

Article

Experimental Study of Buoyancy of Spark-Generated Bubbles Oscillating in Water

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Abstract: The buoyancy of radially oscillating spark-generated bubbles is studied experimentally. Bubble sizes, defined as the maximum bubble radius, range from 26.6 to 52.1 mm, and the bubbles oscillate at a hydrostatic pressure of 127 kPa in a large expanse of liquid. We found that the position of these bubbles is relatively stable during the first two oscillations. They move upwards only in short time intervals when their size is, due to contraction, close to the minimum volume. The vertical movement, therefore, takes place in the form of steps. The relative heights of these steps, defined as the ratio of step heights to bubble size, increase with bubble size and range from 0.15 to 0.29. No significant deformations in the spherical shape of bubbles are observed during the first two oscillations.

Keywords: spark-generated bubbles; buoyancy of oscillating bubbles

1. Introduction

Bubbles oscillating in liquids may either act harmfully, e.g., as in the case of cavitation in hydraulic machinery [1], or they can perform useful functions, e.g., in ultrasonic cleaners [1]. The bubbles that are formed in these technical applications are never single bubbles oscillating in a predetermined way but always appear in large bubble fields and randomly oscillate in a certain sense. These bubbles are also usually small. For these reasons, such bubbles are very difficult to study. In order to better understand the physical and chemical processes taking place in bubbles, thus helping to prevent their harmful action or to support their useful functions, experiments with bubbles generated by exploding wires [2–5], pulsed corona discharges [6–9] and spark discharges through the small gap between electrodes [10–18] are often used. One reason for using the bubbles generated through these techniques in model experiments is, among others, that these bubbles, compared to real cavitation bubbles, have a larger size and can be generated in a relatively controllable manner. Therefore, they are more amenable to experimental studies.

According to Archimedes' principle, all bubbles in liquids are subjected to buoyancy force, and this force grows with bubble size. Therefore, because of their greater buoyancy, larger bubbles may behave somewhat differently than smaller bubbles. It is necessary to verify how significant buoyancy is in the behavior of larger bubbles. In [19], the size of bubbles, at which buoyancy starts to be important, was theoretically determined. In this work, the variation in buoyancy with bubble size is studied experimentally.

Because buoyancy increases with bubble size, it was first studied in connection with the large bubbles generated by underwater chemical explosions [20–25]. However, in these works, the buoyancy of bubbles of only one size was usually described. Therefore, it is not possible to determine the variation in buoyancy with bubble size from the published data. Also, the hydrostatic pressure where the bubbles are located usually differs significantly



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from the atmospheric pressure. And the distances of the oscillating bubbles from free or hard surfaces are not always sufficient to exclude the influence of the surfaces on the bubble motion. These factors will be discussed in more detail in Sections 3 and 6.

The natural bubble shape is spherical (or almost spherical). However, due to the combination of vertical bubble migration with radial bubble oscillations, the spherical shape can be distorted. Such deformations have been observed [3,4,26]. In the experiments analyzed here, in the time interval considered, no distortion in the spherical shape was observed.

This study is organized as follows. In Section 2, the radial oscillatory motion of the bubble and the vertical migration of the bubble are briefly described in general, and the notation used in this paper is introduced. Section 3 reviews the available literature [3,10,18,22–25], studying the buoyancy of freely oscillating bubbles. In Section 4, the experimental set-up used in this work is shortly described; in Section 5, the experimental results are presented; and in Section 6, the experimental data that were obtained are discussed.

2. Bubble Motions and the Notations Used

The bubbles studied in this work perform two basic motions. The first is an oscillatory radial (breathing) motion, and the second is vertical buoyancy migration. The resulting movement of the bubbles is then given by the superposition of these two elementary motions. The radial motion is stable in itself, and if the velocity of the vertical movement is small, the spherical bubble shape is not distorted. However, under certain conditions, the superposition of the two motions leads to bubble shape deformations.

First, the oscillatory motion of a spark-generated bubble will be briefly described. Let us denote an instant when the liquid breakdown between the electrodes occurs as t_0 . At time t_0 , the bubble starts growing explosively and radiating luminescence and pressure waves intensively. As shown in Section 5, the bubbles generated in this work are almost spherical. Therefore, the motion of the bubble wall is described by the bubble wall radius $R(t)$. At time t_1 , the explosively growing spherical bubble attains its first maximum radius R_{M1} . Then, the bubble begins contracting, and at time t_{c1} , it contracts to its first minimum volume. Though little is known about the actual bubble shape at time t_{c1} , for simplicity, it is assumed that it is a sphere of radius R_{m1} . Then, the bubble starts expanding again, and at time t_2 , it attains its second maximum radius R_{M2} . A second contraction follows, and at time t_{c2} , the bubble achieves its second minimum volume (a sphere of radius R_{m2}). After time t_{c2} , the bubble can perform several further oscillations, but these are already out of the scope of the present work. The interval (t_0, t_{c1}) represents the time of the first bubble oscillation T_{o1} , and the interval (t_{c1}, t_{c2}) the time of the second bubble oscillation T_{o2} . When the bubble attains its first maximum radius R_{M1} , the pressure at the bubble wall is P_{m1} ; when the bubble wall during the first contraction phase attains the equilibrium radius R_e , the pressure at the bubble wall is equal to the hydrostatic pressure p_∞ ; and when the bubble contracts to R_{m1} , the pressure at the bubble wall is P_{M1} .

