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Article

Theory and Practice of Heavier Drop-Cones for Index Property and Strength Determinations of Fine-Grained Soils

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Abstract

The laboratory fall-cone (FC) test method is widely adopted for liquid limit LL_{FC} determination, and in undrained shear strength s_u measurement of very soft to firm fine-grained soils. Additionally, it is used to establish the PL_{100} parameter (i.e., the water content w mobilizing a 100-fold s_u increase relative to the LL_{FC} strength), typically determined using LL_{FC} data extrapolation approaches. This paper presents a theoretical and data-reanalysis investigation into the application of drop-cone (DC) methods for s_u measurement of firm to stiff fine-grained soils, and their PL_{100} determination. Here, a 30° or 60° cone of modest mass M, released from a drop height h_d, contacts the test-specimen surface with an impact velocity, and penetrates into the soil, before coming to its rest position for a cone penetration depth d. Based on previously reported experimental FC and DC d:w data for ten different fine-grained soils, the authors examined the effects of (**i**) frictional resistance between the stem of the falling DC and its guide bearing, and (**ii**) the DC's higher soil deformation rate, on the deduced PL_{100} water content. An adjusted cone-factor framework for DC s_u testing, which considers the h_d/d ratio, and higher soil deformation rate, is presented. Bearing friction has the effect of deducing slightly higher s_u and PL_{100} values. Small calibration adjustments to the DC setup, namely marginally increasing (**i**) the cone mass M, (**ii**) the drop height h_d, or (**iii**) both M and h_d, can eliminate systematic overpredictions. Theoretical 30° and 60° DC M:h_d combinations are identified for PL_{100} determination. Finally, LL_{FC} data extrapolation for PL_{100} determination, employing the bi-logarithmic $\log_{10}d$:$\log_{10}w$ model, was found to achieve moderate levels of agreement between predicted PL_{100} values and those directly measured using an 8 kg/30°–0 mm contacting cone for d = 20 mm, raising possible concerns about the generalizability of the extrapolation approach.

Keywords: fine-grained soil; fall-cone liquid limit; drop-cone test; cone mass and penetration; drop height; undrained shear strength

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1. Introduction

The laboratory fall-cone (FC) test method provides a convenient, reliable, and quick means of determining the undrained shear strength s_u and sensitivity (thixotropy) of fine-grained soils [1–5]. Referring to Figure 1a; during FC testing, a smooth metal cone of specified mass M and apex angle β is released from a position where its tip just contacts the surface of the soil test-specimen. The cone is then allowed to penetrate the specimen under its self-weight, and the mobilized s_u value is determined from the resulting final cone penetration depth d, typically measured after 5 s, using the following relationship [1]:

$$s_u[\text{in kPa}] = \frac{KMg}{d^2} \tag{1}$$

where K = FC factor for the contacting cone (CC) (i.e., cone initially stationary at the start of penetration); M and d = mass of the CC assembly (in g) and measured cone penetration depth (in mm), respectively; and g = standard gravitational acceleration (=9.81 m/s^2).

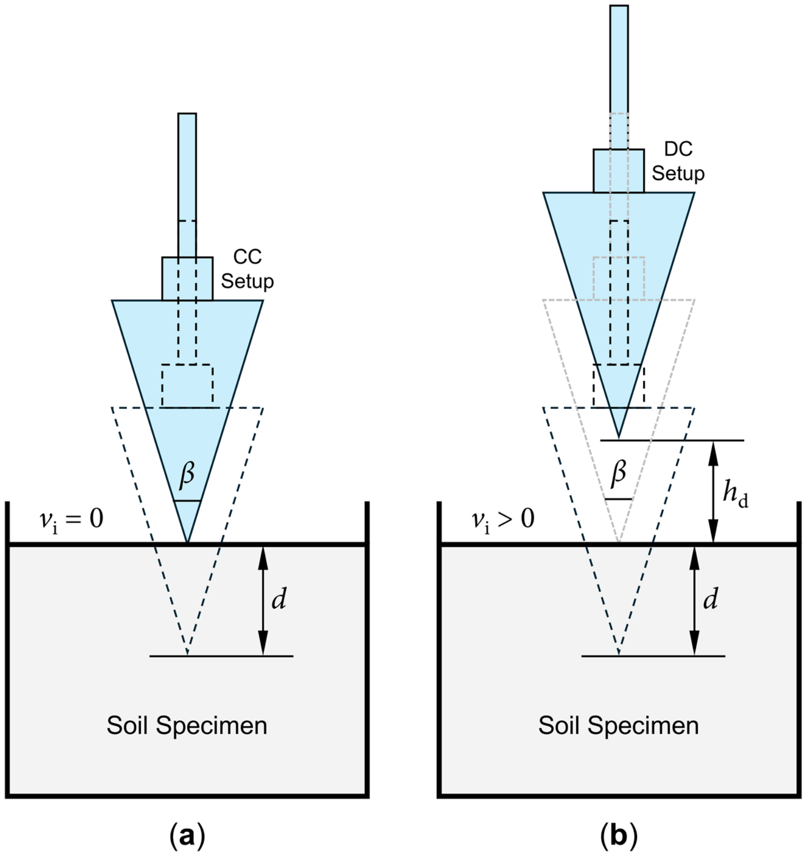


Figure 1. Schematic illustration (not to scale) of FC test setups: (**a**) conventional contacting cone (CC) configuration; and (**b**) alternate drop-cone (DC) arrangement suited for PL_{100} determination and strength testing of stiff fine-grained soils. **Note:** v_i and h_d denote the cone's contact velocity and drop height, respectively.

The FC method, employing standardized 80 g/30° and 60 g/60° CCs, has also been widely adopted in national (e.g., IS 2720–5 [6]; BS 1377–2 [7]; AS 1289.3.9.1 [8]) and international (e.g., ISO 17892–12 [9]) standards for determining the FC liquid limit LL_{FC}. For the 80 g/30° CC setup, the LL_{FC} is defined by a cone penetration depth of d = 20 mm [7,9], with an associated undrained shear strength of $s_{u@LL_{FC}} \approx 1.7$ kPa [4,10,11]. Substituting $s_{u@LL_{FC}}$ = 1.7 kPa, M = 80 g and d = 20 mm into Equation (1) and solving for K, the cone factor for the 30° CC configuration is obtained as K_{30} = 0.866. Similarly, for the standardized 60 g/60° CC, for which the LL_{FC} is defined by d = 10 mm [9], and assuming $s_{u@LL_{FC}} \approx 1.7$ kPa, the corresponding cone factor is determined as K_{60} = 0.289.

Additionally, the FC approach is used to determine the PL_{100} parameter [4,5,12–17], defined as the reduced water content (WC) corresponding to a 100-fold strength increase relative to that mobilized at the LL_{FC} WC. Note that the PL_{100} parameter differs from Atterberg's thread-rolling plastic limit PL_{TR}, in that the former has a strength-based definition, whereas the latter assesses the soil's ductility characteristics, and is defined as the lowest

WC at which the fine-grained soil exhibits plasticity (i.e., the brittle–plastic/ductile state transition) [4,15]. In other words, given its strength-based definition, the PL_{100} WC can be established for any fine-grained soil, whether plastic or non-plastic in nature, whereas the PL_{TR} WC is only obtained for fine-grained soils that exhibit plastic behavior over a certain WC range, as assessed using the standard thread-rolling test [7,9,18].

For the 80 g/30° CC setup (with K_{30} = 0.866), Equation (1) indicates that the PL_{100} condition corresponds to a cone penetration depth of d = 2 mm, obtained by substituting $s_{u@PL_{100}} = 100 \times s_{u@LL_{FC}}$ = 170 kPa into the equation. Since this penetration depth is small, its measurement accuracy can significantly influence the measured PL_{100} value [4]. Consequently, the use of heavier CCs has been advocated, most notably the 8 kg/30° CC setup [10,19]. Applying Equation (1), the PL_{100} parameter for the 8 kg/30° CC is established at d = 20 mm. Furthermore, Koumoto et al. [20] examined the applicability of a 1.5 kg/60° CC for PL_{TR} determination at d = 5 mm, assuming that the PL_{100} value deduced for a given fine-grained soil is equivalent to its PL_{TR} (generally not the case in practice, as elaborated above), and with the LL_{FC} WC established at d = 10 mm. Equally, a 6 kg/60° CC setup can be used for PL_{100} determination at d = 10 mm.

It should also be noted that various extrapolations of the WC–cone penetration depth (i.e., w:d) relationship obtained from standard LL_{FC} testing (with d ranging from 15 to 25 mm for the 80 g/30° CC configuration [7–9]) have been advocated for PL_{100} determination, including the bi-logarithmic $\log_{10}w$:$\log_{10}d$ (or $\log_{10}d$:$\log_{10}w$) model proposed by Feng [21], for which the PL_{100} WC is obtained as $PL_{100} = (2/c)^{\frac{1}{m}}$ [22], where c and m are fitting parameters derived from a power-law trendline fitted to the d:w data. Alongside the PL_{100} WC, other strength-based index parameters have been proposed, namely the PL_{10} [2] and PL_{25} [4], defined as the WCs corresponding to 10-fold and 25-fold strength increases relative to $s_{u@LL_{FC}}$, respectively. From the FC theory, for a given cone mass and apex angle, the corresponding cone penetration depth and mobilized s_u can be computed using Equation (1). For instance, employing the standardized 80 g/30° CC setup utilized for determining the LL_{FC} WC (at d = 20 mm) [7,9], the PL_{10} and PL_{25} WCs are established for d = 6.3 and 4.0 mm, respectively, with corresponding s_u values of 17.0 and 42.5 kPa.

