

Article

The Seismic Response of Two Geotechnically Similar GRS-MB Walls During the Chi-Chi Earthquake: Insights from the Finite Displacement Method

Ching-Chuan Huang 

Department of Civil Engineering, National Cheng Kung University, Tainan City 70101, Taiwan; samhcc@mail.ncku.edu.tw

Abstract

This study re-examines two geologically and geotechnically similar geosynthetic-reinforced soil walls with modular block facings (GRS-MBs) that exhibited markedly different seismic performances during the 1999 Chi-Chi earthquake ($M_L = 7.3$). Integrating a multi-wedge failure mechanism that captures soil–facing–reinforcement interactions with a nonlinear hyperbolic soil model representing shear stress–displacement behavior along the slip surface, the Force–equilibrium-based Finite Displacement Method (FFDM) provides consistent and robust displacement evaluations over a wide range of input seismic inertial forces. A systematic sensitivity investigation confirms that the FFDM framework responds to parameter variations in a physically meaningful manner, and that displacement predictions remain stable with respect to reasonable uncertainties in soil, reinforcement, and facing properties. The analysis clarifies why two similar GRS-MBs responded so differently during strong shaking and demonstrates the broader applicability of FFDM for displacement-based seismic assessment, including under shaking levels (e.g., $k_h \approx 0.3$) that would drive conventional limit–equilibrium calculations to $F_s < 1.0$, a physically impossible state requiring shear resistance greater than the soil’s ultimate strength. A comparative evaluation of seismic displacement predictions using the Newmark method and FFDM shows that FFDM successfully generates displacement-based seismic resisting curves and reproduces field-observed displacements. In contrast, the Newmark method yields order-of-magnitude variability in predicted movements and may be unsuitable for displacement-sensitive engineered slopes where deformations on the order of several 10^{-3} – 10^{-2} m are practically significant. For interaction-rich GRS-MBs with high values of k_{hc} , beyond the predictive capability of Newmark’s equation, FFDM offers a practical and physically grounded tool for seismic displacement assessment of reinforced soil structures.

Keywords: geosynthetic-reinforced soil wall with modular block facing (GRS-MW); seismic displacement; Force–equilibrium-based Finite Displacement Method (FFDM); multi-wedge failure mechanism; hyperbolic soil model



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1. Introduction

The seismic performance of soil retaining structures—including highway and railway embankments, bridge abutments, and slope protection systems—has received increasing attention following major earthquakes, such as the 1995 Hyogo Ken Nambu earthquake in Japan [1]. Similar post-earthquake investigations in New Zealand, Europe, Turkey, and Italy have also demonstrated the importance of integrating field evidence and modern

remote-sensing tools for understanding seismic damage mechanisms [2]. These events have highlighted the need for reliable methods to evaluate the seismic-resisting capacity expressed by the displacement (or deformation) of the structure under earthquake excitation. Performance-based or displacement-based assessments of seismic capacity have a relatively short history compared with traditional force-based approaches. The force-based method evaluates seismic capacity through a safety factor (F_s) against a pseudo-static horizontal seismic force represented by the horizontal seismic coefficient (k_h), defined as $\lambda \cdot \text{HPGA}/g$, where λ is an empirical scaling factor (typically 0.3–0.5), HPGA is the horizontal peak ground acceleration, and g is gravitational acceleration. However, several limitations of F_s -based seismic evaluation are well recognized: (1) increasing F_s beyond 1.5–2.0 often leads to overly conservative and cost-inefficient designs, with unclear implications for actual field performance; (2) the value of λ is empirically determined; and (3) the operational soil strength parameters, c and φ , are also selected empirically, requiring judgment that varies among practitioners.