The pressure wave $p(t)$ radiated by the oscillating bubble and recorded at a distance r_h can be divided into several pressure pulses. The first is a pulse $p_0(t)$ radiated by the bubble during the growth phase (t_0, t_1) , which is followed by a pulse $p_1(t)$ emitted in the interval (t_1, t_2) , and finally, a pulse $p_2(t)$ emitted in the interval (t_2, t_3) . Here, t_3 is the time when the oscillating bubble attains its third maximum volume. Three important instants can be determined in the pressure record $p(t)$ with a precision $\pm 0.1 \mu\text{s}$. These are the time t_0 when the pressure pulse $p_0(t)$ starts explosively growing and the times t_{p1} and t_{p2} when the pressure pulses $p_1(t)$ and $p_2(t)$ attain their peak values p_{p1} and p_{p2} . Due to the propagation of the pressure wave $p(t)$ in the liquid, these times are delayed behind the bubble wall motion, which will be taken into account. In this work, it will be assumed that the time of the first bubble oscillation can be determined as $T_{o1} = t_{p1} - t_0$, and the time of the second

bubble oscillation as $T_{o2} = t_{p2} - t_{p1}$. And it will be also assumed that $t_1 = t_0 + T_{o1}/2$ and $t_2 = t_{p1} + T_{o2}/2$. Examples of the pressure records $p(t)$ and pulses $p_1(t)$ are given in [12]. In this study, the times of the first bubble oscillations T_{o1} and the peak pressures p_{p1} will be used to compute the basic bubble parameters, and the times $t_0, t_1, t_{c1}, t_2, t_{c2}$ will be used in describing the bubble wall motion.

A freely oscillating spherical bubble is described by a set of parameters. First, there are two environmental parameters, the hydrostatic pressure at the place of the sparker p_∞ and the liquid temperature Θ_∞ . In some works dealing with underwater chemical explosions [20,21,23], the authors do not use the hydrostatic pressure p_∞ but the detonation depth h . However, to unify the description, in the following, the hydrostatic pressure p_∞ will be used, and the depth h occurring in the original works will be recalculated into p_∞ , including in the mathematical formulas. Then, there are several liquid parameters, e.g., the liquid density ρ and the viscosity parameter η . In our experiments, the hydrostatic pressure was $p_\infty = 127$ kPa, and the fluid temperature $\Theta_\infty = 21$ °C. The liquid used was water; therefore, $\rho = 10^3$ kg/m³.

Second, two parameters describe the bubble itself. These are the bubble size and intensity of bubble oscillations. Given the almost spherical shape of the studied bubbles, their sizes will be described by the maximum radii R_{M1} . To describe the bubble oscillation intensity is a more difficult task. At present, no universal measure of the bubble oscillation intensity is available. As discussed in [12], the bubble oscillation intensity could be described, e.g., by the ratio of the pressures P_{m1}/p_∞ , or the ratio of radii R_{M1}/R_e . Another intensity measure could be the ratio R_{M1}/R_{m1} , or the ratio of pressures P_{M1}/p_∞ . However, neither $P_{m1}, P_{M1}, R_e, R_{m1}$ can be determined in experiments easily (to our best knowledge, there is no experimental work where these pressures or radii were determined in case of spark-generated bubbles). Therefore, as discussed in [12], a non-dimensional peak pressure in the first pulse $p_1(t)$ can be used as a parameter describing the intensity of bubble oscillation. This quantity is defined as $p_{zp1} = (p_{p1}r_h)/(p_\infty R_{M1})$ and can be relatively easily determined in experiments. The parameter p_{zp1} can be best interpreted as the peak pressure p_{p1} at a distance from the bubble center $r_{M1} = R_{M1}$, divided by the hydrostatic pressure p_∞ . The maximum bubble radii R_{M1} and the corresponding non-dimensional peak pressures p_{zp1} were determined for each studied bubble from the records $p(t)$ using an iterative procedure described in detail in [12]. The bubbles studied in this work had sizes R_{M1} ranging from 26.6 to 52.1 mm and oscillated with intensities p_{zp1} ranging from 22.1 to 120.9.

Let us now briefly mention the buoyant migration of bubbles. According to Archimedes' principle, on a spherical bubble in a liquid, the buoyant force having a magnitude $F_b = (4/3)\pi R^3 \rho g$ acts. Here, ρ is liquid density, and g is the gravity acceleration. The buoyant force F_b has a vertical upward direction. Against the motion of the bubble, a delaying force F_r acts. Due to the combination of the radial bubble oscillations and upward migration, the delaying force is very complex. In its simplest form, it can be described by Stokes' law, where $F_r = 6\pi\eta Rv$. Here, η is the viscosity coefficient of the liquid, and v is the constant velocity of the bubble. Because the vertical velocity v of the oscillating bubbles is not constant, Stokes' law represents just a first approximation of the delaying force and can be used for selected time intervals only.

When increasing the bubble oscillation intensity and vertical buoyant velocity, the spherical shape of the bubble starts deforming. Though these deformations are out of the scope of the present work, they will be briefly mentioned in Section 6. It should be stressed here that neither F_b nor F_r is dependent on p_∞ . But, by changing p_∞ , the bubble size R_{M1} and intensity of oscillations p_{zp1} will change, even if the bubble generating mechanism and the initially delivered energy to the bubble are the same.

To study the pure buoyancy, the observed bubbles must oscillate at a sufficient depth h and at a sufficient distance d from hard surfaces. These depths h and distances d have to be considered in relation to the bubble size R_{M1} , that is, as the relative distance parameters h/R_{M1} and d/R_{M1} . The magnitude of the relative distances for which the assumption of a large expanse of liquid begins to apply can be partially estimated from the results given in [11] to be $h/R_{M1} \approx 4$ and $d/R_{M1} \approx 3$. In experiments performed in this work, the relative distances in the case of the largest bubbles are $h/R_{M1} = 53$ and $d/R_{M1} = 23$. Thus, the bubble oscillations studied here can be considered to take place in a large expanse of liquid.

