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Entry

Extremes of the Edgeworth Box

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Definition

Extremes of the Edgeworth box concern corner allocations and their relationship to the contract curve in a two-good, two-agent exchange economy. In the standard pure-exchange setting with well-behaved preferences, the contract curve comprises all Pareto-efficient allocations, including interior tangencies and boundary corners, where no mutually beneficial trade remains. When money is introduced as a numéraire (a medium of exchange only), real feasibility and preferences are unchanged, so the contract curve remains the benchmark for efficiency. When money provides liquidity services (is valued for holding), agents may rationally abstain from trade even near interior tangencies; short-run outcomes can therefore include inaction at corners. This entry defines these objects, outlines the efficiency conditions at boundaries, and summarizes how monetary interpretations affect short-run behavior in general equilibrium and monetary economics. The Edgeworth geometry remains a real-exchange depiction; when we discuss money as a store of value, we use it as a short-run, reduced-form outside option that proxies intertemporal motives. This does not "fix" the box; it clarifies why no-trade at or near corners can be individually rational when liquidity is valued.

Keywords: Edgeworth box; contract curve; corner solutions; Pareto efficiency; general equilibrium; indifference curves; budget constraint; numéraire; liquidity preference; monetary economy

1. Introduction

The extremes of the Edgeworth box [1–3] are the two boundary allocations in a two-good, two-agent exchange economy where one agent holds all resources and the other holds none. Let agents $i \in \{A, B\}$ have strictly increasing, strictly quasi-concave utilities $U_i(x_i, y_i)$ and endowments satisfying $x_A + x_B = \overline{x}$ and $y_A + y_B = \overline{y}$. An allocation is Pareto-efficient if no feasible reallocation makes someone better off without making the other worse off.

For interior efficient points,

$$\mathrm{MRS}_A^{xy} = \mathrm{MRS}_B^{xy},$$

while at the extremes a non-negativity/boundary condition binds, so any feasible move that benefits the deprived agent necessarily harms the endowed agent; hence the corners are efficient boundary optima.

With strictly positive prices (p_x, p_y), a Walrasian equilibrium selects either a tangency on the contract curve or a boundary optimum when the budget line through the endowment

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supports it. Thus, the extremes are limit cases of price-supported efficient allocations in the Edgeworth geometry.

2. Interpretation

Here, we situate the concept within broader macro reasoning. In microeconomics, where money is a neutral numéraire, the budget-feasible, voluntary-exchange setup implies that, absent mutually desired trade, a feasible allocation is treated as efficient. A short-run Keynesian reading differs: if real balances yield liquidity utility, agents have a genuine outside option, and no-trade at or near the extremes can reflect liquidity preference rather than full efficiency.

To orient the discussion, we first sketch the philosophical and macroeconomic background that motivates the liquidity perspective and then turn to the framing of microeconomics.

Choice theory is applied logic: it states how agents rank options under constraints and derives testable implications for observed choices [4]. In this sense it belongs to applied logic's remit as "the practical art of right reasoning" in domain-specific settings [5]. Decision-theoretic rational choice is one prominent application, and microeconomics adopts it to model individual behavior and derive comparative statics from the preference–constraint–choice triad [6]. Taken together, the familiar apparatus of preferences, utility, and optimization functions as an applied-logic program tailored to economic choice [7].

There is, however, a macroeconomic principle running through this logic: the indestructibility of purchasing power, commonly captured by Say's law [8,9]. It underlies both partial and general equilibrium analysis. In partial equilibrium it appears in the budget constraint, which encodes that demand is limited by the value of what one can supply [10]. In general equilibrium it reappears in Walrasian accounting identities that aggregate those constraints [11].

Read through general equilibrium, Say's structural logic appears as Walras' law: at any strictly positive price vector, the value of aggregate excess demand is identically zero, because each individual budget binds when preferences are increasing. Formally, if $z(p)$ is the aggregate excess demand, then $p \cdot z(p) = 0$; in a two-good economy, this implies $p_x z_x(p) = -p_y z_y(p)$, so an excess demand in one market is matched by an equal-value excess supply elsewhere [12]. This is exactly the accounting backbone that aggregates individual budget constraints and gives operational content to the "supply-creates-demand" reading. Thus, Say's law is invoked in that accounting sense: in a pure-exchange economy, individual budgets restrict demand to the value of endowments, and their aggregation yields Walras' law. "Supply" here means the willingness to relinquish endowed goods for trade, not physical production.

Combined with macro principles, this logic gains empirical bite: budgets, prices, and clearing conditions yield testable predictions. In short, microeconomics' applied logic is not merely formal; anchored by Say's law, it yields predictions that can be estimated, confronted with evidence, and refined.

For clarity, we adopt the standard Edgeworth box geometry: the southwest corner is consumer A's origin, the northeast corner is consumer B's, and each point in the box represents a feasible division of fixed aggregate endowments. The contract curve is the locus where indifference curves are tangent; away from that set, the curves intersect and further mutually beneficial trades exist. This is also the natural stage to define barter equilibria as the voluntary-exchange rest points in the box [12].

As seen, in the classic Edgeworth box of general equilibrium [11], the contract curve is the set of Pareto-efficient allocations, characterized (for interior points) by equality of marginal rates of substitution (MRSs). The two corner allocations, where one agent holds

all the resources and the other holds none, are also Pareto-efficient in a trivial sense: any feasible deviation that benefits the deprived agent necessarily makes the endowed agent worse off. Hence, the corners belong to the contract curve.