A major shift toward displacement-based evaluation began with Newmark's sliding-block method [3], which combines limit-equilibrium analysis with the dynamics of a rigid block sliding on a planar surface. Successful application of the original Newmark method requires calibration of a critical seismic coefficient (k_{hc}), obtained from the F_s – k_h curve at $F_s = 1.0$. This calibration depends on iterative adjustments to identify an operational internal friction angle that produces an acceptable F_s – k_h relationship. Numerous studies have attempted to improve the Newmark method in five main areas: (1) enhancing the accuracy of k_{hc} and horizontal slope displacement (Δ_h) through improved failure mechanisms [4–11]; (2) incorporating lumped-mass or soil-column dynamics to overcome rigid-block assumption [12–16]; (3) improving k_{hc} and Δ_h predictions by allowing evolving slip-surface geometry and/or strength degradation [17–22]; (4) accounting for interactions among multiple soil blocks within the sliding mass [23,24]; and (5) comparing or improving displacement-prediction equations [25–27]. Although these enhancements broaden the applicability of Newmark-type analyses, they introduce new challenges. A fundamental drawback remains: Newmark-type approaches predict zero displacement prior to yielding acceleration, in contrast to observed slope behavior, as emphasized in [28]. The lumped-mass approach requires additional parameters such as spring and dashpot constants, which often demand specialized laboratory or in situ testing not commonly available in routine design. Improvements related to slip-surface geometry are most relevant for large displacements on the order of 10^{-1} m or more, whereas engineered transportation slopes typically require attention at much smaller displacement levels. The Newmark-based displacement-prediction equations simplify the double-integration step of Newmark's method but still produce displacement estimates that vary by more than an order of magnitude, making them unsuitable for displacement-sensitive soil structures.

Other analytical approaches for evaluating seismic performance include: (1) finite-element and discrete-element methods that compute seismic displacements using nonlinear dynamic response analysis [29–33], and (2) energy-based theories for sliding-block displacement estimation [34]. Numerical analyses in item (1) require extensive effort for validation and calibration of input soil properties and remain research-oriented tools rather than practical design methods. The energy-based theory integrates wave-propagation concepts and earthquake energy release to estimate seismic displacement of a simple block with a friction angle φ on a planar slip surface. Because the method idealizes the slope as a simple block sliding on a planar surface, it is inherently unsuitable for evaluating complex, interaction-rich systems, such as GRS-MB walls, and remains oriented toward regional hazard mapping rather than engineered structures.

Given the limitations of existing approaches, this study adopts the Force–equilibrium-based Finite Displacement Method (FFDM) proposed in [35,36] as an efficient, design-oriented tool for re-examining the seismic performance of two geosynthetic-reinforced soil walls with modular block facings (GRS-MBs). FFDM offers several key advantages: (1) it accommodates peak strength and post-peak degradation without requiring iterative derivation of operational strength parameters, consistent with the direction of recent improvements to Newmark-type methods; (2) it directly incorporates peak ground acceleration (HPGA/g) to represent the primary inertial force induced by ground shaking, reducing uncertainties associated with seismographic interpretation; and (3) the multi-wedge framework adopted in the FFDM explicitly incorporates an essential mechanism in interaction-rich GRS-MB systems, namely the interactions among adjacent components, such as soil–soil, soil–facing, and reinforcement–facing. This capability overcomes the common limitation of most improved Newmark-type methods, which still overlook or oversimplify interactions among soil blocks. In addition, GRS-MB walls often exhibit relatively high values of k_{hc} due to the presence of closely spaced reinforcement, which makes Newmark-type displacement-prediction equations unreliable, as demonstrated later in the case study. Building on these strengths, the FFDM-based multi-wedge analytical tool is used here to re-evaluate the two Chi-Chi GRS-MB walls to identify the mechanistic causes of their contrasting seismic responses, and to propose seismic-resisting curves (expressed by Δ_h vs. k_h) that resemble the monotonic pushover curves for displacement-based seismic performance evaluations in structural engineering [37].