1.1. Sivakumar et al.'s [5] 0.727 kg/30°–200 mm Drop-Cone Setup for PL_{100} Determination

Given the practical difficulties associated with using heavy (e.g., 8 kg/30° or 6 kg/60°) CCs in a laboratory setting, Sivakumar et al. [5] proposed an innovative solution whereby a drop-cone (DC) of smaller mass is released from a specified drop height h_d (see Figures 1b and 2). The elevated lighter cone is released, and allowed to fall freely through the air, reaching the surface of the soil test-specimen with an impact energy that, following a further 20 mm of penetration, matches the energy absorbed by an 8-kg CC penetrating the specimen to a depth of d = 20 mm. Specifically, the total energies expended by the CC and DC setups in penetrating the test-specimen to a depth of d = 20 mm are identical. Following an energy-based approach, Sivakumar et al. [5] proposed a modified FC setup, employing a smooth 0.727 kg/30° metal cone with a drop height of h_d = 200 mm (i.e., the 0.727 kg/30°–200 mm setup), for determination of the PL_{100} WC corresponding to d = 20 mm. The required cone mass (of M = 0.727 kg) for the proposed DC setup was obtained from a simple energy-conservation calculation, assigning a cone penetration depth of 20 mm to both the 8-kg CC and 0.727-kg DC setups at the PL_{100} condition—that is, Work (W) = Force (F) × Distance ⇒ 8 kg × 20 mm [for CC] = M × (200 + 20) mm [for DC]. Note that the 0.727-kg mass represents that of the entire cone assembly (i.e., the metal cone and its extended stem). Once released, the DC free-falls under gravitational acceleration, with its tip contacting the surface of the soil test-specimen at a velocity of v_i. Upon penetration, the cone continues to accelerate initially, but then progressively decelerates on account of the soil's growing resistance (as the mobilized s_u zone increases with increasing d), before finally coming to a stationary position. Taking the surface of the soil test-specimen as a reference level, and equating the potential energy (i.e., $PE = Mgh_d$) of the stationary elevated cone (for h_d = 200 mm) to the kinetic energy (i.e., $KE = \frac{1}{2}Mv_i^2$) of the falling cone at the instant its tip just contacts the specimen surface, the contact velocity of the falling 0.727-kg DC is computed as $v_i = \sqrt{2gh_d}$ = 2.0 m/s = 7.2 km/h. It should be noted that these calculations assume full transfer of PE to KE (i.e., no frictional resistance acting between the stem of the falling cone and its guide bearings/bushings). Furthermore, with considerably higher deformation rates ($\dot{\gamma}$) mobilized for the penetrating DC setup on account of the significant contact velocity of v_i = 2.0 m/s compared to the penetrating CC configuration (with v_i = 0), it is important to emphasize that the above work/energy calculations assume that the mobilized s_u is deformation-rate independent (i.e., μ = 0, where μ is the strain-rate dependence of the s_u parameter), which is not the case in practice. This aspect and its implications for PL_{100} and s_u determinations are comprehensively investigated later in this paper.

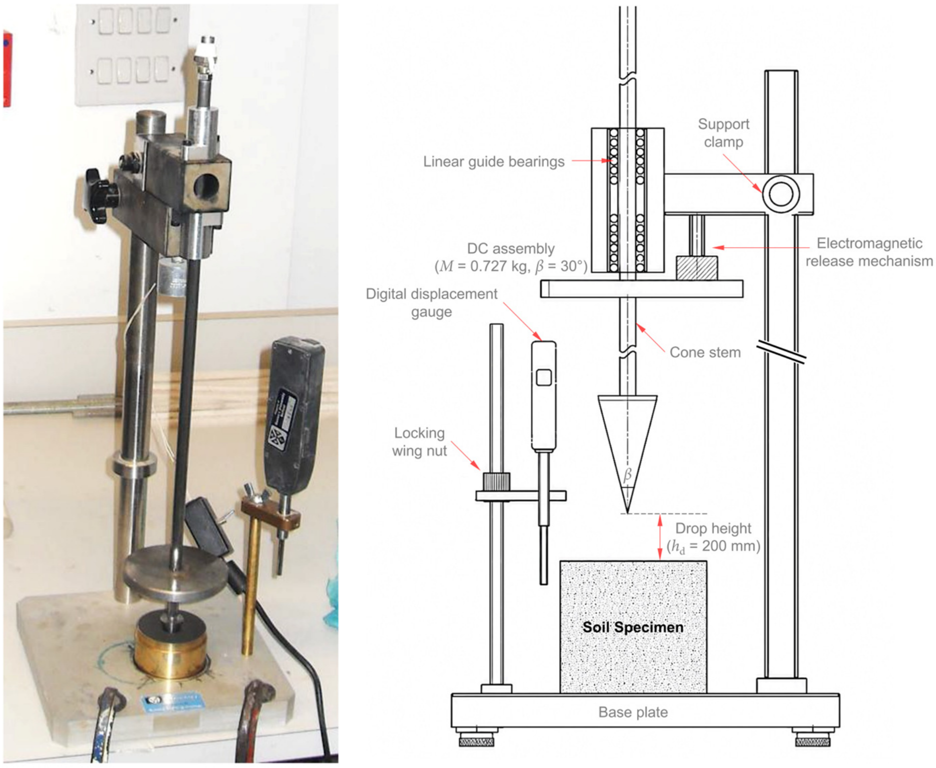
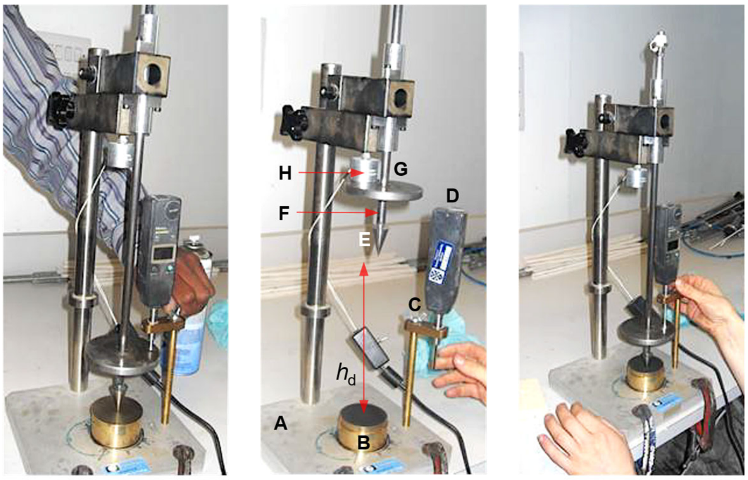


Figure 2. Sivakumar et al.'s 0.727 kg/30°–200 mm DC apparatus for PL_{100} determination: (**a**) photograph (reproduced from NPRA [23]); and (**b**) schematic illustration (not to scale) (modified from NPRA [23]).

In performing the 0.727 kg/30°–200 mm DC test for PL_{100} determination, the specimen cup is placed on the apparatus base plate (see Figure 3a) and, with the cone tip resting on the rim of the cup (i.e., flush with the test-specimen surface), the digital displacement gage is set to zero. The cone is then raised to its starting position, corresponding to a drop height of h_d = 200 mm above the test-specimen surface (see Figure 3b). The cone is subsequently released and allowed to fall freely, before penetrating the soil test-specimen, with the final penetration depth reading recorded after 5 s (see Figure 3c). In the Sivakumar et al. [5] investigation, a 240-N-capacity electromagnet was used to hold the 0.727 and 8 kg cone assemblies in position at the start of testing. This prompted consideration of whether the change in electromagnet polarity, required to release the cone assembly for its free-fall, could impart an additional propulsive force to the departing cone. To investigate this possibility, Sivakumar et al. [5] presented Cartesian d:w plots for Kaolin Clay (KC) and London Clay (LC) test-specimens obtained using the 0.727-kg DC and 8-kg CC setups, both with and without electromagnets used to hold the cone assemblies, before their release, at the start of testing. For the tests conducted without electromagnets, the cone assemblies were initially suspended using a wire hanger, which was cut to release the cones for their free-fall. They reported excellent agreement between the PL_{100} values obtained using the different testing configurations for the KC and LC test-specimens, from which they concluded that the electromagnet did not significantly affect the cone penetration magnitude.



(**a**) (**b**) (**c**)

Figure 3. Procedural stages in performing the 0.727 kg/30°–200 mm DC test for PL_{100} determination (modified from NPRA [23]): (**a**) setting the cone penetration-depth reading to zero; (**b**) cone raised to its starting position (of h_d = 200 mm); and (**c**) recording the final cone penetration depth. **Note:** **A**, base plate; **B**, soil test-specimen in standard cup; **C**, locking wing nut; **D**, digital displacement gage; **E**, β = 30° cone; **F**, cone stem; **G**, DC assembly (M = 0.727 kg); and **H**, electromagnetic release mechanism.

In summary, Sivakumar et al. [5] advocated the use of their 0.727 kg/30°–200 mm DC setup for determining the PL_{100} WC corresponding to d = 20 mm, noting here that the energy-conservation calculation for the cone mass of M = 0.727 kg assumes that s_u is deformation-rate independent (i.e., μ = 0). However, in testing identical test-specimens, it is found that compared to the 8 kg/30° CC (i.e., 8 kg/30°–0 mm) setup, higher s_u values are mobilized in practice for the DC setup since its penetration (deformation) rates are considerably greater. Accordingly, with the PL_{100} WC established for $s_{u@PL_{100}}$ = 170 kPa, the 0.727 kg/30°–200 mm DC setup would be anticipated to systematically underpredict the actual PL_{100} WCs determined using the 8 kg/30°–0 mm CC setup. Conversely, frictional resistance occurring between the stem of the falling cone and its guide bearings would reduce the cone acceleration during its free-fall stage, and hence the actual contact velocity of the cone tip with the soil test-specimen (i.e., v_i < 2.0 m/s), which would tend to produce an overprediction in the PL_{100} WC. In view of these counteracting effects, the accuracy of Sivakumar et al.'s [5] 0.727 kg/30°–200 mm DC setup for PL_{100} determination is investigated later in this paper through a comprehensive re-analysis of previously reported experimental data.

