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Self-Similar Analysis of Start-Up Fluid Flow over Flat Plate

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Abstract

Based on the Lie group method (symmetry transformation groups), an analysis of an unsteady (start-up) flow over a flat surface was performed. This approach enabled reducing the number of independent arguments, which significantly simplifies the process of numerical modeling. An unsteady solution was obtained for the velocity profile in the boundary layer. This enabled estimating the dynamics of the velocity profile transformation and its transition to a steady-state mode. It was shown that in the limit of infinite time of the process, the velocity profile tends to the classical steady-state Blasius profile in the boundary layer. The dynamics of the friction coefficient variation over time were elucidated too.

Keywords: boundary layer; Blasius flow; unsteady flow; start-up flow; Lie group analysis; symmetry groups; self-similar solutions

1. Introduction

Modern theoretical fluid mechanics is based on H. Schlichting's work [1], in which he proposed that the flow around a flat body consists of two regions. The first region is a very thin layer on the surface of the body, called the boundary layer, where friction effects are significant. The thickness of this layer increases along the surface of the body. The second region is the area outside the boundary layer, where friction effects can be neglected. In 1908, Heinrich Blasius proposed a solution for the case of viscous laminar flow over a flat plate [1,2].

Blasius' studies gave a strong impetus to the development of theoretical research on the dynamics of various flow types, which are closely related to practical applications. The emergence of new fields of science (nano, micro-, biotechnology) associated with flow dynamics effects maintains interest in the study of boundary layers.

Currently, boundary layers are investigated analytically, numerically, and experimentally. The methodology of Computational Fluid Dynamics (CFD) is the most suitable approach for simulating fluid flow. Some examples of CFD techniques applied to flow over a flat surface can be found in [3–5]. An example of the numerical solution for the flow and heat transfer of nanofluids, in a case similar to Blasius' problem, is presented in [6]. This study was conducted numerically using the Runge–Kutta method. This method was also implemented in MATLAB R2024b for modeling fluid flow past a flat plate in [7].

Today, the vast majority of modeling tasks in fluid mechanics involve the analysis and solution of differential equations. Examples of analytical solutions for flow over a flat plate using methods such as the homotopy perturbation method (HPM) [7], and the homotopy

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analysis method (HPM) [8,9], show good agreement between analytical modeling results and experimental data.

Applications of symmetry groups (Lie groups) [10] to solve various physical problems based on differential equations are widely presented in studies [11–20].

The classical theory of ordinary differential equations offers a wide variety of specialized methods for solving different kinds of equations. Sophus Lie's discovery showed [10] that all special analytical methods for integrating differential equations are specific cases of a general integration procedure based on the invariance of the differential equation under a particular symmetry group. The Lie group approach unifies and significantly expands the various specialized methods for integrating certain types of equations, such as homogeneous equations, separable equations, and exact differential equations, among others. The same principle applies to systems of equations.

An analysis of unsteady fluid flows on the basis of symmetry groups was performed in the monograph [10]. This work outlines the laws of time-dependent development that allow for the construction of solutions for the boundary layer. In [18], based on the analysis of symmetry groups, self-similar forms were obtained for the independent variables and functions describing the fields of velocity, temperature, and concentration. These forms made it possible to derive generalized self-similar ordinary differential equations for parabolic flows of nanofluids.

The studies [11,12] present the results of theoretical modeling of accelerating flows of incompressible fluids in a microtube and a rectangular microchannel during a sudden pressure drop. The equation of motion was reduced to a self-similar form based on symmetry analysis. These works demonstrated temporal changes in velocity profiles until fully developed flow is achieved. Relations for calculating the hydraulic resistance coefficient were obtained. The instability of sliding flow in a curved channel formed by two concentric cylindrical surfaces was considered in [13]. The authors revealed the dependence of the flow stability criteria on the channel's geometric parameters and the slip velocity coefficient on the walls.

The works [14–17] outline simulations of heat and momentum transfer in nanofluids using Lie group theory. Self-similar solutions for heat transfer in nanofluids were obtained with account for dissipation. Based on the generalized transport equations, the problems of magnetohydrodynamic flow of ferrofluid over a radiating surface [14], film boiling [15], flows of nanofluid with a longitudinal pressure gradient, instability of nanofluid flows [16], and traffic jam in a microchannel [17] were examined.

