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Editorial

Editorial for the Special Issue of Foundations "Recent Advances in Fractional Differential Equations and Inclusions"

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1. Introduction

The subject of fractional calculus addresses the research of asserted fractional derivatives and integrations over complex domains and their utilization. Fractional calculus has gained considerable popularity in recent years. Fractional differential equations have attracted much attention within the literature because certain real-world problems in physics, engineering, design, material science, fluid mechanics, probability theory, image processing, optimal control, and other fields can be described better with the help of fractional differential equations. Fractional differential equations are also regarded as a better tool for the description of hereditary properties of various materials and processes than the corresponding integer-order differential equations.

2. Special Issue Overview

This Special Issue focuses on "Recent Advances in Fractional Differential Equations and Inclusions" and covers some of the recent developments in the theory and application of fractional differential equations, inclusions, inequalities, and systems of fractional differential equations and inclusions. Topics include, but are not limited to, the following:

- Fractional-order differential equations [1,2]
- Fractional-order integral equations [3,4]
- The Nabla fractional difference equation [5]
- Fractional evolution equations [6]
- Fractional boundary value problems [7–10]
- Fractional coupled systems [11,12]
- Fractional integral inequalities [13,14]
- Fractional hybrid q-difference equations [15]
- Λ-fractional differential equations [16]
- Fractional Lyapunov functions [17]
- The generalized Mittag-Leffler function [18]
- Error analysis for fractional-order differential equations [19]

In total, 22 manuscripts were submitted, and 19 papers by 39 authors were successfully published. Authors come from the following 16 countries:

- Algeria (2)
- Egypt (2)
- Ethiopia (3)
- Greece (2)
- India (8)
- Iran (1)
- Lebanon (1)
- Malaysia (1)
- Pakistan (2)
- Romania (1)
- Saudi Arabia (3)
- Spain (2)
- Thailand (1)
- UAE (1)
- UK (2)
- USA (7).

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The Special Issue contains 19 contributions covering different branches of differential equations. Below, we highlight the main results of the papers.

- In [3], fixed-point theorems for the class of $\beta - G$, $\psi - G$ contractible operators of a Darbo type are proved in the context of noncompactness measurements. The obtained results are used to establish the existence of solutions for fractional integral equations satisfying some local conditions in Banach space. Moreover, a numerical example is provided to validate the results.
- The existence and uniqueness of certain solutions are studied in [1] using families of fractional order differential equations and the Banach and Schauder fixed point theorems.Moreover, by using the differential transform method, analytical or semi-analytical results for the proposed problems are computed. Examples illustrating the obtained results are also presented.
- In [5], a two-point boundary value problem for a finite Nabla fractional difference equation with dual non-local boundary conditions is studied. The existence of at least one and at least two positive solutions is verified by using Guo–Kranosel'skiĭ fixed point theorem on cones. Further sufficient conditions for the existence and uniqueness of solutions for the proposed class of boundary value problems are obtained using Brouwer and contraction mapping theorems, respectively. The applicability of the established results is demonstrated with examples.
- The existence of continuous solutions of a ϕ-fractional order quadratic integral equation is established in [4], and the existence of maximal and minimal solutions is proved.
- In [17], fractional Lyapunov functions for epidemic models are introduced and the concept of Mittag-Leffler stability is applied. The global stability of the epidemic model at an equilibrium state is established.
- A coupled system of Hilfer fractional differential inclusions with nonlocal integral boundary conditions is studied in [11]. An existence result is established when the set-valued maps have non-convex values and the set-valued maps are Lipschitz in the state variables, by using a Filippov's technique, thus avoiding the applications of fixed point theorems.
- The objective of [18] is to obtain some fractional integral formulas concerning products of the generalized Mittag–Leffler function and two H-functions. The resulting integral formulas are described in terms of the H-function of several variables. The H-functions associated with fractional calculus have been recognized to play a fundamental role in the probability theory as well as in their applications, including non-Gaussian stochastic processes and phenomena of nonstandard (i.e., anomalous) relaxation and diffusion. Moreover, the H function and the generalized Mittag–Leffler function reduce to a hypergeometric function and polynomials, so it becomes more important from the application viewpoint. Therefore, fractional calculus formulae involving hypergeometric functions and polynomials play an important role in the theory of special functions and mathematical physics.
- In [7], the criteria ensuring the existence and uniqueness of solutions for a ψ-Hilfer generalized proportional fractional differential equation complemented with mixed nonlocal boundary conditions are presented. The boundary conditions considered in this study are more general, as they include multipoint, multiorder fractional integral and fractional derivative operators. After converting the given nonlinear problem into a fixed point problem, Banach's contraction mapping principle, the Krasnosel'skiĭ and Schaefer fixed point theorems as well as the Leray–Schauder nonlinear alternative are applied to derive the desired results. Additionally, two existence results for the ψ-Hilfer generalized proportional fractional differential inclusion with mixed nonlocal boundary conditions are presented, for cases in which the multivalued map takes convex as well as non-convex values by means of the Leray–Schauder nonlinear alternative for multivalued maps and Covitz-Nadler's fixed point theorem for contractive multivalued maps, respectively. Numerical examples illustrating the obtained results are also presented.

