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BioExergoEconomics: A Unified Thermodynamic–Environmental Framework Integrating Exergy Analysis, Circular Economy, Life Cycle Assessment and Biological Resource Recovery

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Abstract

The integration of thermodynamic efficiency, environmental mitigation, and circular resource recovery remains a fundamental challenge in the sustainability assessment of agro-industrial systems. Conventional methodologies—such as Exergoeconomics, Life Cycle Assessment (LCA), and Circular Economy (CE) indicators—typically operate in methodological isolation, limiting their ability to quantify trade-offs between energy-intensive biological recovery and environmental mitigation. To address this gap, this study proposes BioExergoEconomics, a novel unified framework that structurally integrates physical exergy analysis, Exergoeconomic cost allocation, carbon and water footprints, and biological resource recovery into a single F-P-R-I-B functional model. The principal contribution is the formulation of the BioExergoEconomics Index (BEC Index), a dimensionless metric that uniquely synthesizes exergy efficiency, circular resource recovery, biological recovery, carbon mitigation, water conservation, and economic performance. The framework is validated through a comparative case study of beef production (1 kg functional unit). Although biological processing increases electricity demand by 47.8%, this localized energy investment reduces upstream agricultural burdens, resulting in a 17.6% net reduction in energy and exergy demand, a 99.4% reduction in freshwater footprint, and a 51.1% decrease in carbon emissions. With a BEC Index of 0.55, the framework demonstrates that electrical exergy investments in biological nodes can drive systemic sustainability.

Keywords: BioExergoEconomics index; Exergoeconomics; circular economy; life cycle assessment; carbon footprint; water footprint

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1. Introduction

The transition toward more sustainable and circular production systems requires assessment methodologies that simultaneously address the thermodynamic quality of resource use, environmental burdens, economic performance, and the recovery of materials and biological resources. This challenge is particularly relevant to agro-industrial systems, where energy and material flows are strongly interconnected and residual biomass can be transformed into new products, energy carriers, nutrients, or functional biological resources [1]. Conventional sustainability assessments, however, tend to evaluate these

dimensions through separate methodological frameworks. Consequently, improvements in one dimension may remain disconnected from, or even conflict with, performance in others. The development of integrated assessment approaches is therefore essential for determining whether emerging circular production systems genuinely improve overall resource efficiency and environmental performance, rather than merely shifting burdens between stages or impact categories. In parallel, the increasing need to reduce emissions and resource consumption has stimulated the optimization of energy-intensive and biomass-processing technologies, including biomass gasification and steam methane reforming, for which increasingly rigorous thermodynamic indicators have been employed to identify inefficiencies and opportunities for improvement [2,3].

Exergy analysis provides a particularly valuable foundation for such assessments because it evaluates the quality and availability of energy and material resources rather than considering only their quantity. By quantifying the maximum useful work obtainable relative to a specified environment, exergy analysis enables the identification and localization of irreversibilities throughout complex production systems. The theoretical foundations established by Bejan [4], Szargut et al. [5], and Tsatsaronis [6] have supported the application of exergy analysis to a wide range of industrial processes and resource-conversion systems. Subsequent developments in Thermoeconomics and Exergoeconomics incorporated economic considerations into second-law analysis, enabling the allocation of costs associated with exergy flows, resource consumption, and exergy destruction. In particular, the contributions of Antonio Valero and Alicia Valero [7,8] expanded the use of exergy-based methods for evaluating resource depletion and the cumulative thermodynamic cost of natural resources. Nevertheless, although exergy and Exergoeconomic analyses provide a rigorous description of thermodynamic and economic performance, they do not, by themselves, fully capture life-cycle environmental burdens, freshwater consumption, circularity, or the additional value generated through biological recovery pathways.

Environmental assessment methodologies have consequently developed along complementary but largely independent lines. Life Cycle Assessment (LCA), standardized by ISO 14040 [9] and ISO 14044 [10], provides a systematic framework for quantifying environmental impacts associated with products and processes throughout their life cycles. LCA has become a widely established approach for evaluating greenhouse gas emissions, resource use, and other environmental burdens associated with industrial and agricultural systems. Similarly, the Water Footprint methodology developed by Hoekstra and Mekonnen [11,12] introduced a complementary perspective on freshwater consumption by distinguishing blue, green, and grey water components. This distinction is particularly relevant to food and agro-industrial systems, in which water demand may constitute a major component of overall environmental performance. Recent research has emphasized the need to improve the spatial and methodological resolution of water-footprint assessments, including the integration of hydrological and regional-scale approaches [13]. These developments demonstrate that environmental performance cannot be adequately represented by thermodynamic efficiency alone and that resource consumption must also be evaluated within its broader environmental context.

At the same time, Circular Economy (CE) has shifted the sustainability paradigm from minimizing impacts within predominantly linear production chains toward maintaining the value of resources through reuse, recovery, regeneration, and recirculation [14]. The principles introduced by Stahel [15] and subsequently disseminated by the Ellen MacArthur Foundation [16] reduce waste generation. However, the concept of circularity encompasses multiple dimensions and remains subject to methodological debate. Based on an analysis of 221 definitions, Kirchherr et al. [17] highlighted the lack of a universally agreed-upon definition of CE and characterized it as a regenerative system that replaces end-of-life disposal

with strategies such as reducing, reusing, recycling, and recovering resources throughout value chains. More recent standardization efforts, including ISO 59020:2024 [18], have established principles and quantitative approaches for measuring circularity performance, while international guidelines for measuring the Circular Economy [19] have further emphasized the need to evaluate material and value retention across environmental, socioeconomic, and policy dimensions. These developments reinforce the importance of evaluating circularity not as an isolated characteristic but as a property that emerges from interactions among resource flows, environmental performance, and production processes [20].

Recent research has increasingly recognized agro-industrial residues as secondary resources capable of supporting circular bioeconomy pathways rather than being treated exclusively as waste streams. Biological, physicochemical, and electrochemical conversion routes can transform these residues into energy carriers, nutrients, biomaterials, and other value-added products, although their environmental and resource implications depend strongly on the process configuration and system boundaries [1,21].

The need for such an integrated perspective is particularly evident in agro-industrial systems that involve biological resource recovery. Residual biomass generated by one production process can become a feedstock for another process through anaerobic digestion, microbial conversion, nutrient recovery, fermentation, or microalgae cultivation. In this context, the transformation of a residual stream into a useful biological resource can simultaneously modify material flows, energy requirements, environmental impacts, and economic costs. Recent applications of exergy analysis to biotechnological systems have demonstrated the usefulness of second-law analysis for identifying thermodynamic inefficiencies in processes such as microalgal biomass cultivation and extraction [22,23]. For example, operations associated with photobioreactor cultivation and biomass stabilization can exhibit significant irreversibilities that are not adequately characterized through first-law energy balances alone. At the same time, biological recovery pathways may provide benefits extending beyond material recovery, including nutrient recycling, water reuse, carbon mitigation, and reductions in methane emissions. In particular, algal-derived products have been investigated as functional feed components with the potential to reduce enteric methane emissions in livestock systems [24,25]. Thus, a single biological pathway may simultaneously influence thermodynamic efficiency, circularity, environmental impacts, resource consumption, and economic performance.

This multidimensional behavior exposes a fundamental limitation of conventional assessment practices. Exergy analysis primarily characterizes the quality and degradation of resources; Exergoeconomics relates thermodynamic performance to economic costs; LCA quantifies environmental burdens over a defined life-cycle boundary; water-footprint methodologies characterize freshwater consumption; and CE indicators describe the retention, recovery, and recirculation of resources. At the same time, efforts to integrate thermodynamic and environmental assessment methodologies have demonstrated that individual indicators may provide only partial representations of sustainability. Recent integrated approaches combining Exergy Analysis, Emergy Analysis, and LCA have therefore proposed aggregated sustainability indices to support multidimensional decision-making [26]. Each approach provides valuable information, but their independent application can make it difficult to determine a system's overall performance when improvements in one dimension are accompanied by changes in others. This limitation is especially significant when comparing linear and circular agro-industrial configurations, because the latter can introduce additional processing stages and energy requirements while simultaneously recovering materials that would otherwise be lost, reducing environmental burdens, and generating new biological products. Consequently, a higher thermodynamic efficiency in an individual process does not necessarily imply superior system-level sustainability, just as a reduction

in a single environmental indicator does not necessarily demonstrate improved resource performance across the entire production chain.

To contextualize this methodological gap, a balanced comparison between 'separated models' and 'integrated models' is necessary. Separated models, such as isolated Life Cycle Assessment (LCA) or Circular Economy (CE) metrics, offer the advantage of being highly detailed and less data-intensive. For instance, traditional LCA provides rigorous environmental impact assessments but often lacks mechanisms to measure resource circularity and material value retention [27]. Conversely, isolated CE indicators focus heavily on resource recirculation but may overlook the underlying energy demand or environmental burdens of the recovery processes [28]. While integrated models are more data-intensive and more complex to interpret, they are essential for capturing the trade-offs among thermodynamic efficiency, environmental impacts, and circularity [29]. The purpose of integrated approaches is not to replace these detailed, separate methods, but rather to enrich decision-making tools by providing a holistic perspective that prevents environmental burden shifting across sustainability dimensions [30].

Although recent studies have sought to combine exergy analysis with environmental indicators and LCA, these approaches generally retain the different dimensions as parallel or complementary indicators rather than establishing a unified analytical structure in which thermodynamic performance, economic costs, environmental burdens, circular resource recovery, and biological valorization are evaluated under a common functional basis. The methodological challenge is therefore not simply to add additional sustainability indicators to an existing exergy or Exergoeconomic assessment. Rather, it is to establish a consistent framework that represents the interactions among these dimensions and translates them into a comparable measure of overall system performance. In particular, a unified approach should allow conventional and circular production systems to be compared using an equivalent functional unit while accounting simultaneously for resource quality, exergy destruction, economic implications, life-cycle environmental burdens, freshwater consumption, and the recovery of biological resources. To the authors' knowledge, such an integrated framework remains insufficiently developed for circular agro-industrial systems.

To address this gap, this study proposes BioExergoEconomics, a unified thermodynamic–environmental framework that extends the conventional scope of Exergoeconomics by incorporating Circular Economy principles, Life Cycle Assessment, carbon and water footprints, and biological resource recovery into a common methodological structure. The framework is designed not to replace existing assessment methodologies but to establish interfaces among them, enabling consistent evaluation of thermodynamic, environmental, circular, biological, and economic performance within the same production system. Its central premise is that the sustainability of a circular process should be assessed by the combined performance of the resources it consumes, the irreversibilities it generates, the environmental burdens associated with its life cycle, the resources it recovers, and the economic value of those transformations.

A further contribution of the proposed framework is the development of the BioExergoEconomics Index (BEC Index), a dimensionless indicator that synthesizes the principal dimensions of integrated system performance into a single comparative metric. The BEC Index combines exergy efficiency, circular resource recovery, biological recovery, carbon mitigation, water conservation, and economic performance, thereby providing a quantitative basis for comparing alternative production configurations on an equivalent functional unit basis. This formulation addresses a key limitation of multidimensional sustainability assessments: while individual indicators reveal specific strengths and weaknesses, they do not necessarily provide a direct basis for determining the overall relative performance of competing systems. The BEC Index is therefore conceived as a decision-support metric that

complements, rather than substitutes for, the underlying thermodynamic, environmental, circularity, and economic indicators.