Why is there no trade at the corners? In standard micro, either barter holds or money is only a numéraire. If no trade is mutually desired, no exchange occurs, and the model labels the allocation efficient.

We proceed in three steps: pure barter, a budget line with money as a numéraire, and money as a store of value. In the first two, corners are efficient. With liquidity preference, no-trade can be rational. The contract curve can "thicken" into a liquidity region. This framing links Say's long-run structure to Keynes's short-run selection.

Nothing in this step suggests that the Edgeworth model is defective; the corner allocations are fully consistent with Pareto efficiency under its real-exchange assumptions. Our contribution is interpretive: by making explicit the background premise that money is absent or a neutral numéraire, we then show how allowing money to be held for its own sake provides a short-run outside option, so observed no-trade at or near the extremes can reflect liquidity preference rather than a failure of the box.

Thus, pursuing the extremes of the Edgeworth box lets us bring Keynes [13] into the picture without abandoning the micro logic. Start with pure barter. Move to a budget-line world where money is only a numéraire. Then, allow money to be a store of value. At the first two stages, no-trade at the corners is read as efficiency. Once money yields utility, no-trade can instead reflect a preference for liquidity. The corners become short-run rest points rather than definitive optima, and the contract curve can thicken into a liquidity region.

This sequence clarifies the relation between Say and Keynes: Say organizes the long-run structure of exchange, while Keynes describes short-run selection when agents value liquidity. Treating money as a choice variable adds a third dimension to the box, illustrating how prices, expectations, and liquidity can arrest or restart exchange. We treat this as an overlay on the real-exchange geometry: goods feasibility is unchanged, and the extension simply makes explicit an outside option to hold real balances in the short run. In what follows, we use this ladder, from barter to numéraire to store of value, to integrate liquidity preference into a strictly micro foundation.

3. The Extremes

Let us dive deeper into this: the contract curve is the set of Pareto-efficient allocations where the two consumers' indifference curves are tangent, so their marginal rates of substitution are equal. Why, then, do the origin points of each consumer appear on the contract curve? In the Edgeworth box, consumer A's origin is the lower-left corner, the point where A holds everything (both goods) and consumer B holds nothing, while B's origin is the upper-right corner, the point where B holds everything and A holds nothing [8]. At both extremes, the following is obtained: the indifference curves become vertical or horizontal relative to the other consumer because one agent is at a saturation point (has everything). For the agent who has nothing, the MRS is not well defined. She sits at a corner of her indifference map, and the resulting "tangency" with the other consumer's map is a degeneracy arising from the fact that one agent cannot trade at all (has nothing). Technically, the MRS tangency condition can include box corners if they are Pareto-efficient, and giving everything to one agent is Pareto-efficient because any deviation would benefit one party and harm the other. Therefore, when consumer A has everything, any change that gives something to B makes A worse off, hence being Pareto-efficient, and when consumer B has everything, any change that gives something to A makes B worse off, hence being Pareto-efficient. In short, extreme allocations are trivial Pareto efficiencies: there is no way

to make someone better off without making the other worse. So, they belong to the contract curve even though one consumer ends up with zero of each good.

Formally, in the two-agent, two-good pure-exchange economy with aggregate endowment $(\overline{x}, \overline{y})$, agents $i \in \{A, B\}$ choose non-negative bundles (x_i, y_i) with $x_A + x_B = \overline{x}$ and $y_A + y_B = \overline{y}$, and have utilities $U_i \colon \mathbb{R}^2_+ \to \mathbb{R}$ that are strictly increasing and strictly quasi-concave. The extremes of the Edgeworth box are the two boundary allocations $E^A = ((\overline{x}, \overline{y}), (0,0))$ and $E^B = ((0,0), (\overline{x}, \overline{y}))$ in which one agent holds all resources and the other holds none.

The corners are textbook boundary optima. With monotone preferences, feasibility puts one agent at a saturation point and the other at a non-negativity boundary. However, Kuhn–Tucker logic allows tangency to include boundary cases. Indeed, Pareto-efficient allocations solve

$$\max_{0 \le x_A \le \overline{x},\ 0 \le y_A \le \overline{y}} \theta U_A(x_A, y_A) + (1-\theta) U_B(\overline{x} - x_A, \overline{y} - y_A),\ \theta \in (0,1).$$

Let $\mu_x, \nu_x, \mu_y, \nu_y \ge 0$ be multipliers on $x_A \ge 0,\ x_A \le \overline{x},\ y_A \ge 0,\ y_A \le \overline{y}$. The KKT (Karush–Kuhn–Tucker) conditions are

$$\theta U_{A,x} - (1-\theta) U_{B,x} - \mu_x + \nu_x = 0, \theta U_{A,y} - (1-\theta) U_{B,y} - \mu_y + \nu_y = 0,$$

$$\mu_x x_A = 0,\ \nu_x(\overline{x} - x_A) = 0,\ \mu_y y_A = 0,\ \nu_y(\overline{y} - y_A) = 0, \mu_\bullet, \nu_\bullet \ge 0.$$

If $0 < x_A < \overline{x}$ and $0 < y_A < \overline{y}$ then $\mu_\bullet = \nu_\bullet = 0$ and

$$\frac{U_{A,x}}{U_{A,y}} = \frac{U_{B,x}}{U_{B,y}} \ \Leftrightarrow\ \mathrm{MRS}^{xy}_A = \mathrm{MRS}^{xy}_B$$

If a bound binds (e.g., $x_A = \overline{x}$ so $\nu_x \ge 0$), the condition $\theta U_{A,x} \ge (1-\theta) U_{B,x}$ certifies boundary efficiency. Thus, interior efficiency equalizes marginal rates of substitution; at a binding boundary, a non-negative shadow value means a small feasible transfer would harm the bound-side agent at least as much as it helps the other, so no Pareto improvement exists. Of note, if a marginal utility vanishes at the boundary, e.g., $U_{i,x} = 0$, the same KKT conditions certify efficiency without appealing to strict monotonicity. This is the formal content behind calling the corners "trivially" Pareto-efficient [12].

