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Entry

Lorenz's View on the Predictability Limit of the Atmosphere

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Definition: To determine whether (or not) the intrinsic predictability limit of the atmosphere is two weeks and whether (or not) Lorenz's approaches support this limit, this entry discusses the following topics: **(A)**. The Lorenz 1963 model qualitatively revealed the essence of a finite predictability within a chaotic system such as the atmosphere. However, the Lorenz 1963 model did not determine a precise limit for atmospheric predictability. **(B)**. In the 1960s, using real-world models, the two-week predictability limit was originally estimated based on a doubling time of five days. The finding was documented by Charney et al. in 1966 and has become a consensus. Throughout this entry, Major Point A and B are used as respective references for these topics. A literature review and an analysis suggested that the Lorenz 1963 model qualitatively revealed a finite predictability, and that findings of the Lorenz 1969 model with a saturation assumption supported the idea of the two-week predictability limit, which, in the 1960s, was estimated based on a doubling time of five days obtained using real-world models. However, the theoretical Lorenz 1963 and 1969 models have limitations, such as a lack of certain processes and assumptions, and, therefore, cannot represent an intrinsic predictability limit of the atmosphere. This entry suggests an optimistic view for searching for a predictability limit using different approaches and is supported by recent promising simulations that go beyond two weeks.

Keywords: Lorenz models; predictability limit; doubling time; intrinsic predictability

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1. Introduction

Is the predictability limit of the atmosphere two weeks? Has a physical foundation been robustly established and verified for such a (theoretical) predictability limit? The concept of predictability can be defined as the ability to make predictions (Thompson 1957 [1]), and can be further broken down into intrinsic predictability, which is determined by flow itself; and practical predictability, which is influenced by mathematical techniques such as models and data assimilation systems (Lorenz 1963a [2]) The above definitions are consistent with the following in Lorenz (1982 [3]): "*The instability of the atmosphere places an upper bound on the predictability of instantaneous weather patterns. The skill with which current operational forecasting procedures are observed to perform determines a lower bound.*" Therefore, the question becomes whether (or not) the intrinsic predictability of the atmosphere is limited to two weeks and if the upper limit of predictability for most advanced models is also two weeks. These questions have been raised for more than five decades (e.g., Lorenz 1963b [4]; Charney et al., 1966 [5]). However, as implicitly suggested by the title of Lorenz (1996, 2006 [6,7]) "Predictability—A problem partly solved", the predictability problem remains partly unsolved, according to Lorenz, who is known for his contributions to chaos theory. To provide a baseline for future researchers continuing to tackle this partially solved problem using theoretical and/or real-world models, this study presents a brief overview of the current understanding of finite predictability (e.g., Lorenz 1963b [4]; 1993 [8]; Charney et al., 1966 [5]; Reeves 2014 [9]), as well as major features of the Lorenz 1969 model (e.g., Lorenz 1969 [10]; Lilly 1972 [11]), which is often considered to be a major tool for illustrating the two-week predictability limit.

Past studies regarding the complexities of the atmosphere have yielded numerous, different approaches for studying atmospheric predictability as well as dynamics. Major theory-based concepts, including chaos (e.g., Lorenz 1963b [4]), (baroclinic) instability and waves (Tribbia and Baumhefner, 2004 [12]; Lorenz 1984a [13]), and turbulence (Lilly 1972 [11]; Leith 1971 [14]; Leith and Kraichnan 1972 [15]; Lorenz 1969 [10]), have been applied in order to understand atmospheric predictability. For example, in the 1960s, the Lorenz 1963 model (Lorenz 1963b [4]) was proposed in order to rediscover the sensitive dependence of solutions on initial conditions (SDICs), later known as chaos (Li and Yorke, 1975 [16]). Although the Lorenz 1963 model and generalized Lorenz models with many modes have been used to demonstrate a finite intrinsic predictability for the atmosphere (e.g., Lorenz 1993 [8]; Shen 2014, 2019 [17,18]; Shen et al., 2021, 2022a, b [19–21] and references therein), as discussed below, they have not been used to quantitatively determine an upper limit for predictability (Reeves, 2014 [9]). This fact is not well known.