As is shown in Section 5, in the vicinity of the first bubble contraction t_{c1} , a migration movement in the vertical direction due to buoyancy is observed. This movement has the form of a step, and the value of this migration step will be denoted Δh . To enable comparison with some results reported in the literature reviewed in Section 3, the migration step is considered in the interval (t_1, t_2) . Because the value of the migration step Δh varies with the bubble size R_{M1} , in the following, the relative migration step $\Delta h/R_{M1}$ will be considered.

The experiments described in this work were always performed at hydrostatic pressure $p_\infty = 127$ kPa. According to the similarity principle [19], the bubbles generated via the same technique but oscillating under different hydrostatic pressures p_∞ are not similar. The comparison of the data presented here with the data measured at a different hydrostatic pressure p_∞ must, therefore, be performed with the utmost care. Thus, for example, the results presented here cannot be compared directly with the results published in [3], where the bubbles were studied at very decreased pressures p_∞ (as low as 4 kPa) or with the results of underwater explosions research [20,21], where the bubbles oscillated at high hydrostatic pressures p_∞ (as high as 2 MPa).

The technique by which the oscillating bubbles are generated also plays a significant role, which can manifest itself, e.g., in the chemical composition of the bubble interior. The bubbles generated using different techniques (exploding wires, chemical explosions, corona pulsed discharges, spark discharges) behave differently, e.g., they oscillate with different intensities and have different internal temperatures. Therefore, they are not similar. Again, any mutual comparison between bubbles generated using different techniques can only be performed with the utmost care.

The framing rate of our camera was too low to enable observing the bubble shape in the vicinity of the minimum bubble volume (near R_{m1}). In addition to that, we did not use any optical filters to prevent the over-exposition of the films by the bubble luminescence, which is significantly increased in the vicinity of t_{c1} . Therefore, we cannot describe the bubble shape in the interval around the first maximum contraction of the bubble. However, when carefully observing all nine films in the interval (t_0, t_{c2}) , we did not see any bubble shape deformations, similar to those that can be seen in [3,4,26]. The bubble shape during the first and second oscillation was always close to a spherical shape, and no penetrating gravity jets could be seen. The situation is different in later bubble oscillations, where plasma prominences, in a certain sense similar to solar prominences, can be seen. But these phenomena are out of the scope of the present study. Analysis of recorded luminescence pulses $u_1(t)$ was presented in [17]. In these pulses, there were no deformations in the shape that could be attributed to the gravity jets. Assume that the gravity jets occur in the studied bubbles. In that case, the frequency of the deformations in the luminescence pulses caused by the gravity jets should increase with the bubble size because the buoyancy effects also increase with the bubble size. This was not observed. Thus, it can be concluded that, although some deformations in the bubble shape due to buoyancy can occur (we cannot prove or disprove it), such deformations seem to be insignificant in the case of the bubbles studied here.

3. Literature Review

Buoyancy was first studied in the case of large bubbles generated during underwater chemical explosions. Let us now briefly review the works where the buoyancy migrations are described. For example, Menon and Lal [22] studied the oscillations of bubbles generated by the explosion of detonating gas contained in glass spheres having an initial radius $R_{m0} = 31.7$ mm. These researchers, among other things, also determined the upward bubble motion due to buoyancy. The bubbles oscillated at a depth $h = 0.65$ m ($p_{\infty} = 106.5$ kPa), and the distance to the nearest hard surface (to the bottom of the experimental tank) was $d = 0.785$ m. The generated bubbles had a size of $R_{M1} \approx 82$ mm. Thus, the relative distances are $h/R_{M1} \approx 7.9$ and $d/R_{M1} \approx 9.6$. The generated bubbles oscillated with low intensity. From Figure 4 in [22], the value of the ratio $R_{m1}/R_{M1} \approx 0.33$ can be determined, and from Figure 3a in [22], the ratio $P_{M0}/p_{\infty} \approx 9.2$ can be estimated (here, P_{M0} is the maximum pressure in the detonating gas after the detonation has been initiated). As can be verified in [12], both of these values indicate a very low intensity of bubble oscillations. From Figure 3b,c in [22], an estimate of the non-dimensional peak pressure in the first pulse is $p_{zp1} < 3$, again a very low value. Regarding buoyancy, the authors of this work say that “between the explosion and the first bubble maximum, there is negligible upward migration of the bubble. However, as time progresses, the bubble migrates upwards at an almost linear rate” (see also Figure 14 in [22]). Thus, there is no migration step similar to the one shown in Section 5 of this work.

Klaseboer et al. [23] studied bubbles generated by an underwater chemical explosion of explosive compositions. The bubbles were produced at a depth of $h = 3.5$ m (at hydrostatic pressure $p_{\infty} = 135$ kPa). The largest generated bubble had a size $R_{M1} = 0.54$ m; the distance d to the hard surface was 7.5 m. Therefore, the relative distance is $d/R_{M1} = 13.9$. As can be estimated from Figure 11 in [23], depending on the pressure sensor position, the non-dimensional peak pressure p_{zp1} was in a range from 28.2 to 43.3. The vertical bubble motion due to the buoyancy is shown in Figure 20 in [23]. In this case, the bubble oscillates below a resilient horizontal plate at a distance $d = 1.2$ m. Thus, the relative distance is only $d/R_{M1} = 2.2$ now. As can be seen, the bubble center is relatively stable during T_{o1} , but after a steep movement in the vicinity of t_{c1} , the center of the bubble starts to migrate moderately upwards during T_{o2} . The migration step between times t_1 and t_2 can be estimated from Figure 20 in [23] to be approximately $\Delta h \approx 0.4$ m, and, thus, the relative migration step is $\Delta h/R_{M1} \approx 0.74$.