The methodological framework is demonstrated through a comparative assessment of beef production, in which a conventional feeding system is evaluated against a circular bioeconomy configuration incorporating residual brewery biomass and microalgae valorization. The comparison is structured around an equivalent functional unit of 1 kg of beef and integrates mass and energy balances, physical and chemical exergy analysis, Exergoeconomic evaluation, life-cycle environmental indicators, carbon and water footprints, and biological resource recovery. This configuration provides a representative test case because it links residual biomass valorization to biological conversion and livestock production, thereby enabling evaluation of interactions among resource recovery, environmental mitigation, thermodynamic performance, and economic performance within a single production chain.

Accordingly, the main contribution of this study is twofold: (i) the development of BioExergoEconomics as an integrated framework for the simultaneous assessment of thermodynamic, environmental, circular, biological, and economic dimensions of agro-industrial systems; and (ii) the formulation of the BEC Index as a dimensionless metric for comparative sustainability assessment under a common functional unit. By connecting exergy-based resource accounting with life-cycle environmental assessment, circular resource recovery, biological valorization, and economic performance, the proposed approach seeks to provide a more comprehensive basis for evaluating circular bioeconomy systems and for identifying whether resource-recovery strategies generate a net improvement in overall system performance. The framework is intended to support the design, optimization, and comparative assessment of emerging circular technologies in which thermodynamic efficiency, environmental mitigation, resource conservation, biological recovery, and economic viability must be considered simultaneously.

2. Methodology: The BioExergoEconomics Framework

2.1. Conceptual Framework and Methodological Integration

BioExergoEconomics is proposed as an integrated methodological framework for evaluating the thermodynamic, environmental, biological, and economic performance of circular production systems. Unlike conventional assessment methodologies, which independently quantify energy efficiency, exergy destruction, environmental impacts, or economic performance, the proposed framework simultaneously integrates these dimensions within a unified analytical structure. Biological recovery plays a central role in this integration, as resource recovery processes can simultaneously modify exergy flows, carbon and water footprints, and the economic performance of the system.

The methodology combines the principles of Exergy Analysis [4–6], Exergoeconomics [6–8], Life Cycle Assessment (LCA) [9,10], Water Footprint Assessment [18,19], Circular Economy [11–13], and biological resource recovery processes into a single evaluation framework capable of comparing conventional linear production systems with circular bioeconomy alternatives.

As outlined in Table 1, the conceptual foundation of BioExergoEconomics is built on the chronological evolution of five distinct scientific paradigms that have historically operated as independent analytical silos. The first pivotal stage (1970s–1980s) established the thermodynamic baseline through Exergy Analysis. The fundamental contributions of Bejan [4], Szargut et al. [5], and Tsatsaronis [6] enable the engineering community to identify thermodynamic inefficiencies and irreversibilities. This was subsequently expanded in the 1990s with the advent of Exergoeconomics. By integrating economic principles with exergy, authors such as Antonio Valero [8], Alicia Valero et al. [7], Tsatsaronis [6], and Moran et al. [31] created a robust framework for assigning monetary

costs to thermodynamic destruction, fundamentally changing how industrial processes allocate the true cost of resource utilization. While the thermodynamic and economic dimensions matured, the 2000s and 2010s marked a critical shift toward environmental accountability. The standardization of Life Cycle Assessment (LCA) (ISO 14040 [9], ISO 14044 [10], Heijungs et al. [32]) and the development of the Water Footprint Assessment by Hoekstra et al. [12] introduced systemic methodologies to evaluate cradle-to-grave environmental impacts and freshwater consumption (blue, green, and grey water). Concurrently, the 2010s–2020s saw the consolidation of the Circular Economy paradigm, driven by Stahel [15] and the Ellen MacArthur Foundation [16], which formalized the transition from linear extraction models to regenerative systems focused on material recirculation and waste valorization. Despite these monumental advancements, a critical Scientific Gap persists in the current decade. As identified by Nemati et al. [33], these mature methodologies continue to evaluate thermodynamic, environmental, economic, and biological aspects independently. This fragmentation impedes a holistic understanding of complex, multi-scale systems, such as agro-industrial processes that involve bioengineering and environmental mitigation. Consequently, current approaches do not allow determining whether circular interventions generate net improvements at the system level or merely shift thermodynamic, economic, and environmental loads across different dimensions of sustainability. To bridge this gap, the BioExergoEconomics framework proposed in this work acts as the unifying methodological step. It systematically integrates exergy analysis, Exergoeconomics, circular economy principles, LCA, and water footprint into a single, cohesive assessment model, explicitly incorporating biological resource recovery to capture the full spectrum of sustainability performance.

Table 1. Scientific Evolution Leading to BioExergoEconomics.

Period	Scientific Milestone	Key Contributors	Main Contribution
1970s–1980s	1. Exergy Analysis	Bejan [4]; Szargut et al. [5]; Tsatsaronis [6];	Development of exergy analysis for evaluating thermodynamic efficiency and identifying irreversibilities in engineering systems.
1990s	2. Exergoeconomics	Antonio Valero [7]; Alicia Valero et al. [7]; Tsatsaronis [6]; Moran [27]	Integration of exergy analysis with economic evaluation, enabling the allocation of exergy costs to products and industrial processes.
2000s	3. Life Cycle Assessment (LCA)	ISO 14040 [9]; ISO 14044 [10]; Heijungs et al. [32];	Comprehensive assessment of environmental impacts throughout the entire life cycle of products and industrial systems.
2010s	4. Water Footprint Assessment	Hoekstra et al. [11]	Quantification of freshwater consumption (blue, green, and grey water) associated with products and production systems.
2010s–2020s	5. Circular Economy	Stahel [15]; Ellen MacArthur Foundation [16];	Transition from linear production systems toward regenerative models based on resource recovery, waste valorization, and material recirculation.
2020s and Beyond	Identified Scientific Gap	Nemati et al. [33]	Existing methodologies evaluate thermodynamic, environmental, economic, and biological aspects independently, without a unified assessment framework.
This Work	BioExergoEconomics	Current Authors	Unified thermodynamic–environmental framework integrating exergy analysis, Exergoeconomics, circular economy, life cycle assessment, carbon footprint, water footprint, biological resource recovery, and environmental mitigation into a single methodology.

To operationalize the previously detailed conceptual evolution, Table 2 presents a Methodological Integration Matrix that contrasts traditional assessment methods with the proposed BioExergoEconomics framework. This matrix highlights the specific quantitative capabilities and limitations of each discipline. Historically, conventional energy and exergy analyses have been strictly confined to evaluating thermodynamic performance—quantifying energy demand, exergy destruction ($\dot{I}$), and rational efficiencies. Even when Exergoeconomics bridged the gap between thermodynamics and accounting through cost allocation ($\dot{C}$, c), it inherently lacked the capacity to measure environmental loads and biological processes. Conversely, while LCA and Circular Economy frameworks successfully evaluate global warming potential (CF), water footprint (WF), and material recovery rates, they fail to account for the thermodynamic irreversibilities that govern resource transformation. By integrating these diverse quantitative indicators, the framework culminates in a composite metric (the BEC Index), ensuring that environmental mitigation and biological valorization are evaluated with the same mathematical rigor as thermodynamic efficiency.

Table 2. Methodologies of energy process evaluation.

Methodology	Thermodynamic Efficiency	Economic Cost Allocation	Carbon Footprint	Water Footprint	Circularity Recovery	Biological Recovery	Principal Quantitative Indicators
Energy Analysis	✓						η_{en}, Energy Demand (kJ)
Exergy Analysis	✓						$\dot{E}x_F$, $\dot{E}x_P$, Irreversibility ($\dot{I}$)
Exergoeconomics	✓	✓					Exergy Cost ($\dot{C}$), Specific Cost (c)
LCA			✓	✓			GWP ($kgCO_2eq$)
Circular Economy			✓	✓	✓		Material Recovery Rate, ηCE
BioExergoEconomics (this work)	✓	✓	✓	✓	✓	✓	BEC Index, $\dot{E}x_D$, ΔCF, ΔWF, ηB

To apply this comprehensive framework in practice, the analysis is structured into a systematic sequence. Specifically, the proposed methodology consists of three sequential stages:

1. Definition of the conventional production system (Baseline System).
2. Development of the circular production model.
3. Comparative BioExergoEconomic assessment under an equivalent functional unit.

Throughout the methodology, all comparisons are performed using the same functional unit to ensure consistency among thermodynamic, environmental, and economic indicators. This systematic progression ensures a transparent and standardized comparison, which is detailed in the subsequent stages.

2.2. BioExergoEconomic Nomenclature and Variables

To ensure mathematical rigor and alignment with international thermoeconomic standards, specifically the Specific Exergy Costing (SPECO) methodology, and Life Cycle Assessment guidelines, the BioExergoEconomics framework adopts a standardized nomenclature. The variables are categorized into thermodynamic, environmental, biological, and economic dimensions, with their respective primary operational units.

Thermodynamic Variables

- $\dot{E}x_F$: Exergy rate of Fuel, representing the thermodynamic resources consumed to drive the process (kW or kJ/s).

- $\dot{E}x_{Fe}$: Exergy rate of External Fuel introduced into intermediate stages of the supply chain (kW or kJ/s).
- $\dot{E}x_P$: Exergy rate of Product, representing the desired thermodynamic output of the process (kW or kJ/s).
- $\dot{E}x_{P'}$: Exergy rate of the Recovered Co-product integrated into the circular system (kW or kJ/s).
- $\dot{I}$ (or $\dot{E}x_D$): Rate of Exergy Destruction or thermodynamic irreversibility due to entropy generation (kW or kJ/s).

Environmental and Biological Variables

- $\dot{R}$: Mass flow rate of physical Residual streams rejected to the environment (kg/s or t/h).
- *CF*: Carbon Footprint, typically expressed as Global Warming Potential ($kgCO_2eq$).
- *WF*: Water Footprint, accounting for the freshwater volume consumed (m^3 or L).
- *B*: Biological Recovery Efficiency, representing the fraction of the incoming residual stream that is successfully converted into a useful biological product. It is a dimensionless parameter ranging from 0 to 1 (0–100%).

Exergoeconomic Variables

- $\dot{C}$: Exergoeconomic cost rate, accounting for the monetary flow associated with an exergy stream ($/s or $/h).
- *c*: Specific Exergoeconomic cost per unit of exergy ($/GJ or $/kWh)

To quantify the improvements achieved through the transition from a linear to a circular system, the framework defines a set of differential metrics (Δ). These indicators evaluate the net variation between the Baseline (LB) and the Circular Economy (CE) models:

Comparative Sustainability Indicator

- $\Delta\dot{E}x_F$: Reduction in fuel exergy consumption.
- ΔWF: Reduction in freshwater demand.
- ΔCF: Mitigation of greenhouse gas emissions.
- $\Delta\dot{R}$: Decrease in physical residual streams.
- $\Delta\dot{I}$: Reduction in overall thermodynamic irreversibility.
- $\Delta\$P$: Variation in the specific production cost of the final product.