Previous studies [35,36,38] focused on developing prototype versions of the FFDM framework and associated hyperbolic and post-peak soil models. Those efforts validated the FFDM by computing displacement along a known or observed slip surface, either in natural slopes subjected to groundwater-induced movement or in reinforced model slopes tested in a tilting box. No previous FFDM study has attempted to search for a critical failure surface, incorporate multi-wedge mechanisms, or evaluate reinforced slopes under high seismic loading. These earlier works, therefore, represent the first phase of FFDM development, in which the method was verified under controlled or predefined failure geometries. The present study represents a conceptual and methodological expansion into the second phase of FFDM development, implemented in a structured computer program and capable of identifying critical failure mechanisms and evaluating reinforced slopes under realistic seismic and hydraulic loading scenarios.

The remainder of this paper presents the analytical framework, summarizes the case histories, evaluates the FFDM results, and discusses implications for seismic design of reinforced soil structures.

2. Study Sites and Methodology

The 1999 Chi-Chi earthquake provided rare opportunities to uncover structural vulnerabilities that had been masked by facing elements or by inherent misdesign. Among the affected systems were two geosynthetic-reinforced soil walls with modular block facings (GRS-MBs), located on the upstream side of a 40 m wide highway, and constructed using similar backfill materials and reinforcement products. Despite their apparent similarity, the two walls exhibited markedly different seismic performances during the earthquake, prompting detailed post-event investigations.

2.1. GRS-MB Walls at Sites 1 and 3

The studied slopes [23] are steep-faced geosynthetic-reinforced soil walls with modular block facings (GRS-MBs) located in Taichung County, in central Taiwan. The wall at Site 1 experienced severe damage during the 1999 Chi-Chi earthquake ($M_L = 7.3$), as shown

in Figures 1–3. In contrast, a similar GRS-MB at Site 3, located approximately 2 km south of Site 1, exhibited only light damage, with facing displacement almost undetectable (Figures 4 and 5). Although the overall deflection and displacement of the wall were minimal, the slight misalignment at the top of the facing column (Figure 5) suggests that a small amount of movement—on the order of a few centimeters (or tens of millimeters)—is an appropriate description of the post-earthquake condition of the Site-3 wall. At a nearby seismograph station (TCU052, N–S component), a peak horizontal ground acceleration of $HPGA = 0.45\text{ g}$ (g : gravitational acceleration) was recorded.

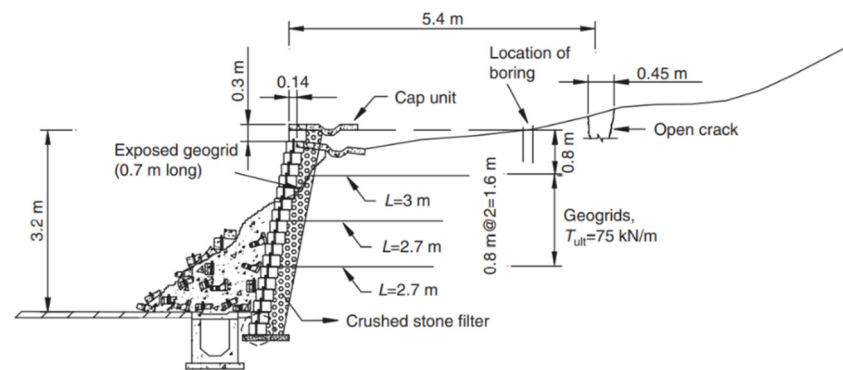


Figure 1. Cross-section 1 of the severely damaged GRS-MB at Site 1 [23].