1.2. DC Setups for Undrained Strength Determination

While the preceding discussion mostly concerns PL_{100} determination, DC setups can also be employed for s_u measurements [24–26], which would be particularly useful for strength testing of firm to stiff fine-grained soils. However, two aspects require careful consideration in the s_u calculations: (**i**) the adopted cone-factor value; and (**ii**) the deformation-rate dependence of s_u. That is, whereas the value of K for CCs is governed by the cone's apex angle β and its surface roughness parameter α [2], for DC setups it is additionally influenced by: (**i**) the cone's drop height relative to its penetration depth (i.e., the h_d/d ratio) [24,25]; and (**ii**) the effect of different deformation rates produced in the DC and CC setups on the mobilized s_u. Accordingly, the K parameter in Hansbo's [1] FC formulation [or Equation (1)] should be appropriately adjusted [i.e., $K_d = f_n(\alpha,\beta,h_d/d,\dot{\gamma})$] to account for these two additional factors in the case of DC setups.

Considering point (**i**) above; the discussion paper by Sivakumar et al. [24] advocated a dynamic analysis of the DC setup, in which the PE reduction for the cone dropped from a height of h_d above the test-specimen surface is equated to the work done by the soil resistance force [i.e., $F = Mg = s_u d^2/K$ from rearranging Equation (1)] in bringing the cone to its rest position at a penetration depth of d. O'Kelly [26] developed this energy-conservation approach, as follows, for computing the value of the cone factor K'_d (i.e., accounting for the h_d/d ratio, but not the deformation-rate effect) from the CC factor K. Note that a prime symbol is included here in the cone-factor notation (i.e., K'_d) to distinguish it from its fully adjusted form K_d, since the underlying energy-conservation calculations assume that the mobilized s_u is deformation-rate independent (i.e., $\mu = 0$). Accordingly, as per point (**ii**) above, K'_d must be further adjusted to account for the deformation-rate dependence of the mobilized s_u, thereby obtaining K_d of the DC setup for use in Equation (1).

Employing the energy-conservation approach, O'Kelly [26] stated that for the CC setup, after it has come to rest, the work done by the penetrating cone is given by $W = Fd = Mgd = s_u d^3/K$ [i.e., $Mg = s_u d^2/K$ from rearranging Equation (1)], whereas for the DC setup, one gets $W = Mg(h_d + d) = s_u d^2(h_d + d)/K'_d$. By equating these expressions, one obtains that for the DC setup, the associated cone factor K'_d is given by Equation (2) [26]. It should be noted that the relationship presented in Equation (2) (i.e., for rate-independent soil) was also deduced using a different approach in the paper by Dastider et al. [25].

$$K'_d = K\left(1 + \frac{h_d}{d}\right) \tag{2}$$

In other words, with K'_d (and hence K_d) for the DC setup dependent on the h_d/d ratio, the (adjusted) cone factor becomes increasingly larger (dominant) with increasing drop height and/or decreasing cone penetration depth.

Combining Equations (1) and (2), the authors of the present paper propose Equation (3a) for s_u determinations using DC setups, with the value of K_d computed using the authors' Equation (3b). In Equation (3b), the deformation-rate independent parameter K'_d [i.e., Equation (2)] is further adjusted to account for the different deformation rates in the DC and CC setups. For the calculations, the commonly assumed average 10% increase/decrease in s_u per 10-fold increase/decrease in the deformation (strain) rate for triaxial compression testing [27,28] (i.e., $\mu = 0.10$) is adopted.

$$s_u[\text{in kPa}] = \frac{K_d Mg}{d^2} \tag{3a}$$

$$K_d = \frac{K'_d}{1 + \mu \log_{10}\left(\frac{\dot{\gamma}_{DC}}{\dot{\gamma}_{CC}}\right)} = \frac{K\left(1 + \frac{h_d}{d}\right)}{1 + \mu \log_{10}\left(\frac{\dot{\gamma}_{DC}}{\dot{\gamma}_{CC}}\right)} \tag{3b}$$

where K = cone factor for the CC setup; K'_d and K_d = adjusted cone factors for the DC setup, assuming deformation-rate independent (i.e., μ = 0) and deformation-rate dependent (i.e., $\mu > 0$) mobilized s_u, respectively; M, h_d and d = mass (in g), drop height (in mm) and measured cone penetration depth (in mm) of the DC assembly, respectively; $\dot{\gamma}_{DC}$ and $\dot{\gamma}_{CC}$ = average cone penetration (deformation) rates for the DC and CC setups, respectively; and μ = strain-rate dependence of s_u, herein adopted as μ = 0.10.

Equations (1) and (3a) provide a universal basis for comparing s_u of fine-grained soils employing both standard and non-standard CC and DC setups. Furthermore, when combined with the LL_{FC} and PL_{100} parameters (i.e., $s_{u@LL_{FC}}$ = 1.7 kPa and $s_{u@PL_{100}}$ = 170 kPa), and assuming a bi-logarithmic s_u:w relationship, the value of $\log_{10}s_u$ corresponding to any WC within the plastic range can be interpolated as follows [16]:

$$\log_{10} s_u[\text{in kPa}] = \log_{10} 1.7 + \frac{2\log_{10}\left(\frac{LL_{FC}}{w}\right)}{\log_{10}\left(\frac{LL_{FC}}{PL_{100}}\right)} \quad (4)$$

In other words, the predicted value of s_u is given as follows:

$$s_u[\text{in kPa}] = 1.7 \times 100^{\left[\frac{\log_{10}\left(\frac{LL_{FC}}{w}\right)}{\log_{10}\left(\frac{LL_{FC}}{PL_{100}}\right)}\right]} \quad (5)$$

1.3. Aims and Scope of the Present Investigation

As described above, for Sivakumar et al.'s [5] 0.727 kg/30°–200 mm DC setup, the deformation-rate independence (of s_u) implicit in the energy-conservation calculations results in the cone mass of M = 0.727 kg being slightly too large, such that the PL_{100} WC (corresponding to d = 20 mm) is systematically underpredicted. Whereas frictional resistance occurring between the stem of the falling cone and its guide bearings would tend to produce an overprediction in the PL_{100} WC. In view of these counteracting effects, the first objective of the present paper is to assess the accuracy of Sivakumar et al.'s [5] 0.727 kg/30°–200 mm DC setup for PL_{100} determination relative to the 8 kg/30°–0 mm CC setup, based on a comprehensive re-analysis of the Cartesian d:w data plots originally reported in their paper for ten different mineral fine-grained soils tested using both apparatus configurations. The second objective of this paper is to investigate the equivalence of the DC and CC approaches in producing experimental s_u:w data plots for these soils. A comprehensive review of the research literature reveals that only two other journal papers have reported experimental investigations utilizing Sivakumar et al.'s [5] 0.727 kg/30°–200 mm DC test method (these concerned PL_{100} determinations), namely a paper by Saxena and Arora, which was subsequently retracted by the publisher because parts of the work were plagiarized from [5], and the study by Ko and Lee [29]. In addition, there is a Master's dissertation by Saxena [30], from which the retracted Saxena and Arora journal paper was based. All these investigations focused on the correspondence of the measured Atterberg PL_{TR} WC with the PL_{100} WC deduced using the 0.727 kg/30°–200 mm DC setup, with a large body of work in the wider research literature showing that, when considering an extensive range of dissimilar fine-grained soils, any apparent agreement between these parameters is merely coincidental. Furthermore, of these investigations, i.e., [5,29,30], only Sivakumar et al. [5] reports both DC and CC d:w results, presented in Cartesian data plots, from testing identically prepared soil test-specimens for ten different fine-grained soils (although they did not examine the agreement between their corresponding PL_{100} values). Moreover, none of these investigations considered the potential use of the DC setup for s_u measurement.

Applying the presented theory [i.e., Equations (1)–(3)], the third objective of this paper is to identify suitable combinations of cone mass M and drop height h_d for 30° and 60° DCs, establishing the PL_{100} for d = 20 and 10 mm, respectively (i.e., the same cone penetration depths employed for LL_{FC} determinations). As the required DC mass decreases with increasing drop height, a range of practical M:h_d combinations is possible.

Finally, with the Sivakumar et al. [5] study reporting both 80 g/30°–0 mm LL_{FC} and 8 kg/30°–0 mm PL_{100} datasets, the fourth objective of the present paper is to assess the accuracy of LL_{FC} d:w data extrapolation, employing Feng's [21] bi-logarithmic $\log_{10}d$:$\log_{10}w$ model, for determination of the PL_{100} WC. To the authors' knowledge, this aspect has not been investigated previously.

2. Reappraisal of Sivakumar et al.'s [5] 0.727 kg/30°–200 mm DC Setup for PL_{100} Determination

Figure 4 presents semi-logarithmic $\log_{10}d$:w scatter plots for ten mineral fine-grained soils of intermediate to very high plasticity (i.e., LL_{FC} = 36.0–77.0%, PL_{TR} = 17.5–33.8% and $PI = LL_{FC} - PL_{TR}$ = 18.5–45.5%; see Table 1) investigated using both the 8 kg/30°–0 mm CC and 0.727 kg/30°–200 mm DC setups. The experimental data points were obtained by digitizing the Cartesian d:w plots presented as *Figs. 7* and *10* in Sivakumar et al. [5]. The extracted d:w data are presented in Figure 4 of the present study using the semi-logarithmic plotting format (with d in logarithmic scale) prescribed by ISO 17892–12 [9] for LL_{FC} determination. According to the Unified Soil Classification System (USCS) [31], the ten soils are categorized as nine clay (C) soils and one silt (M) soil. The PL_{TR} parameter for each soil was determined in accordance with BS 1377–2 [7] by four independent geotechnical testing laboratories, with the findings also summarized in Table 1.