2.3. Stage I: Baseline Functional Structure (Linear Model)

The first stage establishes the conventional production system, hereafter referred to as the Baseline (LB). This structure represents a traditional linear production process in which raw materials are transformed into final products while generating irreversibilities and residual streams.

To ensure a rigorous thermodynamic evaluation, a defined control volume is established for the entire supply chain operating under steady-state conditions. The methodology adopts the Functional Structure shown in Figure 1, where each process is represented by the exergy rate of fuel ($\dot{E}x_F$), exergy rate of product ($\dot{E}x_P$), residual material flows ($\dot{R}$), and the rate of exergy destruction ($\dot{I}$), respectively.

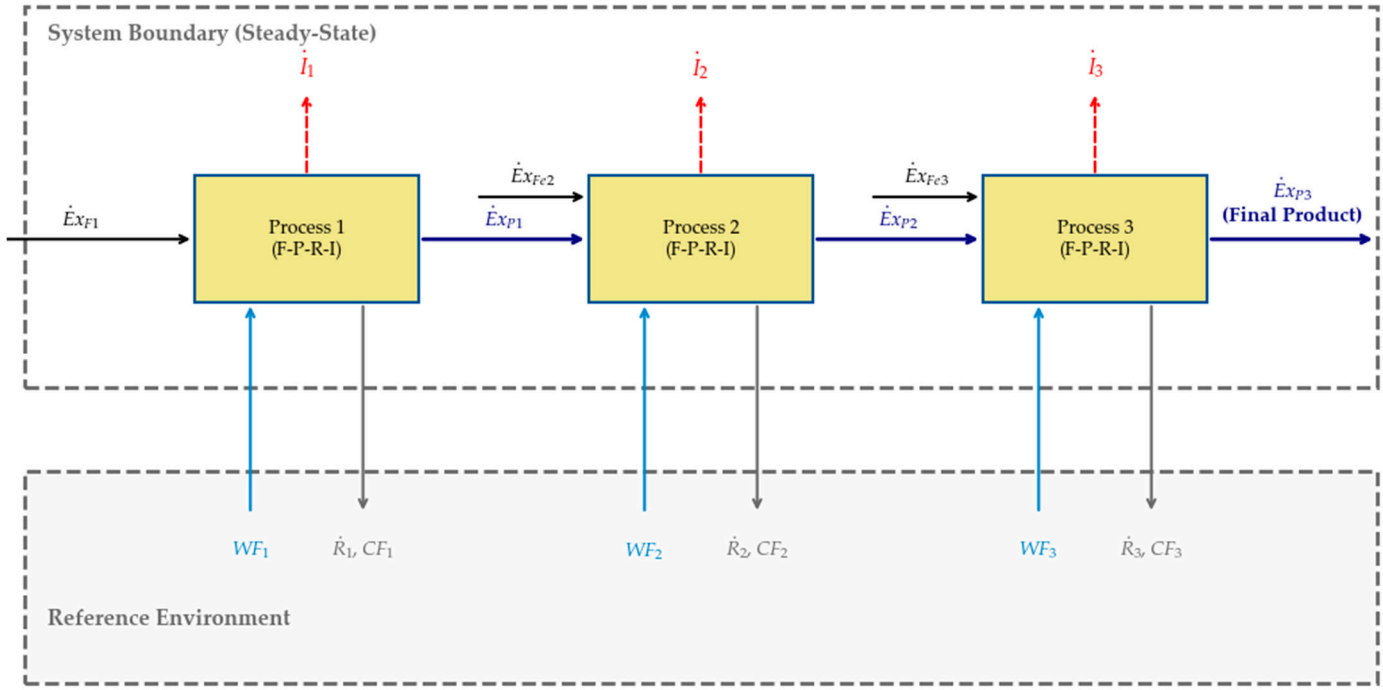


Figure 1. Baseline functional structure for the linear productive process. The control volume delineates the conversion of fuel exergy ($\dot{E}x_F$) into product exergy ($\dot{E}x_P$), isolating thermodynamic irreversibilities ($\dot{I}$) from physical residues ($\dot{R}$) and environmental footprints (CF,WF).

For each process included within the production chain, the following analyses are performed:

- Mass balance and energy balance.
- Physical and chemical exergy balance.
- Exergy destruction quantification.
- Exergoeconomic cost allocation.
- Carbon footprint and water footprint assessment.

As illustrated in Figure 1, the proposed Baseline Functional Structure strictly adheres to physical and thermodynamic conservation laws. The control volume (dashed line) delineates the boundary where external resources are transformed under steady-state conditions. For each sequential process i, the thermodynamic driver is represented by the exergy of fuel ($\dot{E}x_{F,i}$), which is partially converted into the exergy of product ($\dot{E}x_{P,i}$). According to the Gouy-Stodola theorem [34], this transformation inevitably generates thermodynamic irreversibilities ($\dot{I}_i$) due to entropy generation, which are visually depicted as exergy destroyed within the system. Crucially, the BioExergoEconomics model structurally uncouples these thermodynamic inefficiencies from the physical environmental burdens. The interactions with the Reference Environment are depicted based on actual mass and physical flow directions: freshwater resources (WF_i) are extracted from the ecosphere and enter the control volume to sustain the operation, whereas physical residual streams ($\dot{R}_i$) and greenhouse gas emissions (CF_i) are rejected from the processes back into the environment. This rigorous physical distinction ensures that subsequent calculations of the sustainability indicators accurately distinguish between the depletion of freshwater and the emission of pollutants (carbon and waste discharge). By systematically quantifying these parameters, the Baseline System constitutes the reference condition against which the circular production model will subsequently be evaluated.

2.4. Stage II: Circular Economy Functional Structure (Bio-Circular Model)

The second stage introduces the Circular Economy (CE) configuration, in which residual streams generated by external industrial processes or upstream stages are incorporated as secondary resources. This transition evolves the baseline functional structure into an F-P-R-I-B model, where *B* represents biological transformation processes—including anaerobic digestion, microalgae cultivation, aerobic biodegradation, microbial fermentation, and nutrient recovery.

A fundamental premise of this stage is the allocation of environmental and Exergoeconomic burdens. In accordance with standardized Life Cycle Assessment approaches [6,7], recovered co-products and residual biomass are considered inputs whose primary environmental burdens and historical exergy destruction costs have already been allocated to the original production system. Consequently, these secondary resources enter the biological processes (B) with a "zero-burden" assumption regarding their upstream generation.

As illustrated in Figure 2, the biological units (B_1,B_2,B_3) act as regenerative interfaces. Instead of rejecting the physical residual streams ($\dot{R}_1, \dot{R}_2, \dot{R}_3$) directly into the Reference Environment, these streams are routed into the bioprocesses. Through biological transformation, a significant fraction of this waste is converted into recovered product exergy streams ($\dot{E}x_{P'1}, \dot{E}x_{P'2}, \dot{E}x_{P'3}$). Because the upstream environmental impacts are mathematically reset ("Reset Env. Impacts"), the integration of these recovered streams into subsequent downstream stages (e.g., $\dot{E}x_{P'1}$ into Process 2, and $\dot{E}x_{P'2}$ into Process 3) displaces the need for primary external fossil fuels ($\dot{E}x_{Fe}$), thereby creating a direct mechanism for environmental mitigation.

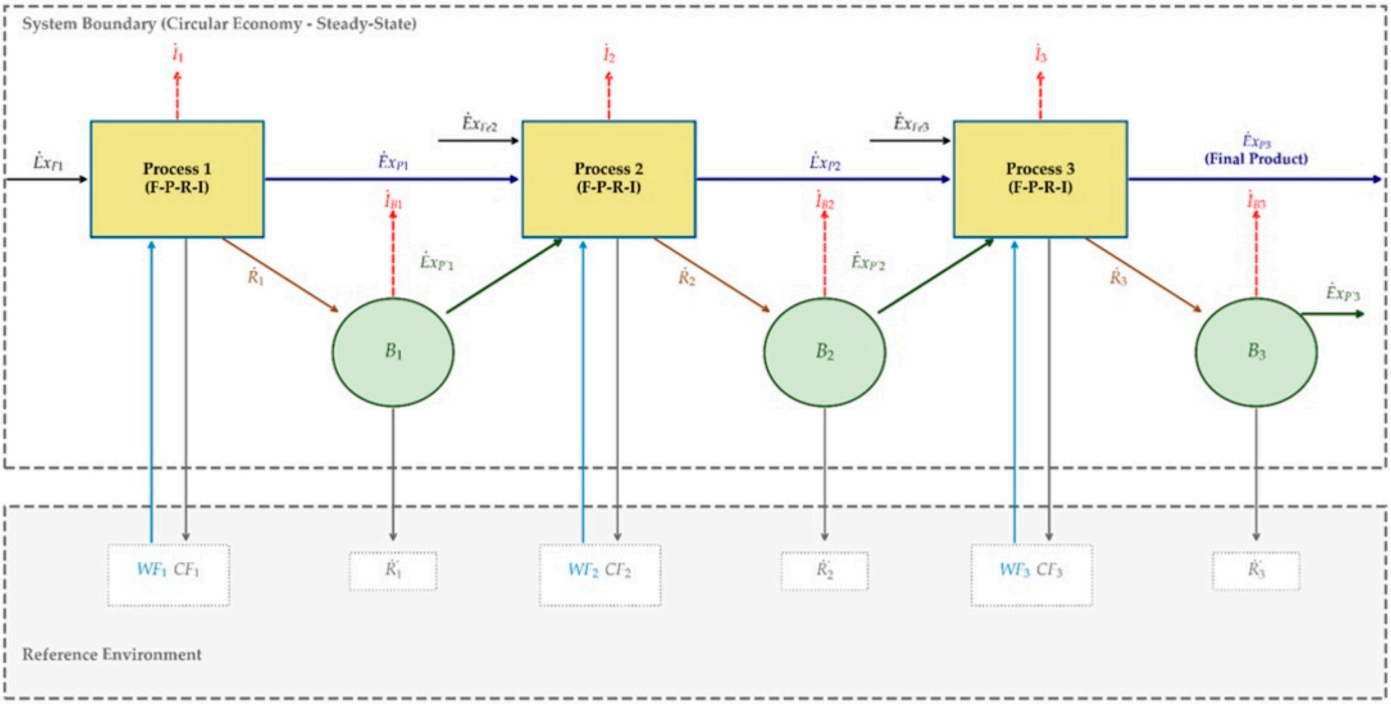


Figure 2. Functional structure for the Circular Economy model (F-P-R-I-B). The biological processes (B_1,B_2,B_3) valorize intermediate residual streams ($\dot{R}$) into recovered product exergy ($\dot{E}x_{P'}$).

From a strict thermodynamic perspective, these biological valorization stages are real physical transformations subject to the Second Law of Thermodynamics. Therefore, the BioExergoEconomics framework dictates that each biological node must account for its own exergy destruction ($\dot{I}_{B1}, \dot{I}_{B2}, \dot{I}_{B3}$) due to entropy generation during the biological conversion. Furthermore, any unconverted matter is finally rejected to the environment as a minimized,

stabilized secondary residual ($\dot{R}'_1, \dot{R}'_2, \dot{R}'_3$). This structural integration can reduce the net physical residual streams and, depending on the specific process configuration and system boundaries, greenhouse gas emissions (CF) of the overall supply chain. From a thermodynamic perspective, biological valorization changes the exergy-flow configuration of the integrated system by allowing part of the exergy associated with residual streams to be converted into useful co-products and subsequently incorporated into the productive cycle. This does not imply the creation of exergy or a modification of fundamental thermodynamic limits; rather, it represents a reallocation and recovery of available exergy within an expanded system boundary, subject to the exergy requirements and irreversibilities of the biological conversion process.

