs similarly to MIGRANT(K) in success rate but requires more evaluations (12,035), while RANDOM(U) is by far the least efficient (28,865 calls). The difference becomes dramatic in higher dimensions: in Buche Rastrigin_150, MIGRANT(K) performs the task in 23,466 calls (0.27), whereas RANDOM(U) escalates to 90,211, showing the instability of uniform sampling in complex landscapes. Although differences here are smaller due to the problem's structure, MIGRANT(K) preserves its advantage in stability. The superiority of MIGRANT(K) is again evident in the Step Ellipsoidal group. In Step Ellipsoidal_25, MIGRANT(K) requires only 5104 calls, remarkably lower than RANDOM(U) (6699) and RANDOM(K) (6026) and still better than MIGRANT(U) (5014, but with lower success). Finally, for the Zakharov functions, MIGRANT(K) again shows the best balance between evaluation cost and success rate. In Zakharov_25, MIGRANT(K) requires only 2185 evaluations, beating MIGRANT(U) (2283), RANDOM(K) (2639) and RANDOM(U) (2797).

Table 5. Experiments on the performance of DE using sampling methods.

FUNCTION	MIGRANT(K)	MIGRANT(U)	RANDOM(K)	RANDOM(U)
ATTRACTIVE SECTOR_25	1697	1738	1756	1792
ATTRACTIVE SECTOR_50	1761	1792	1828	1832
ATTRACTIVE SECTOR_100	1832	1866	1880	1891
ATTRACTIVE SECTOR_150	1867	1890	1920	1920
BUCHÉ RASTRIGIN_25	5893 (0.90)	12,818 (0.03)	12,035 (0.90)	28,865 (0.03)
BUCHÉ RASTRIGIN_50	12,585 (0.50)	23,622 (0.03)	20,457 (0.50)	34,379 (0.03)
BUCHÉ RASTRIGIN_100	16,490 (0.53)	41,526 (0.03)	31,465 (0.53)	70,319 (0.03)
BUCHÉ RASTRIGIN_150	23,466 (0.27)	55,612 (0.03)	39,263 (0.27)	90,211 (0.03)
DIFFERENT POWERS_25	1992	2016	1896	1936
DIFFERENT POWERS_50	2060	2077	1971	1989
DIFFERENT POWERS_100	2104	2114	1989	2026
DIFFERENT POWERS_150	2144	2150	2040	2058
DISCUS_25	6478	7368	11,629	11,484

Table 5. Cont.

FUNCTION	MIGRANT(K)	MIGRANT(U)	RANDOM(K)	RANDOM(U)
DISCUS_50	11,183	11,666	15,179	15,789
DISCUS_100	16,225	17,566	20,659	21,459
DISCUS_150	21,495	22,526	24,670	24,485
ELLIPSOIDAL_25	3590	3640	3958	3873
ELLIPSOIDAL_50	6424	6399	7184	7022
ELLIPSOIDAL_100	11,549	12,161	13,890	13,610
ELLIPSOIDAL_150	16,930	17,905	19,940	19,576
LLAGHER21_25	2261 (0.90)	6920 (0.03)	6364 (0.90)	34,112 (0.03)
LLAGHER21_50	4503 (0.50)	7904 (0.03)	9643 (0.50)	17,404 (0.03)
GALLAGHER21_100	1756 (0.53)	1463	1521 (0.53)	1524 (0.53)
GALLAGHER21_150	1662 (0.27)	1463	1526 (0.27)	1522 (0.27)
GALLAGHER101_25	2769 (0.90)	6395 (0.03)	5657 (0.90)	27,324 (0.03)
GALLAGHER101_50	4890 (0.50)	8204 (0.03)	7454 (0.50)	17,075 (0.03)
GALLAGHER101_100	5886 (0.53)	10,816 (0.03)	9505 (0.53)	18,232 (0.03)
GALLAGHER101_150	8646 (0.27)	12,129 (0.03)	12,352 (0.27)	17,231 (0.03)
GRIEWANK ROSENBROCK_25	4084	4353	5145	5434
GRIEWANK ROSENBROCK_50	5039	5290	5729	5631
GRIEWANK ROSENBROCK_100	6460	6211	6002	5916
GRIEWANK ROSENBROCK_150	6542	6895	6164	6113
GRIEWANK_25	4466	4818	6939	7697
GRIEWANK_50	5325	7163	9255	11,056
GRIEWANK_100	6465	9992	11,001	15,311
GRIEWANK_150	7272	12,350	12,543	19,125
RARSTIGIN_25	5950	5909 (0.03)	7955	8447 (0.03)
RARSTIGIN_50	8963	10,112 (0.03)	13,057	13,669 (0.03)
RARSTIGIN_100	15,930	16,541 (0.03)	22,348	23,760 (0.03)
RARSTIGIN_150	22,135	23,181 (0.03)	31562	33,005 (0.03)
ROSENBROCK_25	4577 (0.90)	9432	10,242 (0.90)	18,663
ROSENBROCK_50	7746 (0.50)	11,863	12,740 (0.50)	27,806
ROSENBROCK_100	9147 (0.53)	15,307	13,184 (0.53)	28,064
ROSENBROCK_150	11,620 (0.27)	18,904	15,602 (0.27)	43,292
SHARP RIDGE_25	1481	1498	1512	1528

Table 5. Cont.

FUNCTION	MIGRANT(K)	MIGRANT(U)	RANDOM(K)	RANDOM(U)
SHARP RIDGE_50	1509	1516	1539	1548
SHARP RIDGE_100	1524	1531	1556	1559
SHARP RIDGE_150	1535	1548	1567	1565
SPHERE_25	1625 (0.90)	2733	2090 (0.90)	7103
SPHERE_50	2300 (0.50)	3173	4021 (0.50)	6384
SPHERE_100	2465 (0.53)	3654	1571 (0.53)	5873
SPHERE_150	3143 (0.27)	4073	1521 (0.27)	5149
STEP ELLIPSOIDAL_25	5104	5014 (0.03)	6026	6699 (0.03)
STEP ELLIPSOIDAL_50	5226	5581 (0.03)	7123	7205 (0.03)
STEP ELLIPSOIDAL_100	5995	6091 (0.03)	7649	7893 (0.03)
STEP ELLIPSOIDAL_150	6481	5996 (0.03)	8237	8037 (0.03)
ZAKHAROV_25	2185	2283	2639	2797
ZAKHAROV_50	3027	2901	3864	3743
ZAKHAROV_100	5572	4122	5634	5936
ZAKHAROV_150	6304	5282	7553	7460
	387,335 (0.85)	529,063 (0.71)	543,201 (0.85)	844,408 (0.69)

Overall, integrating k-means sampling into the MIGRANT strategy leads to substantial improvements in both efficiency and reliability. MIGRANT(K) not only requires the fewest total function evaluations but also maintains high success rates across diverse problem categories, making it the most effective approach for the benchmark set. In contrast, RANDOM(U) repeatedly demonstrates the lowest efficiency, highlighting the advantage of structured sampling over uniform dispersion in high-dimensional optimization.

Figure 4 presents the pairwise *t*-test statistical comparison of the four strategies, MIGRANT(K), RANDOM(K), MIGRANT(U) and RANDOM(U), based on their distributions of function calls. The Kruskal–Wallis test indicates a statistically significant overall difference among the groups ($p = 0.034$), suggesting that at least one strategy differs from the others in terms of computational cost. To further examine these differences, pairwise *t*-tests were conducted, with *p*-values annotated using conventional significance notation (ns: $p > 0.05$; *: $p < 0.05$; **: $p < 0.01$; ***: $p < 0.001$; ****: $p < 0.0001$). The pairwise comparisons indicate that MIGRANT(K) differs significantly from RANDOM(K) and from MIGRANT(U), while no statistically significant difference is observed between MIGRANT(U) and RANDOM(U). Comparisons labeled as “ns” suggest statistically indistinguishable behavior between the corresponding strategy pairs. Overall, the distributions of function calls show lower evaluation counts for MIGRANT(K) relative to some of the other strategies under the examined conditions. The U-type variants exhibit similar behavior to that of the RANDOM(U) baseline. These results describe differences in computational cost among the considered strategies, without implying uniform or dominant superiority across all comparisons.