In theoretical work, Wang [24] modeled the explosion of a detonator at a depth of $h = 1.5$ m (at a hydrostatic pressure of $p_{\infty} = 115$ kPa). In this case, the bubble size was $R_{M1} = 0.171$ m, the migration step between t_1 and t_2 can be estimated from Figure 7a in [24] to be $\Delta h \approx 82$ mm, and, therefore, the relative migration step is $\Delta h/R_{M1} \approx 0.48$.

Li et al. [25] studied the buoyancy of bubbles generated by the underwater explosion of TNT charges theoretically. According to the data in Table I in [25], the computations were done for hydrostatic pressure p_{∞} ranging from 135 to 520 kPa and for bubble sizes R_{M1} ranging from 0.58 to 6.78 m. In Figure 11 in [25], the trajectories of three centroids' migration are shown for times between the explosion initiation and an instant shortly after t_{c1} . In this interval, the centroids migrate upwards continuously, with increasing velocity as time progresses and with an interesting hump shortly before t_{c1} . According to the authors, this hump is due to the gravity jet impact. Such a hump has not been reported in any other work.

Buogo and Cannelli [10] experimentally studied a spark-generated bubble oscillating at ambient pressure of 125 kPa below a rigid paraboloidal surface. The paraboloidal surface was at a distance $d = 100$ mm above the spark center. The bubble had a size of $R_{M1} = 35$ mm and oscillated with an intensity of $p_{zp1} \approx 86$. The relative distance is now

only $d/R_{M1} = 2.3$. The observed migration step was $\Delta h \approx 20$ mm, so the relative migration step is $\Delta h/R_{M1} \approx 0.57$. The bubble radial motion was probably influenced by the presence of relatively thick tungsten electrodes with a diameter of 10 mm.

Zhang et al. [3] studied bubble shape deformations due to buoyancy. In this work, the bubbles were generated by exploding wires in water. In the case of experiments in an infinite liquid, the bubbles were generated at a depth $h = 0.25$ m, and the distance to the hard surface of the cylindrical tank was $d = 0.4$ m. As the descriptor of buoyancy, the authors use the buoyancy parameter δ . Therefore, the particular values of R_{M1} and p_∞ are not given. The hydrostatic pressure p_∞ was usually very low (as low as 4 kPa), and the size of the generated bubble, R_{M1} , could be as large as 75 mm. Then, the relative distances can be estimated to be $h/R_{M1} > 3.3$ and $d/R_{M1} > 5.3$. The intensity of bubble oscillation cannot be estimated from the presented data. The dimensionless displacements of the top and bottom of bubble surfaces are shown in Figure 6 in [3] for three bubbles. During the first oscillation, the trajectories are similar to those given in Section 5 in this work, but they are different during the second oscillation. It is also interesting that no significant bubble luminescence and no presence of plasma packets floating in water adjacent to the bubble wall were described in [3]. This indicates that in [3], the composition of the substances inside the bubble substantially differs from the composition of substances inside the bubbles studied in this work or in bubbles studied by Klaseboer et al. [23]. For example, in Figure 4 in [23], strange formations (objects) can be seen in the bubble neighborhood. The authors call these formations multi-directional “spikes”. No similar objects can be seen in [3].

In a recent paper, Zhang et. al. [18] studied the buoyancy of bubbles generated by underwater explosions and spark discharges. Both experimental and theoretical data are presented. The bubble generated by the underwater explosion had a size $R_{M1} = 158$ mm. It was formed at a detonation depth $h = 300$ mm; thus, $h/R_{M1} = 1.9$. According to Figure 3c in [26], there was no upward buoyancy motion but migration downwards. The theoretical computations were carried out for a bubble of this size both at a depth h and in a large expanse of liquid. In a large expanse of liquid, the theoretical migration step is approximately $\Delta h \approx 50$ mm; thus, the relative migration step is $\Delta h/R_{M1} \approx 0.32$. The spark-generated bubble had a size $R_{M1} = 16.6$ mm. In experiments, this bubble was situated at a distance $d = 44$ mm from a rigid wall; thus, $d/R_{M1} = 2.6$. The migration step displayed in Figure 4c in [26] is approximately $\Delta h \approx 9$ mm now and, therefore, $\Delta h/R_{M1} \approx 0.54$. In theoretical computations in a large expanse of liquid, the migration step of this bubble is approximately $\Delta h \approx 1.5$ mm, and the relative migration step is, therefore, $\Delta h/R_{M1} \approx 0.09$. The buoyancy in a large volume of liquid is very small for a bubble of this size, as can be expected.

4. Measuring Set-Up

Freely oscillating bubbles were generated by discharging a capacitor bank via a sparker submerged in a laboratory water tank having dimensions 6 m (length), 4 m (width), and 5.5 m (depth). The sparker consisted of two electrodes made of tungsten wires having a diameter of 1.2 mm and a length ranging from 30 to 50 mm. The tungsten electrodes were fastened in conical brass holders and were connected via a secondary air-gap sparker and a very-low-resistance cable to a capacitor bank. The angle between the electrodes was 110 degrees in the horizontal plane, and there was a gap of about 2 mm between the tips of electrodes. The sparker was submerged in water at a depth of $h = 2.75$ m, and the distance of the sparker gap to the nearest tank wall (in the experiments described here to the glass observation window) was $d = 1.2$ m. The capacitance of the capacitor bank was 80 μF (for generating small bubbles), 200 μF (medium bubbles), and 360 μF (large bubbles). The capacitors were charged from a high-voltage source 2.35–2.5 kV. The liquid breakdown between the electrodes was triggered by switching on the air-gap sparker.

The generated bubbles were filmed using a high-speed camera (Photec 16 mm film full-frame rotating prism camera, Photec Co. Ltd., Higashi-ku, Japan) with framing rates ranging from 2800 to 3000 frames/s. The camera was positioned behind a glass window in a chamber adjacent to the water tank. The bubbles were illuminated continuously using a submerged inclined mirror (positioned at a distance of about 1.5 m from the sparker) and two lamps clamped above the water surface. The recorded films were later digitized so that the frames could be analyzed on a computer.