2.5. Stage III: Comparative Assessment Matrix

The final stage of the methodology systematically compares the Baseline (LB) and the Circular Economy (CE) systems under an identical functional unit. This standardization is paramount to ensure that all thermodynamic, environmental, biological, and economic indicators are evaluated on a strictly equivalent basis, avoiding distortions caused by differing production scales.

The comparison integrates the comprehensive set of variables evaluated in the previous stages: mass and energy balances, exergy analysis, Exergoeconomic assessment, carbon footprint, water footprint, biological resource recovery, waste recovery, and final product economic cost. To structure this comparative analysis, Table 3 establishes the determination matrix for the total impacts of both production models.

Table 3. Comparative Framework of the BioExergoEconomics Methodology.

Impact Category	Indicator (Symbol)	Baseline Total (LB)	Circular Total (CE)	Variation (Δ)
Exergy consumption	Fuel Exergy ($\dot{E}x_F$)	$\sum \dot{E}x_{F,i}$	$\sum \dot{E}x'_{F,i}$	$\Delta\dot{E}x_F = \dot{E}x_{F,LB} - \dot{E}x_{F,CE}$
Exergoeconomic cost	Cost Rate ($\dot{C}$)	$\sum \dot{C}_i$	$\sum \dot{C}'_i$	$\Delta\dot{C} = \dot{C}_{LB} - \dot{C}_{CE}$
Carbon footprint	Carbon (CF)	$\sum CF_i$	$\sum CF'_i$	$\Delta CF = CF_{LB} - CF_{CE}$
Water footprint	Water (WF)	$\sum WF_i$	$\sum WF'_i$	$\Delta WF = WF_{LB} - WF_{CE}$
Residuals (wastes)	Residuals ($\dot{R}$)	$\sum \dot{R}_i$	$\sum \dot{R}'_i$	$\Delta\dot{R} = \dot{R}_{LB} - \dot{R}_{CE}$
Irreversibility	Exergy Destruction ($\dot{I}$)	$\sum \dot{I}_i$	$\sum \dot{I}'_i$	$\Delta\dot{I} = \dot{I}_{LB} - \dot{I}_{CE}$
Biological recovery	Bio-Recovery (B)	—	$\sum B'_i$	$\Delta B = B_{CE} - 0$
Economic cost	Product Cost ($\$P$)	$\sum \$P_i$	$\sum \$P'_i$	$\Delta\$P = \$P_{LB} - \$P_{CE}$
Resource recovery	Co-products ($\dot{E}x_{P'}$)	—	$\sum \dot{E}x_{P',i}$	$\Delta\dot{E}x_{P'} = \dot{E}x_{P',CE} - 0$

Note: The subscripts i denote the i-th flow stream of each process.

As detailed in Table 3, the total impact for each category is calculated by mathematically aggregating the individual contributions of all processes within the respective supply chain (Baseline or Circular). The ultimate objective of this stage is to determine the net variation (Δ) for each indicator. These differential metrics quantify the absolute sustainability improvements, such as the reduction in total exergy destruction ($\Delta\dot{I}$), the decrease in environmental burdens (ΔCF, ΔWF), and the mitigation of operational costs ($\Delta\$P$), achieved through the implementation of bioengineering and circular economy strategies. This comparative matrix serves as the foundational data structure for the subsequent mathematical formulation of the BioExergoEconomics Index (BEC Index).

2.6. Thermodynamic and Environmental Mathematical Formulation

The mathematical formulation of the BioExergoEconomics framework relies on the simultaneous resolution of thermodynamic, economic, and environmental balances for each control volume k (representing either a conventional industrial process or a biological unit) under steady-state conditions.

2.6.1. Thermodynamic Balances

Following the principles of the First and Second Laws of Thermodynamics, the general exergy balance distinguishes between the exergy of fuel ($\dot{E}x_{F,k}$), exergy of product ($\dot{E}x_{P,k}$), exergy associated with physical residual streams ($\dot{E}x_{R,k}$), and the rate of exergy destruction ($\dot{I}_k$) due to internal irreversibilities (entropy generation) [4,5]:

$$\dot{E}x_{F,k} = \dot{E}x_{P,k} + \dot{E}x_{R,k} + \dot{I}_k \quad (1)$$

To quantify the thermodynamic performance of the k-th process, the rational exergy efficiency ($\eta_{ex,k}$) is defined as the ratio of the useful product exergy to the driving fuel exergy:

$$\eta_{ex,k} = \frac{\dot{E}x_{P,k}}{\dot{E}x_{F,k}} = 1 - \frac{\dot{E}x_{R,k} + \dot{I}_k}{\dot{E}x_{F,k}} \quad (2)$$

2.6.2. Exergoeconomic Cost Allocation

The Exergoeconomic assessment applies the Specific Exergy Costing (SPECO) methodology [35]. The general cost balance equation assigns a monetary value to all exergy streams, incorporating the levelized capital investment, operating, and maintenance costs ($\dot{Z}_k$) [6]:

$$\sum \dot{C}_{in,k} + \dot{Z}_k = \sum \dot{C}_{out,k} \quad (3)$$

Expressed in terms of the functional structure (F–P–R) and specific costs per unit of exergy (c), Equation (3) expands to:

$$c_{F,k}\dot{E}x_{F,k} + \dot{Z}_k = c_{P,k}\dot{E}x_{P,k} + c_{R,k}\dot{E}x_{R,k} \quad (4)$$

Under the BioExergoEconomics Circular Economy approach (Stage II), the residual stream ($\dot{R}$) entering a biological process (B) adopts a zero-burden assumption. Consequently, its specific upstream cost is considered zero ($c_{R,in} = 0$), meaning the Exergoeconomic cost of the biologically recovered co-product ($c_{P',B}$) is driven entirely by the bioprocess investment ($\dot{Z}_B$) and its specific operational exergy inputs.

2.6.3. Environmental and Biological Quantification

The environmental performance was evaluated using selected LCA-related indicators, specifically Carbon Footprint (CF) and Water Footprint (WF), to quantify greenhouse-gas emissions and water consumption associated with the proposed system. These indicators were selected according to the scope and objectives of the study and do not represent a comprehensive LCA covering all environmental and health impact categories. These are calculated by multiplying the physical mass ($\dot{m}$) or volume ($\dot{v}$) flow rates by their specific characterization factors (cf and wf) [7,8]:

$$CF_k = \sum \left(\dot{m}_j \cdot cf_j \right) \quad (5)$$

$$WF_k = \sum \left(\dot{v}_j \cdot wf_j \right) \quad (6)$$

For the circular economy integration, the biological recovery efficiency (η_B) must be quantified. This indicator characterizes the physical fraction of the incoming residual stream ($\dot{m}_R$) successfully converted into useful biological co-products ($\dot{m}_{P'}$):

$$\eta_{B,k} = \frac{\dot{m}_{P',k}}{\dot{m}_{R,k}} \tag{7}$$

2.7. Differential BioExergoEconomic Indicators

To quantitatively evaluate the transition from the linear Baseline (LB) system to the Circular Economy (CE) model, the BioExergoEconomics framework introduces a set of comparative differential indicators (Δ). As previously outlined in Table 3, these metrics calculate the net variation across thermodynamic, environmental, and economic dimensions under an equivalent functional unit.

The overall thermodynamic improvement is measured by the reduction in primary fuel exergy consumption ($\Delta\dot{E}x_F$) and the mitigation of total exergy destruction ($\Delta\dot{I}$):

$$\Delta\dot{E}x_F = \sum \dot{E}x_{F,LB} - \sum \dot{E}x_{F,CE} \tag{8}$$

$$\Delta\dot{I} = \sum \dot{I}_{LB} - \sum \dot{I}_{CE} \tag{9}$$

From an environmental perspective, the mitigation of external impacts is quantified by the net reduction in Greenhouse Gas emissions (ΔCF) and the decrease in freshwater depletion (ΔWF):

$$\Delta CF = \sum CF_{LB} - \sum CF_{CE} \tag{10}$$

$$\Delta WF = \sum WF_{LB} - \sum WF_{CE} \tag{11}$$

The economic implications of the circular transition are assessed by quantifying the variation in the specific production cost of the final product ($\Delta\$P$—USD/kg of product) following the integration of biological recovery processes. This indicator measures the economic impact of the proposed integration relative to the baseline production system, identifying potential cost reductions or additional production costs.

$$\Delta\$P = \$P_{LB} - \$P_{CE} \tag{12}$$

Then:

$\Delta\$P > 0$: The integrated system reduces the specific production cost.

$\Delta\$P = 0$: The integration does not change the specific production cost.

$\Delta\$P < 0$: The integration increases the specific production cost.

2.8. The Composite BioExergoEconomics Index (BEC Index)

The BioExergoEconomics Index (BEC) is proposed as a dimensionless sustainability indicator that integrates thermodynamic, environmental, biological, circular, and economic performance into a single metric. Unlike conventional exergy efficiency, which evaluates only thermodynamic performance, the BEC Index simultaneously quantifies the contribution of resource recovery, environmental mitigation, biological valorization, and economic competitiveness.

The general formulation of the proposed index is defined as the weighted sum of its normalized performance factors:

$$BEC = \sum (w_i \cdot \eta_i) \tag{13}$$

where $\sum w_i = 1$ and $0 \leq \text{BEC} \leq 1$. The normalized performance factors are categorized into six specific efficiencies:

- η_{Ex}: Exergy efficiency
- η_{CE}: Circularity efficiency
- η_C: Carbon mitigation efficiency
- η_H: Water conservation efficiency
- η_B: Biological recovery efficiency
- η_E: Economic efficiency

Therefore, the complete BioExergoEconomics Index can be expressed as:

$$BEC = w_1\eta_{Ex} + w_2\eta_{CE} + w_3\eta_C + w_4\eta_H + w_5\eta_B + w_6\eta_E \quad (14)$$

To ensure mathematical consistency across the framework, each efficiency is calculated based on the thermodynamic, physical, and economic parameters defined in the previous stages.