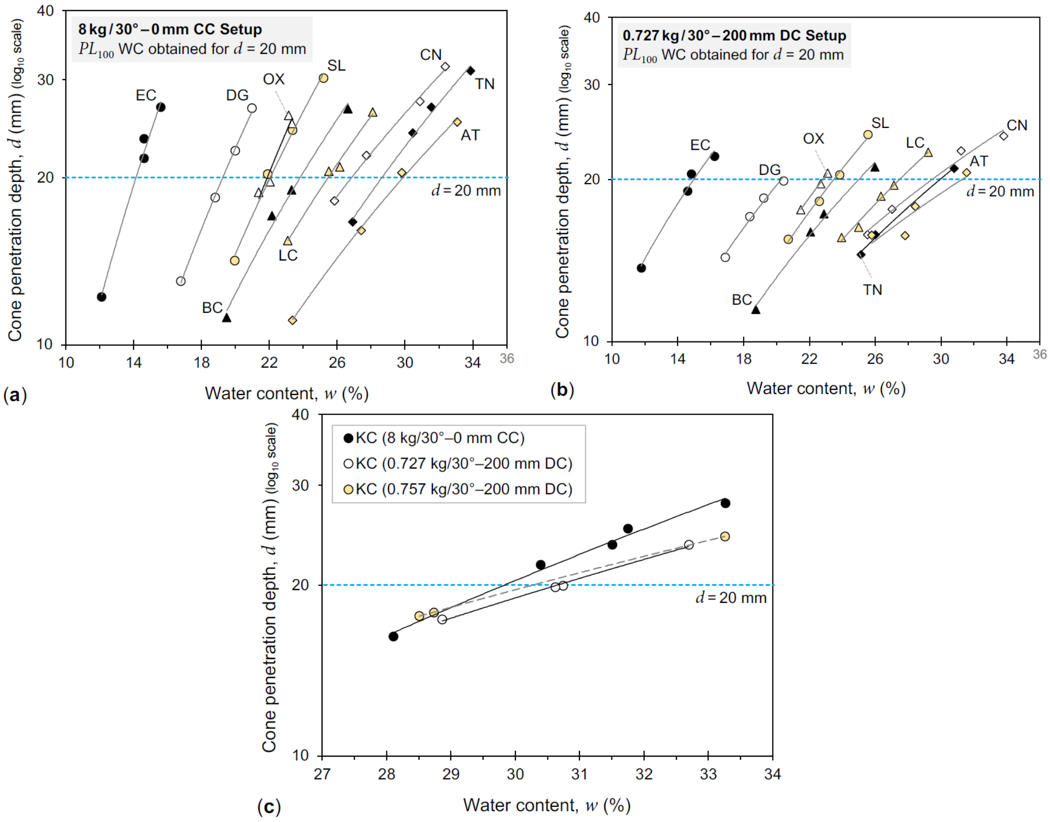


Figure 4. Semi-logarithmic $\log_{10}d$:w scatter plots for the ten fine-grained soils investigated in Sivakumar et al. [5]: (**a**) 8 kg/30°–0 mm CC setup; (**b**) 0.727 kg/30°–200 mm DC setup; and (**c**) both FC setups for KC soil. **Note:** soil identification (ID) labels are defined in Table 1.

Table 1. Index properties of the ten mineral fine-grained soils investigated. **Note:** entries in regular and italic fonts indicate data reported in Sivakumar et al. [5] and results deduced in the present study, respectively.

Soil Type	LL_{FC} (%)	PL_{TR} (n = 4) (%)			PI (%)	USCS	PL_{100} (%)		
		Range	Mean	SD			A: 8 kg/30°–0 mm	B: 0.727 kg/30°–200 mm	Diff. (A − B) †
Enniskillen (EC)	36	16–19	17.5	*1.1*	*18.5*	*CI*	*14.0*	*15.0*	*−1.0*
Donegal Clay (DG)	43	20–21	20.3	*0.4*	*22.7*	*CI*	*19.1*	*20.3*	*−1.2*
Sleech (SL)	50	23–25	24.3	*0.8*	*25.7*	*CH*	*21.9*	*23.4*	*−1.5*
Belfast Clay (BC)	55	23–26	24.8	*1.3*	*30.2*	*CH*	*23.6*	*25.1*	*−1.5*
Oxford Clay (OX)	55	20–24	22.3	*1.5*	*32.7*	*CH*	*21.8*	*22.8*	*−1.0*
Kaolin (KC)	70	29–37	33.8	*3.1*	*36.2*	*MV*	*29.7*	*30.6*	*−0.9*
London Clay (LC)	71	27–30	28.0	*1.2*	*43.0*	*CV*	*25.3*	*27.4*	*−2.1*
Tennessee (TN)	72	28–35	31.5	*2.7*	*40.5*	*CV*	*28.5*	*29.9*	*−1.4*
Canadian Clay (CN)	73	27–30	28.5	*1.5*	*44.5*	*CV*	*26.8*	*29.4*	*−2.6*
Ampthill (AT)	*77*	30–33	31.5	*1.1*	*45.5*	*CV*	*29.6*	*31.2*	*−1.6*

Note: n = number of inter-laboratory replicates; PI = plasticity index (=LL_{FC} − PL_{TR}); SD = standard deviation; CI, CH and CV = clay with *intermediate*, *high* and *very high* plasticity, respectively; MV = silt with *very high* plasticity; and † R^2 (coefficient of determination) = 0.991, average error = −1.5% WC, RMSE (root-mean-squared-error) = 1.55% WC, NRMSE (normalized RMSE) = 6.4%, and MAPE (mean absolute percentage error) = 6.2%.

The authors used least-squares regression to obtain the best-fit lines to the ten d:w data series shown in the semi-logarithmic plots of Figure 4, with their corresponding fitting parameters [i.e., a and b, where $d = a \times \exp(bw)$] and fit-measure metrics for both the 8 kg/30°–0 mm and 0.727 kg/30°–200 mm setups listed in Table A1 of Appendix A. Accordingly, the authors employed the fitted equations of these soils to deduce their PL_{100} values (i.e., the WCs corresponding to d = 20 mm), calculated as $PL_{100} = [\ln(20/a)]/b$. Additionally, the d:w data for each soil were replotted by the authors in Cartesian format (as per BS 1377–2 [7]), and analyzed using linear regression, with the corresponding fitting parameters (i.e., a' and b', where $d = a' + b'w$) and PL_{100} WCs [i.e., calculated as $PL_{100} = (20 - a')/b'$] also reported in Table A1. Note that the PL_{100} WCs listed in Table 1 are the values deduced by the authors from the Cartesian plot trendlines of $d = a' + b'w$, with Figure 5 of the present paper plotting the deduced 0.727 kg/30°–200 mm PL_{100} WCs against the 8 kg/30°–0 mm PL_{100} WCs.

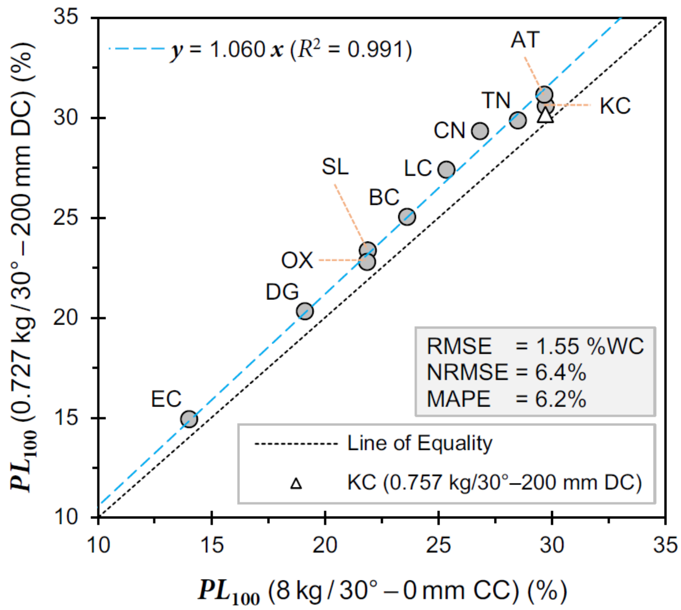


Figure 5. Comparison of PL_{100} WCs obtained using the 0.727 kg/30°–200 mm DC and 8 kg/30°–0 mm CC setups for the ten fine-grained soils.

Referring to Table A1 and Figures 4 and 5, several important observations are made, as follows. Considering any given soil, the gradients of the semi-logarithmic and Cartesian $d{:}w$ plots (i.e., b and b' for $\log_{10}d{:}w$ and $d{:}w$, respectively) obtained for the 0.727 kg/30°–200 mm DC setup are substantially smaller than those produced for the 8 kg/30°–0 mm CC setup. Additionally, the PL_{100} values determined using the 0.727 kg/30°–200 mm DC are consistently, albeit marginally, greater than those obtained for the 8 kg/30°–0 mm CC (i.e., 6.0% larger according to the regression equation presented in Figure 5, or by 1.5% WC when averaged across all ten soils). Furthermore, the R^2 (coefficient of determination), root-mean-squared-error (RMSE), normalized RMSE (NRMSE) and mean absolute percentage error (MAPE) associated with the ten PL_{100} data pairs presented in Figure 5 were calculated [using Equations (A1)–(A4) of Appendix A] by the authors as 0.991, 1.55% WC, 6.4% and 6.2%, respectively.

As identified earlier in this paper, PL_{100} measurement errors arise for the DC setup due to (**i**) deformation-rate (of s_u) effects having not been accounted for in deciding on the $M{:}h_d$ combination of 0.727 kg–200 mm (i.e., underpredicting the PL_{100} WC), and (**ii**) frictional resistance occurring between the stem of the falling cone and its guide bearings (i.e., overpredicting the PL_{100} WC). Here, the systematic overprediction of the PL_{100} parameter by Sivakumar et al.'s [5] 0.727 kg/30°–200 mm apparatus indicates that of these two factors, the frictional-resistance effect is the dominant one, outweighing the deformation-rate effect. Some frictional resistance within the cone-guidance system is inevitable, even when using smooth linear bearings to guide the ∼220-mm vertical movement of the DC, as was the case in the Sivakumar et al. [5] 0.727 kg/30°–200 mm setup.