On the other hand, the meteorology community has cited Lorenz's 1969 model (Lorenz 1969 [10]) and follow-up studies by Lilly (Lilly 1972, 1973, 1990 [11,22,23]; Rotunno and Snyder 2008 [24]; Palmer et al., 2014 [25]; Durran and Gingrich 2014 [26]; Lloveras et al., 2022 [27]) for providing answers to the question of the intrinsic predictability limit being two weeks. Therefore, as of 2023, the following statement is implicitly or explicitly accepted by the meteorology community:

The intrinsic predictability limit of two weeks was reported in Lorenz (1969) [10].

As discussed later in Section 2, the content of the above statement is not supported by a review of studies, including Lorenz (1993 [8]) and Reeves (2014 [9]). As such, the above statement is referred to as the "hypothesis for the intrinsic predictability limit". In fact, as we discuss in the text below, the above statement is not accurate and will be revised.

An idealized model or concept may effectively and qualitatively reveal the fundamental dynamics, and the mechanism, for a targeted phenomenon. On the other hand, Turing (1952 [28]) reminded us that an idealized model is *"a simplification and an idealization, and consequently a falsification"*. This paper argues that inconsistencies between idealized models and new results from different approaches may indicate a need to revisit a model's realism and assumptions to improve our understanding of concepts. Previous, promising 30-day simulations (Shen et. al., 2010, 2011 [29,30]) provided such a motivation for revisiting the validity of the two-week predictability limit, which is presumably supported by Lorenz's studies (e.g., Lorenz 1969 [10]).

The paper is organized to present Lorenz's perspective on predictability limits and major features of the Lorenz 1969 model and is followed by a review and analysis of relevant studies.

2. A Review of Lorenz's View and the 1969 Model

2.1. Lorenz's View of the Predictability Limit

As demonstrated by Lighthill (1986 [31]), the notion of a finite predictability limit for chaotic systems is widely accepted. In the application of chaos to meteorology (Lorenz 1963b [4]; Zeng et al., 1993 [32]), the finite-dimensional chaotic nature of the atmosphere has been illustrated through various means such as laboratory experiments using rotating annulus experiments in the laboratory (Ghil et al., 2010 [33]), an analysis of weather maps (Read 1993 [34]), and numerical simulations based on sophisticated models (Legras and Ghil, 1985 [35]; Washington 2000 [36]). Due to the chaotic nature the predictability of the atmosphere has been proposed to be finite (Lorenz 1993 [8]).

To estimate the predictability limit of the atmosphere, both doubling time and saturation time have been utilized. The term "doubling time" denotes the duration required for a quantity, such as an error, to increase twofold in value. On the other hand, the "saturation time" is defined as the period for a disturbance, such as one occurring at specific scales, to reach a stable state or constant value.

Lorenz's seminal study on chaos was published 30 years prior to his 1993 book *The Essence of Chaos*. The book aimed to provide a review of the origin of the butterfly effect and the history of the two-week predictability limit (Lorenz 1993 [8]). Recently, a review was conducted that suggested three kinds of butterfly effects within Lorenz models (Shen et al., 2022c [37]). Here, drawing from Lorenz's book and an interview conducted in 2007 (Reeves, 2014 [9]), this article provides a review of the history of the two-week predictability limit. To aid in the discussion, Lorenz's view on the predictability limit, summarized as Major Point A and B in the Definition section, is discussed below, along with relevant excerpts.

Since the Lorenz 1963 model only indicated the existence of the limit(s) without specifying duration, when asked about the predictability limit in 2007, Lorenz expressed his desire to determine the precise limit, as indicated by the following excerpt (Reeves, 2014 [9]):

> *I was hoping to get a better idea what the limits were because "this simple model" said there were limits but it didn't tell you whether they were a week or year or what.*

The statement aligns with Point A. Lorenz had also previously expressed a similar sentiment in 1982 (Lorenz 1982 [3]), stating that:

> *The lack of complete periodicity in the atmosphere's behavior is sufficient evidence for instability (Lorenz, 1963b) [4], but it does not reveal the range at which the uncertainty in prediction must become large.*

Further information is presented below.