The results of the analysis (using the symmetry method) of unsteady flow and heat transfer of a viscous fluid on a porous surface are presented in [19]. The unsteadiness of the process was caused by the time-dependent oncoming flow. The influence of process parameters on velocity and temperature profiles was indicated. Solutions for heat transfer in an unsteady three-dimensional laminar boundary layer of viscous incompressible electrically conductive fluid flow over an inclined permeable surface in the presence of a uniform magnetic field were obtained using the Lie group method in [20]. The effects of heat generation/absorption and surface porosity on velocity and temperature profiles were demonstrated.

On the basis of this analysis, substantiation of the Blasius series was found in [21] for the case of a boundary layer development in sudden start-up of motion. A body suddenly subjected to start-up of motion in a quiescent fluid attains its ultimate constant speed very fast. Hence, it starts up in fact instantly. In this case, in a coordinate system rigidly associated with the body, the potential flow velocity is equal to

$$\begin{array}{ll} t \le 0, & \mathrm{U}_\infty(x,t) = 0, \\ t \ge 0, & \mathrm{U}_\infty(x,t) = \mathrm{U}_\infty(x). \end{array} \quad (1)$$

The boundary conditions are

$$\begin{array}{ll} y = 0, & u = 0, \\ y \to \infty, & u = U_\infty(x). \end{array} \quad (2)$$

Here t is time, x is the marching (streamwise) coordinate, y is the transverse coordinate orthogonal to the surface (value $y = 0$ corresponds to the surface of the body), and U_∞ is velocity on the outer boundary of the boundary layer.

Based on this approach, parameters at the point of separation of fluid flow over a round [1] and elliptic [22] cylinders were predicted. Similar computations for a case of gradual speed-up were performed in [23,24].

In a series of investigations, particular types of flows were studied, which possess full symmetries and, hence, also full self-similar forms. For example, in works [25–27] cases were investigated where the potential flow velocity complies with the law $U_\infty \sim 1/t$.

One more opportunity to apply symmetries consists of use of transformations which allow reducing three independent variables x, y, t to two transformed (at least partially) variables. Authors of work [28] in this case speak about so-called "semi-self-similar" solutions. For example, it is possible to reduce primary variables x, y, t to transformed variables y and x/t. The resulting solutions were named "pseudo-stationary" [29]. A wide variety of such solutions is considered in [30].

The objective of the present research is to model an unsteady Blasius flow over a flat plate in the case where conditions (1) and (2) hold. In doing so, U_∞ = const.

The present research includes results of a theoretical study of unsteady incompressible fluid flow over a flat surface (Blasius flow). The novelty of this study consists of the following.

1. The analysis of the system of differential equations was performed for the first time using the classical symmetry group (Lie group) approach. An algorithm for mathematical transformations is presented using an infinitesimal generator (unlike other studies), whose coefficients depend only on independent variables and functions. It was shown how the infinitesimal generator enables finding self-similar variables. This approach reduced the number of independent arguments, significantly simplifying the numerical modeling process.

2. The symmetry group method enabled deriving unsteady self-similar momentum transfer equations describing the time-dependent dynamics of boundary layer development. Based on the derived equation, the velocity profile of the Blasius flow was analyzed. The conditions under which the velocity profile reaches a steady-state flow regime were identified.

3. It was shown that if the boundary conditions satisfy self-similarity requirements, the results of symmetry transformations and numerical calculations are independent of the choice of the type of the marsh variable (time or longitudinal coordinate).

4. The temporal variation in the friction coefficient was quantified and analyzed. The time limit at which the friction coefficient asymptotically reaches the classical value of Blasius's law was found. It was shown that the present solution provides an asymptotic approximation to the steady-state equation for the friction coefficient, which is consistent with the physics of the process. At the same time, it was shown that the transient solution proposed by Blasius [1] does not fulfill the requirement of transition to the asymptotic steady-state solution.

5. This approach is of an evaluative nature and can be developed further to solve more complex problems of unsteady flows arising in different types of industrial equipment.

2. Mathematical Model and Symmetry Group Analysis

The boundary layer equations for unsteady flow with zero streamwise pressure gradient can be written as follows

$$\begin{array}{l} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \\ \frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0, \end{array} \tag{3}$$

where u is the streamwise velocity component, v is the transverse velocity component, and ν is the kinematic viscosity. Initial and boundary conditions for system (3) are given by Equations (1) and (2). It was shown in work [10] that the basic Lie algebra appropriate to system (3) is

$$\begin{array}{l} \mathrm{q} = [C_1 + 2C_2 t]\partial_t + [C_3 x + \kappa_1(t)]\partial_x + [C_2 y + \kappa_2(t) + \Phi(x)]\partial_y \\ + \left[(C_3 - 2C_2)u + \kappa_1'(t)\right]\partial_u + [-2C_2\mathrm{v} + \kappa_2'(t) + \Phi'(x)u]\partial_\mathrm{v}, \end{array} \tag{4}$$

where constants C_1, C_2, C_3 correspond to independent symmetries, whereas κ_1, κ_2, Φ are arbitrary functions. The constants C_1, C_2, and C_3 have no physical meaning. They represent independent Lie subalgebras, each of which performs symmetry transformations independently of the other.