In summary, although Sivakumar et al.'s [5] 0.727 kg/30°–200 mm DC setup provides a good measure of the PL_{100} WC established using the 8 kg/30°–0 mm CC for the ten investigated soils, some minor adjustments to this DC setup could reduce the minor systematic overprediction of PL_{100} caused by the overriding bearing frictional-resistance effect. Suitable adjustments include the following: (**i**) marginally increasing the cone mass M; (**ii**) increasing the drop height h_d; or (**iii**) increasing both M and h_d accordingly [5,26]. For example, in the case of the KC soil, increasing the mass of the 30° DC from 0.727 to 0.757 kg (i.e., +30 g) for the same drop height of h_d = 200 mm was found to improve the PL_{100} measurement accuracy [5], i.e., producing deduced WCs of 30.6% and 30.2%, respectively, as compared to the measured PL_{100} WC of 29.7% determined using the 8 kg/30°–0 mm CC setup (see Figures 4c and 5). An alternative calibration strategy proposed by the authors for the DC apparatus involves periodically measuring the cone's contact velocity v_i with the test-specimen surface between testing programs. The same rationale also applies for DC test setups manufactured differently from the Sivakumar et al. [5] design presented in Figure 2. After accelerating from $v = 0$ (at the start of the DC test) to a measured contact velocity of $v = v_i$ (after falling a distance of h_d), the actual KE of the cone assembly at the instant of the cone tip contacting the test-specimen surface is given by $KE = \frac{1}{2}Mv_i^2$. Accordingly, the effective potential energy [i.e., PE', defined as PE (= Mgh_d) minus the energy losses resulting from bearing frictional resistance] of the cone assembly at the start of the DC test ($v = 0$) can be determined from $KE = PE' = Mgh'_d$, with the cone's effective (reduced) drop height computed as $h'_d = KE/Mg = v_i^2/2g$. Considering the 20-mm penetration depth achieved by both the CC and DC setups at the PL_{100} condition, an energy-conservation calculation gives 8 kg × 20 mm [for CC] = $M \times (h'_d + 20)$ mm [for DC], such that the required adjustment (increase) to the 30° DC mass is determined as M [in kg] = $160/(h'_d + 20)$. An additional iteration is then necessary to account for the difference in penetration (deformation) rates between the CC and DC setups on the mobilized strengths for the PL_{100} WC.

3. Undrained Shear Strength Measurement Using the 0.727 kg/30°–200 mm DC Setup

In this section, the authors investigate the equivalence of the 0.727 kg/30°–200 mm DC and 8 kg/30°–0 mm CC setups by comparing their experimental s_u:w plots deduced from digitized d:w data reported for ten different fine-grained soils in *Figs. 7* and *10* of Sivakumar et al. [5]. Assuming frictionless guide bearings; in the case of the 8 kg/30°–0 mm CC, the mobilized s_u (in kPa) is computed herein from the extracted cone penetration depth (d in mm) measurement using Equation (1), with M = 8 kg and K_{30} = 0.866 (i.e., the same K value associated with the 80 g/30° CC setup used for LL_{FC} determination, since their cone characteristics and soil deformation rates are identical). Whereas in the case of the 0.727 kg/30°–200 mm and 0.757 kg/30°–200 mm setups, the mobilized s_u (in kPa) is calculated herein from the extracted d (in mm) value using Equation (3a), inputting M = 0.727 and 0.757 kg, respectively, with K_d obtained using Equation (3b). However, to apply Equation (3b), one needs a representative average value of the DC-to-CC cone penetration (deformation) rate ratio (i.e., $\dot{\gamma}_{DC}/\dot{\gamma}_{CC}$ ratio), approximately evaluated herein as the ratio of average cone penetration velocities measured for the two setups. This is a reasonable approach since the cone characteristics (α, β) are identical, and the angle of the heaved specimen surface and cone penetration depth ranges are similar. Figure 6 presents experimental cone penetration velocity against elapsed time (i.e., v:t) data plots for the 8 kg/30°–0 mm and 0.727 kg/30°–200 mm setups, investigating a specimen WC close to the PL_{100} (obtained for d = 20 mm). The data points in this figure were computed by the authors from the digitized d:t data presented in *Fig. 9* of Sivakumar et al. [5], with the cone penetration velocity computed herein as $v = d/t$. Referring to Figure 6, the 0.727-kg DC and 8-kg CC setups produce significantly different v:d (i.e., deformation rate) profiles. With initial and peak velocities of $v_i \approx 2.0$ m/s and $v_p \approx 2.3$ m/s (i.e., initially the cone velocity continues to increase until such time that the soil penetration resistance builds up sufficiently to cause its progressive reduction), the 0.727-kg DC comes to rest for $t \approx 14$ ms. Whereas the 8-kg CC, with $v_i = 0$ and $v_p \approx 0.4$ m/s, comes to rest for $t \approx 80$ ms. Moreover, considering the cone's penetration of the test-specimen over the depth ranges of d = 0–5 and 5–19 mm; compared to the 8-kg CC, the penetration velocity of the 0.727-kg DC was ∼14-fold and ∼6-fold greater, respectively (and, on average, 8-fold greater over d = 0–19 mm). In other words, based on the commonly adopted 10% increase/decrease in s_u per 10-fold increase/decrease in the deformation rate (i.e., μ = 0.10) [27,28], an 8% reduction in s_u is required for the 0.727 kg/30°–200 mm setup, with the associated cone factor computed herein according to Equation (3b) as $K_d = K(1 + h_d/\mathrm{d})/1.08$, resulting in $K_d = 0.802 + 160.4/d$ (for K_{30} = 0.866 and h_d = 200 mm).

Figure 7 presents the s_u:w data and their best-fit correlations deduced by the authors from the 8 kg/30°–0 mm CC and 0.727(0.757) kg/30°–200 mm DC setups for each of the ten fine-grained soils. Overall, the CC and DC setups produced reasonably good agreement in their computed s_u values, with particularly close correspondence achieved for the EC, DG, BC, KC, TN and AT soils. It should be noted that, for a given WC, the 0.727 kg/30°–200 mm DC invariably produced higher s_u values than the 8 kg/30°–0 mm CC setup, particularly for the SL, OX, LC and CC soils. This can be attributed, at least in part, to previously highlighted energy losses resulting from bearing frictional resistance, which are not accounted for in the s_u calculations using Equations (1) and (3a). While both setups experience bearing friction, its effect is expected to be greater for the 0.727(0.757) kg/30°–200 mm DC, owing to its much longer travel distance. For the DC setup, the targeted KE at the instant of the cone tip initially just contacting the test-specimen surface can be achieved using a slightly heavier (0.757 kg) cone. For instance, compared to the 0.727 kg/30°–200 mm DC setup, the 0.757 kg/30°–200 mm DC exhibits better agreement of mobilized s_u with the 8 kg/30°–0 mm CC for the KC soil (see Figure 7a). Other potential, albeit lesser, contributing factors include the following: (**i**) soil-dependent deformation-rate effects; and (**ii**) small procedural inconsistencies in the WC determinations. In other words, the 8% s_u reduction applied herein for the DC calculations (to account for the higher deformation rates produced by the 0.727 kg/30°–200 mm setup) is based on the 10% increase/decrease in s_u per 10-fold increase/decrease in deformation rate commonly adopted in engineering practice. However, this 10% value represents an average, with its actual value dependent on the specific soil under investigation, potentially ranging from 5% to 15% when considering a wide range of dissimilar fine-grained soils [4,27,28]. Secondly, it is observed from Figure 7 that the s_u values measured for each individual soil cover narrow WC ranges. As such, in performing the WC tests for the DC and CC test-specimen sets, minor procedural differences (e.g., in employing different oven-drying periods for the specimen sets) could produce some horizontal (i.e., w-axis) translations of the s_u:w datasets [32], thereby effecting the relative proximity of their correlations.

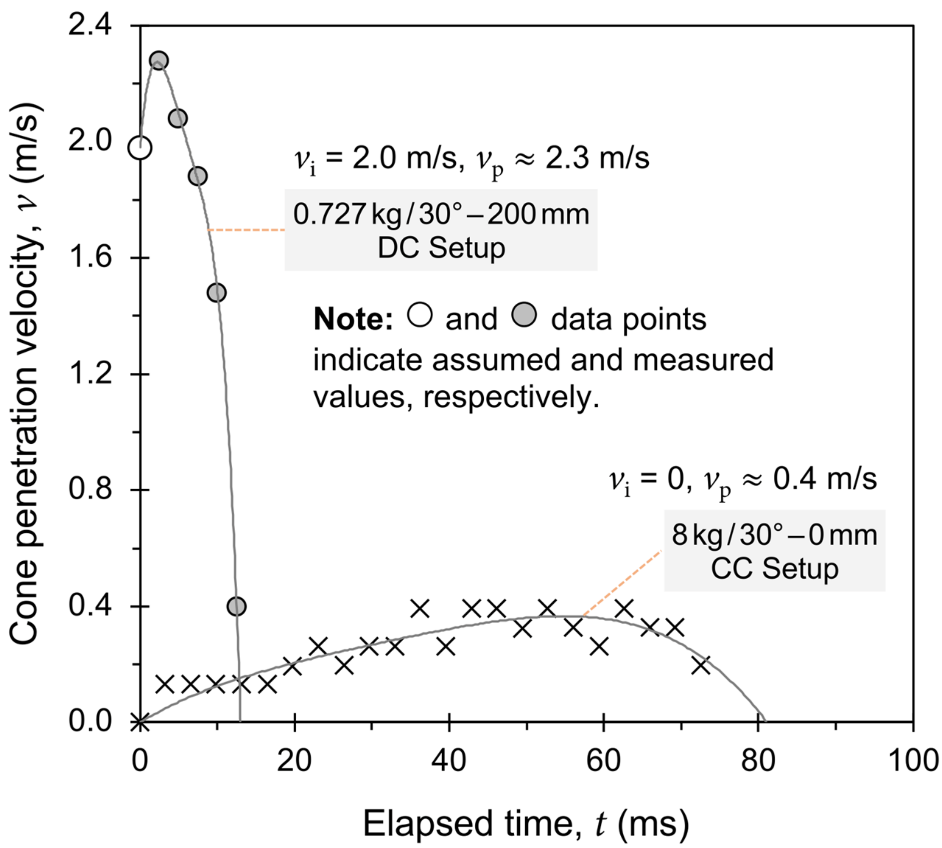


Figure 6. Cone penetration velocity plotted against elapsed time for the 8 kg/30°–0 mm CC and 0.727 kg/30°–200 mm DC setups close to PL_{100} WC.