At the same depth as the sparker, a broadband hydrophone (Reson Type TC 4034, Reson, Slangerup, Denmark) was submerged. In experiments with smaller bubbles, the sensitive element of the hydrophone was positioned at a distance $r_h = 0.1$ m from the sparker gap; in experiments with the three largest bubbles, the distance was $r_h = 0.5$ m. The output voltage of the hydrophone, which corresponds to the pressure wave radiated by the oscillating bubble, was digitized and stored on a computer for later analyses. Further details concerning the experimental set-up can be found in [12].

5. Experimental Results

A total of nine films of oscillating bubbles were taken. Oscillations of the largest bubbles were recorded on three films, medium-sized bubbles on another three films, and the smallest bubbles also on three films. Examples of several frames selected from a film with a medium-sized bubble are given in Figure 1.

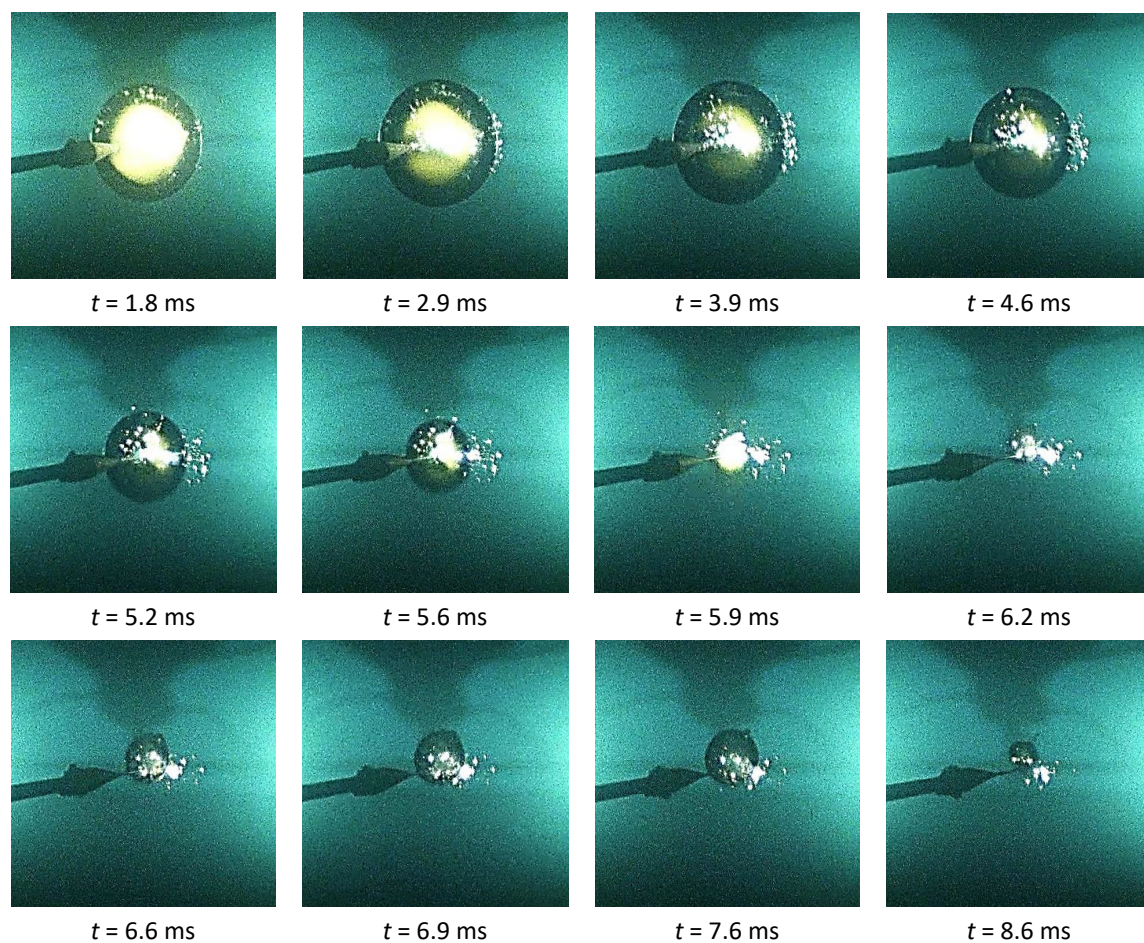


Figure 1. Selected frames from a film record showing a bubble during the first two oscillations. The bubble has a size $R_{M1} = 36.6$ mm, and the non-dimensional peak pressure is $p_{zp1} = 90.7$. The times below each frame refer to the time origin, which is set at the instant of the liquid breakdown. The corresponding bubble sizes at these times are given in Figure 3.

In the following, the rectangular coordinates x, y, z are used, and the x -axis is aligned with the optical axis of the camera. In six experiments, the sparker was positioned so that the angle between the tungsten electrodes and the x -axis was 35 degrees. The position of tungsten electrodes can be seen in Figure 1. In this case, the discharge channel lies on the x -axis. In the following three experiments, the sparker was turned by 90 degrees, and the angle between the tungsten electrodes and the x -axis was 55 degrees. Selected frames from the film record showing the bubbles with this sparker position are displayed in Figure 5 in [12]. In this case, the discharge channel lies on the y -axis. Thanks to the rotation of the sparker by 90 degrees, the generated bubbles could be observed from two mutually perpendicular directions, and, thus, the symmetry could be checked.