As we delve into Point B, it is worth noting that while Reeves (2014 [9]) briefly touched upon the issue, more comprehensive details can be found in Lorenz's works, particularly in Lorenz (1993) [8] and Lorenz (1996, 2006 [6,7]). Between pages 103 and 106 in Lorenz's 1993 book, the discussion on the predictability limit began with the following question:

> *What is the basis for choosing two weeks as a time after which the forecasts might differ significantly?*

To address this, Lorenz documented the following: (1) During the early 1960s, the Global Atmospheric Research Program (GARP) required significant funding, and "selling points" had to be established. (2) The GARP Chair, Charney, managed to alter the focus to determine the feasibility of forecasts. Charney's committee used numerical models and concluded that a reasonable estimate for the average doubling time of small errors in temperature or wind patterns was five days. (3) The five-day doubling time suggested a promise for one-week forecasts, but little hope for one-month forecasts, making two-week forecasts appear to be borderline. Lorenz's documentation aligned with Charney et al.'s (1966) [5] report, which include the title "The feasibility of a global observation and analysis experiment" and a conclusion that:

> *We may summarize our results in the statement that, based on the most realistic of the general circulation models available, the limit of deterministic predictability for the atmosphere is about two weeks in the winter and somewhat longer in the summer.*

In subsequent studies, Lorenz (1996, 2006 [6,7]) also cited Charney et al. (1966 [5]) and echoed similar information. Lorenz (1984a [13]) provided additional information, as follows:

> *Predictability experiments were soon made with the few large global circulation models then in existence (Smagorinsky 1963 [38], Mintz 1964 [39], Leith 1965 [40]); . . . . . . As might have been anticipated, the models were sufficiently dissimilar to one another for the predictability studies performed with them to give contradicting results. Leith's model indicated no growth of errors at all; Smagorinsky's indicated a 10-day doubling time, while Mintz's showed a 5-day doubling time. For various reasons Mintz's result came to be the most generally accepted one (see Charney et al., 1966) [5].*

By 1970, the doubling time appeared to be around three days and, as noted by Lorenz in various publications (e.g., Lorenz 1993, 1996, 2006 [6–8]), became even shorter in the early 1980s. Initially, as highlighted in his 1993 publication, Lorenz held a pessimistic view on the feasibility of making two-week predictions. However, in 2007, Lorenz's perspective shifted to optimism. This optimism was noted by Reeves (2014) [9], who reported Lorenz's statement, shown in the following excerpt, that the upper limit for useful day-to-day forecasts may be around two weeks in another 20 years:

> *Now it begins to look as if the upper limit may be somewhere around two weeks, and I get the feeling that another 20 years or so we may actually be making useful day-to-day forecasts up to the two-week range, though I don't think we are doing it now. But we got up to one week, which I didn't really expect at the time.*

The discussions mentioned not only provide support for PointB but also indicated a change in Lorenz's view on the feasibility of two-week predictions. Recent studies (Judt 2018, 2020 [41,42]; Zhang et al., 2019 [43]), using the most advanced models, have reported a similar order of magnitude for the two-week predictability limit.

As stated in Point B, during the 1960s, a doubling time of five days and a consensus on the predictability limit of two weeks were established (Charney et al., 1966 [5]). To demonstrate this predictability limit, Lorenz (1969) [10] proposed an idealized system of ordinary differential equations (ODEs) and an empirical formula. This work inspired Leith (1971) [14] and Leith and Kraichnan (1972) [15] to propose improved modeling approaches (as reviewed in Shen et al., 2022a [20]) that supported a predictability limit of one or two weeks, using adjustable model parameters such as dissipation coefficients and an instability function. In comparison, Lorenz (1969) [10] compared his model to real-world models, highlighting advantages such as resolving a wide range of scales but also limitations, such as not accounting for baroclinic processes, dissipative effects, radiation, homogeneity, and isotropy assumptions, among others. He suggested that further model improvements would likely result in quantitative changes to predictability estimates.