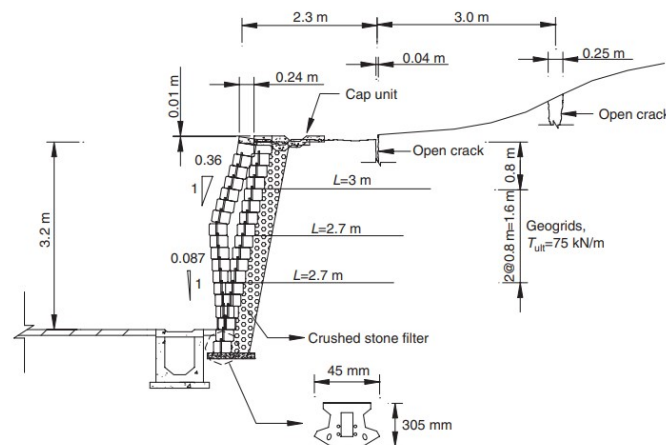


Figure 2. Cross-section 2 of severely deformed GRS-MB at Site 1 [23].



Figure 3. Severely damaged GRS-MB at Site 1 [23].

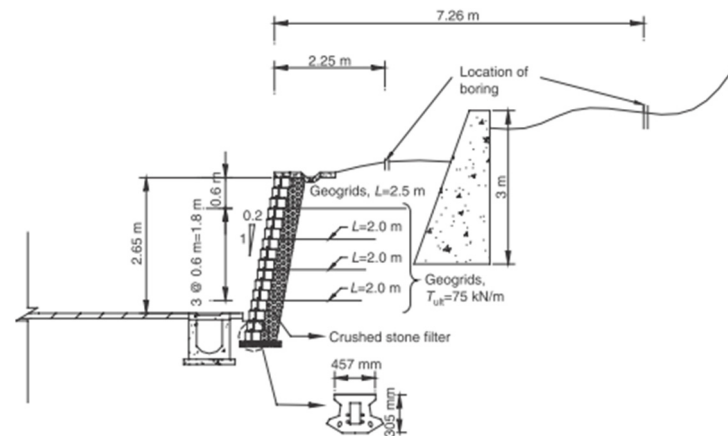


Figure 4. Lightly damaged (or lightly displaced) GRS-MB at Site 3 [23].

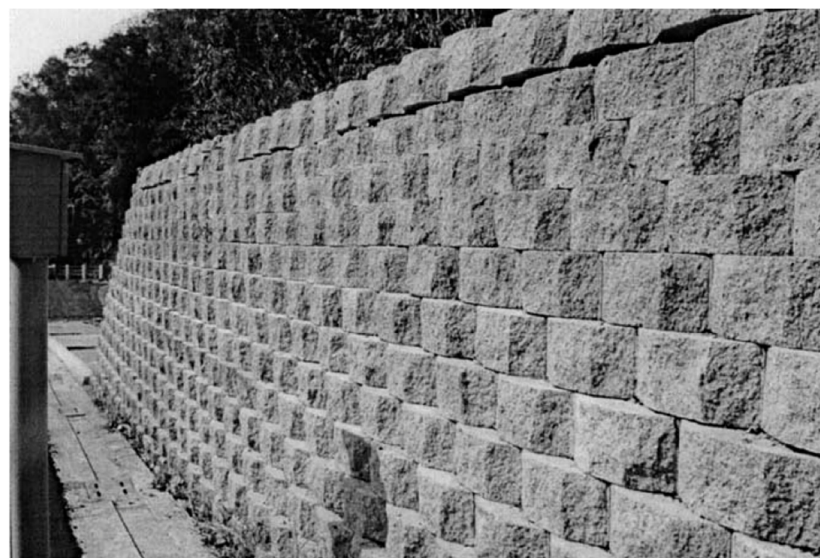


Figure 5. Lightly displaced GRS-MB at Site 3 [23].