Frames from all films were analyzed on a computer, and the coordinates $y_1, z_1, y_2, z_2, y_3, z_3$, and y_4, z_4 of the four points on the bubble wall were determined at each image (see the sketch in Figure 2). From the y and z coordinates of the four selected points, the horizontal and vertical diameters of the bubble were determined as $D_y = (y_2 - y_1)$ and as $D_z = (z_3 - z_4)$. The studied bubbles are prolate spheroids with z as the symmetry axis. However, as the ratio of diameters D_y/D_z is approximately only 0.92, it will be assumed that the bubbles are spheres with the same volume as the prolate spheroids. The bubble radius is then calculated using the formula $R = (D_y^2 D_z)^{1/3} / 2$. The vertical coordinates of the bubble upper wall, center, and bottom wall positions are $z_u = z_3 - z_c(1)$, $z_c = (z_3 + z_4) / 2 - z_c(1)$, $z_b = z_4 - z_c(1)$. Here, $z_c(1)$ is the vertical coordinate of the bubble center in the first frame.

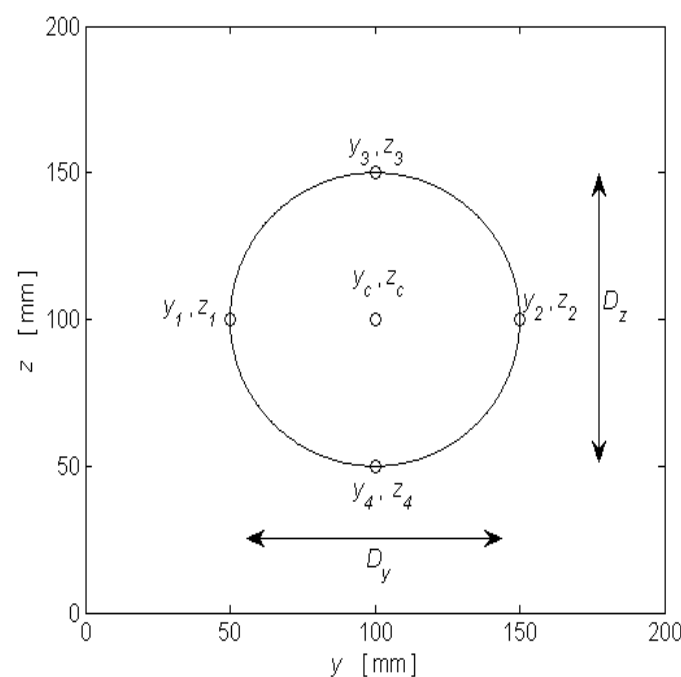


Figure 2. A sketch showing the points on the wall of the bubble whose coordinates were read from the frames. Bubble center coordinates y_c and z_c and bubble diameters D_y and D_z are also displayed.

An example of the variation in the bubble radius R with time determined in this way is given in Figure 3.

An example of the variation of the bubble upper wall position z_u , center position z_c , and the bottom wall position z_b with time is given in Figure 4. The displayed data correspond to the bubble shown in Figure 1.

It can be seen in Figure 4 that during almost the whole first bubble oscillation T_{o1} , the bubble center z_c is approximately at the same depth h , and the bubble performs almost no upward motion. However, when approaching the time of the first bubble contraction t_{c1} ,

the bubble center starts migrating upwards and travels a migration step Δh in the vertical direction. During almost the whole second oscillation T_{o2} , the bubble center z_c is again at approximately the same depth, which means that the bubble performs almost no upward motion, even in the second oscillation. Similar bubble behavior was seen in all nine films.

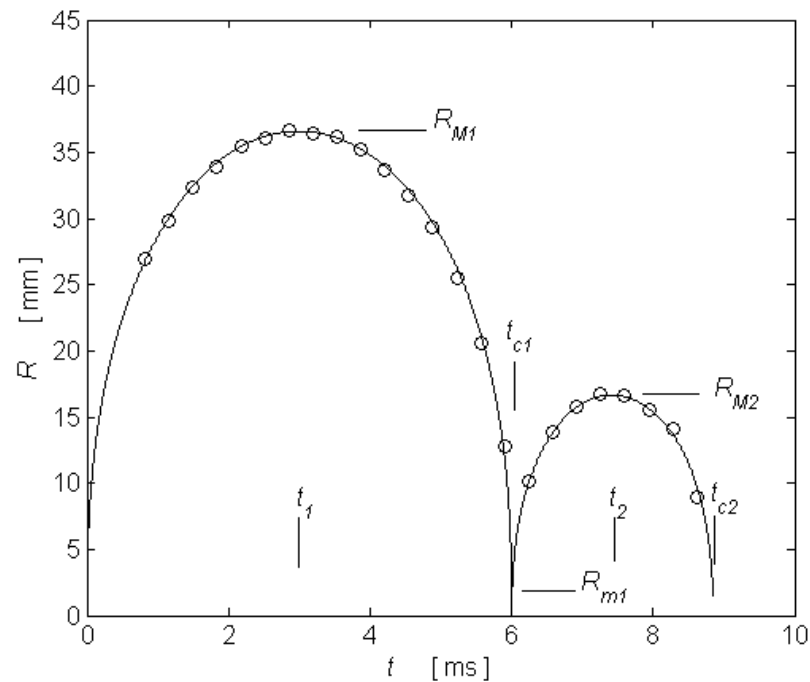


Figure 3. Variation in the bubble radius R with time t . Experimental data are fitted with a regression polynomial. The displayed experimental data correspond to the bubble shown in Figure 1.