Despite being frequently cited within the meteorology community, the original study by Lorenz (1969) [10] and related turbulence studies by Lilly, Leith, and Kraichnan (Lilly, 1972, 1973, 1990; Leith 1971 [14]; Leith and Kraichnan 1972 [15]) were not referenced in Lorenz's later studies on predictability limits (e.g., Lorenz 1993 [8]; Reeves, 2014 [9]). Instead, Lorenz (1996, 2006 [6,7]) used a different chaotic system for predictability estimates and only briefly acknowledged that additional assumptions were required to close the equations, similar to Lorenz (1969) [10], as indicated in the following excerpt:

> *I have confined my quantitative discussions to results deduced from pairs or ensembles of numerical solutions of mathematical models with various degrees of sophistication, but alternative approaches have also been exploited. Some studies have been based on equations whose variables are ensemble averages of error magnitudes. These equations have been derived from conventional atmospheric models, but, to close the equations, i.e., to limit the number of variables to the number of equations, it has been necessary to introduce auxiliary assumptions of questionable validity (see, for example, Thompson, 1957 [1]; Lorenz, 1969 [10]). Results agree reasonably well with those yielded by more conventional approaches.*

Lorenz's studies in 1996 and 2006 did not delve into the particular methods used in Lorenz's 1969 study or into the dynamics of turbulence. Furthermore, while estimating predictability horizons, Lorenz (1996) [6] utilized a time unit of five days, which differs from the time unit of six days that required a velocity scale of 17.2 m/s in Lorenz's 1969 study. Predictability estimates display a dependence on the time unit (i.e., time scale) that may be determined by the choice of the velocity scale (e.g., Lorenz 1969) [10]. Below, we further explore the reliability of the Lorenz 1969 model in establishing an inherent predictability boundary.

2.2. Major Features of the Lorenz 1969 Model

Within the scientific community, an overlooked fact is the reality that the Lorenz 1969 model does not qualify as a turbulence model, according to a recent study by Shen et al. (2022a [20]). The model is built upon a straightforward partial differential equation (PDE) that conserves vorticity and lacks baroclinic and dissipative processes. Consequently, the Lorenz 1969 model, which comprises a set of ordinary differential equations (ODEs), cannot be considered as a turbulence model. Moreover, despite being derived from a modified quasi-normal approximation, the model's closure leads to inconsistent characteristics (e.g., Leith 1971 [14]) and produces unphysical outcomes (Orszag 1977 [44]; Aurell et al., 1996 [45]). Shen et al. concluded that the Lorenz 1969 model is a closure-based, physically multiscale, mathematically linear, and numerically ill-conditioned system. Therefore, caution should be exercised when interpreting predictability estimated using the 1969 model, whether with or without saturation assumptions, since they may have limitations.

The saturation time is defined as the time for a perturbation to reach a constant value. In Lorenz (1969) [10], the saturation time, which is a function of wavenumber, determines the predictability horizon for the specific scale. While a saturation assumption was applied in order to constrain the growth of unstable modes, a system with a saturation assumption should be viewed as a linear, homogeneous equation with nonhomogeneous conditions between the time interval during which two successive modes become saturated. A system with piecewise linearity includes nonlinearity.

Our analysis is also consistent with that of Lorenz (1984b) [46], who called the Lorenz 1969 model *"a system of second-order linear ordinary differential equations"*. To illustrate why the Lorenz 1969 model is not a chaotic system, an important concept is presented. A linear system with an unstable solution of $y = y_o e^{\sigma t}$, where $y'' = \sigma^2 y$ and $\sigma > 0$, represents the simplest version of the Lorenz 1969 model with a system of 21 second-order ODEs. Although the exponential "function" ($e^{\sigma t}$) is a nonlinear function of time, it represents a solution to the "linear" system. Therefore, when a solution exponentially varies with time, it does not necessarily mean that the system is nonlinear. As illustrated in Shen (2021) [47] and Shen et al. (2022a) [20], the above linear system can be expanded to become a nonlinear ODE which is comparable to the non-dissipative Lorenz 1963 model, the inviscid Pedlosky model (Pedlosky, 1971, 1972, 1987 [48–50]), an epidemic model, etc. In the case of the Lorenz 1969 model, the inclusion of one or more assumptions to incorporate nonlinear effects for constraining growth makes computing Lyapunov exponents (Wolf et al., 1985 [51]; Jordan and Smith, 2007 [52]) challenging. Lyapunov exponents are long-term averaged quantities within a model. To date, no study has ever illustrated the existence of a chaotic attractor within the original or modified Lorenz 1969 model. As stated, when Lorenz (1993) [8] discussed chaos, he did not reference Lorenz (1969) [10]. In contrast, the chaotic attractor within the Lorenz 1963 model has been intensively studied (e.g., Lorenz 1963b [4]; Tucker 2002 [53]; Stewart 2000 [54]). In addition to the Lorenz 1963 model, Lorenz's chaotic models and turbulence models have been reported in separate publications (e.g., chaotic models in Lorenz 1996 [6] and 2005 [55], turbulence models in Lorenz 1972 a, b [56,57]). A recent review of Lorenz's models from 1960 to 2008 discussed the dependence of chaotic and nonchaotic solutions on different types of Lorenz models (Shen et al., 2023 [58]).