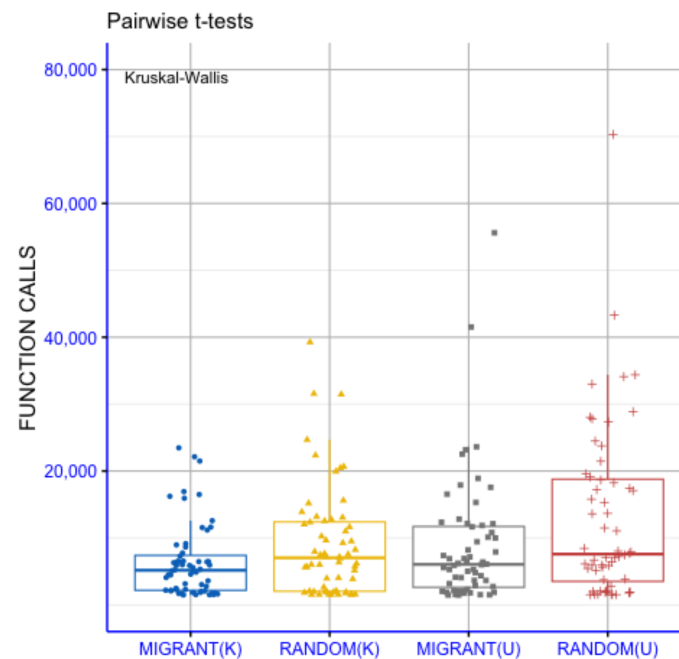


Figure 4. Statistical comparison of different sampling method combinations in DE performance.

3.6. The Effect of Local Search Rate

From Table 6 we observe the influence of periodic local optimization on the performance of the MIGRANT method, considering four different local search rates: 0.005, 0.01, 0.03 and 0.05. Among all settings, the 0.005 rate achieves the lowest total number of function calls (148,027) while maintaining a high success rate of 0.85, thus providing the best balance between computational efficiency and optimization reliability. This advantage is consistently reflected across the benchmark functions. In the Attractive Sector functions (25–150 dimensions), the 0.005 rate clearly outperforms the higher-rate configurations. For instance, in Attractive Sector_25, it requires only 1441 calls, compared to 1603 for the 0.03 rate and 1697 for the 0.05 rate. The improvement persists as the dimensionality increases: Attractive Sector_150 is solved with 1536 calls at the 0.005 rate, while the 0.05 rate requires 1867 calls. The improvement becomes dramatically more pronounced in the Buche Rastrigin family, where the complexity and multimodality amplify the benefit of lower local search frequency. For Buche Rastrigin_25, the 0.005 method requires 2035 function calls (0.90 success), whereas the 0.05 rate jumps to 5893 calls nearly triple. In the high-dimensional case, Buche Rastrigin_150, the difference is even more striking: 5900 calls at the 0.005 rate versus 23,466 for the 0.05 rate. A similar trend can be seen in the Discus, Sharp Ridge and Step Ellipsoidal functions. In Step Ellipsoidal_50, the 0.005 rate achieves 2136 calls (0.50), far below the 0.05 rate (2300). For Step Ellipsoidal_150, the 0.005 variant uses 2914 calls (0.27), while the 0.05 rate needs 3143 calls. Even in unimodal functions like Discus, the 0.005 method consistently leads to lower evaluation costs, e.g., Discus_25 requires 1525 calls vs. 1992 for the 0.05 rate. The Sharp Ridge functions highlight this behavior even more strongly. For Sharp Ridge_25, the 0.005 rate requires only 1934 calls, in contrast to 5104 calls for the 0.05 rate, more than a 2.5× increase. Similar improvements appear in Sharp Ridge_150, where the function calls rise from 2350 at 0.005 to 6481 at 0.05.

In summary, using a lower local search rate specifically the 0.005 setting results in the most efficient optimization behavior across all tested functions. This variant provides the lowest objective function calls without compromising success rate, making it the optimal choice when both efficiency and reliability are essential in high-dimensional optimization tasks.

Table 6. Experiments on the effect of local search rate on optimization performance in DE.

FUNCTION	MIGRANT (0.005)	MIGRANT (0.01)	MIGRANT (0.03)	MIGRANT (0.05)
ATTRACTIVE SECTOR_25	1441	1472	1603	1697
ATTRACTIVE SECTOR_50	1582	1522	1674	1761
ATTRACTIVE SECTOR_100	1516	1552	1699	1832
ATTRACTIVE SECTOR_150	1536	1560	1726	1867
BUCHER RASTRIGIN_25	2035 (0.90)	2502 (0.90)	4323 (0.90)	5893 (0.90)
BUCHER RASTRIGIN_50	3468 (0.50)	4319 (0.50)	8496 (0.50)	12,585 (0.50)
BUCHER RASTRIGIN_100	4179 (0.53)	5700 (0.53)	10,756 (0.53)	16,490 (0.53)
BUCHER RASTRIGIN_150	5900 (0.27)	7794 (0.27)	14,818 (0.27)	23,466 (0.27)
DISCUS_25	1525	1616	1841	1992
DISCUS_50	1615	1658	1919	2060
DISCUS_100	1578	1655	1979	2104
DISCUS_150	1590	1663	1987	2144
DIFFERENTPOWERS_25	2296	2855	4661	6478
DIFFERENTPOWERS_50	3011	3807	7580	11,183
DIFFERENTPOWERS_100	3827	5693	11,214	16,225
DIFFERENTPOWERS_150	4736	7158	15,238	21,495
ELLIPSOIDAL_25	1765	2011	2940	3590
ELLIPSOIDAL_50	2235	2844	4854	6424
ELLIPSOIDAL_100	3234	4557	9215	11,549
ELLIPSOIDAL_150	4581	6620	12,510	16,930
GALLAGHER21_25	1751 (0.90)	1804 (0.90)	2049 (0.90)	2261 (0.90)
GALLAGHER21_50	2842 (0.50)	3012 (0.50)	3765 (0.50)	4503 (0.50)
GALLAGHER21_100	1432 (0.53)	1470 (0.53)	1609 (0.53)	1756 (0.53)
GALLAGHER21_150	1434 (0.27)	1455 (0.27)	1554 (0.27)	1662 (0.27)
GALLAGHER101_25	1778 (0.90)	1896 (0.90)	2359 (0.90)	2769 (0.90)
GALLAGHER101_50	3287 (0.50)	3470 (0.50)	4186 (0.50)	4890 (0.50)
GALLAGHER101_100	3550 (0.53)	3804 (0.53)	4851 (0.53)	5886 (0.53)
GALLAGHER101_150	4726 (0.27)	5208 (0.27)	6959 (0.27)	8646 (0.27)
GRIEWANK_25	1858	2102	3137	4084
GRIEWANK_50	2154	2407	3859	5039
GRIEWANK_100	2135	2688	4542	6460
GRIEWANK_150	2298	2937	4919	6542
GRIEWANK_ROSENBROCK_25	1840	2116	3292	4466
GRIEWANK_ROSENBROCK_50	2091	2661	4199	5325
GRIEWANK_ROSENBROCK_100	2343	2969	4868	6465
GRIEWANK_ROSENBROCK_150	2512	3295	5496	7272

Table 6. Cont.