Economic Efficiency (η_E): Evaluates the cost reduction of the final product in the circular model compared to the baseline.

$$\eta_E = 1 - \left(\frac{\$P_{CE}}{\$P_{LB}}\right) \quad (15)$$

Exergy Efficiency (η_{Ex}): Quantifies the rational thermodynamic performance of the overall system.

$$\eta_{Ex} = \frac{\sum \dot{E}x_P}{\sum \dot{E}x_F} \quad (16)$$

Circularity Efficiency (η_{CE}): Measures the physical fraction of the residual streams successfully recovered and re-integrated into the system.

$$\eta_{CE} = \frac{\dot{m}_{Recovered}}{\dot{m}_{Residual}} \quad (17)$$

Carbon Mitigation Efficiency (η_C): Determines the relative reduction in greenhouse gas emissions.

$$\eta_C = 1 - \left(\frac{CF_{CE}}{CF_{LB}}\right) \quad (18)$$

Water Conservation Efficiency (η_H): Determines the relative reduction in freshwater consumption.

$$\eta_H = 1 - \left(\frac{WF_{CE}}{WF_{LB}}\right) \quad (19)$$

Biological Recovery Efficiency (η_B): Evaluates the effectiveness of the biological transformation processes.

$$\eta_B = \frac{\dot{N}_{Recovered}}{\dot{N}_{Residual}} \quad (20)$$

where $\dot{N}$ represents the mass flow rate of recovered nutrients or biologically converted organic matter. Depending on the specific application, this parameter may be expressed using recovered protein, Chemical Oxygen Demand (COD) removal rates, nutrient recovery, or algal biomass production. In the present study, the highest weighting factor was assigned to Exergy ($w_1 = 0.25$) because it provides a direct measure of the quality and effective utilization of energy and material resources, including the identification of thermodynamic inefficiencies and irreversibilities. The relevance of exergy-based indicators for assessing efficiency and sustainability has been extensively discussed in the energy and sustainability literature.

Carbon Mitigation ($w_2 = 0.20$) was assigned the second-highest weight because reducing greenhouse-gas emissions represents a primary environmental objective of circular and resource-efficient agro-industrial systems. Circularity ($w_3 = 0.15$), Water Conservation

($w_4 = 0.15$), and Biological Recovery ($w_5 = 0.15$) were assigned equal weights to represent the three complementary resource-management dimensions of the proposed framework: material recirculation, water-resource efficiency, and biological valorization, respectively. Finally, Economic Performance ($w_6 = 0.10$) was included as an essential enabling dimension, but with a lower weight because the proposed index is primarily intended to characterize thermodynamic, environmental, and circular performance rather than economic profitability alone.

The operational priority of the proposed framework is normalized for weighting vector:

$$\sum_{i=1}^{6} w_i = w_1 + w_2 + w_3 + w_4 + w_5 + w_6 = 1.00 \quad (21)$$

The methodology establishes that these weights may be adjusted according to expert judgment, multi-criteria decision analysis (MCDA), or specific sensitivity analyses, depending on the regional context.

Based on the calculated BEC score, a proposed four-level performance scale is used to facilitate the interpretation of the normalized circular BioExergoEconomic performance of the evaluated system:

- $BEC < 0.40$: Low performance
- $0.40 \leq BEC < 0.60$: Moderate performance
- $0.60 \leq BEC < 0.80$: High performance
- $BEC \geq 0.80$: Excellent performance

These ranges are proposed as interpretive performance bands within the present BEC framework. The composite indicators are sensitive to methodological choices such as normalization, weighting, aggregation, and classification criteria, the BEC score is reported together with its individual component indicators. The proposed classification is therefore intended to facilitate comparison and communication of results rather than to establish absolute limits of sustainability.

3. Case Study: Beef Production Assessment

3.1. System Boundaries and Functional Unit

To validate the proposed BioExergoEconomics framework, a comparative case study was conducted using beef production as a representative and resource-intensive agro-industrial system. The assessment established 1 kg of beef as the common functional unit, enabling a direct and standardized comparison between a conventional production chain and an alternative circular bioeconomy model under strictly equivalent system boundaries. Two production scenarios were evaluated using the comprehensive framework developed in Section 2: a Conventional Baseline System (LB) and a Circular BioExergoEconomics System (CE).

The design of both scenarios is grounded in specific methodological assumptions chosen to illustrate contrasting production paradigms. For the Baseline System (LB), the underlying assumption is a highly intensive industrial feedlot model. This assumes that all agricultural feed (maize, alfalfa, etc.) is cultivated externally using conventional synthetic inputs, processed in industrial mills, and transported to the farm. This scenario was purposefully designed to represent an extreme, fully linear resource-depletion boundary. Conversely, the Circular System (CE) assumes the localized availability of wet brewery spent grain and the operational feasibility of integrating microalgae cultivation within or near the livestock facility. While recognizing that real-world agricultural systems operate on a diverse spectrum between these two models, these specific assumptions were selected to create a clear analytical space. This allows the BioExergoEconomics framework to rigorously

test and quantify the thermodynamic and environmental trade-offs when shifting from a purely linear extraction model to localized circular regeneration.

3.2. Scenario I: Conventional Production System (Baseline System, LB)

The first scenario corresponds to a reference configuration of conventional livestock production, which considers the stages of agricultural production, food processing, livestock feeding, and final meat processing, as well as the associated material and energy flows. This configuration serves as the baseline for evaluating, through the BioExergoEconomics methodology, the changes resulting from the incorporation of circular economy strategies. In this setup, animal feed formulation relies heavily on conventional agricultural commodities, primarily maize, alfalfa, barley, oats, and soybean meal. It is important to note that this baseline models a specialized industrial supply chain in which agricultural crop production and livestock rearing are physically separate operations. Therefore, harvested crops must be transported to centralized industrial fodder mills for formulation and processing, and the finished feed is subsequently transported to specialized livestock feedlots. Consequently, the system boundaries encompass all life-cycle stages associated with feed production: land preparation, crop cultivation, intensive irrigation, synthetic fertilization, harvesting, transportation, feed manufacturing (milling and pelletizing), livestock feeding, and final meat processing. Because these conventional agricultural inputs require substantial quantities of freshwater, fossil fuels, and electricity, this scenario establishes the Baseline System (LB) for the comparative assessment.

The comprehensive Life Cycle Inventory presented in Figure 3 highlights the highly resource-intensive nature of conventional linear agriculture. The production of standard animal feed, relying heavily on the cultivation of maize and alfalfa, inherently demands massive volumes of freshwater—totaling approximately 22.0 m^3 per kilogram of beef—alongside significant fossil diesel inputs for heavy machinery, tillage, and transport. Within this conventional boundary, the agricultural stage is the primary driver of freshwater depletion (blue and green water footprints) and of abiotic resource consumption. Subsequently, the livestock farming stage dominates the system's global carbon footprint (1.39 kg CO_2e per kg of beef) primarily due to direct biogenic emissions, particularly enteric methane (CH_4) from rumination and uncontrolled emissions from manure management. Furthermore, this linear configuration generates approximately 12.0 kg of physical agricultural residues (such as maize and oat stover) per kilogram of final meat product. In this modeled baseline, advanced biological resource recovery nodes are not integrated into the system boundaries, meaning these material flows predominantly follow a linear extraction-to-disposal trajectory. While in actual agricultural practice, such as in European mixed-farming systems, these residues are frequently utilized for basic management like soil incorporation or animal bedding, within the strict boundaries of this baseline model, unmitigated gaseous emissions and unrecovered physical residues are treated as environmental burdens and a loss of potentially useful secondary resources. This establishes a clear methodological contrast with advanced bio-circular valorization technologies.

3.3. Scenario II: Circular BioExergoEconomics System

The second scenario integrates Circular Economy, biological engineering, and environmental mitigation through the valorization of brewery spent grain (BSG) as an illustrative case study to assess the potential effects of selected resource-valorization strategies on the overall system performance. The selection of BSG is based on its potential for use as animal feed and as a source of nutrients for microalgae cultivation [36–39]. This scenario makes it possible to illustrate the integration of different resource recovery processes through the BioExergoEconomics methodology, although it is not intended to represent all circular

livestock production alternatives. The recovered solid fraction is dried and pelletized to produce animal feed, while the nutrient-rich liquid fraction is used for microalgae cultivation. The harvested microalgae are reincorporated into the feed formulation as a functional ingredient with the potential to reduce enteric methane emissions. The system also incorporates water recovery, nutrient recycling, and integral biomass valorization.

The operational shift detailed in Figure 4 demonstrates a profound reconfiguration of the production system, highlighting a clear engineering trade-off between localized energy demand and systemic environmental mitigation. By replacing conventional agricultural crops with recovered wet brewery spent grain (BSG) and cultivated microalgae, the circular system entirely bypasses the massive water and land requirements traditionally associated with feed cultivation. The inventory reveals that the introduction of specialized solid-liquid separation and algae cultivation equipment inherently increases the electricity demand, shifting from 1.86 kWh in the baseline to 2.75 kWh. However, this electrical penalty enables powerful environmental mitigation mechanisms: microalgae cultivation not only operates primarily on residual water streams but also biologically captures ambient CO_2 during its growth phase. More critically, when this high-protein algal biomass is reincorporated into the livestock feed as a functional bio-ingredient, it actively modifies ruminal fermentation, suppressing enteric CH_4 generation by up to 60%. Consequently, the systemic carbon footprint drops to 0.68 kg CO_2e, and the freshwater footprint is practically eliminated (0.14 m^3). This process flow demonstrates how biological integration transforms physical waste streams into high-value secondary resources, setting the foundation for the rigorous thermodynamic and Exergoeconomic evaluation detailed in the subsequent section.

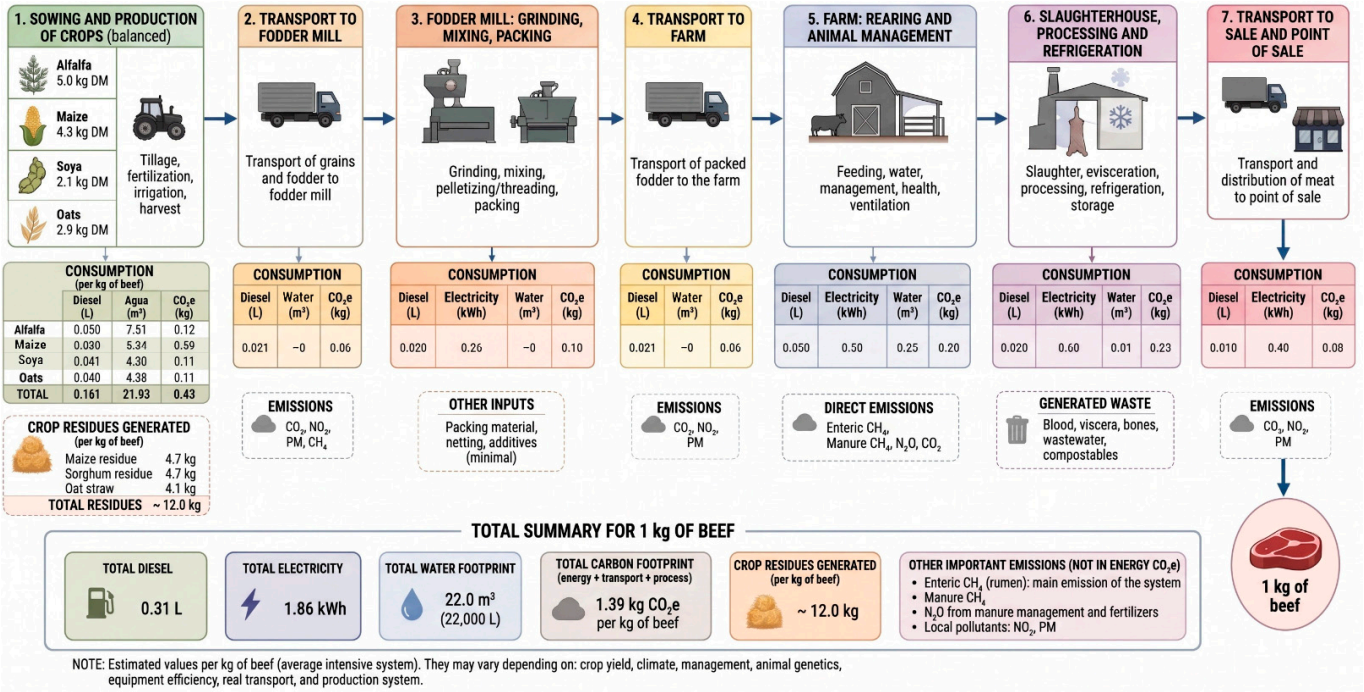


Figure 3. Detailed Process Flow and Life Cycle Inventory for the Conventional Baseline System (LB). The infographic details the stage-by-stage consumption of diesel, electricity, and water, alongside the generation of greenhouse gas emissions for 1 kg of beef.