- New Simpson's type integral inequalities for the class of functions whose derivatives of absolute values are s-convex via generalized proportional fractional integrals, are given in [13].
- In [8], single and multi-valued (k, ψ)-Hilfer type fractional boundary value problems of order $(1, 2]$ involving nonlocal integral boundary conditions are studied. For the single-valued case, existence and uniqueness results are established by applying the Banach contraction mapping principle, Krasnosel'skiĭ fixed point theorem and the Leray–Schauder nonlinear alternative. The first existence result dealing with the convex-valued multi-valued map involved in the inclusion is established by applying the Leray–Schauder nonlinear alternative for multivalued maps, while the existence result for the non-convex valued multivalued map in the inclusion is obtained by applying the Covitz–Nadler fixed point theorem for contractive multivalued maps. The obtained theoretical results are illustrated using numerical examples.
- Ref. [15] is concerned with the study of a new class of hybrid fractional q-integro-difference equations involving Caputo type q-derivatives and Riemann–Liouville q-integrals of different orders with a nonlocal q-integro-initial condition. An existence result is obtained by means of Krasnosel'skiĭ's fixed point theorem, whereas the uniqueness of its solutions is shown by applying the Banach contraction mapping principle. Additionally, the stability of solutions is discussed and it is found that it depends on the nonlocal parameter in contrast to the initial position of the domain. To demonstrate the application of the obtained results, examples are constructed.
- In [9], the existence of multiple positive solutions to boundary value problems of a nonlinear fractional differential equation with integral boundary conditions and parameter dependences is studied, by using a functional-type cone expansion–compression fixed point theorem and the Leggett–Williams fixed point theorem. Examples are included to illustrate the results.
- Λ-fractional differential equations are discussed in [16]. A summary of the Λ-fractional analysis is presented, with a discussion of its influence on the sources of differential equations, such as fractional differential geometry, field theorems, and calculus of variations.
- A predictor-corrector method for solving Caputo–Hadamard fractional differential equations is proposed in [19]. The smoothness properties of various aspects of such equations are analyzed and the effect of these properties on the convergence order of the numerical method is explained.
- Caputo fractional differential equations in the sequence Lebesgue spaces $\ell_p(\mathbb{N}_0)$ with $p \geq 1$ are considered in [6]. The associate operator is obtained by using a convolution with a sequence derived from Banach algebra which belongs to $\ell_1(\mathbb{Z})$. Techniques from functional analysis and Banach algebra are used to obtain more information about the problem.
- In [10] existence results are studied for a three-term fractional boundary value problem using Schauder's fixed point theorem. The Green's functions are constructed using spectral theory.
- Sufficient criteria for the existence and uniqueness of solutions for a coupled system of nonlinear (k, ψ)-Hilfer fractional differential equations complemented with coupled (k, ψ)-Riemann–Liouville fractional integral boundary conditions are presented in [12]. The Leray–Schauder alternative, Krasnosel'skiĭ's fixed-point theorem and Banach's contraction mapping principle are used to establish the desired results.
- In [2], it is illustrated that linear nq-order sequential Caputo fractional differential equations, which are sequential of the order q where $q < 1$ with fractional initial conditions and/or boundary conditions, can be solved. The reason for choosing sequential fractional dynamic equations is that linear non-sequential Caputo fractional dynamic equations with constant coefficients cannot be solved in general. The Laplace transform method is used to solve sequential Caputo fractional initial value problems,

while fractional boundary conditions are used to compute Green's function for sequential boundary value problems. In addition, the solution of the sequential dynamic equations yields the solution of the corresponding integer-order differential equations as a special case, as $q \to 1$.

- In [14], by using the Jensen–Mercer inequality, the authors obtain Hermite–Hadamard–Mercer type inequalities for a convex function employing left-sided (k, ψ)-proportional fractional integral operators involving a continuous strictly increasing function.

We hope this Special Issue will be of interest to readers, and that the ideas and works included will inspire further studies on fractional differential equations and inclusions.

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