Furthermore, every supply logically creates its own demand. In the abstract microeconomic, purely voluntarist view of exchange, this means that trade occurs only if both sides want it, that the supply of a good makes sense only if someone is willing to deliver another good in return, and that if there is no desire to trade the allocation remains as it is and is, by construction, treated as "efficient." Linking this to the contract curve, at the extreme points (the origin of A or the origin of B), where one agent holds 100 percent of the goods while the other holds 0 percent, there is no desire to trade because the fully endowed agent would lose by trading, and therefore, the endowed agent does not supply, and no trade occurs. At A's origin: A has everything, B has nothing, A does not want to trade, and so A does not supply, which implies no trade. At B's origin: B has everything, A has nothing, B does not want to trade, and so B does not supply, and this implies no trade. Hence, if the party controlling the goods does not wish to supply them, no demand is generated, and no trade takes place. The allocation persists, and it is treated as Pareto-efficient due to the absence of a mutual desire to trade.

Formally, in a two-good exchange economy, an interior allocation $((x_A, y_A), (x_B, y_B))$ is Pareto-efficient if and only if

$$\text{MRS}_A^{xy}(x_A, y_A) = \text{MRS}_B^{xy}(x_B, y_B) \text{ where } \text{MRS}_i^{xy} = \frac{\partial U_i/\partial x_i}{\partial U_i/\partial y_i}.$$

At the extremes $E^A = ((\overline{x}, \overline{y}), (0,0))$ or $E^B = ((0,0), (\overline{x}, \overline{y}))$, at least one non-negativity constraint binds. Under monotonic preferences ($\partial U_i/\partial x_i > 0, \partial U_i/\partial y_i > 0$), any feasible transfer that makes the deprived agent better off necessarily reduces the endowed agent's utility; hence, the corner is a Pareto-efficient boundary optimum. If preferences admit local satiation, boundary efficiency can also arise when a marginal utility vanishes at the boundary ($\partial U/\partial x = 0$ or $\partial U/\partial y = 0$).

Observe that this is exactly in the spirit of Say's law. In classical microeconomics, corner allocations are efficient due to Say's law. In contrast, Keynesian liquidity preference implies that corner allocations may represent short-run inaction rather than efficiency. The intuition is: trade happens only if both sides want it; at the extremes, the fully endowed agent is satisfied and will not supply anything; without supply, there is no trade; the situation is stable, and it is classified as efficient. Thus, this efficiency by absence of trade is coherent with Say's world, where there is no excess supply without demand because supply appears only when there is an interest in demanding something in return. The conclusion is that the contract curve presupposes a world of voluntary exchanges in which markets and efficiency emerge from the subjective willingness to offer and demand rather than from prior distributive criteria. This is deeply Say-ian: there is no trade without an offer and no offer without a simultaneously desired demand, and if an agent does not want to trade, the market accepts that position as efficient even if the other agent ends up with nothing.

It may seem unusual to invoke Say's law in microeconomics, yet partial equilibrium already embeds it implicitly in the budget constraint, as we have argued elsewhere [10]. By the same token, general equilibrium carries the same logic, with the Edgeworth box providing a clear instance: exchange is modeled as mutually voluntary, so supply appears only as a vehicle to demand something else. That is precisely why Say's law functions as a foundational organizing principle in these settings. Bringing it to the foreground is not just exegetical; it also opens a practical bridge to a Keynesian perspective within microeconomics, clarifying when the standard exchange geometry holds and when short-run liquidity considerations can disrupt it. Importantly, there is no fundamental theoretical divide between partial and general equilibrium in their reliance on Say's law. The distinction is methodological. Add up individual budget constraints in partial equilibrium and Walras's law in general equilibrium follows immediately [11], illustrating that both frameworks rest on the same logic and emphasizing that Say's mechanism is built into each.

Say's law does not claim that "there can be no unemployment" or "no failures" [8,9]. It says that in an economy of voluntary exchanges, supply is motivated by the intention to demand something else, and therefore, in equilibrium there is no "idle" supply without corresponding demand because agents only supply when they intend to trade. Keynes accepts this logic as structurally valid in the long run [10], but he argues that in the short run, agents may withhold their supply (money or goods), staying out of the exchange market, and may fail to turn supply into demand because of uncertainty, preference for liquidity, and related reasons [13]. So, the will to trade can seize up and the market may fail to reach the Pareto-efficient state implied by voluntary exchange.