As known from the literature [1], for a steady-state fluid flow over a flat plate, a solution of system (3) under condition (2) takes the form of a self-similar solution of Blasius. This solution is based on the use of a self-similar variable

$$\eta = y\sqrt{\frac{\mathrm{U}_\infty}{\nu x}}, \tag{5}$$

which is a consequence of the Lie sub-algebras C_2 and C_3 of the infinitesimal generator (4).

Let us present the solution for the unsteady Blasius flow as

$$u = \mathrm{U}_\infty \frac{\partial f(\eta, \varsigma)}{\partial \eta}, \tag{6}$$

where

$$\varsigma = \frac{t\mathrm{U}_\infty^2}{\nu} \tag{7}$$

denotes a dimensionless time. Substitution of Equation (6) into system (4) yields the following ordinary differential equation

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{1}{2} f \frac{\partial^2 f}{\partial \eta^2} = \mathrm{Re}\frac{\partial f}{\partial \eta \partial \varsigma}, \tag{8}$$

where

$$\mathrm{Re} = \frac{\mathrm{U}_\infty x}{\nu}. \tag{9}$$

It is evident that Equation (8) is not self-similar. This is in line with expectations, because variables η and ζ were obtained only on the basis of the symmetry corresponding to the steady-state equation. For a correct representation of the self-similar solution it is necessary to take into account the unsteady nature of the initial system of equations, i.e., it is necessary to use the following Lie sub-algebra

$$\mathrm{q} = 2C_2 t\partial_t + C_3 x\partial_x + C_2 y\partial_y + (C_3 - 2C_2)u\partial_u - 2C_2\mathrm{v}\partial_\mathrm{v}, \tag{10}$$

On the basis of Equation (10), equations for defining self-similar variables can now be constructed. We need the solution of the unsteady equation to become asymptotically a steady-state form, so one variable must have the form of a Blasius variable, Equation (5). Therefore, the equation for one variable should be taken in the following form

$$C_3 x \frac{\partial \eta}{\partial x} + C_2 y \frac{\partial \eta}{\partial y} = 0. \tag{11}$$

One can find from here

$$\eta = \frac{U_\infty y}{\nu} \mathrm{Re}^{-C_2/C_3}. \tag{12}$$

To separate influence of the initial conditions (1) and boundary conditions (2), the second self-similar variable should include only streamwise variables. Based on that, Equation (10) yields

$$\xi = \varsigma \mathrm{Re}^{-2C_2/C_3}. \tag{13}$$

To construct the self-similar function, it is necessary to choose a parametrical variable. For this purpose, a streamwise variable can be chosen. If the streamwise coordinate x is used, the infinitesimal generator (10) results in the following relation:

$$u = U_\infty \frac{\partial f(\eta, \xi)}{\partial \eta} \mathrm{Re}^{1-2C_2/C_3}. \tag{14}$$

If time serves as a parametrical variable, one can find from Equation (10)

$$u = U_\infty \frac{\partial f(\eta, \xi)}{\partial \eta} \varsigma^{1-C_3/(2C_2)}. \tag{15}$$

Using variables (12)–(14), one can obtain from Equation (3) the following self-similar equation:

$$\frac{\partial^3 f}{\partial \eta^3} + \left[\left(\frac{C_2}{C_3} - 1 \right) f - \frac{2C_2}{C_3} \xi \frac{\partial f}{\partial \xi} \right] \frac{\partial^2 f}{\partial \eta^2} + \left(\frac{2C_2}{C_3} - 1 \right) \left(\frac{\partial f}{\partial \eta} \right)^2 = \left(1 - \frac{2C_2}{C_3} \xi \frac{\partial f}{\partial \eta} \right) \frac{\partial f}{\partial \eta \partial \varsigma}. \tag{16}$$