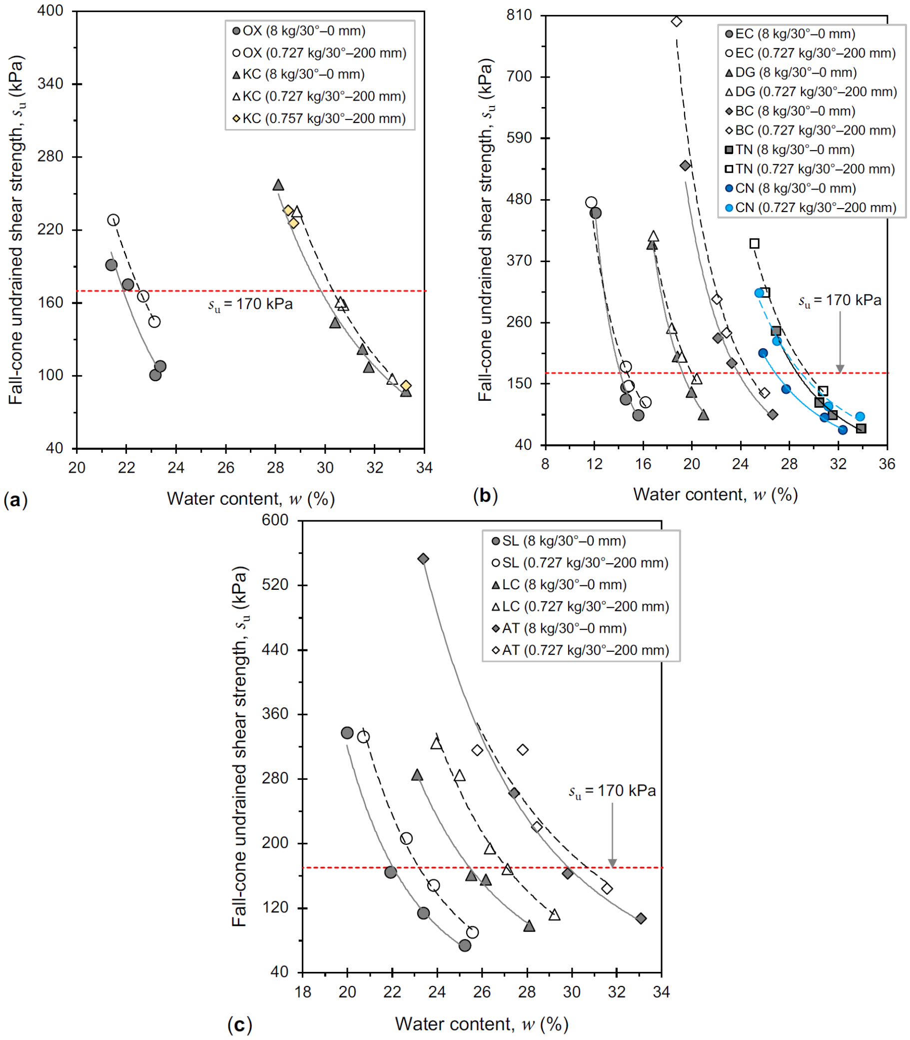


Figure 7. (**a**–**c**) s_u:w scatter plots obtained using the 8 kg/30°–0 mm CC and 0.727(0.757) kg/30°–200 mm DC setups for the ten fine-grained soils. **Note:** solid and dashed curves indicate correlations for the CC and DC setups, respectively.

4. Alternative Practical DC Setups for PL_{100} Determination

Following the suggestion of O'Kelly [26], suitable theoretical combinations of practical cone mass (M_{30} in kg) and drop height ($h_{d_{30}}$ in mm) for 30° DC setups are identified herein for PL_{100} WC determinations (at d = 20 mm; i.e., the same penetration depth used for establishing the LL_{FC} WC). For a rate-independent soil, and assuming frictionless guide bearings, the theoretical M_{30}:$h_{d_{30}}$ combinations are obtained from a simple energy-conservation calculation with respect to the 20-mm penetration of the 8 kg/30°–0 mm CC at the PL_{100} condition—that is, $M_{30} \times (h_{d_{30}} + 20) = 8 \times 20 = 160$ kg.mm [see Equation (6a)]. Alternatively, the same relationship can be obtained by substituting $s_{u@PL_{100}}$ = 170 kPa for d = 20 mm, together with $K'_d = K_{30}(1 + h_d/d)$, where K_{30} = 0.866, into Equation (3a). Repeating these steps for 60° cones (K_{60} = 0.289), with the associated LL_{FC} and PL_{100} WCs determined for d = 10 mm using 60 g and 6 kg cones, respectively, the theoretical M_{60}:$h_{d_{60}}$ (M_{60} in kg and $h_{d_{60}}$ in mm) combinations for a rate-independent soil can be given by Equation (6b).

$$M_{30} = \frac{160}{h_{d_{30}} + 20} \quad \text{or} \quad h_{d_{30}} = \frac{160}{M_{30}} - 20 \tag{6a}$$

$$M_{60} = \frac{60}{h_{d_{60}} + 10} \quad \text{or} \quad h_{d_{60}} = \frac{60}{M_{60}} - 10 \tag{6b}$$

Figure 8 presents plots of Equation (6a,b), together with a data point representing the Sivakumar et al. [5] 0.727 kg/30°–200 mm DC setup. It is important to emphasize that the above calculations assume rate-independent soil behavior and for frictionless bearings guiding the cone movement. Using the available experimental data for the 0.727 kg/30°–200 mm DC setup [5], the authors of the present paper demonstrated that an 8% adjustment (reduction) in s_u (for v_i = 2.0 m/s with h_d = 200 mm) was necessary to account for the difference in deformation rates compared to the 8-kg CC setup, based on an averaged μ value of 0.10 [27,28]. Accordingly, to deduce the same PL_{100} WCs, the mass of the 30° DC assembly needs to increase from 0.727 to 0.785 kg (no adjustment needed for the 8-kg CC setup). In terms of deformation-rate effects, greater drop heights (i.e., for h_d > 200 mm) require larger cone-mass adjustments (> 8%), noting that the frictional resistance occurring between the stem of the falling cone and its guide bearings also increases due to the associated longer travel distance. Overall, however, while important to consider the deformation-rate dependency; in terms of s_u measurement and PL_{100} determination, it has a relatively minor effect when compared to the counteracting bearing frictional-resistance effect.

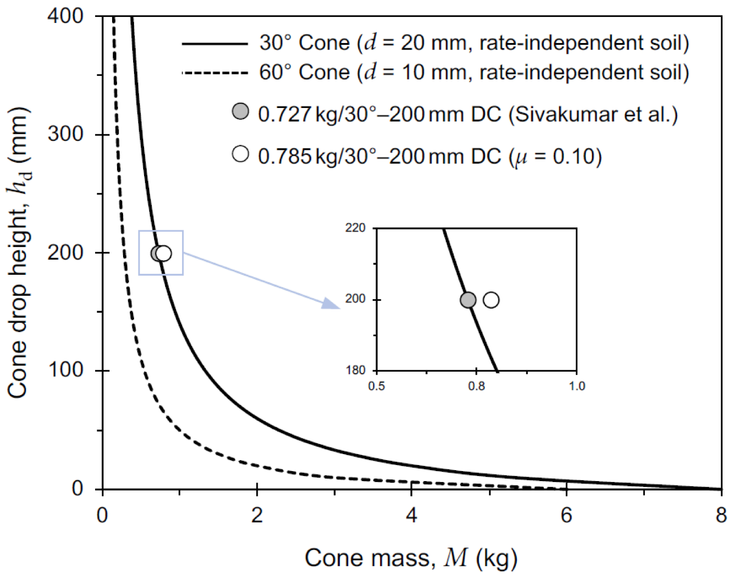


Figure 8. Theoretical cone mass, apex angle and drop height combinations for PL_{100} determination (frictionless guide bearings), together with data points representing the 0.727 kg/30°–200 mm [5] and 0.785 kg/30°–200 mm (μ = 0.10) DC setups.

Another important observation concerns the 60° DC setup, which requires more modest theoretical M_{60}:$h_{d_{60}}$ combinations for PL_{100} determination (e.g., a 400-g cone dropped from a height of $h_{d_{60}}$ = 140 mm, penetrating into the soil test-specimen to a depth of exactly d = 10 mm, assuming rate-independent soil behavior and frictionless guide bearings). Accordingly, it could be argued that the 60° DC configuration can offer clear practical advantages over the 30° DC configuration in geotechnical laboratory practice, since theoretically the former offers (**i**) enhanced operator health and safety, due to its lighter DC mass and shorter falling distance; (**ii**) likely lower bearing-frictional resistance on account of the reduced travel distance; and (**iii**) a more compact (overall shorter) DC apparatus.

5. Accuracy of LL_{FC} Data Extrapolation for PL_{100} Determination

Finally, in this section, the Sivakumar et al. [5] 80 g/30°–0 mm and 8 kg/30°–0 mm d:w datasets for WCs around the LL_{FC} and PL_{100}, respectively, are employed by the authors of the present paper to assess the accuracy of LL_{FC} d:w data extrapolation—using Feng's [21] bi-logarithmic $\log_{10}d$:$\log_{10}w$ model—for determination of the PL_{100} WC. Following Feng's [21] methodology, the PL_{100} WC value for each of the ten fine-grained soils is obtained as $PL_{100} = (2/c)^{\frac{1}{m}}$ [22], where c and m denote fitting parameters derived from a power-law trendline fitted to the LL_{FC} d:w data [i.e., $d = c(w)^m$]. The measured (by the 8 kg/30°–0 mm CC setup) and extrapolated/predicted PL_{100} values for these soils are listed in Table 2, and also plotted against each other in Figure 9.

Table 2. Summary of measured (by the 8 kg/30°–0 mm CC setup) and extrapolated/predicted (from the 80 g/30°–0 mm LL_{FC} data) values of PL_{100} WC for the ten fine-grained soils investigated.