Observe that Keynes would say the extreme points may not be stable in the short run: the contract curve assumes free and immediate trades whenever they are mutually desired, yet Keynes maintains that the desire to trade may not manifest even when potential gains from trade exist. At an extreme, one agent holds everything and offers nothing, so no trade occurs (which microeconomics labels Pareto-efficient). But Keynes would add that the fact

that a trade does not occur (because no offer/demand is expressed) does not imply that it would not be desirable from a macro perspective, for employment or income. Thus, this implies a critique of the normative use of microeconomic efficiency, because an absence of trade is not the same as an economically desirable or short-run stable situation.

In microeconomics, if you do not supply, you do not wish to demand; therefore, the situation is deemed efficient. Arguably, Keynes would counter that you may fail to supply out of fear or uncertainty, so the absence of trade is not evidence of efficiency but of inaction [14]. Consequently, the extreme points of the contract curve are efficient under Say's voluntarist logic, yet Keynes would question their immediate macroeconomic relevance. He does not deny Say; he denies its short-run descriptive applicability in environments with uncertainty and a preference for liquidity.

Pure microeconomics is a world without real money. In the contract curve analysis, agents cannot retain money as an independent good. They only exchange goods for goods. Even if one good is labeled "money," it is only a numéraire, not a store of value. The consequence is straightforward: if someone does not wish to trade, we infer that the person is satisfied with their current holdings. One cannot hide in liquidity, because that option does not exist. Hence, extreme allocations are deemed efficient. The agent who has everything has no incentive to trade. The agent who has nothing cannot offer anything. There is no pent-up demand because demand equals the offer of some other good. This is an implicit application of Say's law in its pure form: producing or offering is the only way to demand, and there is no autonomous demand without a corresponding offer.

Keynes breaks here by introducing money as an asset with a store-of-value function. With true money one can decide not to offer goods now, not because of satiation but because of a desire to hold liquidity. In the short run, liquidity preference can separate demand from supply: agents may plan to buy more later while producing nothing today, creating demand without contemporaneous supply; or they may produce today but hoard the proceeds in cash, creating supply without contemporaneous demand while they wait for better opportunities. The macro result is excess idle capacity. It is involuntary unemployment. It is situations in which many prefer liquidity, so no one offers goods even when gains from trade exist, which the contract curve does not display. Thus, the absence of trade does not imply efficiency. It signals a preference for liquidity that micro would misread as Pareto efficiency. A recent clarification separates exchange liquidity (dealer-supplied tradability at bid–ask terms) from redemption liquidity, the par, on-demand reflux of an issuer's own liabilities. This distinction frames our "outside option"' precisely: money's value in short-run inaction reflects redemption liquidity rather than mere ease of trade [15].

Recent heterogeneous-agent New Keynesian (HANK) evidence clarifies why short-run inaction can coexist with potential gains from trade. When households update expectations sluggishly, models can simultaneously match the "micro jumps" in spending after transitory income shocks and the "macro humps" in aggregate consumption after monetary shocks. In such environments the direct intertemporal-substitution channel is quantitatively small; most of the consumption response arrives indirectly through income, and informational frictions dampen the immediate, price-guided movement to tangency. In short, short-run "no trade" may reflect informational inertia and a preference for liquidity rather than efficiency [16].

Here, we offer interpretation rather than a modification of the Edgeworth model. A synthesis is useful: in the micro view, with Edgeworth and implicitly with Say, exchange is good for good, money is a neutral numéraire, when there is no trade the allocation is efficient and the contract curve accepts it, supply and demand are simultaneous and intertwined, and at the extreme origin of the box, everything appears fine because the agent simply does not want to trade. In the Keynesian view with non-neutral money, exchange

is good for money as a liquid asset, money is a store of value and a refuge, when there is no trade we can face a macro failure such as unemployment or a liquidity trap, supply and demand can come apart over time because of liquidity, and at the extreme origin the agent may wish to trade but is frozen by the desire to keep cash, so the outcome is not true efficiency. The conclusion is clear: general equilibrium micro assumes Say's law by removing money as a store of value; Keynes questions the short-run relevance of that assumption because liquidity allows the symmetry of supply and demand to break. On the contract curve the extreme is efficient, and with Keynes, the extreme can be a collapse of exchange driven by an absolute preference for liquidity. Of note, this monetary step does not "justify" the corners: they are already Pareto-efficient under the real-exchange assumptions. The liquidity layer only explains why no-trade may rationally persist at or near the extremes in the short run when real balances carry utility.

In this light, treating money as a store of value should be understood as a short-run behavior rather than a permanent feature of exchange. In the long run, money either is not needed or reduces to a means of exchange because Say's law reasserts the principle that only production, and the corresponding act of supplying goods or services, creates demand. Purchasing power is not destroyed by being held; it is conserved and ultimately redirected into expenditure, a point consistent with the idea that purchasing power is effectively indestructible in competitive exchange [8,9]. Credit does not create net purchasing power; it merely anticipates it, since today's credit is tomorrow's debt, and tomorrow's debt can only be discharged by tomorrow's production. By the same logic, hoarding does not erase demand; it postpones it. People do not wish to hoard forever, and intertemporal incentives steadily push idle balances back into goods, services, and productive claims as relative prices, yields, and opportunities adjust. Thus, liquidity preference can dominate the short run, but over longer horizons the system gravitates toward a configuration where supply is offered precisely to demand something else, money's role returns to that of a transactional conduit, and the Edgeworth geometry regains descriptive force.