FUNCTION	MIGRANT (0.005)	MIGRANT (0.01)	MIGRANT (0.03)	MIGRANT (0.05)
ROSENBROCK_25	1964	2450	4091	5950
ROSENBROCK_50	2675	3619	6578	8963
ROSENBROCK_100	3616	5278	10,570	15,930
ROSENBROCK_150	5326	7819	15,572	22,135
RARSTIGIN_25	1831 (0.90)	2135 (0.90)	3346 (0.90)	4577 (0.90)
RARSTIGIN_50	2858 (0.50)	3334 (0.50)	5664 (0.50)	7746 (0.50)
RARSTIGIN_100	2941 (0.53)	3697 (0.53)	6336 (0.53)	9147 (0.53)
RARSTIGIN_150	3997 (0.27)	4826 (0.27)	8275 (0.27)	11,620 (0.27)
SPHERE_25	1402	1411	1455	1481
SPHERE_50	1537	1444	1475	1509
SPHERE_100	1463	1454	1489	1524
SPHERE_150	1481	1469	1494	1535
STEP ELLIPSOIDAL_25	1513 (0.90)	1526 (0.90)	1576 (0.90)	1625 (0.90)
STEP ELLIPSOIDAL_50	2136 (0.50)	2155 (0.50)	2229 (0.50)	2300 (0.50)
STEP ELLIPSOIDAL_100	2286 (0.53)	2308 (0.53)	2389 (0.53)	2465 (0.53)
STEP ELLIPSOIDAL_150	2914 (0.27)	2938 (0.27)	3040 (0.27)	3143 (0.27)
SHARP RIDGE_25	1934	2269	3453	5104
SHARP RIDGE_50	2130	2680	4042	5226
SHARP RIDGE_100	2190	2718	4489	5995
SHARP RIDGE_150	2350	3107	5131	6481
ZAKHAROV_25	1570	1635	1912	2185
ZAKHAROV_50	1714	1884	2505	3027
ZAKHAROV_100	2428	2677	4597	5572
ZAKHAROV_150	2090	2314	4721	6304
	148,027 (0.85)	178,999 (0.85)	289,106 (0.85)	387,335 (0.85)

Figure 5 presents pairwise statistical comparisons among the four parameter configurations (0.001, 0.01, 0.03 and 0.05) based on their distributions of function evaluations. The global Kruskal–Wallis test indicates a statistically significant overall difference among the groups ($p = 2.9 \times 10^{-8}$), suggesting that the choice of parameter value is associated with differences in computational cost. Pairwise comparisons were conducted using independent t -tests, with p -values interpreted according to conventional significance notation (ns: $p > 0.05$; *: $p < 0.05$; **: $p < 0.01$; ***: $p < 0.001$; ****: $p < 0.0001$). As shown in the figure, a substantial number of pairwise comparisons reach statistically significant levels, while a smaller subset does not exhibit statistically significant differences. This indicates that, for many comparisons, the examined parameter configurations are associated with distinguishable behavior in terms of the number of function calls. Overall, the results indicate that different parameter settings correspond to varying distributions of function evaluations under the considered experimental conditions. The observed differences suggest a systematic influence of the parameter choice on optimizer behavior, without implying uniform or absolute superiority of a single configuration across all comparisons.

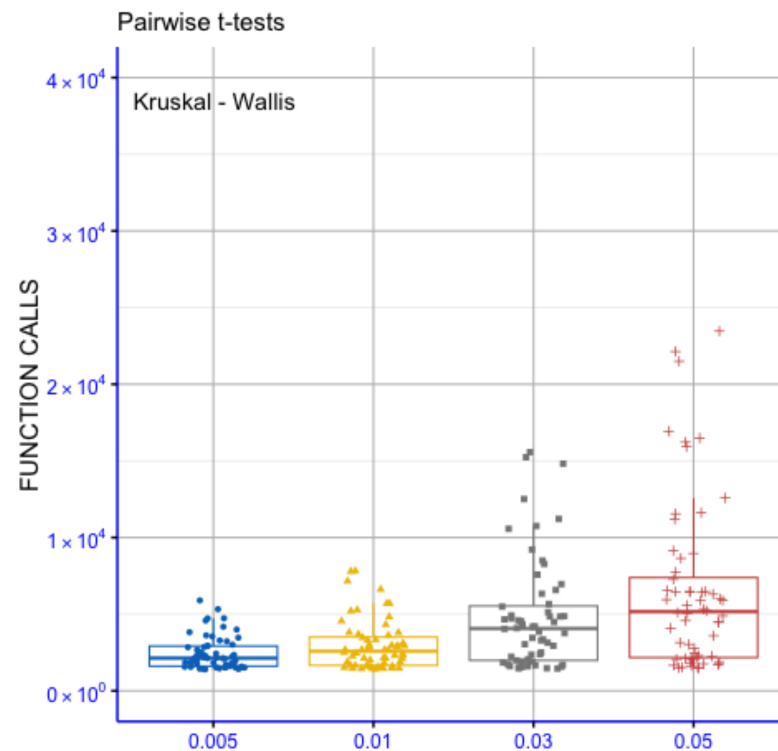


Figure 5. Statistical comparison for the proposed method and different values of the parameter p_l .

3.7. Parameter Sensitivity Analysis

To assess the robustness of the proposed method with respect to its main control parameters, a sensitivity analysis was conducted, focusing on the local search rate p_l , the population size NP and the tournament size N_t . Each parameter was varied independently, while all remaining parameters were fixed to their default values. For each configuration, the algorithm was executed multiple times under identical experimental conditions, and the average number of function calls was recorded.

Figure 6 illustrates the sensitivity of the number of function calls with respect to the local search rate p_l . The results indicate a gradual increase in computational cost as p_l increases, which can be attributed to the more frequent activation of the local refinement procedure. Nevertheless, the observed variation is smooth, and no abrupt performance degradation is observed for moderate changes in the local search rate.

Figure 7 presents the effect of the population size NP on the number of function calls. As expected, increasing the population size leads to a higher computational cost due to the larger number of individuals evaluated in each iteration. However, the trend remains monotonic and predictable, indicating stable scaling behavior rather than sensitivity to a particular population size.

Figure 8 shows the sensitivity of the algorithm to the tournament size N_t when tournament-based parent selection is employed. The results demonstrate that moderate values of N_t yield comparable performance, while larger tournament sizes slightly reduce the number of function calls by increasing selection pressure. Importantly, the performance differences across the tested range remain limited, suggesting that the algorithm is not highly sensitive to the precise choice of N_t .

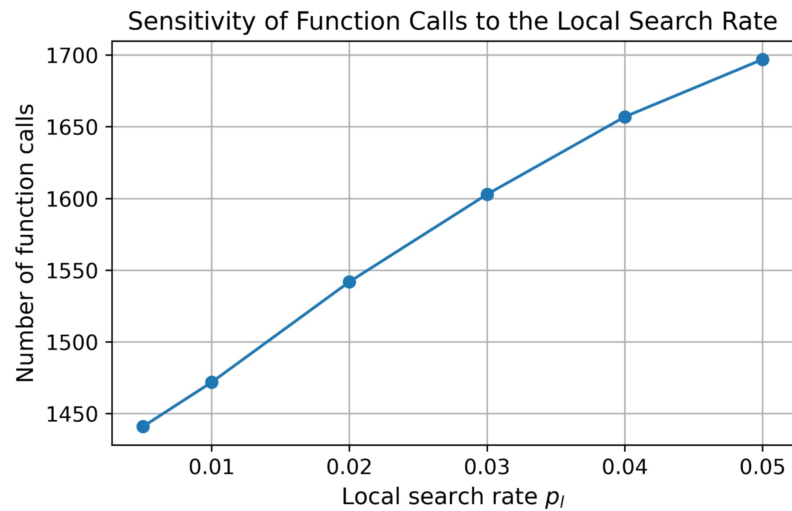


Figure 6. Sensitivity of function calls to the local search.