Soil ID	80 g/30° LL_{FC} Data			PL_{100} (%)		
	c	m	R^2	A: Extrapolated (80 g/30°)	B: Measured (8 kg/30°–0 mm)	Diff. (A − B) †
EC	4.27×10^{-3}	2.314	0.998	14.3	14.0	+0.3
DG	4.02×10^{-4}	2.879	0.994	19.2	19.1	+0.1
SL	8.62×10^{-4}	2.566	0.963	20.5	21.9	−1.4
BC	2.72×10^{-4}	2.793	0.977	24.2	23.6	+0.6
OX	1.29×10^{-3}	2.404	0.982	21.2	21.8	−0.6
KC	1.36×10^{-3}	2.248	0.887	25.6	29.7	−4.1
LC	2.77×10^{-5}	3.147	0.940	35.0	25.3	+9.7
TN	8.33×10^{-5}	2.868	0.998	33.7	28.5	+5.2
CN	6.39×10^{-4}	2.415	0.983	28.0	26.8	−1.2
AT	1.56×10^{-3}	2.176	0.969	26.8	29.6	−2.8

Note: † R^2 = 0.615, RMSE = 3.87% WC, NRMSE = 16.1%, and MAPE = 9.9%.

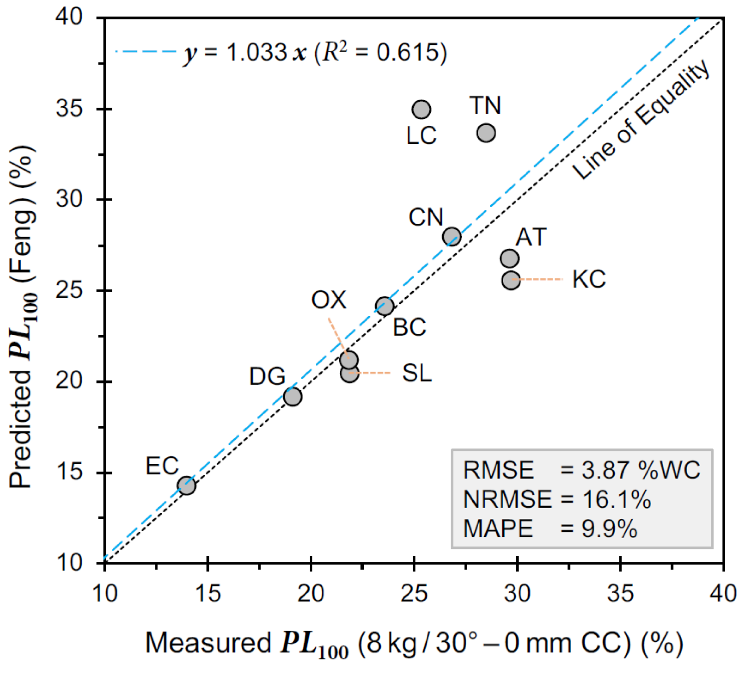


Figure 9. Comparison of PL_{100} WCs obtained using the 8 kg/30°–0 mm CC setup with those extrapolated/predicted from the 80 g/30°–0 mm LL_{FC} data [22] for the ten fine-grained soils.

The R^2, RMSE, NRMSE and MAPE values associated with the ten PL_{100} data pairs presented in Figure 9 were calculated [employing Equations (A1)–(A4) of Appendix A] by the authors as 0.615, 3.87% WC, 16.1% and 9.9%, respectively. These fit-measure metrics indicate a moderate level of agreement between the measured and extrapolated PL_{100} values, raising concerns about the generalizability of the extrapolation approach. Referring to Table 2; *excellent*, *good*, *moderate*, and *poor* agreements (< 1%, 1–3%, 3–5% and > 5% WC disparities) were achieved between the extrapolated and measured PL_{100} for 4, 3, 1 and 2 of the ten fine-grained soils, respectively. With similar cone characteristics (same α and β), the gradients of the $\log_{10}d$:$\log_{10}w$ relationships obtained from investigating a given soil using the 80 g and 8 kg variants of the 30° CCs are expected to be similar, thereby resulting in closely corresponding PL_{100} values. However, close inspection of the $\log_{10}d$:$\log_{10}w$ data for these CCs indicates that this was not consistently the case (i.e., compare the fitted m values for each of the ten fine-grained soils in Table A2 of Appendix A). An obvious explanation for the variability is that the extrapolated PL_{100} value is inherently sensitive to the accuracy of the LL_{FC} d:w measurements (and hence the obtained fitting parameters c and m). In other words, small experimental errors in measured LL_{FC} penetration depths and/or rounding of the measured wet and dry specimen masses used in the WC calculations (e.g., as reflected by the comparatively lower R^2 values of the LL_{FC} d:w regression lines for the KC, LC and AT soils; see Table 2) may have only minor influences on the determination of the LL_{FC} itself. However, their effects can become amplified during extrapolation, thereby affecting the predicted PL_{100} value. Furthermore, the authors raise the possibility that the assumption of a linear relationship between $\log_{10}d$ and $\log_{10}w$, inherent in Feng's [21] extrapolation methodology, may not be strictly valid over the full 2–20-mm penetration-depth range, particularly when extrapolating to PL_{100} WCs below Atterberg's thread-rolling plastic limit PL_{TR}. Consequently, even modest departures from linearity in the 80 g/30° LL_{FC} $\log_{10}d$:$\log_{10}w$ data could lead to appreciable errors when linearly extrapolating beyond the measured penetration range (of d = 15–25 mm) to the PL_{100} WC (at d = 2 mm).

Given the challenges associated with achieving uniform preparation of stiff soil specimens for DC testing about the PL_{100} WC [24], together with the inconsistencies observed herein for predicting the PL_{100} parameter by bi-logarithmic extrapolation of measured LL_{FC} d:w data, the authors of the present paper recommend the following alternative approach for determining the PL_{100} WC of fine-grained soil. Workable test samples of firm and firm-to-stiff consistencies (i.e., within the plastic WC range) are prepared in the specimen cups, and tested using the DC method. Then, with s_u computed using Equation (3a), the PL_{100} parameter is identified as the WC corresponding to $s_{u@PL_{100}}$ = 170 kPa from bi-logarithmic extrapolation of the measured s_u:w data.

6. Future Research

As an alternative to miniature handheld strength-testing devices, such as the pocket (soil) penetrometer and torvane (vane-shear) apparatus [33–36], which test only a small portion of the investigated soil, the authors propose upscaling the laboratory DC setup for field (in-situ) s_u measurements in fine-grained soil deposits. For instance, according to the FC theory presented herein, a smooth 1.2 kg/30°–700 mm DC setup, with recorded cone penetration depths of d = 35–75 mm, would cover an undrained shear strength measurement range of $s_u \approx$ 19–175 kPa. As with the laboratory DC test, the cone mass–drop height combination for the field DC setup can be adjusted depending on the desired cone penetration depth range for the mobilized s_u values. In this manner, in-situ s_u measurement can be achieved quickly and reliably from a straightforward DC test. Accordingly, comprehensive field validation studies involving a range of dissimilar fine-grained soil deposits are recommended.

7. Summary and Conclusions

The FC test method is employed for LL_{FC} and PL_{100} determinations, sensitivity assessments, and s_u measurement for very soft to firm fine-grained soils. PL_{100} determination typically relies on LL_{FC} data extrapolation, e.g., using Feng's bi-logarithmic $\log_{10}d$:$\log_{10}w$ model. Heavier CCs required for direct PL_{100} determination and s_u measurement in stiffer soils present practical difficulties, including health and safety concerns. Accordingly, this paper advocates DC setups, with the major research findings summarized as follows.

A DC framework for s_u determination [i.e., Equation (3a,b)] was proposed, including (**i**) computation of the associated cone factor K_d, and (**ii**) accounting for the effect of different deformation rates in DC and CC setups on the mobilized s_u. For instance, compared to the 8 kg/30°–0 mm CC, the velocity of the penetrating cone for the 0.727 kg/30°–200 mm DC was, on average, approximately 8-fold greater, necessitating an 8% s_u reduction based on the commonly adopted μ value of 0.10. From the authors' reanalysis of previously published d:w data for ten different fine-grained soils, slightly higher s_u values were deduced for the DC setup, owing to the larger bearing friction effect arising from its substantially longer cone travel distance, although, overall, reasonably good s_u agreement was obtained between the CC and DC setups. Accordingly, the 0.727 kg/30°–200 mm PL_{100} values were consistently, albeit marginally (i.e., 1.5% WC when averaged across all ten soils), greater than those obtained for the 8 kg/30°–0 mm CC setup, again largely explained by the bearing friction effect. Some frictional resistance within the cone-guidance system is inevitable, reducing the DC's contact velocity v_i with the test-specimen surface. Hence, periodic calibration, routine cleaning of the apparatus, and regular lubrication of its cone-guidance system are essential. Small adjustments to the DC setup, namely (**i**) marginally increasing the cone mass M, (**ii**) increasing the drop height h_d, or (**iii**) increasing both M and h_d appropriately, can eliminate the minor systematic overprediction of the PL_{100} parameter. For instance, from periodic measurements of a 30° DC's contact velocity with the test-specimen surface, the effective drop height can be computed as $h'_d = v_i^2/2g$, such that the adjustment (increase) in cone mass is determined as M [in kg] = $160/(h'_d + 20)$ (assumes rate-independent soil behavior).

A range of theoretical M:h_d combinations [i.e., given by Equation (6a,b), assuming rate-independent soil behavior and frictionless bearings] were identified for 30° and 60° DCs with the PL_{100} parameter determined for d = 20 and 10 mm, respectively. It was found that the 60° DC setup requires more modest M_{60}:$h_{d_{60}}$ combinations, theoretically giving it clear advantages over 30° DCs in geotechnical laboratory settings, in terms of enhanced operator health and safety, a more compact test apparatus, and likely lower bearing friction (on account of reduced cone travel distance).

Finally, moderate levels of agreement were found between the PL_{100} values predicted using Feng's bi-logarithmic LL_{FC} data extrapolation model and those measured using the 8 kg/30°–0 mm CC, raising possible concerns about the generalizability of the extrapolation approach. While small experimental errors in measured LL_{FC} penetration depths and/or rounding of the measured wet and dry specimen masses used in the WC calculations may have only minor influences on the determination of the LL_{FC} itself, it was postulated that their effects can become amplified during d:w data extrapolation, thereby affecting the accuracy of the PL_{100} WC predictions.