Introducing a budget line changes the picture: it encodes relative prices through $p_x x + p_y y = m$, where m is the immediate purchasing power that must be spent now. In this setup, money cannot be retained as an asset; it only enables exchange at the moment and is not itself a good in the Edgeworth space. The result is an economy with instantaneous monetary intermediation, where money enters only as a vehicle of trade and not as an autonomous alternative to trade. Keynes departs here by making money a strategic alternative to exchange: under the budget line, the agent must spend m on goods now and cannot simply hold cash for a better future, yet Keynes allows exactly that, so agents may rationally suspend trade at given relative prices because they prefer liquidity. Put differently, the micro budget line imposes trade, while Keynesian money lets agents remain off the line. In Edgeworth's terms the contract curve presumes all agents operate along some budget line, whereas Keynes asks what happens if they choose to stay off it and hold money instead of trading. The upshot is that adding a budget line already breaks barter and introduces relative prices, but keeps money neutral because it is not a choice variable. The contract curve remains valid because agents are forced to trade while respecting the budget constraint. And the Keynesian break arises when money becomes a third good outside the Edgeworth box, a good that can be preferred to other goods rather than a mere instrument of exchange.

When money is treated as a composite good, it is embedded inside the budget, not outside it as a strategic choice. The agent cannot stop at just money; they pass through money to swap one good for another. There is no option to "stay in cash until further notice," so the budget constraint forces trade. Money does not enter utility and cannot remain with positive utility. Keynes breaks exactly at the point where this rule is suspended,

allowing money to be held with positive liquidity utility, creating a real choice between buying goods or holding money and temporarily undoing the equivalence "supply equals demand." Liquidity preference makes money a good with its own indifference surface. On this basis one can construct a "broken" Edgeworth box with money as a third refuge good, in which the contract curve loses stability because agents can park in liquidity rather than continue trading to the Pareto tangency. Next, we will move to this task.

4. Exploding the Box

So far, we have established that, read structurally, Say's law says that supply is motivated by the intention to demand something else: goods are offered only when agents want to acquire other goods. With voluntary exchange and no autonomous role for money, "no trade" is tantamount to "no mutually beneficial offer," which legitimizes corners as efficient (see Figure 1 for illustration). Crucially, Keynes does not pronounce Say "false." Rather, he treats it as irrelevant in the short run: agents can rationally withhold supply, not because they are satiated in consumption goods, but because they prefer to hold liquidity. Formally, adding liquidity utility augments the intertemporal optimality condition with a positive marginal utility of wealth. Geometrically, this steepens the relationship between spending and interest rates implied by the consumption Euler equation, so short-run dynamics do not collapse at the zero lower bound on nominal interest rates [17].

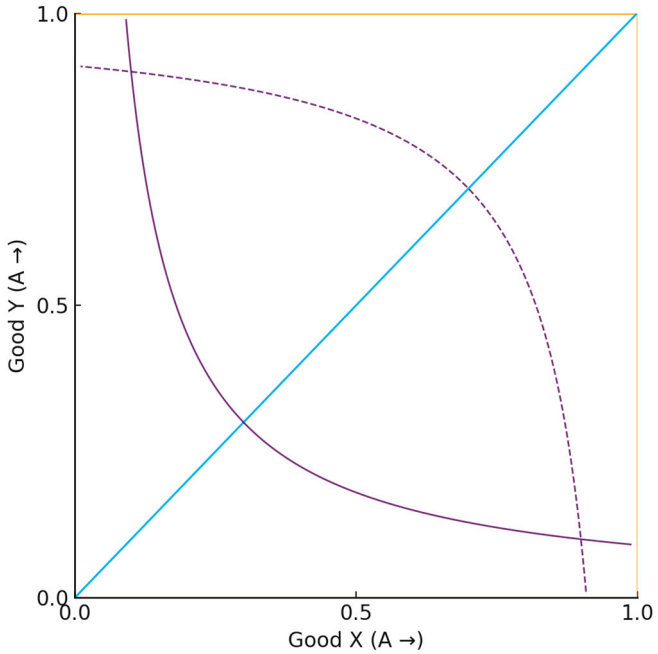

Figure 1. The Edgeworth box, illustrating the contract curve with minimal geometry. The solid diagonal is the contract curve for symmetric, strictly convex preferences, where the marginal rates of substitution are equal. A single solid indifference contour for agent A (read from the lower-left origin) and a single dashed indifference contour for agent B (read from the upper-right origin) illustrate how tangency to the diagonal delivers interior Pareto efficiency. The axes are labeled in A's orientation (Good X and Good Y increasing right and up); B's quantities are read from the opposite corner as usual. The figure deliberately shows only one contour per agent and omits corner labels to keep attention on the logic of tangency and feasibility inside the box.

Moving beyond pure barter, we introduce prices and a budget line. Money appears as a composite good (numéraire) that facilitates trade at given relative prices but is not a store of value that competes with goods in utility. Agents must spend their purchasing power on goods now. The budget line constrains and coordinates trade but does not allow "staying in cash" as an end in itself (see Figure 2 for illustration).