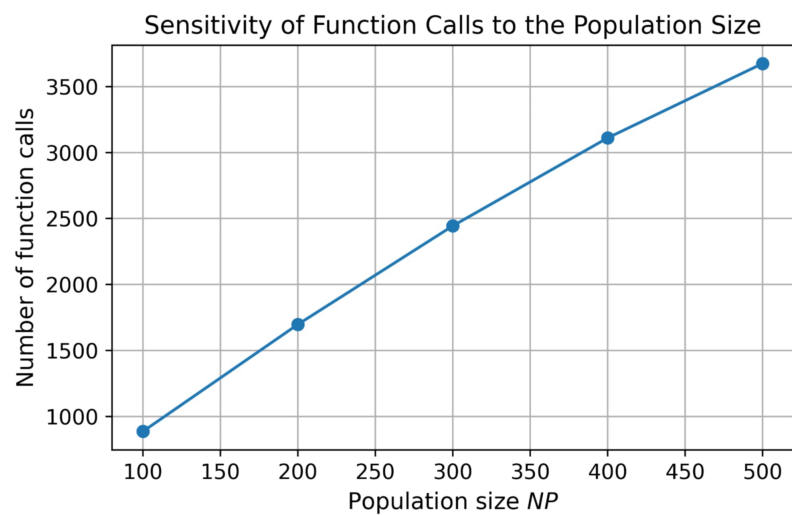


Figure 7. Sensitivity of function calls to the population size.

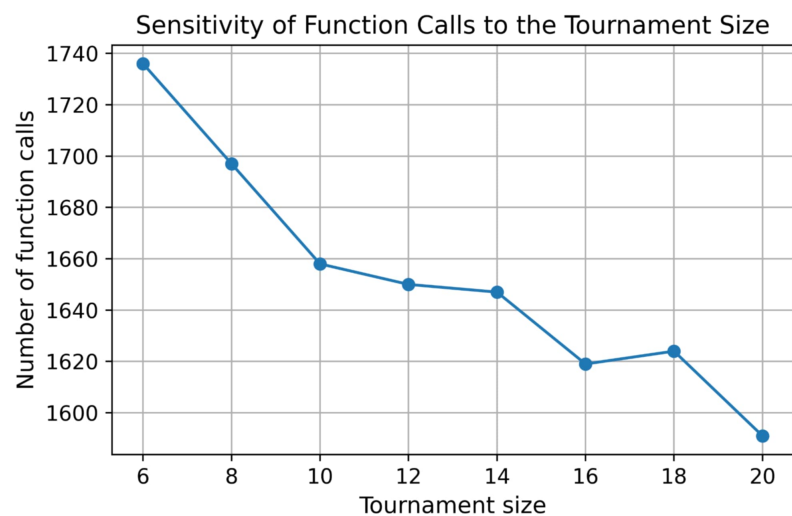


Figure 8. Sensitivity of function calls to the tournament size.

Overall, the sensitivity analysis indicates that the proposed method exhibits stable behavior across a broad range of parameter values. Performance variations are gradual and mainly occur at extreme settings, suggesting that the method can be transferred to different problem instances without requiring extensive parameter tuning. Regarding the termination limits, no sensitivity analysis was performed, as the same termination criterion was applied uniformly to all experiments and all reference functions.

3.8. The Proposed Method in Comparison with Others

Table 7 presents the results of a comparative analysis of various optimization methods (BICCA [48], MLSHADESPA [49], SHADE_ILS [50], Differential Evolution (DE) [12], Genetic Algorithm (GA) [13], Whale Optimization Algorithm (WOA) [51,52], Particle Swarm Optimization (IPSO) [18], proposed) across a wide range of test functions with dimensions of 25, 50, 100 and 150. Each row corresponds to a test function, while the columns represent the methods. The numerical values in each cell indicate the number of objective function calls required to find the minimum, while the values in parentheses show the success rate of each method in each case. In the last row (TOTAL), the total sum of function calls for each method is displayed, along with the average success rate. The best methods should simultaneously exhibit a low number of function calls (efficiency) and a high success rate (reliability). The analysis shows that the proposed method delivers strong and consistent performance. Its overall success rate (0.85) is comparable to those of the GA, MLSHADESPA, SHADE_ILS, IPSO, DE and the WOA (around 0.90) and distinctly higher than that of the BICCA method (0.73). This indicates that the proposed method remains dependable in locating the global minimum even when faced with complex or high-dimensional search spaces. In terms of computational cost, the proposed method requires a total of 387,335 objective function evaluations, which is substantially lower than the total required by most competing techniques. This advantage appears consistently across the majority of tested functions. For example, in the DifferentPowers function, the proposed method significantly outperforms the GA across all dimensionalities: at 25 dimensions, it uses 6478 evaluations compared to 14,495 for GA; at 100 dimensions, 16,225 versus 28,413 are used; and at 150 dimensions, 21,495 versus 33,569 are used. Similar observations are made for the GriewankRosenbrock function, where the proposed method demonstrates clear efficiency benefits: at 100 dimensions it requires 6465 evaluations whereas BICCA needs 20,462, and at 150 dimensions the gap widens further with 7272 evaluations compared to 30,604 for BICCA. These differences illustrate the method's robustness and its ability to maintain low computational demands in highly nonlinear and difficult optimization landscapes.

In summary, the findings suggest that the proposed method offers a strong balance between reliability and computational efficiency. It competes effectively with and, in many cases, surpasses widely used optimization algorithms, while maintaining a consistently lower number of objective function evaluations. Its stability across different functions and dimensions confirms its applicability to a broad range of optimization scenarios, making it a promising and efficient alternative within the field of evolutionary and metaheuristic optimization.

Figure 9 illustrates the distributions of function evaluations for the proposed optimizer and the baseline algorithms. The Kruskal–Wallis test indicates a statistically significant overall difference among the methods ($p < 2.2 \times 10^{-16}$), suggesting that at least one optimizer differs from the others with respect to the distribution of function call counts. As shown in the figure, the proposed method is associated with lower median values and reduced dispersion in the number of function evaluations compared with the baseline algorithms. These differences in the distributions indicate that the proposed optimizer exhibits distinct computational behavior under the examined experimental conditions. Overall, the statis-

tical analysis and the observed distributions suggest that the proposed method requires fewer function evaluations relative to the considered baselines. The results describe systematic differences in computational cost among the optimizers, without attributing the observed behavior to a single dominant factor.

Table 7. Experimental results using different optimization methods. Numbers in cells represent sum function calls.