When Lorenz (1984b) [46] revisited his 1969 study in order to address the disparity in predictability between his theoretical model and a weather forecast model, he reported different estimated levels of predictability. To facilitate discussions, it is worth noting that Charney et al. (1966) [5] suggested that a doubling time of five days could result in a reasonable predictability of seven days (one week) up to fourteen days (two weeks), implying a scale factor of 1.4 or 2.8 between estimated predictability and the doubling time. For example, Lorenz (1984a) [13] provides an illustration demonstrating how a doubling time of 4 days can result in a predictability of eight days, as follows: "*Let us see what a four-day doubling time would imply regarding practical weather forecasting. A typical observational error in temperature may be as low as l °C; it is probably not much less. In eight days, such an error would grow to 4 °C, which would usually be considered tolerable. Reasonably good forecasts a week* *in advance should therefore be possible. In twenty days, however, the error would grow to 32 °C, which would presumably be intolerable.".*

In Lorenz (1984b) [46], the ECMWF model with a wavenumber of 40 had a doubling time of 2.0 to 2.5 days. Hence, if we roughly apply a scale factor of two, we can expect a predictability of approximately four to five days. By referring to Table 1 (derived from Table 3 of Lorenz 1969 [10]), we can determine the value of n using the relationship wavenumber $40 \approx 2^{n-1}$ from Lorenz (1969) [10], resulting in a range of $6 < n < 7$. As demonstrated in Table 1, the corresponding saturation times are approximately 15.7 h and 1.1 days, which are shorter than both the above "estimated predictability of 4 to 5 days" and a doubling time of 2.0 to 2.5 days in the ECMWF model.

Table 1. Estimated predictability as a function of n. The first and third columns are taken from Table 3 of Lorenz (1969) [10], while the second column for wavelengths ($\lambda = 40,000$ km/k) is from Table 1 of Lorenz (1969). Here, t_n indicates the saturation time for the perturbation at wavenumber $k = 2^{n-1}$. The red numbers correspond to information obtained at wavelengths of 625 and 1250 km, as discussed in the main text.

n	λ	t_n
21	38 m	2.9 min
20	76	3.1
19	153	4
18	305	5.7
17	610	8.4
16	1221	13
15	2441	20.3
14	4883	32.1
13	9766	51.1
12	19,531	1.3 h
11	39 km	2.2
10	78	3.6
9	156	5.8
8	312	9.5
7	625	15.7
6	1250	1.1 day
5	2500	1.8
4	5000	3.2
3	10,000	5.6
2	20,000	10.1
1	40,000	16.8

Lorenz (1984b) [46] acknowledged the contradiction and attributed it to the model that produced Table 1 Figure 1. In response, Lorenz (1984b) [46] deliberated on necessary enhancements for the model but opted to employ a different spectrum to enhance predictability. Two experiments were conducted: the first experiment was a control run that utilizes an unaltered spectrum, while the second was a parallel run that incorporates a modified spectrum featuring a spectral gap.

Figures 1 and 2a provide evidence that the control run in the study yielded predictability outcomes comparable to those originally documented in Lorenz (1969) [10]. The red box in Figure 2a,b indicates a scale band of 625–1250 km. In Figure 2a, which represents the control run, the predictability results at these scales suggest that the system exhibits reasonable predictability at 1/2-day, slight predictability at 1 day, and unpredictability at 1.5 days (Lorenz 1984b [46]). Conversely, in the parallel run depicted in Figure 2b, the system demonstrates moderate predictability at four days, slight predictability at five days, and unpredictability at six days. Therefore, at scales ranging from 625 to 1250 km, the two cases with different spectra, respectively, yielded estimated predictability of one and four days. The enhanced predictability appears to align more realistically with a weather forecast model. The above comparison with a better predictability limit reported in Lorenz

(1984b) [46] provides another example that the findings in the original Lorenz 1969 study cannot represent an intrinsic limit for atmospheric predictability.