If function (15) is used, one can obtain the following equation:

$$\frac{\partial^3 f}{\partial \eta^3} + \left[\frac{C_2}{C_3} \xi^{\frac{C_3}{2C_2}-1} f - \frac{2C_2}{C_3} \xi^{\frac{C_3}{2C_2}} \frac{\partial f}{\partial \xi} \right] \frac{\partial^2 f}{\partial \eta^2} + \left(1 - \frac{2C_2}{C_3} \right) \frac{1}{\xi} \frac{\partial f}{\partial \eta} = \left(1 - \frac{2C_2}{C_3} \xi^{\frac{C_3}{2C_2}} \frac{\partial f}{\partial \eta} \right) \frac{\partial f}{\partial \eta \partial \varsigma}. \tag{17}$$

As may be seen from Equations (14) and (15), in order for the boundary condition at $y \longrightarrow \infty$ to be independent of the longitudinal coordinate, it is necessary to assume that $2C_2 = C_3$. In this case, both Equations (16) and (17) can be reduced to

$$\frac{\partial^3 f}{\partial \eta^3} + \left[\frac{1}{2} f - \xi \frac{\partial f}{\partial \xi} \right] \frac{\partial^2 f}{\partial \eta^2} = \left(1 - \xi \frac{\partial f}{\partial \eta} \right) \frac{\partial f}{\partial \eta \partial \xi}. \tag{18}$$

Initial and boundary conditions should also be presented using the self-similar variables

$$\begin{array}{l} \xi \leq 0 \ \frac{\partial f(\eta, \xi)}{\partial \eta} = 0, \\ \xi \geq 0 \ \frac{\partial f(\infty, \xi)}{\partial \eta} = 1, \eta = 0 \ \ f(\eta, \xi) = \ \frac{\partial f(\eta, \xi)}{\partial \eta} = 0 \end{array}. \tag{19}$$

3. Numerical Results and Discussion

The numerical solution of Equation (18) under conditions (19) was performed in the present study with the help of the MATLAB software. Results of this solution are depicted in Figure 1.

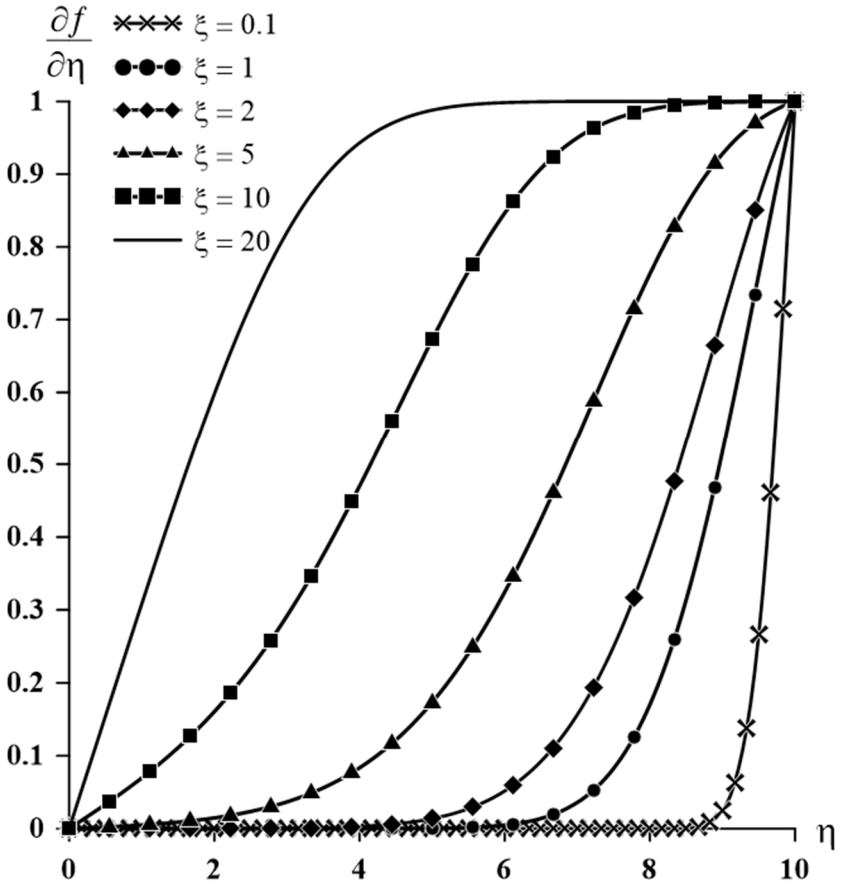


Figure 1. Blasius flow velocity profiles.