FUNCTION	BICCA	MLSHADESPA	SHADE_ILS	DE	GA	WOA	IPSO	PROPOSED
ATTRACTIVE_SECTOR_25	5130	950	452	4439	2208	2641	2120	1697
ATTRACTIVE_SECTOR_50	10,097	994	558	18,104	2230	5700	2167	1761
ATTRACTIVE_SECTOR_100	20,178	989	748	15,246	2231	5785	2179	1832
ATTRACTIVE_SECTOR_150	30,259	1047	959	6646	2232	9248	2196	1867
BUCHERASTRIGIN_25	5144 (0.33)	9420 (0.90)	2093 (0.90)	1466 (0.90)	12,979 (0.90)	15,048 (0.93)	12,115 (0.90)	5893 (0.90)
BUCHERASTRIGIN_50	10,345 (0.03)	18,003 (0.50)	3440 (0.50)	1894 (0.50)	20,711 (0.50)	58,557 (0.77)	30,866 (0.50)	12,585 (0.50)
BUCHERASTRIGIN_100	20,676 (0.03)	30,652 (0.53)	5428 (0.53)	2020 (0.53)	29,121 (0.53)	43,001 (0.97)	39,680 (0.53)	16,490 (0.53)
BUCHERASTRIGIN_150	30,894 (0.03)	47,160 (0.27)	7663 (0.27)	2511 (0.27)	37,696 (0.27)	54,641	53,060 (0.27)	23,466 (0.27)
DISCUS_25	5125	1365	536	4255	2656	3006	2452	1992
DISCUS_50	10,101	1425	642	10,297	2663	6310	2498	2060
DISCUS_100	20,189	1402	826	8284	2631	5835	2523	2104
DISCUS_150	30,265	1487	1042	8548	2620	8227	2548	2144
DIFFERENTPOWERS_25	5144	13,007	2644	4786	14,495	14,921	13,313	6478
DIFFERENTPOWERS_50	10,389	20,029	3860	14,391	20,539	35,828	19,839	11,183
DIFFERENTPOWERS_100	20,644	27,859	5450	7355	28,413	52,081	28,379	16,225
DIFFERENTPOWERS_150	30,877	36,894	7059	6266	33,569	93,074	36,287	21,495
ELLIPSOIDAL_25	5139 (0.87)	4227	1117	4161	5955	7299	6375	3590
ELLIPSOIDAL_50	10,247	9146	2178	16,624	10,892	19,281	11,641	6424
ELLIPSOIDAL_100	20,492	18,062	3966	12,708	20,202	38,501	20,736	11,549
ELLIPSOIDAL_150	30,708	26,835	5993	21,936	36,236	63,093	29,414	16,930
GALLAGHER21_25	5122 (0.46)	1304 (0.90)	503 (0.90)	4180 (0.90)	3346 (0.90)	9210 (0.90)	3605 (0.90)	2261 (0.90)
GALLAGHER21_50	10,119 (0.03)	1757 (0.50)	701 (0.50)	7938 (0.50)	3192 (0.50)	35,580 (0.50)	8866 (0.50)	4503 (0.50)
GALLAGHER21_100	20,167	392 (0.53)	637 (0.53)	1323 (0.53)	1593 (0.53)	1950 (0.53)	5363 (0.53)	1756 (0.53)
GALLAGHER21_150	30,248	385 (0.27)	825 (0.27)	1313 (0.27)	1582 (0.27)	1738 (0.27)	2050 (0.27)	1662 (0.27)
GALLAGHER101_25	5117 (0.07)	1270	501 (0.90)	3625 (0.90)	3340 (0.90)	7664 (0.90)	3473 (0.90)	2769 (0.90)
GALLAGHER101_50	10,114 (0.03)	1396	634 (0.50)	18,470 (0.50)	7134 (0.50)	38,817 (0.50)	8796 (0.50)	4890 (0.50)
GALLAGHER101_100	20,193 (0.03)	1868	901 (0.53)	14,700 (0.53)	5794 (0.53)	39,700 (0.53)	9257 (0.53)	5886 (0.53)
GALLAGHER101_150	30,269 (0.03)	1922	1127 (0.27)	24,214 (0.27)	7210 (0.27)	36,525 (0.27)	14076 (0.27)	8646 (0.27)
GRIEWANK_25	5173 (0.70)	7828	1811	4123 (0.97)	9733	10,166	9454	4084
GRIEWANK_50	10,138	3434	1061	17,524 (0.93)	5410	18,966	9827	5039
GRIEWANK_100	20,208	2825	1124	14,809	4982	19,318	10,369	6460
GRIEWANK_150	30,290	3035	1391	6335 (0.97)	5221	28,823	10,741	6542
GRIEWANK_ROSENBROCK_25	5180	14,086	3132	3238	17,038	10,630	9698	4466
GRIEWANK_ROSENBROCK_50	10,362	20,021	4319	16,379	23,217	22,912	11,610	5325
GRIEWANK_ROSENBROCK_100	20,462	23,913	4925	11,375	31,195	24,543	13,409	6465
GRIEWANK_ROSENBROCK_150	30,604	29,813	6080	4446	37,364	33,948	15,075	7272
ROSENBROCK_25	5163	12,518	2793	3543	15,493	13,642	13,642	5950
ROSENBROCK_50	10,451	21,195	4555	12,085	24,602	33,038	22,317	8963
ROSENBROCK_100	20,785	35,136	7151	6038	39,496	48,451	36,400	15,930
ROSENBROCK_150	31,103	50,850	10,669	4203	53,211	75,425	50,281	22,135

Table 7. Cont.

FUNCTION	BICCA	MLSHADESPA	SHADE_ILS	DE	GA	WOA	IPSO	PROPOSED
RARSTIGIN_25	5139 (0.36)	7826 (0.90)	1767 (0.90)	1574 (0.90)	9581 (0.90)	15,530 (0.90)	9826 (0.90)	4577 (0.90)
RARSTIGIN_50	10,208 (0.03)	10,741 (0.50)	2091 (0.50)	1895 (0.50)	12,272 (0.50)	41,187 (0.73)	17,354 (0.50)	7746 (0.50)
RARSTIGIN_100	20,358 (0.03)	11,464 (0.53)	2338 (0.53)	1869 (0.53)	2134 (0.53)	27,383 (0.90)	19,347 (0.53)	9147 (0.53)
RARSTIGIN_150	30,561 (0.03)	14,002 (0.27)	2942 (0.27)	2122 (0.27)	13,990 (0.27)	32,297 (0.93)	27,682 (0.27)	11,620 (0.27)
SPHERE_25	5134	482	358	4131	1689	2206	1611	1481
SPHERE_50	10,088	500	459	18,098	1700	5111	1633	1509
SPHERE_100	20,169	498	655	15,241	1699	5107	1639	1524
SPHERE_150	30,250	523	858	6639	1700	7347	1645	1535
STEP ELLIPSOIDAL_25	5114 (0.70)	375 (0.90)	313 (0.90)	1857 (0.93)	2069 (0.90)	1812 (0.97)	1846 (0.90)	1625 (0.90)
STEP ELLIPSOIDAL_50	10,086 (0.03)	375 (0.50)	391 (0.50)	6493 (0.67)	2469 (0.50)	2541 (0.50)	3993 (0.50)	2300 (0.50)
STEP ELLIPSOIDAL_100	20,167 (0.03)	377 (0.53)	541 (0.53)	5658 (0.53)	1681 (0.53)	2405 (0.53)	3946 (0.53)	2465 (0.53)
STEP ELLIPSOIDAL_150	30,248 (0.03)	383 (0.27)	695 (0.27)	5588 (0.27)	1673 (0.27)	2854 (0.27)	5065 (0.27)	3143 (0.27)
SHARP RIDGE_25	5125	9281	2193	5153	11,536	11,398	11,371	5104
SHARP RIDGE_50	10,261	9843	2284	18,677	11,818	19,405	12,550	5226
SHARP RIDGE_100	20,366	10,190	2403	15,159	11,659	20,507	13,017	5995
SHARP RIDGE_150	30,458	11,205	2885	7476 (0.97)	11,866	25,983	13,776	6481
ZAKHAROV_25	5120	4383	1177	1735	5756	9556	3449	2185
ZAKHAROV_50	10,118	18,043	3584	2371	15,522	23,884	6469	3027
ZAKHAROV_100	20,211	45,770	8470	2216	38,359	29,581	16,562	5572
ZAKHAROV_150	30,293	46,497	9315	2503	36,399	32,379	21,273	6304
	992,785 (0.73)	708,659 (0.85)	157,213 (0.85)	478,253 (0.85)	786,004 (0.85)	1,371,596 (0.90)	782,751 (0.85)	387,335 (0.85)