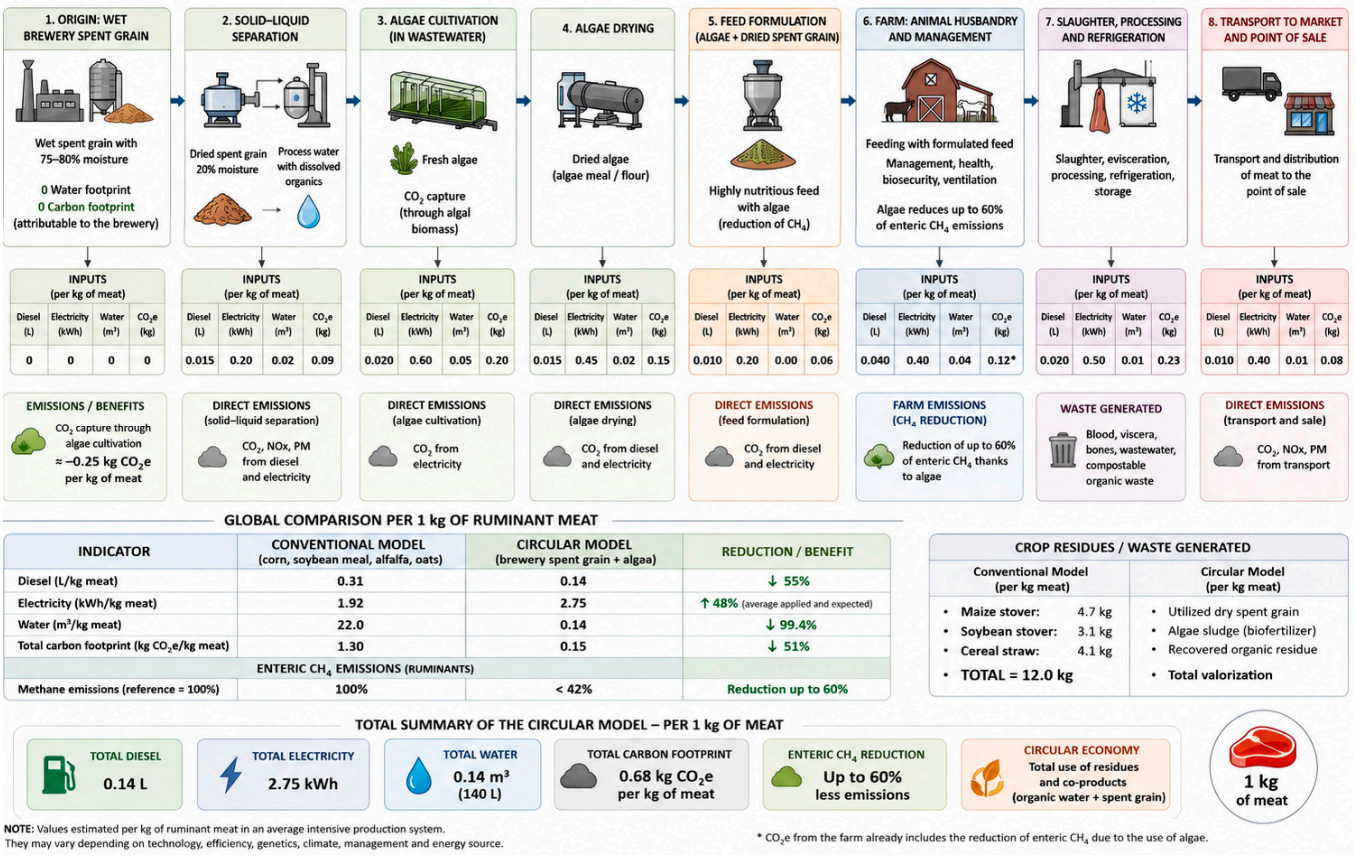


Figure 4. Detailed Process Flow and Life Cycle Inventory for the Circular BioExergoEconomics System (CE). The infographic quantifies the operational shifts, demonstrating how biological processing eliminates agricultural water demand and mitigates enteric methane.

4. Results

4.1. Thermodynamic and Environmental Flow Analysis

The comparative assessment of the conventional and circular production systems fundamentally relies on analyzing how exergy and physical mass flow through the established control volumes. Figure 5 illustrates the thermodynamic and environmental topology of the Baseline System (LB) through the F-P-R-I functional structure. In this linear configuration, external fuel exergy ($\dot{E}x_F$) is continuously consumed at each sequential stage—particularly during crop cultivation and feed milling—to drive the exergy of the product ($\dot{E}x_P$) forward. Due to the absence of recirculation loops, the thermodynamic irreversibilities ($\dot{I}$) generated by entropy production at each node represent absolute exergy losses. Concurrently, the physical interactions with the reference environment are strictly unidirectional and detrimental: massive volumes of freshwater (WF) are extracted primarily during the agricultural phase (21.93 m^3), and significant carbon emissions (CF) are rejected into the ecosphere, predominantly as enteric methane during the livestock farming stage (0.28 kg CO_2e). In this baseline model, residual streams are treated as waste, finalizing their life cycle without any secondary exergetic valorization.

In contrast, the transition to the Circular BioExergoEconomics System (CE) structurally reconfigures these flows into the advanced F-P-R-I-B model, as depicted in Figure 6. The fundamental thermodynamic shift occurs at the system boundary: the wet brewery spent grain (BSG) enters the control volume under a "zero-burden" assumption ($\dot{R}_{in}$), effectively displacing the exergy destruction and environmental footprint previously associated with conventional agriculture. The integration of the biological nodes (B_1 for solid-liquid separation and B_2 for microalgae cultivation) introduces a new dynamic. While

these biological transformations inherently generate their own thermodynamic irreversibilities ($\dot{I}_{B1}$, $\dot{I}_{B2}$) and require localized external electrical exergy ($\dot{E}x_F$), they act as powerful regenerative interfaces.

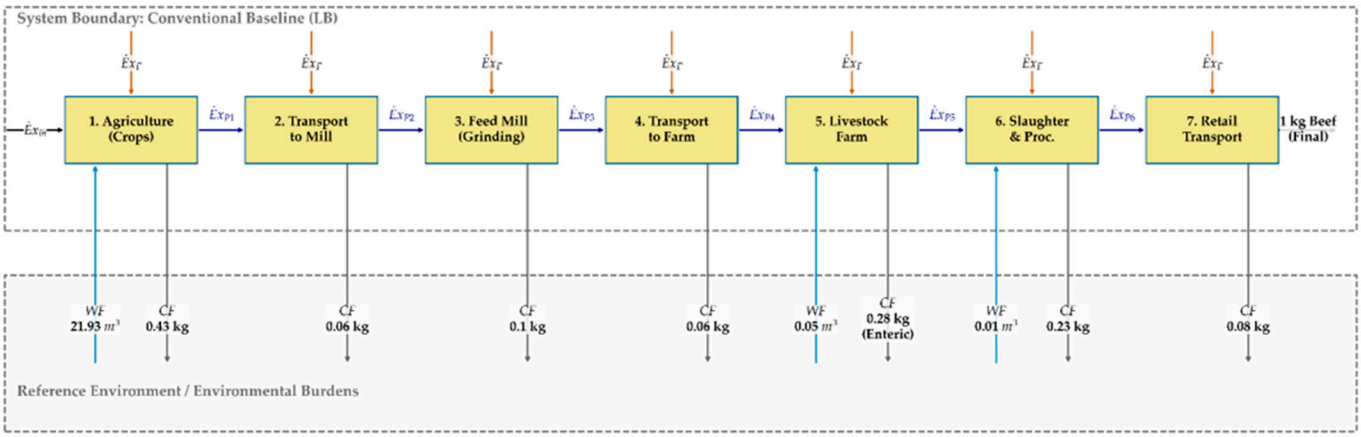


Figure 5. Functional Structure (F-P-R-I) of the Conventional Baseline System (LB). This diagram translates the physical inventory from Figure 3 into rigorous thermodynamic exergy flows ($\dot{E}x_F$) and environmental burdens (WF,CF) interacting with the reference environment.

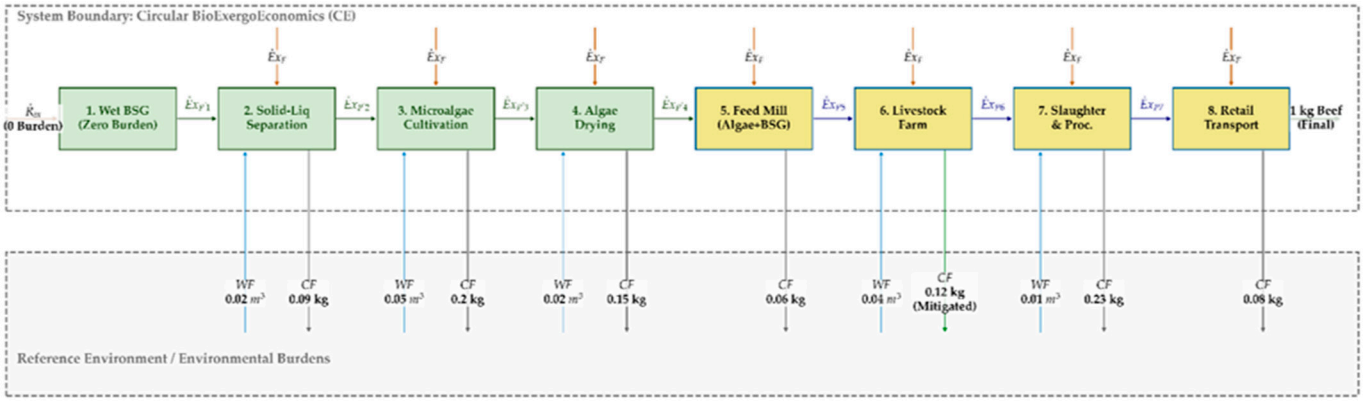


Figure 6. Functional Structure (F-P-R-I-B) of the Circular System. The diagram mathematically represents the "zero-burden" entry of residual biomass ($\dot{R}_{in}$) and the biological nodes (B_1,B_2) that actively mitigate the environmental footprints.