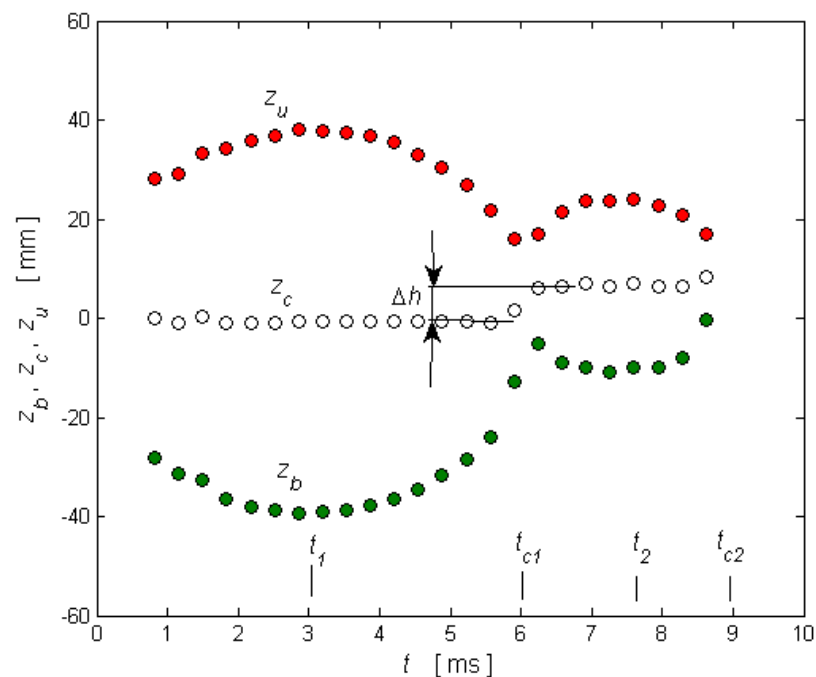


Figure 4. Experimental data showing the variation of the bubble upper wall position z_u (red ringlets), center position z_c (empty ringlets), and bottom wall position z_b (green ringlets) with time t .

The magnitude of the migration step Δh depends on the bubble size R_{M1} . Therefore, in the following, the relative migration step $\Delta h/R_{M1}$ will be used. The relative migration step was determined for all nine studied bubbles. The variation in the relative migration step

$\Delta h/R_{M1}$ with the bubble size R_{M1} is given in Figure 5. The regression quadratic polynomial fit to the experimental data in Figure 5 has a form $\Delta h/R_{M1} = c_1 R_{M1}^2 - c_2 R_{M1} + c_3$ [m], where $c_1 = 10^2 \text{ m}^{-2}$, $c_2 = 2.8 \text{ m}^{-1}$, $c_3 = 0.1729$. This fit can only be considered valid in the range of bubble sizes shown in Figure 5, and no extrapolation to larger bubble sizes is currently possible.

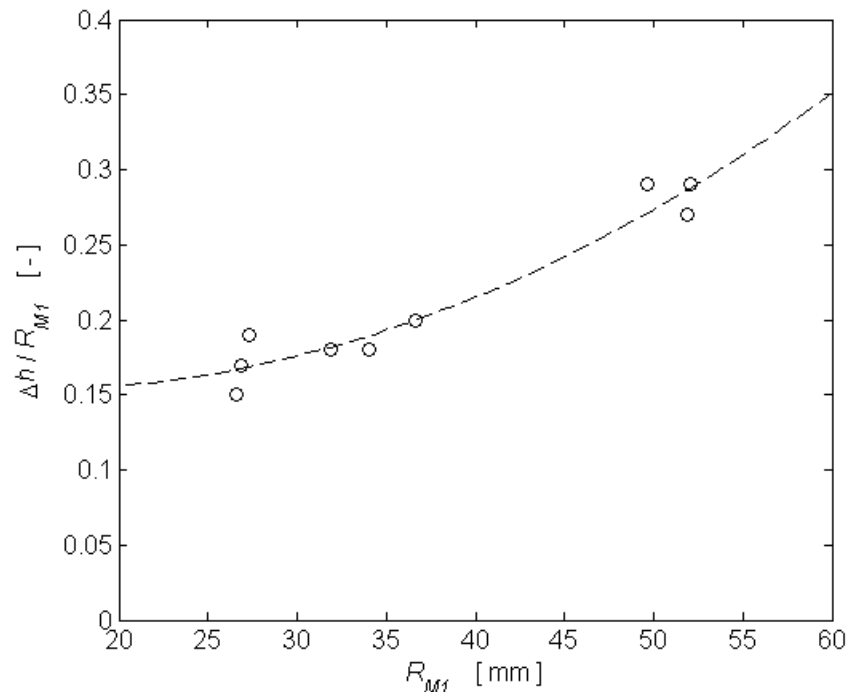


Figure 5. Variation in the relative migration steps $\Delta h/R_{M1}$ with the bubble size R_{M1} .

To describe the effect of buoyancy, some authors [3,23–25] use the buoyancy parameter $\delta = (\rho g R_{M1}/p_\infty)^{1/2}$. This buoyancy parameter contains the basic bubble descriptors R_{M1} and p_∞ . However, as will be discussed in Section 6, the parameters R_{M1} and p_∞ should be given in the description of each experiment separately. The buoyancy parameter δ is, therefore, redundant in a certain sense, and we are not using it. Just let us say that in the case of the bubbles studied here, the magnitude of the buoyancy parameter is $\delta = 0.044$ for a bubble of size $R_{M1} = 25$ mm and $\delta = 0.063$ for a bubble of size $R_{M1} = 50$ mm.

As already mentioned, the ratio of diameters D_y/D_z for the studied bubbles was approximately 0.92 in all experiments. The reason why the studied bubbles have the shape of a prolate spheroid is not clear at present. From a physical point of view, a more acceptable bubble shape during the first oscillation would be an oblate spheroid, e.g., because the initial discharge channel has the shape of a horizontal tube [27]. This point will require further research. However, it is evident that the departure from the spherical shape is relatively small, and the assumption of a spherical bubble introduces no significant error.