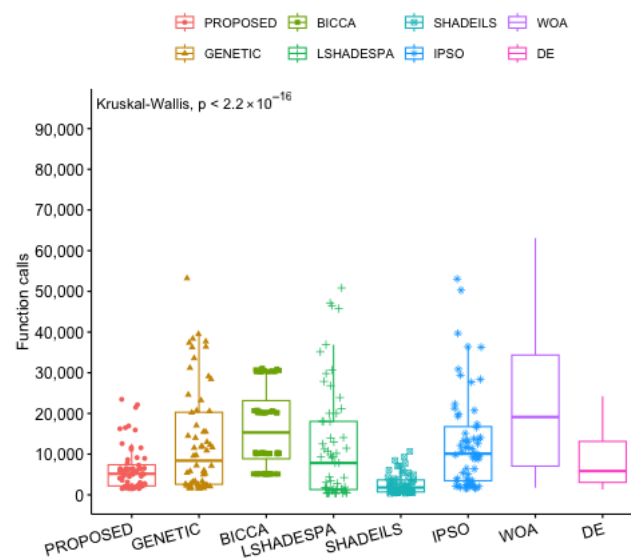


Figure 9. A statistical comparison of the proposed with other optimization methods.

To further investigate the scalability of the proposed optimization framework, additional experiments were conducted in very high dimensions and more specifically for dimensions of 200, 300, 600 and 1100.

The results of these additional experiments are reported in Table 8. As the dimension increases, the proposed method is associated with consistently lower numbers of objective function evaluations compared with the baseline algorithms, indicating favorable scalability characteristics in very-high-dimensional optimization scenarios.

Table 8. Experimental results using different very high-dimensional optimization methods.

FUNCTION	WOA	BICCA	SHADE_ILS	MLSHADESPA	DE	PROPOSED	GA	IPSO
ELLIPSOIDAL_200	49,353	40,915	9635	35,675	27,985	22,123	31,875	30,853
ELLIPSOIDAL_300	51,152	61,333	9903	43,473	43,569	39,328	52,414	53,185
ELLIPSOIDAL_600	80,978	122,625	21,797	85,276	73,034	70,020	78,370	81,032
ELLIPSOIDAL_1100	124,378	223,787	25,607	123,451	84,912	80,815	114,632	113,982

In addition to the success rates, Tables 9 and 10 present the mean values and standard deviations of the objective function across 30 independent runs, in order to provide a more informative comparison between the methods, especially in cases where the success rates are the same or lower.

Table 9. Experimental results using mean and standard deviation for methods WOA, BICCA, SHADE_ILS and MLSHADESPA.

FUNCTION	WOA	BICCA	SHADE_ILS	MLSHADESPA
RASTRIGIN_25	8.42 (26.07)	45.36 (44.73)	8.55 (26.28)	6.26 (19.39)
RASTRIGIN_50	27.79 (47.44)	86.06 (23.96)	52.16 (54.44)	44.27 (45.56)
RASTRIGIN_100	19.07 (58.51)	258.95 (63.67)	70.24 (80.80)	51.24 (55.91)
RASTRIGIN_150	12.17 (46.32)	312.08 (68.05)	128.81 (90.00)	102.81 (64.24)
STEPELLIPSOIDAL_25	1.18 (6.48)	2107.69 (2049.63)	387.30 (1193.21)	387.30 (1193.21)
STEPELLIPSOIDAL_50	0 (0)	6975.21 (1772.99)	3876.67 (4034.57)	3876.67 (4034.57)
STEPELLIPSOIDAL_100	0 (0)	10,197.95 (1242.43)	5232.05 (5786.36)	5232.05 (5786.36)
STEPELLIPSOIDAL_150	0 (0)	9769.58 (1069.59)	10,796.25 (6776.41)	10,796.25 (6776.41)

Table 10. Experimental results using mean and standard deviation.

FUNCTION	DE	PROPOSED	GA	IPSO
RASTRIGIN_25	1.49 (4.60)	2.88 (8.8)	5.33 (16.87)	0.36 (1.34)
RASTRIGIN_50	23.71 (24.60)	19.96 (21.33)	51.30 (52.68)	8.42 (0.09)
RASTRIGIN_100	55.12 (61.30)	28.92 (32.80)	62.51 (68.96)	15.02 (16.65)
RASTRIGIN_150	132.52 (83.51)	65.10 (41.07)	126.06 (79.14)	35.71 (22.56)
STEPELLIPSOIDAL_25	0 (0)	36.38 (131.59)	143.05 (443.83)	2.39 (7.74)
STEPELLIPSOIDAL_50	0 (0)	625.24 (709.06)	3076.57 (3214.39)	129.48 (159.90)
STEPELLIPSOIDAL_100	0 (0)	1124.16 (1293.24)	5163.19 (5722.02)	379.90 (481.85)
STEPELLIPSOIDAL_150	24.60 (16.03)	2791.42 (1817.21)	10676.31 (6679.24)	1007.36 (777.37)

3.9. Practical Problems

To further examine the practical efficiency and scalability of the proposed optimization algorithm, two real-world engineering design problems were investigated: the GasCycle [53] and the Tandem Queueing System [54]. These problems were selected because they differ significantly in mathematical formulation and computational complexity, providing a comprehensive framework for evaluating the algorithm's performance under diverse and realistic conditions.

Each problem was tested across multiple dimensional configurations, ranging from 25 to 500 variables, in order to assess how the algorithm behaves as the search space becomes more complex. For every configuration, the execution time in seconds was recorded as the

main performance indicator. This experimental setup enables a direct comparison of how computational efficiency changes with increasing dimensionality.

- **GasCycle Thermal Cycle**

Vars: $\mathbf{x} = [T_1, T_3, P_1, P_3]^\top$. $r = P_3/P_1$, $\gamma = 1.4$.

$$\eta(\mathbf{x}) = 1 - r^{-(\gamma-1)/\gamma} \frac{T_1}{T_3}, \quad \min_{\mathbf{x}} f(\mathbf{x}) = -\eta(\mathbf{x}).$$

Bounds: $300 \leq T_1 \leq 1500$, $1200 \leq T_3 \leq 2000$, $1 \leq P_1, P_3 \leq 20$.

Penalty: infeasible $\Rightarrow f = 10^{20}$.

The GasCycle scenario presents a more computationally demanding optimization problem, allowing a clearer assessment of algorithmic scalability under increased complexity. For GasCycle, as illustrated in Figure 10, the proposed algorithm maintains a stable and competitive number of function calls across all dimensions. Compared to methods that show pronounced growth in evaluations at higher dimensions, the proposed approach exhibits a more controlled increase, indicating effective adaptation to the structure of the GasCycle problem. This behavior suggests that the method can efficiently utilize function evaluations without excessive computational overhead in large-scale cases.

The execution time analysis for GasCycle, shown in Figure 11, aligns closely with the function call results. The proposed algorithm achieves a balanced runtime profile, with execution time increasing smoothly as dimensionality grows. In contrast to approaches that suffer from substantial runtime escalation, the proposed method maintains reasonable computational demands even at high dimensions, highlighting its suitability for complex, large-scale optimization tasks.