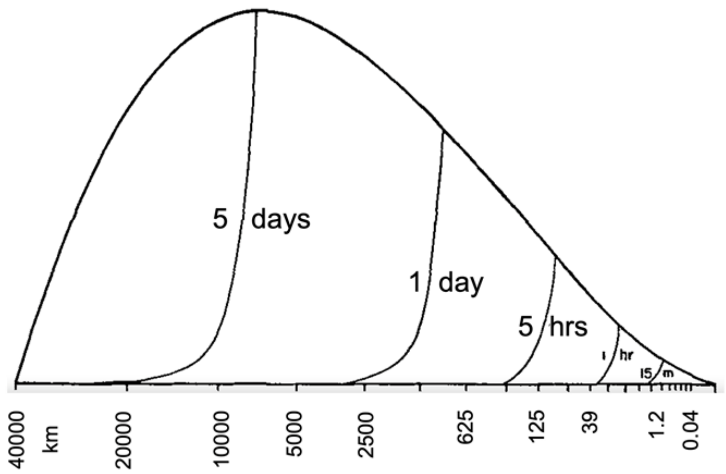

Figure 1. The dependence of estimated predictability on scales (reproduction of Figure 2 in Lorenz 1969) [10].

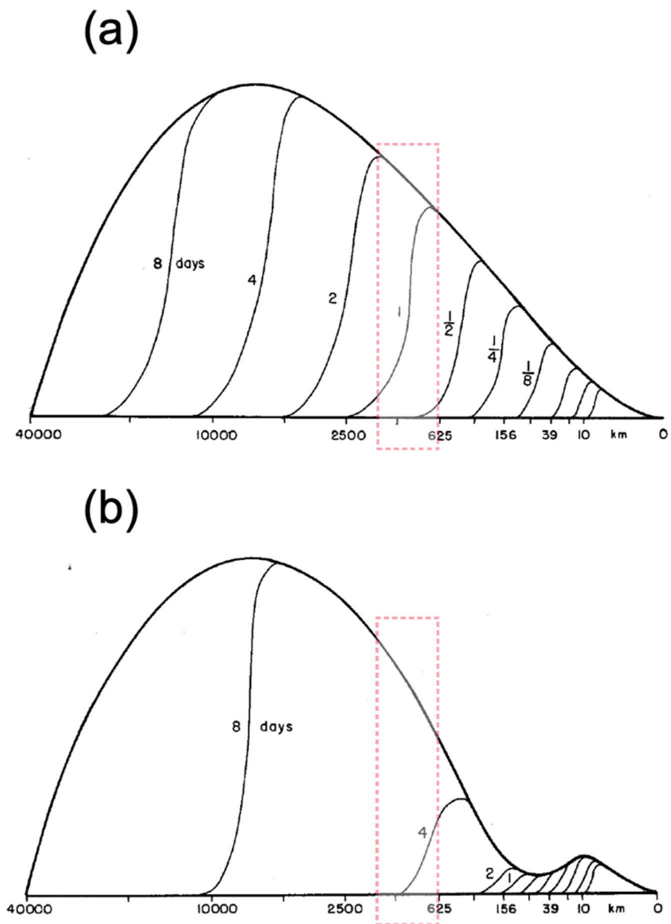

Figure 2. The dependence of deduced predictability on the assumed atmospheric spectrum presented by Lorenz (1984b). Panel (**a**) used an unmodified spectrum that produced results comparable to those in Figure 1 reported in Lorenz (1969) [10], while panel (**b**) applied a modified spectrum with a spectral gap. The red box in Figure 2a,b indicates a scale band of 625–1250 km. Adapted with permission from [46]. Copyright 1984 American Institute of Physics.