The problem was solved using the marching method along the coordinate ξ with a step of 0.1 in the range of ξ from 0 to 20. In this case, for each step of the ξ-coordinate η varied in the range from 0 to 10. This range was divided into 200 steps.

It is evident that during the initial stage of the speed-up the velocity profile has the form characteristic for wake flows, i.e., it has an inflection point. This point disappears at approximately $\xi \approx 13$. Then, following the flow development, fullness of the velocity profiles increases. Finally, the velocity profile accepts the classical form of the steady-state flow of Blasius.

Surface friction can be calculated using the following equation:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \rho U_\infty^2 \left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0} \mathrm{Re}^{-C_2/C_3} = \rho U_\infty^2 \left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0} \mathrm{Re}^{-1/2}, \quad (20)$$

where μ is the dynamic viscosity, and ρ is the density.

Next, we determine the friction coefficient using the following relation,

$$c_f = \frac{2\tau_w}{\rho U_\infty^2} = \frac{2}{\sqrt{\mathrm{Re}}}\left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0}. \quad (21)$$

Numerical calculations performed on the basis of velocity profiles (Figure 1) enabled approximating the data for the friction coefficient in the following form:

$$\frac{c_f}{c_{f0}} = [1 - \exp(-0.15\xi)]^{-1}, \tag{22}$$

where

$$c_{f0} = \frac{0.664}{\sqrt{\mathrm{Re}}} \tag{23}$$

is the classical equation for the Blasius friction coefficient for steady-state laminar boundary layer flow on a flat plate.

Equation (22) was obtained by approximating the numerical values of the derivative of $\left(\frac{\partial^2 f}{\partial \eta^2}\right)_{\eta=0}$ with respect to time (variable ξ) using the least squares method with an accuracy of 3%. The form of this equation itself was chosen in such a way that the time-dependent friction coefficient tended to a steady-state value under the condition $\xi \to \infty$.

As known [1], boundary layer flows have a singularity point at $x = 0$ and $t = 0$. Therefore, to describe the friction coefficient, approximation (22) was chosen, which takes into account both types of singularity.

Equation (22) shows that the friction coefficient for accelerating flow varies from infinity at $\xi = 0$ to the classical form of Equation (23) at the value of ξ, which with 99% accuracy is equal to

$$\xi = \frac{t\mathrm{U}_\infty}{x} \approx 20 \tag{24}$$

A similar solution for an unsteady boundary layer was proposed by Blasius [21], which accordingly to [1] has the following form

$$u = \mathrm{U}_\infty \int_0^\chi \exp\left(-\chi^2\right) d\chi = \mathrm{U}_\infty \mathrm{erf}(\chi), \tag{25}$$

where

$$\chi = \frac{y}{2\sqrt{\nu t}}. \tag{26}$$

This yields the following equation for the friction coefficient

$$c_f = \frac{2}{\sqrt{\pi}} \sqrt{\frac{\nu}{\mathrm{U}_\infty^2 t}}. \tag{27}$$

As can be seen from here, Equation (27), in contrast to the present solution (22), does not provide an asymptotic approximation to the stationary solution (23) at $t \to \infty$. This does not correspond to the physics of the process.

Thus, on the basis of the symmetry groups for the unsteady boundary layer equation, self-similar variables were obtained, which allowed for reducing the number of independent variables. This simplifies the procedure of numerical integration and the analysis of its results. Obviously, this approach can also be extended for more complicated unsteady flows.

4. Conclusions

This paper presents results of a theoretical investigation of unsteady flow of an incompressible fluid over a flat surface (Blasius flow). The analysis of the system of differential equations was performed using the classical symmetry group approach. An algorithm

is presented for mathematical transformations using an infinitesimal generator, whose coefficients depend only on the independent variables and functions.

The symmetry group method enables deriving unsteady self-similar solutions to the system of momentum transfer equations that describe the time-dependent dynamics of the boundary layer development thus enabling reducing the number of independent variables. Based on the derived equations, the influence of unsteady conditions on the velocity profile of the Blasius flow was analyzed. The conditions under which the velocity profile reaches a steady flow regime were revealed.

It was shown that if the self-similarity requirements for the boundary conditions are satisfied, then the results of symmetry transformations and numerical calculations do not depend on the choice of the type of marching variable (time or longitudinal coordinate).

Evidently, this approach is of an evaluative nature and can be extended to solve more complex problems of unsteady flows that occur in various types of technological equipment.

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Abbreviations

CFD	Computational Fluid Dynamics
HAM	Homotopy Analysis Method
HPM	Homotopy Perturbation Method

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