Specifically, the microalgae node (B_2) absorbs the organic load from the residual water, capturing CO_2 and converting physical waste into recovered product exergy ($\dot{E}x_{P'3}$). As shown in Figure 6, when this biologically recovered flow is incorporated into the downstream livestock feed, it actively mitigates the final carbon footprint by suppressing ruminal methanogenesis. To quantitatively visualize this systemic shift, Figure 7 presents a normalized comparative analysis of the principal thermodynamic and environmental indicators derived from the operational inventory.

As observed in Figure 7, normalizing the Baseline System to 100% reveals the magnitude of the operational trade-offs. The circular configuration achieves extraordinary environmental mitigation, reducing the water footprint by 99.4% (0.14 m^3) and the total carbon footprint by 51.1% (0.68 kg CO_2e). Thermodynamically, fossil diesel consumption is reduced by 54.8%. However, these reductions are achieved at the expense of a 47.8% increase in electricity demand (from 1.86 kWh to 2.75 kWh), driven entirely by the mechanical and biological processing requirements of the algae and BSG nodes. Despite this localized electrical penalty, the systemic thermodynamic efficiency is enhanced, evidenced by a 17.6% net reduction in the total combined Energy and Exergy demand (decreasing

from 18,476 kJ to 15,220 kJ). This confirms that the biological nodes successfully recover more useful exergy than the electrical fuel exergy they consume to operate.

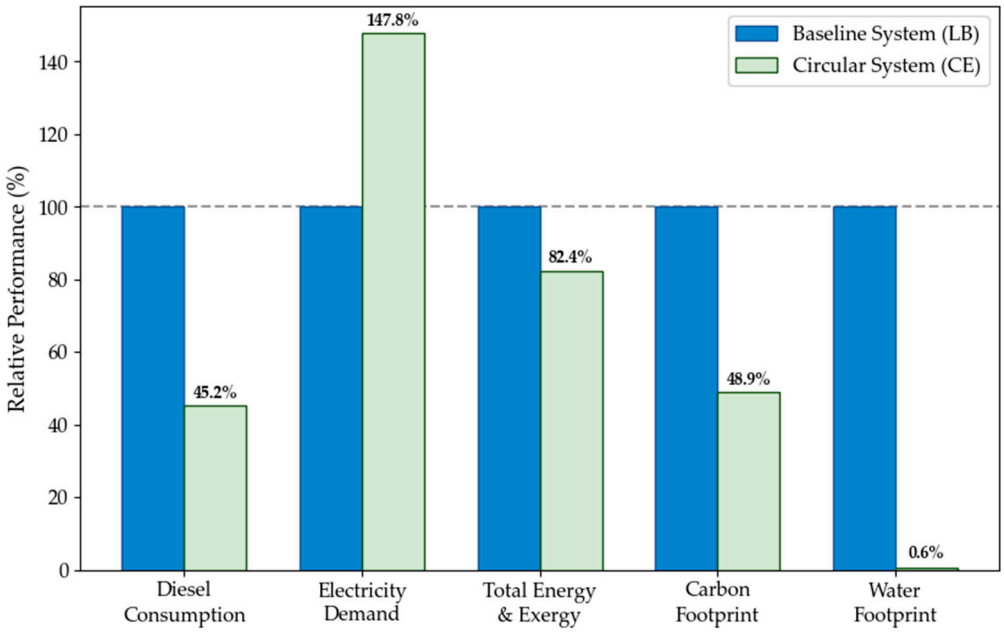


Figure 7. Normalized comparison of thermodynamic and environmental indicators between the Baseline and Circular systems.

4.2. Exergetic and Environmental Trade-Off Analysis

A critical aspect of transitioning from linear to circular bioeconomy systems is the emergence of operational trade-offs, in which improvements in environmental indicators may entail localized thermodynamic penalties.As illustrated in Figure 7, the Circular System (CE) imposes a significant 47.8% increase in specific electricity demand, rising from 1.86 kWh per kilogram of beef in the baseline to 2.75 kWh per kilogram of beef. Thermodynamically, electricity represents pure work (an exergy-to-energy ratio of 1.0); thus, this increment signifies a substantial local injection of high-quality exergy. This penalty is directly attributable to the mechanical and biological processing requirements of the regenerative nodes (B_1 and B_2), specifically the energy-intensive solid-liquid separation of the brewery spent grain, the pumping operations for microalgae cultivation, and the subsequent drying processes.

However, evaluating this localized electrical penalty in isolation misrepresents the systemic thermodynamic performance. The BioExergoEconomics framework dictates that the entire supply chain must be analyzed under the equivalent functional unit. The analysis reveals that investing high-quality electrical exergy in the biological nodes serves as an "exergetic lever". By utilizing recovered residual biomass (with a zero-burden assumption) instead of cultivating primary agricultural crops (such as maize and alfalfa), the circular model completely avoids the massive upstream exergy destruction inherent in conventional agriculture. Specifically, reliance on heavy agricultural machinery, synthetic fertilizers, and extensive irrigation pumping is bypassed, resulting in a 54.8% reduction in fossil diesel consumption (from 0.31 L to 0.14 L).

Consequently, the localized increase in electrical exergy is largely offset by upstream savings, resulting in a net 17.6% reduction in the total integrated Energy and Exergy demand, from 18,476 kJ in the baseline to 15,220 kJ in the circular configuration. This net decrease confirms that the biological nodes operate with sufficient thermodynamic efficiency to justify their electrical requirements. Furthermore, this optimized thermodynamic routing

directly translates into profound environmental mitigation. The electrical exergy invested in the microalgae node (B_2) enables a biological transformation that practically eliminates the freshwater footprint, achieving a 99.4% reduction (0.14 m^3 vs. 22.0 m^3) by shifting from irrigation-dependent crops to residual water utilization. Simultaneously, the functional properties of the recovered algal biomass actively suppress ruminal methanogenesis, driving a 51.1% reduction in the final carbon footprint (0.68 kg CO_2e vs. 1.39 kg CO_2e). These results are directly related to the F-P-R-I-B structure, where exergy demand is associated with the fuel (F) and irreversibility (I) components. The reduction in the carbon footprint is linked to resource recovery (R) and biological recovery (B), while the reduction in the water footprint is associated with the process (P). For its part, circularity is represented by resource recovery (R).

Therefore, the trade-off analysis demonstrates that the BioExergoEconomics circular model exchanges a moderate, localized increase in electrical exergy for substantial system-level reductions in primary resource depletion and environmental burdens by avoiding energy- and resource-intensive upstream agricultural processes.

4.3. Breakdown of the BioExergoEconomics Index (BEC Index)

The ultimate objective of the proposed framework is to translate the multidimensional operational trade-offs into a unified, comparative decision-support metric. As detailed in the methodology, the BioExergoEconomics Index (BEC) integrates six normalized performance factors weighted according to their systemic relevance. For the Circular BioExergoEconomics System (CE), the application of Equation 13 resulted in a global BEC Index of 0.55. This BEC value places the circular system within the moderate sustainability category ($0.40 \leq \text{BEC} < 0.60$), which indicates that the combined improvements in circularity, biological recovery, and environmental performance partially offset the lower energy and economic performance of the system. To rigorously understand this result, it is necessary to deconstruct the index into its individual thermodynamic, environmental, and economic efficiencies. Figure 8 illustrates the multidimensional performance of the circular system through a radar chart of the six normalized BEC components.

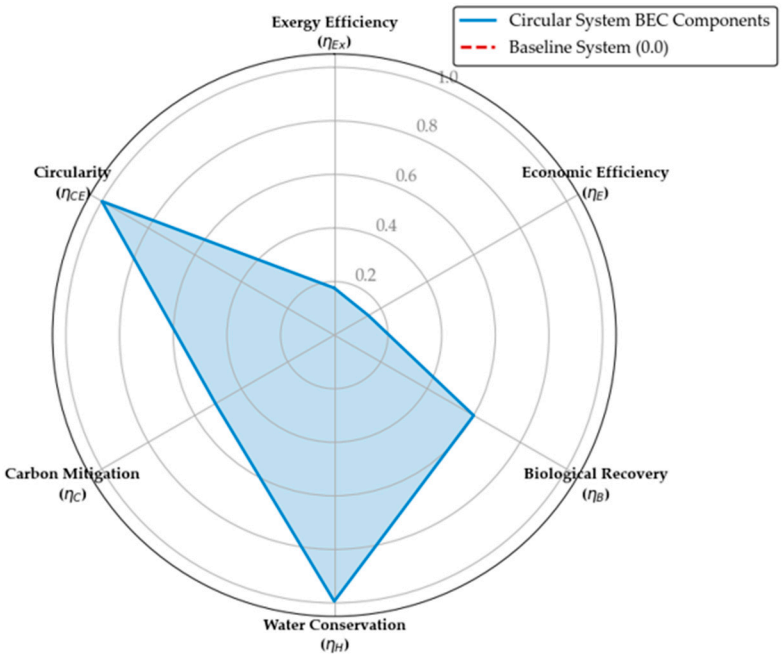


Figure 8. Multidimensional radar chart of the BioExergoEconomics Index components, illustrating the robust environmental performance against thermodynamic and economic constraints.

As depicted in Figure 8, the circular system exhibits extraordinary performance in its environmental and circular dimensions. The Circularity Efficiency (η_{CE}) reaches the maximum theoretical value of 1.000, reflecting the complete utilization of the residual brewery spent grain with a zero-waste-to-landfill approach. Similarly, the Water Conservation Efficiency ($\eta_H = 0.994$) dominates the environmental axis due to the near-total elimination of primary agricultural irrigation. The Biological Recovery ($\eta_B = 0.600$) and Carbon Mitigation ($\eta_C = 0.511$) efficiencies further pull the index outward, validating the effectiveness of the microalgae integration in suppressing ruminal methanogenesis and recycling nutrients.

Crucially, the BEC Index demonstrates its analytical robustness by preventing these high environmental scores from artificially inflating the overall sustainability assessment. The framework inherently penalizes thermodynamic irreversibilities and capital constraints. The Exergy Efficiency ($\eta_{Ex} = 0.176$) remains inherently low, reflecting the fundamental thermodynamic reality of biological and agricultural systems, where large portions of solar and chemical exergy are inevitably destroyed as entropy during biomass conversion and animal metabolism. Furthermore, the Economic Efficiency ($\eta_E = 0.147$) indicates a positive but modest 14.7% reduction in final production costs, acknowledging the high capital expenditure required for algae cultivation and biomass processing equipment.

By mathematically synthesizing these dimensions ($0.25\eta_{Ex} + 0.15\eta_{CE} + 0.20\eta_C + 0.15\eta_H + 0.15\eta_B + 0.10\eta_E$), the final BEC score of 0.55 classifies the evaluated bio-circular integration within the "Moderate sustainability" threshold ($0.40 \leq BEC < 0.60$). Compared with the conventional baseline, which inherently scores a BEC of 0.00 (as it provides no differential mitigation, recovery, or circularity), this score represents a substantial, quantitatively verified progression toward a functional circular bioeconomy.

4.4. Sensitivity Analysis of Weighting Factors

The formulation of the BioExergoEconomics Index incorporates a weighting vector (w_1 to w_6) to reflect specific regional or operational priorities. The baseline assessment, which resulted in a BEC Index of 0.55, utilized a balanced weighting scheme designed for general agro-industrial evaluation. However, to validate the robustness of the proposed framework and demonstrate its applicability as a decision-support tool, a sensitivity analysis was conducted by evaluating the system under three distinct prioritization scenarios.