Additionally, Lorenz hinted in an unpublished manuscript from 1970 (Lorenz 1970 [59]) that the presence of a spectral gap could enhance predictability. Subsequently, he finalized a manuscript for the American Meteorological Society in 1972 (Lorenz 1972c [60]). Although this 1972 report remained unpublished, certain findings from it were incorporated into Lorenz's work in 1985. In the two papers (Lorenz 1972c, 1985 [60,61]), Lorenz documented the influence of a spectral gap on predictability estimates at scales ranging from tens to hundreds of kilometers. He demonstrated a predictability limit of 20.6 days with a spectral gap for a wavelength of 256,000 km, in contrast to a limit of 16.3 days without a spectral gap, specifically detailed in Table 1 of Lorenz's 1972c [60] or Table 6 of Lorenz's 1985 [61] publication.

3. Discussion

Within the Lorenz 1969 study, the predictability limit of weather was estimated based on the conservative momentum equations. However, driven by the external energy source and sink, atmospheric circulations, including eddies and cyclones, involve the transformation of both kinetic and potential energy and the interconversion between them. Furthermore, atmospheric circulations and clouds affect external energy sources and sinks through radiation and surface fluxes from the underlying surface. Evident is the fact that a model that incorporates the nonlinear feedback between clouds (or atmospheric circulations) and external energy fluxes differs from models solely based on momentum equations such as the 1969 model (Zeng 2023 [62]). Consequently, the use of different models is likely to yield varying estimates of the predictability limit.

As discussed above and in Section 2, we illustrated that the predictability limit of two weeks determined by the approaches of Lorenz (1969) [10] cannot represent an intrinsic predictability limit of the atmosphere. Findings with such a predictability limit may be viewed as additional support to the consensus (Major Point B). Our suggestion makes it easier for scientists to accept why reasonable predictions at time scales of larger than two weeks are possible, even though they may be exceptional cases (e.g., Smagorinsky et al. 1969 [63]; Sonechkin et al. 1995 [64]; Mukougawa et al. 2005 [65]; Liu et al. 2009 [66]; Shen et al. 2010, 2011 [29,30]; Krishnamurthy and Sharma 2017 [67]; Krishnamurthy 2019 [68]; Judt 2018, 2020 [41,42]; Mishra et al. 2021 [69]).

For example, Mukougawa et al. (2005) [65] applied ensemble forecasts to confirm the extended-range predictability of stratospheric sudden warming with a lead time of more than two weeks in the case studied by Mukougawa and Hirooka (2004) [70]. Specifically, based on a family of minor and major warming, Quiroz (1986) [71] reported a strong association between stratospheric warmings and tropospheric blocking, resulting in a stagnation of weather patterns that are more predictable. While such an association may contribute to the predictability of stratospheric sudden warming studied by Mukougawa et al. (2005), a significant sensitivity of prediction to initial conditions has also been shown during the onset of warming. By analyzing difference kinetic energy and the root-mean-square error of the 500 hPa geopotential height, Judt (2018) [41] reported that *the predictability limit of the troposphere was estimated to be around 2–3 weeks,* while Judt (2020) [42] suggested that *the tropics have longer predictability than the middle latitudes and polar regions (tropics > 20 days; middle latitudes and polar regions, a little over weeks)*. By applying an atmosphere–ocean coupled model (or a standalone model), Mishra et al. (2021) [69] reported a predictability limit of 22 days (or 20 days) for Indian monsoon rainfall, which aligns with previous predictability studies utilizing nonlinear time series analysis (e.g., Dwivedi 2012 [72]).

As also reported in recent studies (Magnusson and Kallen 2013 [73], Krishnamurthy 2019 [68]; Zagar and Szunyogh, 2020 [74]), the saturation time may be longer than two weeks. For example, by extending the Lorenz (1965) 28-variable model (Lorenz 1965 [75]) to a 1640-variable model, Krishnamurthy (2019) [68] produced a saturation time of about 100 days (e.g., Figure 1 of Krishnamurthy 2019 [68]).

In the meteorology community, it is often assumed that the more complex a model's dynamics and physics, the larger the predictability limit. However, we argue that simpler models, such as Lorenz-type models, may be effective for estimating intrinsic and/or practical predictability. Several reasons are possible. First, as demonstrated by a dynamical core model that produced a saturation of approximately 65–70 days compared to 20–25 days for the full model (Sheshadri et al., 2021 [76]), models with simpler parameterized physics may have a longer saturation time.