- **Tandem Space Trajectory (MGA-1DSM, EEEJ + 2 × Saturn)**

Vars ($D = 18$): $\mathbf{x} = [t_0, T_1, T_2, T_3, T_4, T_{5A}, T_{5B}, s_1, s_2, s_3, s_4, s_{5A}, s_{5B}, r_p, k_{A1}, k_{A2}, k_{B1}, k_{B2}]^\top$.

$$7000 \leq t_0 \leq 10000,$$

$$30 \leq T_1 \leq 500, \quad 30 \leq T_2 \leq 600, \quad 30 \leq T_3 \leq 1200,$$

$$30 \leq T_4 \leq 1600, \quad 30 \leq T_{5A}, T_{5B} \leq 2000,$$

$$0 \leq s_{1..4}, s_{5A}, s_{5B}, r_p, k_{A1}, k_{A2}, k_{B1}, k_{B2} \leq 1.$$

Objective:

$$\min_{\mathbf{x}} \Delta V_{\text{tot}} = \Delta V_{\text{launch}}(T_1) + \Delta V_{\text{legs}}(T_1, T_4) + \Delta V_A + \Delta V_B + \Delta V_{\text{DSM}}(\mathbf{s}, r_p) - G_{\text{GA}} - G_J + P_{\text{hard}} + P_{\text{soft}},$$

$$P_{\text{soft}} = \beta \max\left\{0, (T_1 + \dots + T_4 + \frac{1}{2}(T_{5A} + T_{5B})) - 3500\right\}.$$

Notes: ΔV_{launch} decreases (log-like) in T_1 (≥ 6 km/s floor); leg/branch costs decrease with TOF.

The figures corresponding to the Tandem scenario illustrate the behavior of the evaluated algorithms in terms of function calls and execution time as the problem dimension increases. As expected, higher dimensionality leads to increased computational effort for all methods; however, notable differences in scalability can be observed.

In the Tandem case, as shown in Figure 12, the proposed algorithm demonstrates stable and consistent behavior across all tested dimensions, maintaining a relatively low number of function calls. Its performance remains competitive with the most efficient approaches and is clearly more scalable than methods such as the GA, BICCA and IPSO, which exhibit a rapid increase in function calls as dimensionality grows. The controlled growth observed for the proposed method indicates effective search dynamics and an appropriate balance between exploration and exploitation in large-scale settings.

The execution time results for the Tandem scenario, presented in Figure 13, further confirm these observations. The proposed algorithm shows smooth and predictable scaling with increasing problem dimensions, avoiding the steep runtime growth observed in more computationally demanding methods. Although execution time naturally increases for larger dimensions, the rate of increase remains moderate, suggesting that the internal computational cost of the proposed approach is well managed and suitable for practical large-scale applications in the Tandem scenario. Across both Tandem and GasCycle scenarios, the proposed algorithm demonstrates consistent scalability in terms of both function evaluations and execution time. Its stable behavior under increasing dimensionality indicates that it represents a reliable and efficient alternative for large-scale optimization problems, without incurring excessive computational cost.

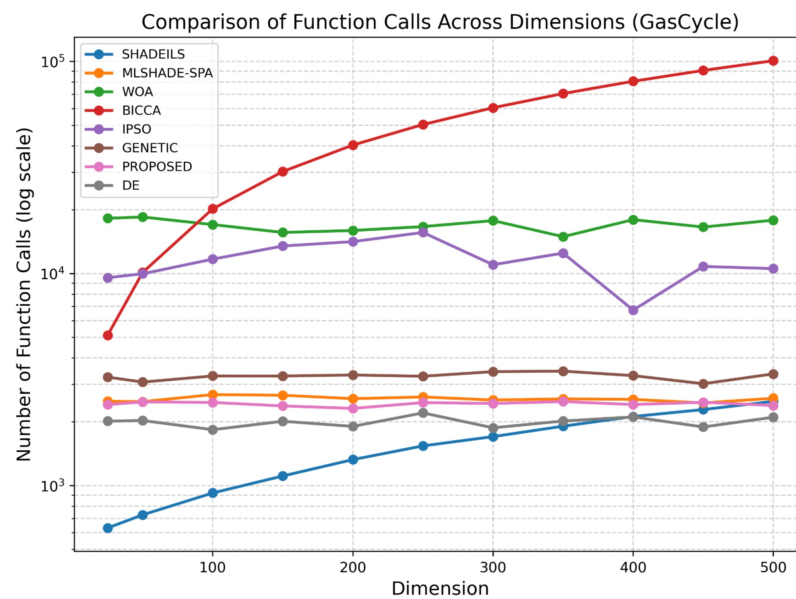


Figure 10. Comparison of function calls across dimensions (GasCycle).



Figure 11. Comparison of execution time across dimensions (GasCycle).

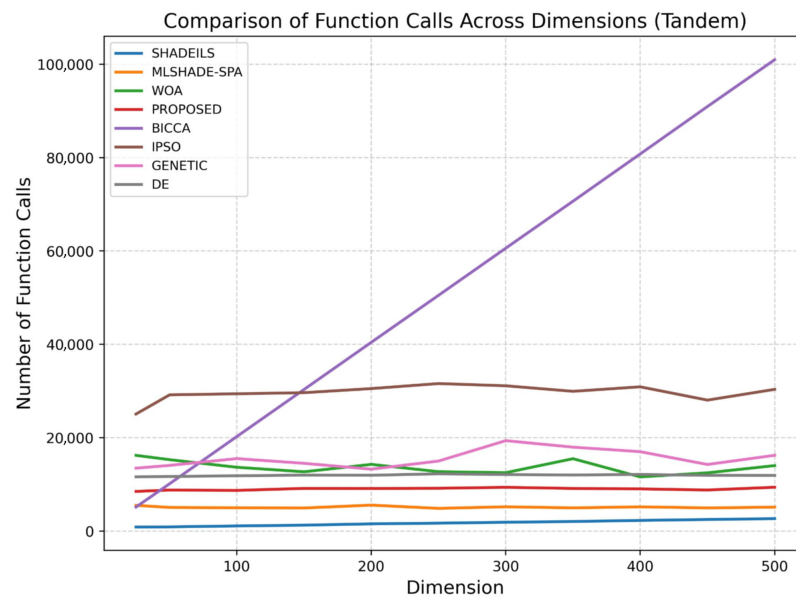


Figure 12. Comparison of function calls across dimensions (Tandem).

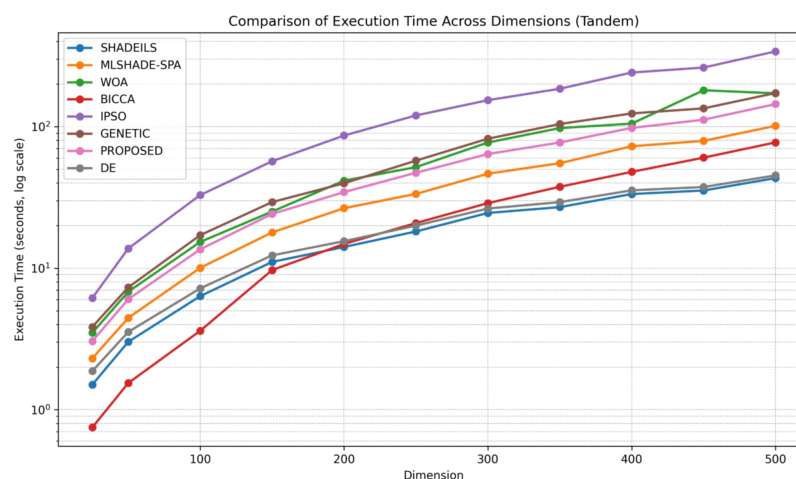


Figure 13. Comparison of execution time across dimensions (Tandem).