The scenarios modify the weight distribution to reflect different industrial constraints:

1. Balanced Baseline Scenario: The original distribution ($w_1 = 0.25$, $w_2 = 0.15$, $w_3 = 0.20$, $w_4 = 0.15$, $w_5 = 0.15$, $w_6 = 0.10$).
2. Techno-Economic Priority Scenario: Simulates a strict industrial environment where thermodynamic efficiency and cost reduction are paramount. Weights are heavily skewed toward Exergy ($w_1 = 0.40$) and Economic Efficiency ($w_6 = 0.30$), with environmental factors reduced (w_2,w_3,w_4,$w_5 = 0.075$ each).
3. Environmental and Climate Priority Scenario: Simulates a highly regulated framework focused on climate mitigation and resource conservation. Weights are heavily skewed toward Carbon Mitigation ($w_3 = 0.40$) and Water Conservation ($w_4 = 0.30$), with technical factors reduced (w_1,w_2,w_5,$w_6 = 0.075$ each).

As illustrated in Figure 9, applying Equation (14) to these scenarios reveals the conditional sustainability of the bio-circular transition. Under the Environmental Priority Scenario, the system's exceptional capacity to mitigate enteric methane and eliminate freshwater demand pushes the overall BEC Index to 0.65 (High Sustainability). This supports the conclusion that in regions with severe water scarcity or carbon pricing mechanisms, biological integration is highly justified. Conversely, under the strict Techno-Economic Priority Scenario, the BEC Index drops to 0.35 (Low Sustainability). In this context, the heavy

penalization of the low exergy efficiency ($\eta_{Ex} = 0.176$) intrinsic to biomass conversion and the modest economic cost reduction ($\eta_E = 0.147$) overshadow the environmental benefits.

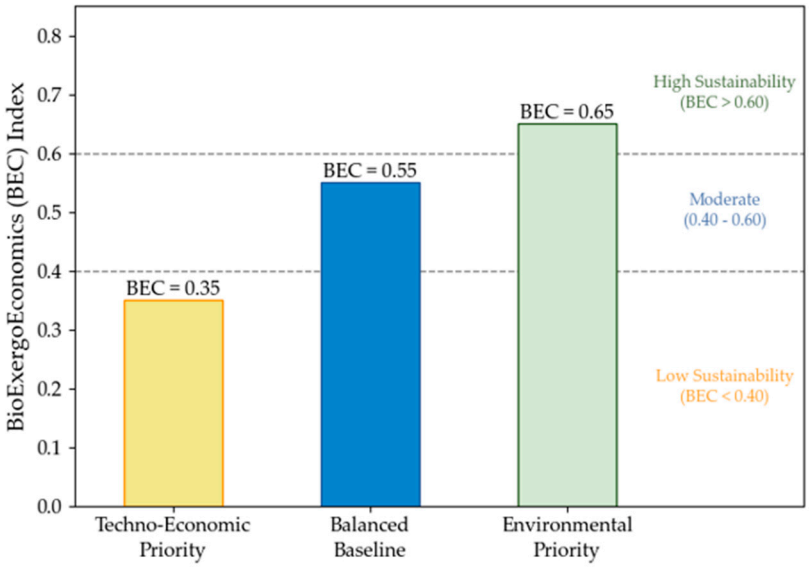


Figure 9. Sensitivity analysis of the BEC Index across different prioritization scenarios.

To complement the prioritization analysis and further evaluate the robustness of the BEC Index, a sensitivity analysis was performed (Figure 10), in which each parameter was modified individually while the others were held constant. The four parameters analyzed were electricity demand, carbon footprint, biological recovery, and production cost, each varied by −20%, −10%, 0%, +10%, and +20%. The results show that electricity demand had the greatest influence on the BEC, with values ranging from 0.5233 to 0.5768. The carbon footprint had the second most important effect, with values between 0.5304 and 0.5696. Production costs, in turn, ranged from 0.5330 to 0.5671. Biological recovery showed an opposite trend (with values between 0.5680 and 0.5320), since an increase in this parameter improves system performance. Despite these variations, all scenarios remained within the moderate sustainability interval defined for the BEC index.

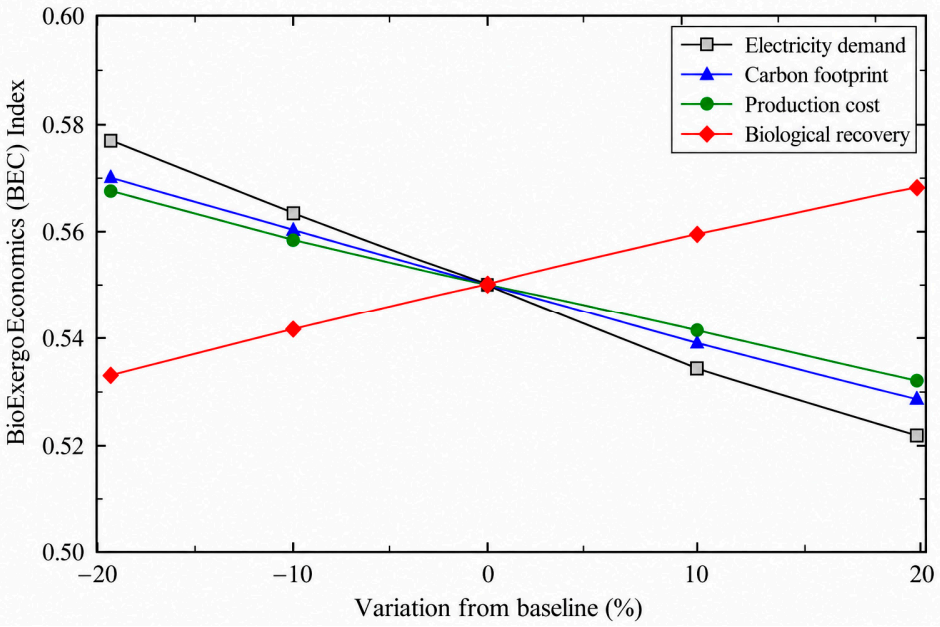


Figure 10. Sensitivity analysis of the BioExergoEconomics (BEC) Index to variations in key parameters.

This sensitivity behavior perfectly encapsulates the methodological necessity of BioExergoEconomics, as identified in the scientific gap (Table 1). If the proposed circular system were evaluated solely through conventional Exergoeconomics (approximating Scenario 2), it would likely be rejected due to high thermodynamic irreversibilities and capital costs. By providing a multidimensional framework, the BEC Index mathematically demonstrates that the system's true value lies in its ability to leverage moderate investments in electrical exergy to achieve profound systemic environmental mitigation.

4.5. Limitations of the BioExergoEconomics Framework

Despite the advantages of the proposed framework, it is important to highlight the limits of the current model. Integrated methods are inherently data-intensive, and their accuracy relies heavily on high-resolution inventory data on mass and energy flows, which can complicate their application and interpretation [28]. To maintain operability, the model relies on a selected set of representative indicators rather than an exhaustive list. For example, the LCA component primarily focuses on carbon and water footprints, whereas detailed, separate environmental LCA models can cover a wider array of impact categories, such as terrestrial ecotoxicity or ozone depletion [27]. Similarly, the circular economy indicator within the BEC Index focuses strictly on physical material circularity and lacks broader systemic circularity metrics [29]. Finally, the current model does not integrate the social aspects of sustainability—such as public acceptance, social equity, human toxicity, and the level of operational disruption—which are critical elements that often require specialized multi-criteria decision-making frameworks for adequate assessment [30]. Additionally, while this study uses 1 kg of beef as the standard functional unit to ensure broad comparability with the existing literature, this mass-based metric does not capture the product's nutritional value. Future assessments could adopt a nutrition-based functional unit, such as 1 kg of protein, to better reflect the true ecological, energetic, and economic efficiency of food production systems. These constraints represent key areas for future refinement to further enrich the framework without compromising its thermodynamic foundation. Likewise, the baseline scenario does not contemplate other alternatives for waste utilization, such as the commercialization of straw or the production of biogas from manure, which could influence the comparative results. Nevertheless, this limitation corresponds to the case study and not to the formulation of the BEC index, whose application allows the evaluation of different production configurations.

5. Conclusions

This study formally introduces BioExergoEconomics, an advanced thermodynamic and environmental framework designed to overcome the limitations of isolated sustainability assessments. By structurally integrating Exergy Analysis, Exergoeconomics, Life Cycle Assessment, and Circular Economy principles, the methodology enables the simultaneous evaluation of resource degradation, economic cost, and environmental mitigation under a rigorously equivalent functional unit. The application of this framework to a bio-circular livestock production system yielded several critical quantitative and methodological conclusions:

The transition from a linear agricultural baseline to a circular bioeconomy model demonstrated a profound operational trade-off. While the integration of biological nodes (microalgae cultivation and biomass processing) resulted in a localized 47.8% increase in electricity consumption, it completely displaced the substantial thermodynamic and physical burdens of conventional crop cultivation. This structural shift resulted in a net 17.6% reduction in total integrated Energy and Exergy demand (from 18,476 kJ to 15,220 kJ), demonstrating that biological resource recovery operates with sufficient systemic efficiency to justify its local electrical penalties.

The substitution of primary feed with zero-burden residual brewery biomass, coupled with the functional properties of cultivated microalgae, fundamentally altered the system's environmental interaction. The circular configuration practically eliminated dependence on agricultural water (achieving a 99.4% reduction in water footprint) and actively suppressed ruminal methanogenesis, resulting in a 51.1% reduction in the final carbon footprint.

The formulation of the BEC Index successfully translated these multidimensional trade-offs into a comparative, dimensionless metric. By mathematically penalizing thermodynamic irreversibilities while rewarding environmental conservation and biological recovery, the circular system achieved a BEC score of 0.55, classifying it within a moderate-to-high sustainability threshold.

The sensitivity analysis validated the BEC Index as a highly adaptable decision-support tool. Under environmental prioritization, the index rose to 0.65, highlighting the system's superior climate mitigation capabilities. Conversely, under strict techno-economic constraints, the score dropped to 0.35, accurately reflecting the inherent thermodynamic challenges and capital costs associated with biomass conversion technologies.

Ultimately, BioExergoEconomics represents a necessary evolution in engineering assessment methodologies. It demonstrates that evaluating bio-circular systems through thermodynamic efficiency alone is insufficient, as it fails to capture the immense value of environmental mitigation and biological valorization. Future research will focus on optimizing the BEC Index weighting factors through multi-criteria decision analysis (MCDA) and extending this unified framework to highly coupled thermochemical conversion systems, such as advanced biomass gasification, steam methane reforming, and solid oxide fuel cell networks.

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Abbreviations

The following abbreviations are used in this manuscript:

BEC	BioExergoEconomic
BSG	Brewery Spent Grain
CE	Circular Economy
CF	Carbon Footprint
COD	Chemical Oxygen Demand
LB	Baseline System
LCA	Life Cycle Assessment
MCDA	Multi-criteria Decision Analysis
SPECO	Specific Exergy Costing
WF	Water footprint

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