Second, the Lorenz 1969 model covers a wide range of scales (e.g., from 38 m to 40,000 km), and an infinite series was constructed in order to project the impacts of unresolved scales. Thus, from a perspective of multiscale interaction, an estimate of predictability using Lorenz 1969 approaches may be more realistic than estimates using doubling times from advanced real-world models. (However, on the other hand, as mentioned above, the Lorenz 1969 model has its own limits (e.g., a lack of certain processes.)

Third, theoretical models have been used to qualitatively illustrate different types of solutions with distinct intrinsic predictability (e.g., limit cycle vs. chaotic solutions) (Shen et al. 2021 [19], Zeng 2023 [62]). For example, the well-known Lorenz 1963 model reveals chaos with a qualitatively finite predictability, yielding the conventional view of *"weather is chaotic"*. In contrast, by generalizing the Lorenz 1963 model into a generalized Lorenz model, we recently proposed a revised view in order to illustrate qualitatively distinct predictability, as follows: *"The atmosphere possesses chaos and order; it includes, as examples, emerging organized systems (such as tornadoes) and time varying forcing from recurrent seasons"* (Shen et al. 2021, 2022b [19,21]). Additionally, a recent reanalysis of the Lorenz 1969 model indicated various types of solutions within the model, including stable, unstable, and oscillatory solutions which possess distinct predictability (Shen et al., 2022a [20]). Recently, an idealized tropical model was developed to consider the interactions of clouds and radiation. This model aims to provide support for the notion that the predictability limit could extend to the lifespan of certain systems in tropical regions, such as the Madden–Julian oscillations spanning 30–60 days and the El Niño–Southern oscillation lasting 3–7 years (Zeng 2023 [62]).

Other than the above methods, machine learning methods have shown promise for improving weather predictions. For example, by applying deep convolutional neural networks, CNNs), Weyn et al. (2019) [77–79] reported lead times of 14 days (in ensemble runs or some deterministic runs). Additionally, reservoir computing has been applied for replicating chaotic solutions of Lorenz models or predicting sea surface temperature (Pathak et al., 2017 [80]; Lu et al., 2018 [81]; Tomizawa and Sawada, 2021 [82]; Walleshauser and Bollt, 2022 [83]).

4. Summary

This study, which extended our recent predictability studies (e.g., Shen et al. 2021; 2022a, b, c [19–21,37]), addressed questions of whether (or not) the intrinsic predictability limit of the atmosphere is two weeks and whether (or not) such a limit is or is not supported by Lorenz's approaches. We first reviewed Lorenz's view on this topic and then provided an insightful analysis of the Lorenz 1969 model. Based on a literature review and our analysis, Lorenz's view on the predictability limit can be summarized as follows:

A. The Lorenz 1963 model qualitatively revealed the essence of a finite predictability within a chaotic system such as the atmosphere. However, it did not determine a precise limit for the predictability of the atmosphere.
B. In the 1960s, the two-week predictability limit was originally estimated based on a doubling time of five days in real-world models. Since then, this finding has been documented in Charney et al. (1966 [5]) and has become a consensus.

The Lorenz 1969 model (Lorenz 1969 [10]) is closure-based, physically multiscale, mathematically linear, and numerically ill-conditioned. The 1969 model with or without the saturation assumption is neither a turbulence model nor a chaotic system because the original PDE does not include dissipative terms. Other limitations include the lack of baroclinic processes, radiation, and thermodynamic processes as well as the homogeneity and isotropy assumptions. As a result, a predictability limit of two weeks, obtained using the 1969 model, cannot represent an intrinsic predictability limit of the atmosphere. Our suggestion is consistent with the reanalysis of Lorenz (1984b) [46] which reported the discrepancy of "predictability" at scales of 625–1250 km between the Lorenz 1969 model and a weather forecast model. At the scales of 625–1250 km, a predictability difference of three days was reported using the Lorenz 1969 model with original and modified atmospheric motion spectra. Our interpretation of Lorenz (1969)'s [10] findings is also consistent with the Major Point A and B (i.e., Lorenz's view on the predictability limit).

Our analysis explicitly indicates an optimistic view for searching a predictability limit using different approaches. Such a view is supported by recent promising simulations that go beyond two weeks (Shen et al., 2010; 2011 [29,30]; Buizza and Leutbecher 2015 [84]; Bretherton and Khairoutdinov 2015 [85]; Judt 2018, 2020 [41,42]).

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