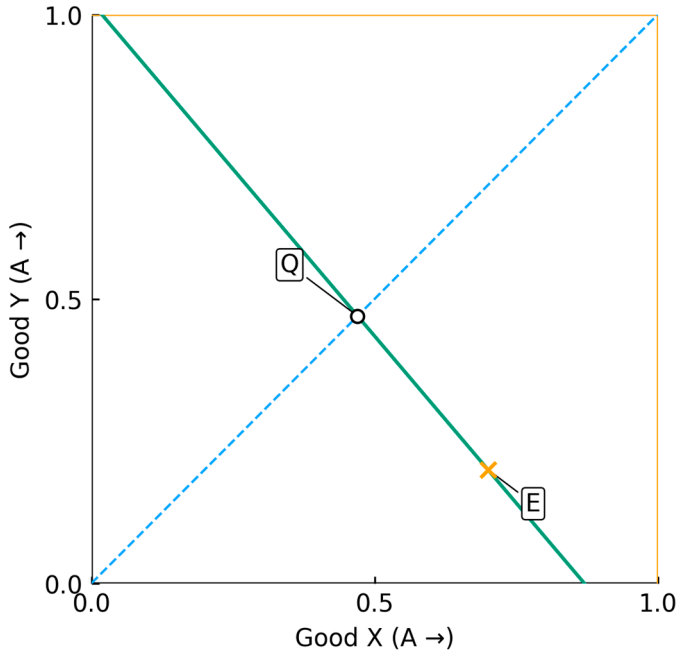

Figure 2. Budget-mediated exchange in the Edgeworth box with money treated as a composite medium of exchange. The dashed diagonal is the contract curve; the solid line is the budget line at given relative prices, passing through the initial endowment E. The competitive allocation Q lies at the intersection of the budget line and the contract curve, illustrating how prices select a Pareto-efficient point from the feasible set when money is not a store of value. Only the essential marks (E, Q, the budget line, and the contract curve) are retained to keep the figure clean while emphasizing the price-line logic.

At the price-mediated stage, the budget set $B = \{x \geq 0 : p \cdot x \leq y\}$ is compact. With strictly monotone, strictly quasi-concave preferences, there exists a unique maximizer on the boundary, so income is fully spent "now." The first-order conditions imply that marginal utilities are proportional to prices and, in two goods, the indifference-curve slope equals $-p_x/p_y$ at the optimum [12]. This is the precise sense in which treating money as a numéraire forces trade along a budget line without admitting an autonomous "stay in cash" option.

Keynes adds money as a store of value. If real balances yield utility, agents may prefer liquidity to trade, and the budget line no longer forces exchange. A missing trade is then no proof of efficiency; it may simply reveal precaution under uncertainty. The geometry also shifts. What was a two-good plane now acquires a money axis, and with it a real outside option that can divert behavior away from the classical tangency. Calibrated one-asset models that match total wealth deliver quarterly marginal propensities to consume of only 3–5 percent, whereas two-asset models with liquid and illiquid assets match empirical marginal propensities to consume without destroying the wealth distribution [18], suggesting that the short-run failure to reach tangency is a liquidity, not a barter, phenomenon.

Of note, even in a goods-only Edgeworth box with symmetric endowments and identical Bernoulli utilities, the symmetric price $p = (1, 1)$ can be unstable; by the index theorem, instability of this symmetric point implies at least two additional stable equilibria, so the economy generically exhibits three equilibria [19]. This shows that non-uniqueness and instability arise even before introducing monetary frictions.

Once liquidity has value, the contract curve need not be the unique rest set. An inaction region (a liquidity region) can open, where moving along the goods plane requires giving up liquidity. The result, especially in the short run, is a Pareto-undominated stasis that does not coincide with interior tangency, even when such a tangency exists (see Figure 3

for illustration). Our treatment is reduced-form rather than a full dynamic program; it is designed to capture the comparative static effect of liquidity preference on short-run selection without altering the underlying Edgeworth feasibility set. In this liquidity-augmented geometry, movement occurs along the liquid balances dimension. Empirically, marginal propensity to consume from liquid windfalls is an order of magnitude larger than from illiquid wealth changes (≈16% vs. ≈1–2%) [18], matching the idea that agents may rationally remain off the goods plane when liquidity is scarce.

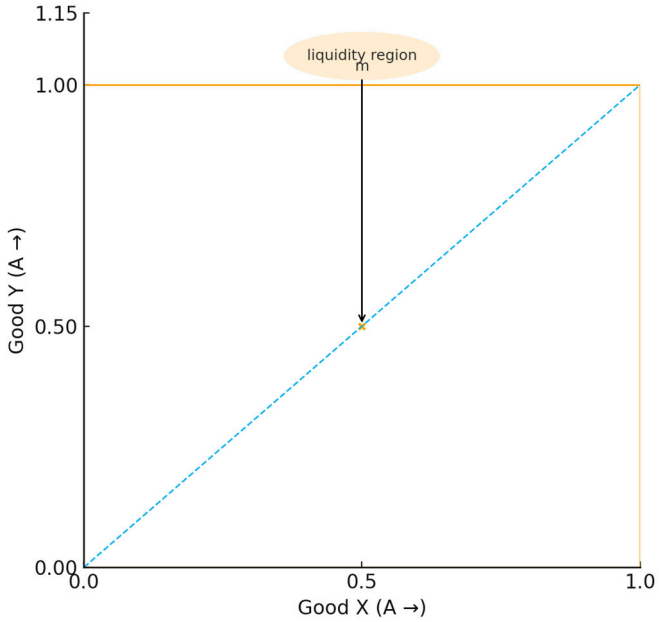

Figure 3. "Exploded" Edgeworth box with a liquidity dimension. The dashed diagonal again marks the goods-only contract curve. The arrow labeled m indicates movement out of the goods plane toward real money balances, and the small translucent ellipse labeled "liquidity region" represents allocations where agents rationally hold liquidity rather than continue trading to tangency. This illustrates the short-run Keynesian outside option: with money valued for its liquidity, the efficient set is no longer confined to the goods plane, and trade can stall away from the classical tangency. The design is intentionally minimal: only the diagonal, a single m-axis, and a compact liquidity region to emphasize the conceptual shift without visual clutter.