6. Discussion

In this work, the buoyancy of spark-generated bubbles during the first and second oscillations was experimentally studied. The bubbles oscillated at a hydrostatic pressure $p_\infty = 127$ kPa. The bubble size, R_{M1} , ranged from 26.6 to 52.1 mm, and the observed relative migration step $\Delta h/R_{M1}$ ranged from approximately 0.15 to 0.29. As can be seen in Figure 4, the bubble center migrates upward significantly only during the interval around the maximum bubble contraction, i.e., in the vicinity of t_{c1} , when the bubble wall radius is close to R_{m1} . During both the first and second bubble oscillations T_{o1} and T_{o2} , the bubble center remains relatively stable. Such behavior was observed for all nine studied

bubbles, regardless of their size. Only the magnitude of the migration steps Δh changes with the bubble size. This observation differs from the results published by other authors. In [3,22–25], the bubble center is relatively stable only during the first oscillation T_{o1} (or during its first part). However, during the second oscillation, the bubble center travels upwards continuously. In these works, there is no migration step similar to that observed in our experiments. Another interesting fact that we observed is that the bubble oscillation intensity p_{zp1} does not influence the magnitude of the migration step Δh . This means that in the case of the bubbles studied in this work, the magnitude of the ratio R_{M1}/R_{m1} (which increases with increasing p_{zp1} [12]) plays no role in the upward bubble migration.

A point not discussed in this work concerns energy partition in oscillating bubbles subjected to buoyancy. The buoyancy motion takes over a part of the bubble energy, and this taken-over part grows with the bubble size. Therefore, the intensity of bubble oscillations decreases with the bubble size, even if other conditions are the same. A theoretical analysis of this problem was given in [19]. Unfortunately, energy partition analysis based on experimental data is not available yet.

Before closing this section, we want to show that ambient pressure, under which the bubbles oscillate, must always be explicitly given as an independent parameter. The influence of p_∞ on bubble oscillation intensity can be best illustrated in the case of underwater chemical explosions. In the case of TNT explosives, the bubble size is given in [23] by a formula $R_{M1} = 72.8(w)^{1/3}/(p_\infty)^{1/3}$ [m, kg, Pa]. Here, w is the mass of the TNT explosive. The peak pressure p_{p1} in the pulse $p_1(t)$ is given in [21] by a formula $p_{p1} = 6.9 \times 10^4(w)^{1/3}(p_\infty)^{1/3}/r$ [Pa, kg, Pa, m]. By substituting these two relations into the definition formula for the non-dimensional peak pressure p_{zp1} , one obtains $p_{zp1} = 945/(p_\infty)^{1/3}$ [- , Pa]. It follows from these relations that by increasing p_∞ , both R_{M1} and p_{zp1} are decreasing. When the bubble size R_{M1} decreases, the buoyancy force and, thus, the magnitude of the vertical motion also decrease. And lower p_{zp1} means that the bubble oscillation intensity is lower, too. At larger pressures p_∞ , the bubble generated by the same amount of explosives w behaves more quietly, with smaller upward motion and less intensive radial oscillations. On the other hand, when p_∞ is decreased, both R_{M1} and p_{zp1} are increased, and the bubble behaves more violently, which may lead to shape deformations. Further remarks on this topic can be found in [20,21].

Shape deformations were also observed when examining upward-floating gas bubbles excited for free oscillations by a sudden increase in pressure in a shock tube [26]. In these experiments, gas bubbles (containing N_2) with an initial radius $R_i = 1.67$ mm were floating upward in a shock tube in 85% diluted glycerin at a velocity of 0.06 m/s. The initial pressure in the liquid was $p_{\infty i} = 10^5$ kPa. The shock tube had an inner diameter of 56 mm (thus, $d = 28$ mm, and the relative distance is $d/R_i = 17$). The bubbles were excited into free oscillations by pressure steps Δp and oscillated under increased pressure $p_{\infty s} = p_{\infty i} + \Delta p$. In the case of this excitation technique, the intensity of bubble oscillations grows with the pressure step strength Δp . It was found that no deformations of the spherical bubble shape occurred at low values of the pressure steps. For stronger pressure steps, the observed deformations gradually increased. In this case, when the ambient pressure $p_{\infty s}$ is increasing, the intensity of bubble oscillations increases, and the bubble shape deformation is also increasing. The influence of ambient pressure is now opposite when compared with underwater chemical explosions.

It can be seen that the upward bubble migration observed in different experiments [3,22,23] differs from the results reported here. With our present state of knowledge, it is difficult to say whether this diversity is due to different experimental hydrostatic pressures p_∞ , due to different relative depths h/R_{M1} and distances d/R_{M1} , or due to different techniques used to generate oscillating bubbles (detonating gas, solid detonators, exploding wires, spark

discharges). Further experiments are needed to shed more light on these complex processes. It is evident from the presented discussion that, to describe the effect of buoyancy on the bubble motion in a large expanse of liquid, as a minimum, three parameters must always be given. These are the bubble size, the bubble oscillation intensity, and the ambient pressure. Without knowing these three parameters, any progress in understanding the influence of buoyancy on the bubble motion cannot be achieved.

7. Conclusions

Experimentally, the buoyancy of spark-generated bubbles with sizes ranging from 26.6 to 52.1 mm that oscillated at a hydrostatic pressure of 127 kPa in a large expanse of liquid was studied. It was verified that migration due to buoyancy increases with the bubble size. However, even in the case of the largest bubbles studied here, no deformations of their spherical shape were observed during the first two oscillations. Therefore, the studied bubbles can be used as model bubbles for smaller cavitation bubbles. Unfortunately, experimental data on the buoyancy of the large bubbles are still very limited. At present, therefore, it is not possible to more precisely determine the energy losses from bubbles caused by buoyancy and the limits beyond which the buoyancy effects become significant.

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