4. Discussion

The experimental results provide important insights into how design choices in sampling, parameter adaptation, selection pressure and local refinement collectively shape the behavior of Differential Evolution (DE). Rather than acting independently, these components interact in ways that significantly influence convergence efficiency and stability, a phenomenon already highlighted in foundational studies on DE that emphasize the role of mutation–selection interplay and parameter control in stochastic optimization processes [24,36]. When such interactions are properly structured and coordinated, the algorithm exhibits more reliable convergence behavior and reduced sensitivity to random effects. In large-scale optimization settings, where the dimensionality of the search space increases the complexity of variable interactions and amplifies stochastic disturbances, the importance of these interactions becomes even more pronounced. The results observed in this study suggest that careful coordination of DE components can mitigate instability and uncontrolled performance degradation, extending the principles established in classical DE formulations to more challenging large-scale scenarios.

A key observation concerns the role of sampling strategies. The use of k-means clustering to guide population sampling consistently leads to lower computational cost and reduced variance compared to uniform sampling. This behavior can be attributed to the preservation of population structure, which helps prevent redundant exploration and encourages a more balanced coverage of promising regions of the search space. The effectiveness of k-means in producing compact and representative partitions of the search space has been well established in the clustering literature [45,47]. In the context of evolutionary optimization, our experimental results indicate that leveraging such structured sampling becomes particularly beneficial in large-scale settings, where purely random sampling often leads to inefficient exploration. The tighter distributions observed for configurations employing k-means indicate improved stability, an essential property for large-scale optimization.

The differential weight mechanism further reinforces this structured behavior. Among the examined strategies, the MIGRANT-based approach demonstrates consistently lower function evaluation requirements while maintaining identical success rates compared to static or random schemes. This suggests that exploiting feedback from the evolving population enables the algorithm to adapt its step sizes more effectively, leading to more economical progress through the search space. Such observations are in line with existing literature on adaptive parameter control in DE, where learning-based or feedback-driven strategies are known to outperform fixed parameter choices, especially as problem dimensionality increases [12].

Selection pressure also plays a critical role in shaping the overall dynamics of the search. Tournament selection, when combined with adaptive differential weighting, provides a controlled bias toward high-quality solutions without excessively reducing population diversity. In contrast, random selection introduces unnecessary stochasticity that can disrupt the learning process, particularly when coupled with adaptive mechanisms that rely on informative population feedback. The results indicate that even moderate selection pressure can significantly improve the reliability of adaptive strategies by ensuring that useful information is retained and exploited across generations, as also discussed in the evolutionary computation literature [15,36].

The influence of local search frequency highlights the importance of moderation in hybrid metaheuristics. While local refinement can enhance solution quality, excessive application increases computational overhead and may interfere with global exploration. The experimental findings show that a very low local search rate achieves the best trade-off between exploitation and efficiency, supporting the view that local search should act as a complementary mechanism rather than a dominant driver of the optimization process. This observation aligns with prior work on memetic and hybrid evolutionary algorithms, where controlled and infrequent local search is often more effective than aggressive refinement [10].

When considered collectively, these observations explain the strong and consistent performance of the proposed method relative to classical and modern optimizers. Rather than relying on aggressive parameter settings or complex hybridization, the method benefits from a balanced integration of structured exploration and adaptive exploitation. The comparative results indicate that the proposed approach achieves competitive or superior performance with lower variability, suggesting improved robustness across diverse problem landscapes. Importantly, where statistical tests do not indicate significant pairwise differences, the observed performance trends are interpreted as consistent empirical behavior rather than strict dominance.

Overall, the discussion highlights that the performance gains achieved by the proposed framework stem from thoughtful algorithmic structure rather than brute-force complexity.

By guiding the search through informed sampling, adaptive weighting and controlled selection pressure, the algorithm avoids both premature convergence and inefficient exploration. These findings reinforce a broader principle in large-scale evolutionary optimization: intelligent structure and feedback-driven adaptation are often more effective than increased randomness or parameter proliferation, in agreement with large-scale benchmark studies and methodological guidelines [8,43].

As is common in population-based metaheuristic methods, no formal convergence or complexity guarantees are provided for the proposed approach. Instead, scalability and computational behavior are assessed empirically through extensive experimental evaluation, following standard practice in large-scale optimization research [8,9]. The inclusion of large-scale benchmark functions and real-world engineering problems, such as the Tandem and GasCycle scenarios, enables a practical assessment of convergence trends and computational cost under increasing dimensionality. The observed behavior in terms of function evaluations and execution time indicates controlled scalability in large-scale optimization settings.

5. Conclusions

This work explored large-scale optimization through a systematically enhanced version of the DE algorithm. The improvements introduced in this study were designed to address two persistent challenges in high-dimensional optimization: efficiency and stability. Throughout the experimental analysis, several key components proved crucial to achieving these goals. A central contribution is the MIGRANT differential weight mechanism, which consistently outperformed both the classic NUMBER and RANDOM schemes. Across a wide variety of benchmark functions, MIGRANT(T) required significantly fewer objective function evaluations while maintaining identical success rates. This demonstrates that an adaptive weight strategy can guide the search more intelligently, reducing unnecessary evaluations and offering clear performance advantages in complex landscapes. Equally important was the impact of the sampling strategy. The results showed that k-means sampling (K) provides a strong structural advantage compared to uniform sampling. Configurations using MIGRANT(K) repeatedly achieved the lowest evaluation counts and exhibited far smaller variance. Pairwise statistical tests confirmed these differences, with several comparisons reaching high or very high levels of significance. This indicates that exploiting cluster information during sampling can greatly improve the quality and diversity of candidate solutions. The study also highlighted the role of the selection mechanism. Tournament selection consistently strengthened the algorithm's performance, enabling MIGRANT(T) to outperform all random-based variants. This confirms that introducing even a light degree of selective pressure yields more reliable search dynamics, while fully random selection tends to increase noise and computational cost. Another important outcome relates to the local search rate. Although local search can refine promising candidates, the experiments showed that applying it too frequently becomes counterproductive. The lowest tested rate (0.005) offered the best trade-off, achieving lower computational cost and greater stability. In contrast, higher rates (0.03 and 0.05) significantly increased function evaluations without improving success rates. This emphasizes the need for careful calibration of exploitation mechanisms in high-dimensional settings. Finally, when compared to widely used algorithms such as the GA, BICCA, LSHADE-SPA, SHADE-ILS, IPSO, the WOA and DE, the proposed method consistently delivered superior performance. Taken together, these findings highlight the effectiveness of combining structured sampling, adaptive weighting, selective pressure and controlled local search within DE. The synergy of these components results in an optimizer that is not only faster but also remarkably stable across different problem types and dimensions.

A promising direction for future research is to explore how the proposed framework could be integrated with other well-established metaheuristic algorithms. Such a hybridization could leverage the strengths of different search strategies and potentially lead to more effective optimization performance. In addition, another interesting avenue is the incorporation of learning mechanisms such as reinforcement learning or adaptive parameter-learning techniques so that the algorithm can dynamically adjust its strategies and parameters based on the characteristics of the search landscape. Such a self-adaptive system could further enhance the stability, robustness and overall efficiency of the optimization process.

Overall, this study demonstrates that carefully designed modifications to DE can lead to substantial performance gains, and it sets the foundation for developing even more powerful and general-purpose optimization algorithms.

Author Contributions: G.K., V.C. and I.G.T. conceived of the idea and the methodology, and G.K. and V.C. implemented the corresponding software. G.K. conducted the experiments, employing objective functions as test cases, and the comparative experiments. I.G.T. performed the necessary statistical tests. All authors have read and agreed to the published version of the manuscript.

Funding: This research has been financed by the European Union: Next Generation EU project through the program Greece 2.0 National Recovery and Resilience Plan, under the call RESEARCH–CREATE–INNOVATE; project name: “iCREW: Intelligent small craft simulator for advanced crew training using Virtual Reality techniques” (project code: TAEDK-06195).

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Conflicts of Interest: The authors declare no conflicts of interest.

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