Heterogeneous-agent models match this geometry. With borrowing limits, spending depends on liquid resources, so marginal propensities to consume are high and dispersed. In incomplete-market models with borrowing limits, households' spending depends strongly on liquid resources ("cash on hand"), yielding high and heterogeneous marginal propensities to consume. Two-asset frameworks, which distinguish liquid from illiquid wealth, show that agents can be asset-rich yet liquidity-poor. Wealthy hand-to-mouth agents hold illiquid wealth but little cash, so they behave as constrained and display high marginal propensities to consume [20]. In our terms, they sit in the liquidity region: further movement along the goods plane requires giving up the liquidity that they value more at the margin, so short-run inaction is individually rational even when interior tangencies exist.

Recent evidence points to liquidity, not net worth: households rich in illiquid assets but short on cash have very high marginal propensities to consume. They display marginal propensities to consume near 30 percent and lift the average marginal propensity to consume well above representative-agent benchmarks, precisely when liquidity preference is salient [18].

A minimal formalization helps. Consider an economy with fixed aggregate quantities of two goods and of money. Each agent's utility has a standard goods component plus a liquidity component weighted by a parameter that reflects the agent's preference for holding money. Feasibility requires that, for each good and for money, individual holdings add up to the fixed totals. When the marginal value of liquidity for at least one agent is sufficiently high relative to the marginal value of the goods, there exist Pareto-efficient allocations in which agents hold positive money balances even though the marginal rates of substitution in the goods space are not equalized at a tangency. In this case the classical contract curve is no longer a single line in the Edgeworth box; it becomes a higher-dimensional efficient set that includes liquidity-rich points. In the short run, this opens the possibility that trade stalls before the economy reaches the tangency that would prevail without liquidity preference. Formal proofs of the boundary-efficiency conditions and of the liquidity-augmented ("exploded box") dimensional claims are provided in the Supplementary Materials.

Normalize the price of real balances to one and let prices be $p = (p_x, p_y, 1)$. For agent i with utility $U_i(x_i, y_i, m_i) = u_i(x_i, y_i) + \alpha_i v(m_i)$, the KKT conditions for a competitive choice given income M_i are as follows:

$$u_{i,x}(x_i, y_i) \leq \lambda_i p_x,\ u_{i,y}(x_i, y_i) \leq \lambda_i p_y,\ \alpha_i v'(m_i) = \lambda_i,$$

with complementary slackness and $p_x x_i + p_y y_i + m_i \leq M_i$. A liquidity rest point is a feasible allocation $((x_A, y_A, m_A), (x_B, y_B, m_B))$ for which $m_i > 0$ for at least one agent and the KKT system holds for both agents. The liquidity region is the set of such feasible allocations. Projected onto the goods plane, it consists of allocations where, for at least one agent, the inequalities

$$\frac{u_{i,x}(x_i, y_i)}{p_x} \leq \alpha_i v'(m_i),\ \frac{u_{i,y}(x_i, y_i)}{p_y} \leq \alpha_i v'(m_i)$$

prevent further mutually beneficial movement along the goods plane. In contrast, the classical contract curve requires $m_i = 0$ and $\mathrm{MRS}_A = \mathrm{MRS}_B = p_x/p_y$.

The boundary of the liquidity region (in the goods plane projection) is characterized by at least one agent being exactly indifferent between trading one more unit of a good and holding liquidity:

$$\frac{u_{i,x}}{p_x} = \alpha_i v'(m_i) \text{ or } \frac{u_{i,y}}{p_y} = \alpha_i v'(m_i),$$

while the interior has strict inequalities and thus rational inaction. As $\alpha_i v'(\cdot) \to 0$ (liquidity preference fades), the region collapses to the classical contract curve; as $\alpha_i v'$ rises, the region thickens, and no-trade at or near the extremes becomes individually optimal even when a goods plane tangency exists. Figure 3 shows the geometry.

For the corners, the message is straightforward: in a barter or numéraire world, they are trivially Pareto-efficient because any trade would make at least one agent worse off, so Say's structural logic holds. With liquidity preference, however, corners or highly unequal endowments can persist not because they are normatively desirable but because agents value liquidity more than the marginal consumption they would gain by trading, which makes short-run inaction rational though not necessarily efficient in a strong welfare sense. This is precisely how Keynes does not deny Say's law but brackets it, since Say's structural identity reasserts itself in the long run when liquidity preference recedes or policy relaxes the constraint, while in the short run a liquidity region can dominate behavior.

The welfare theorems help separate geometry from normative content. First, any Walrasian equilibrium allocation is Pareto-efficient (First Welfare Theorem). Second, for any Pareto-efficient allocation, there exist prices that support it as a Walrasian equilibrium after

a suitable redistribution of endowments (Second Welfare Theorem) [21]. In our context, this means interior tangencies and the two corners can all be price-supported; what changes under liquidity preference is not feasibility or support but the short-run selection of rest points [12].