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Abbreviations

The following abbreviations are used in this manuscript:

AT	Ampthill (soil)
BC	Belfast Clay (soil)
CC	Contacting-cone
CN	Canadian Clay (soil)
DC	Drop-cone
DG	Donegal Clay (soil)
EC	Enniskillen (soil)
FC	Fall-cone
KC	Kaolin Clay (soil)
LC	London Clay (soil)
OX	Oxford Clay (soil)
SL	Sleech (soil)
TN	Tennessee (soil)
USCS	Unified Soil Classification System
WC	Water content

Notations

The following notations are used in this manuscript:

a, b	Fitting parameters for semi-logarithmic ln$d$:w relationship/plot
a', b'	Fitting parameters for Cartesian d:w relationship/plot
c, m	Fitting parameters for bi-logarithmic $\log_{10}d$:$\log_{10}w$ relationship/plot
d	Cone penetration depth [mm]
F	Force [N]
g	Standard gravitational acceleration [= 9.81 m/s^2]
h_d	Drop height (DC setup) [mm]
$h_{d_{30}}$	Drop height for 30° DC [mm]
$h_{d_{60}}$	Drop height for 60° DC [mm]
i	Summation index
K	Cone factor for CC setup [–]
K_{30}	Cone factor for 80 g/30° CC [–]
K_{60}	Cone factor for 60 g/60° CC [–]
K'_d	Adjusted cone factor for DC setup assuming deformation-rate independent s_u [–]
K_d	Adjusted cone factor for DC setup assuming deformation-rate dependent s_u [–]
KE	Kinetic energy [kJ]
LL_{FC}	FC liquid limit [% WC]
M	Mass of cone assembly [g or kg]
M_{30}	Mass of 30° DC assembly [g or kg]
M_{60}	Mass of 60° DC assembly [g or kg]
MAPE	Mean absolute percentage error [%]
N	Number of soils investigated [–]
NRMSE	Normalized root-mean-squared-error [%]
PE	Potential energy [kJ]
PE'	Effective potential energy [kJ]
PI	Plasticity index (= $LL_{FC} - PL_{TR}$) [% WC]
PL_{10}	WC corresponding to 10-fold s_u increase relative to $s_{u@LL_{FC}}$ [% WC]
PL_{25}	WC corresponding to 25-fold s_u increase relative to $s_{u@LL_{FC}}$ [% WC]
PL_{100}	WC corresponding to 100-fold s_u increase relative to $s_{u@LL_{FC}}$ [% WC]
PL_{TR}	Atterberg (thread-rolling) plastic limit [% WC]
R^2	Coefficient of determination [–]
RMSE	Root-mean-squared-error [% WC]
s_u	Undrained shear strength [kPa]

$s_{u@LL_{FC}}$	Undrained shear strength at LL_{FC} WC [kPa]
$s_{u@PL_{100}}$	Undrained shear strength at PL_{100} WC [kPa]
t	Elapsed time during cone's penetration into soil test-specimen [ms]
v	Cone (penetration) velocity [m/s]
v_i	Cone velocity on initial contact with specimen surface [m/s]
v_p	Peak cone velocity [m/s]
w	Water content [% WC]
W	Work [10^{-3} N.mm = 10^{-6} J]
α	Cone surface roughness parameter [–]
β	Cone apex angle [°]
$\dot{\gamma}$	Average cone penetration (deformation) rate [s^{-1}]
$\dot{\gamma}_{CC}$	Average cone penetration (deformation) rate for CC setup [s^{-1}]
$\dot{\gamma}_{DC}$	Average cone penetration (deformation) rate for DC setup [s^{-1}]
μ	Strain-rate dependence (of s_u) parameter [–]

Appendix A

Table A1 presents the PL_{100} WCs for the ten fine-grained soils investigated. The authors deduced these values from regression analyses of digitized experimental *d*:*w* data for the 8 kg/30°–0 mm CC and 0.727 kg/30°–200 mm DC setups, extracted from *Figs. 7* and *10* of the Sivakumar et al. [5] paper. In assessing the degree of correspondence/correlation between the PL_{100} WCs deduced using the 8-kg CC and 0.727-kg DC setups (i.e., y_i and y'_i, respectively), the following fit-measure metrics [i.e., Equations (A1)–(A4)] were adopted. These indices quantify the average deviation between the two PL_{100} WC data sets, thereby providing an overall measure of their similarity.

$$R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - y'_i)^2}{\sum_{i=1}^{N}(y_i - \overline{y})^2} \ni \overline{y} = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{A1}$$

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - y'_i)^2} \tag{A2}$$

$$\text{NRMSE} = \frac{\text{RMSE}}{\overline{y}} \times 100\% \tag{A3}$$

$$\text{MAPE} = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{y_i - y'_i}{y_i}\right| \times 100\% \tag{A4}$$

where RMSE = root-mean-squared-error (in % WC); NRMSE and MAPE = normalized RMSE and mean absolute percentage error, respectively (both dimensionless, expressed in terms of %); N = number of soils investigated (here N = 10); and i = summation index.

Table A2 presents the authors' regression analysis results for these ten fine-grained soils, obtained by fitting a power-law trendline to the 80 g/30°–0 mm and 8 kg/30°–0 mm *d*:*w* datasets obtained for WCs around the LL_{FC} and PL_{100}, respectively, in Sivakumar et al. [5].

Table A1. Summary of PL_{100} values for the ten fine-grained soils investigated, determined from regression analyses of their d:w data obtained using the 8 kg/30°–0 mm CC and 0.727 kg/30°–200 mm DC setups.

Soil ID	Fitting Model: $d = a \times \exp(bw)$ [d in mm and w in % WC]									Fitting Model: $d = a' + b'w$ [d in mm and w in % WC]								
	A: 8 kg/30°–0 mm CC				B: 0.727 kg/30°–200 mm DC				Diff. in PL_{100} (A − B) †	A: 8 kg/30°–0 mm CC				B: 0.727 kg/30°–200 mm DC				Diff. in PL_{100} (A − B) ‡
	a	b	R^2	PL_{100} (%)	a	b	R^2	PL_{100} (%)		a'	b'	R^2	PL_{100} (%)	a'	b'	R^2	PL_{100} (%)	
EC	0.755	0.231	0.972	14.2	3.739	0.112	0.963	15.0	−0.8	4.167	−38.28	0.985	14.0	1.937	−8.96	0.980	15.0	−1.0
DG	0.751	0.170	0.999	19.3	3.030	0.093	0.976	20.3	−1.0	3.197	−41.07	0.986	19.1	1.580	−12.13	0.989	20.3	−1.2
SL	0.835	0.144	0.987	22.1	2.276	0.092	0.998	23.5	−1.4	3.053	−46.75	0.999	21.9	1.798	−22.08	0.988	23.4	−1.5
BC	1.142	0.120	0.984	23.9	2.415	0.085	0.982	25.0	−1.1	2.154	−30.81	0.999	23.6	1.336	−13.48	0.999	25.1	−1.5
OX	0.484	0.170	0.933	21.9	2.308	0.095	0.999	22.8	−0.9	3.760	−62.16	0.924	21.8	1.792	−20.90	0.998	22.8	−1.0
KC	0.844	0.106	0.979	29.9	1.780	0.079	0.998	30.7	−0.8	2.266	−47.32	0.992	29.7	1.607	−29.18	0.990	30.6	−0.9
LC	1.357	0.106	0.991	25.5	2.770	0.072	0.993	27.5	−2.0	2.136	−34.15	0.984	25.3	1.348	−16.97	0.992	27.4	−2.1
TN	1.444	0.092	0.983	28.7	3.003	0.063	0.996	30.0	−1.3	2.103	−39.90	0.999	28.5	1.123	−13.58	0.998	29.9	−1.4
CN	2.166	0.083	0.996	26.9	4.275	0.052	0.966	29.6	−2.7	2.011	−33.92	0.993	26.8	1.031	−10.25	0.982	29.4	−2.6
AT	1.514	0.086	0.989	30.0	4.229	0.050	0.890	31.2	−1.2	1.473	−23.65	0.995	29.6	0.899	−8.04	0.874	31.2	−1.6

Note: † R^2 = 0.988, average error = −1.3% WC, RMSE = 1.45% WC, NRMSE = 6.0%, MAPE = 5.5%; and ‡ R^2 = 0.991, average error = −1.5% WC, RMSE = 1.55% WC, NRMSE = 6.4%, MAPE = 6.2%.

Table A2. Summary of regression analysis outputs for the ten fine-grained soils investigated, derived from power-law trendlines fitted to the 80 g/30°–0 mm and 8 kg/30°–0 mm d:w datasets for WCs around the LL_{FC} and PL_{100}, respectively.

Soil ID	80 g/30°–0 mm LL_{FC} Data			8 kg/30°–0 mm CC Data			A/B Ratio
	c †	A: m †	R^2	c †	B: m †	R^2	
EC	4.27×10^{-3}	2.314	0.998	4.67×10^{-3}	3.158	0.979	0.733
DG	4.02×10^{-4}	2.879	0.994	1.64×10^{-3}	3.182	0.998	0.905
SL	8.62×10^{-4}	2.566	0.963	8.92×10^{-4}	3.239	0.993	0.792
BC	2.72×10^{-4}	2.793	0.977	3.20×10^{-3}	2.757	0.993	1.013
OX	1.29×10^{-3}	2.404	0.982	1.62×10^{-4}	3.800	0.931	0.633
KC	1.36×10^{-3}	2.248	0.887	3.34×10^{-4}	3.240	0.985	0.694
LC	2.77×10^{-5}	3.147	0.940	3.37×10^{-3}	2.684	0.990	1.173
TN	8.33×10^{-5}	2.868	0.998	1.84×10^{-3}	2.770	0.991	1.035
CN	6.39×10^{-4}	2.415	0.983	7.43×10^{-3}	2.401	0.995	1.006
AT	1.56×10^{-3}	2.176	0.969	5.78×10^{-3}	2.399	0.996	0.907

Note: † fitting parameters derived from power-law trendline, i.e., $d = c(w)^m$, where d is in mm and w is in % WC.

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