Stepping to the large-economy limit clarifies how our pieces fit together: by the Debreu–Scarf core-equivalence result, replicating the economy makes the core contract to the Walrasian set [22]. Here, "replication" just means taking many identical copies of the original economy. As the number of traders grows, each becomes negligible, so no coalition can profitably block an allocation unless it is already supported by a single price vector. Hence, the core collapses to the Walrasian set. And the "Walrasian set" means the set of feasible allocations that can be supported as competitive equilibria for some strictly positive price vector and (if needed) some redistribution of endowments: each agent maximizes given those prices, and all markets clear. In the Edgeworth box, it is the subset of the contract curve that a price line can support as an equilibrium point (including boundary cases when prices make them optimal). In Edgeworth geometry, the long-run competitive selections are exactly the price-supported points on the contract curve (including boundary cases when prices support them) [12]. Our short-run Keynesian layer explains the observed departures: when money carries a liquidity premium and minimum cash positions bind, rational rest points can sit off the tangency; the "liquidity region" persists even though gains from trade exist. As liquidity preference recedes or policy lowers the liquidity premium, that region shrinks, and selection returns to the contract curve. Complementarily, if agents value wealth or real balances directly, competitive selections in the long run can themselves depend on monetary policy via the altered Euler condition, so the price-supported points that survive in large economies need not be invariant to policy once liquidity carries utility [23].

Policy and empirical implications follow. First, monetary and fiscal interventions that lower the marginal utility of liquidity (or raise the expected returns to trade) can shrink the liquidity region, restoring movement toward tangency points. Two-asset calibrations [18] show that narrowing the liquid–illiquid return gap or easing rebalancing reduces the share of hand-to-mouth households and the average marginal propensity to consume, thereby restoring movement toward tangency; the converse widens the liquidity region.

Moreover, measurement matters: short-run inactivity is not a proof of micro-efficiency; it may signal liquidity preference, precaution, or expectations of future price changes. Finally, welfare assessment must treat money as a real argument of utility in the short run; simple tangency tests inside the goods plane risk overstating efficiency. Table 1 summarizes the main discussion, contrasting the classical Edgeworth box with the liquidity-augmented geometry.

Table 1. Classical Edgeworth box vs. liquidity-augmented geometry (short-run).

	Classical	**Liquidity-Augmented**
Role of money	Numéraire only (no utility from cash)	Store of value with liquidity utility
Budget line	Channels all trade "now"; no outside option	No longer compels trade; cash is an outside option
Geometry	Two-good plane; interior tangencies on contract curve	Adds a money axis; a liquidity region can dominate
Contract set/rest points	Thin contract curve (tangencies plus corners)	"Thickened" efficient set including liquidity-rich points
Corners	Efficient by construction (no mutually desired trade)	May reflect rational short-run inaction under liquidity preference
Selection principle	Price line selects on the contract curve	Short-run selection can stall off tangency when liquidity is valued
Policy relevance	Not central at this layer	Liquidity/uncertainty-shifting policy can shrink the liquidity region and restore movement

5. Conclusions

In the Edgeworth box, the corners lie on the contract curve because, in a voluntary exchange economy where money is only a composite medium of exchange, the budget constraint confines choices to a price line. So when no trade is mutually desired, the allocation is, by construction, Pareto-efficient. Once money is allowed to serve as a store of value, agents can rationally hold liquidity, the two-good geometry breaks, and the contract curve thickens into a liquidity region in which trade can stall away from tangency.

This framework reconciles Say and Keynes. Say's law is a long-run structural principle: production, together with the act of offering, creates demand and preserves purchasing power. Keynes explains the short run, where uncertainty and liquidity preference can break the simultaneity of supply and demand. In that setting, the absence of trade is not evidence of efficiency.

The practical upshot is that measurement should treat money as an argument in utility when liquidity premia are high, and policy that lowers the marginal utility of liquidity or raises expected returns to trade can shrink the liquidity region and restore movement toward tangency. As the liquidity premium fades, money reverts to a vehicle of exchange, the Edgeworth geometry regains descriptive force, and the corners recover their familiar, purely microeconomic meaning.

Corner allocations in the Edgeworth box are not "unsatisfactory" outcomes that need justification. They are standard boundary cases of Pareto efficiency in a pure-exchange model, and general equilibrium theory already explains why no Pareto-improving reallocation exists at those extremes under the usual assumptions. The role of liquidity preference in this entry is interpretive, not corrective. The contribution is to make the usual numéraire premise explicit and to map the standard Edgeworth geometry to a short-run monetary reading when liquidity services matter. In most microeconomic treatments, money is absent or neutral, and budget constraints together with Walras' law tie demand to the value of what agents can offer, which is the exchange-economy sense in which a Say-type accounting discipline operates. Keynes's point then enters as a short-run qualification: if money provides utility as a store of value, agents can postpone exchange even when relative prices are given. This postponement does not create or destroy purchasing power. Credit mainly shifts purchasing power across time, because today's credit is tomorrow's repayment, and repayment ultimately requires future production. Hoarding likewise delays spending rather than eliminating it, since holding money is a choice to wait, not a plan to abstain forever. With that outside option, no-trade at or near the extremes can reflect liquidity preference in the short run rather than any limitation of the Edgeworth framework. When the liquidity motive recedes, the familiar Edgeworth interpretation of the extremes remains unchanged.

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