For the two Chi-Chi sites examined here, the simultaneous occurrence of vertical and horizontal accelerations was analyzed comprehensively [23], using recordings from nearby seismographs during the 1999 earthquake. Their study showed that the ratio of vertical to horizontal peak ground acceleration around the major pulses was approximately $k_v/k_h = 0.2$ (k_v and k_h : vertical and horizontal seismic coefficients positive for upward and outward inertial forces, respectively). This ratio has been adopted directly in the present work, and the corresponding vertical inertial force is explicitly included in the FFDM calculations in the same manner as the horizontal inertial force. In reinforced soil retaining structures, the vertical component primarily modifies the normal stress acting on potential failure wedges. In the FFDM framework, using a positive k_v/k_h ratio means that upward and outward inertial forces are applied simultaneously to the wedges during strong shaking. As noted in [23], this upward inertial force represents the more critical condition because it reduces the effective normal stress and thus the mobilized shear resistance along the slip surface. The adopted $k_v/k_h = 0.2$, therefore, provides a realistic representation of the vertical shaking intensity for the studied sites and ensures that its influence on the stress state and displacement response is appropriately accounted for in the FFDM analysis.

2.2. Hyperbolic Soil Stress–Displacement Model

As illustrated in Figure 6, a hyperbolic model describing the normalized shear stress (τ/τ_f)–shear displacement (Δ) response along the potential failure surface was proposed in [35]. This model is an adaptation of the well-known hyperbolic relationship for shear stress–strain behavior developed in [39].

$$\tau/\tau_f = \Delta/(a + b \times \Delta) \quad (1)$$

where $a = \tau_f/k_{\text{initial}}$, $b = R_f$, and $R_f = \tau_f/\tau_{\text{ult}}$

k_{initial} : Initial shear stiffness of soils.

τ_{ult} : Asymptote strength at infinite displacement.

τ_f : Shear strength of soil according to the Mohr–Coulomb failure criterion.

R_f : Failure ratio ($=\tau_f/\tau_{\text{ult}}$).

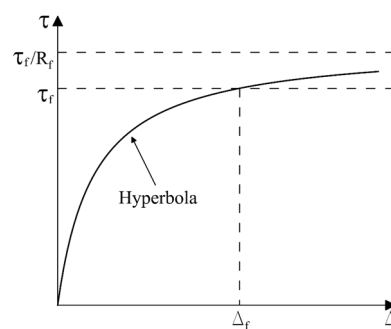


Figure 6. Hyperbolic normalized shear stress–displacement curve used in FFDM.

The shear strength of soils (τ_f) is defined by the Mohr–Coulomb failure criterion:

$$\tau_f = c + \sigma'_n \times \tan \varphi \quad (2)$$

σ'_n : Effective normal stress.

c : Cohesion intercept.

φ : Internal friction angle of soils.

Equation (1) can be viewed as a reciprocal formulation of the local safety factor FS_i at the base of the soil block (or slice) i :

$$FS_i = \tau_{fi}/\tau_i \quad (3)$$

The initial shear stiffness k_{initial} is given as a power function of effective normal pressure, proposed in [38]:

$$k_{\text{initial}} = K \times G \times (\sigma'_n/P_a)^n \quad (4)$$

K : Initial shear stiffness number (dimensionless).

P_a : Atmospheric pressure ($=101.3$ kPa).

G : Reference shear stiffness ($=101.3$ kPa/m).

n : Pressure dependency exponent.

2.3. Multi-Wedge Failure Mechanism

Figure 7 illustrates the multi-wedge failure mechanism adopted in this study. This mechanism evolved from the classical “two-wedge” or “bi-linear” failure model, which has long been used to simulate the failure of engineered slopes with steep faces, such as mechanically stabilized earth (MSE) walls and geosynthetic reinforced slopes with rigid or

near vertical facings. An enhanced version of the two-wedge mechanism was proposed in [23], comprising two active wedges (Wedges 1 and 2), a facing column (Wedge 3), and—when present—a passive wedge located in front of the toe of the facing (Wedge 4). This multi-wedge formulation provides deeper insight into the contribution of the facing system to overall slope stability. Its advantages are further amplified when combined with the FFDM, which enables displacement-based performance evaluation of the slope. Figure 8 presents the body forces and reaction forces acting on a generalized polygon wedge configuration. Triangular wedges, exemplified by Wedges 1 and 4, constitute special cases where the soil strength and reaction forces along one boundary are assumed to be negligible or zero.

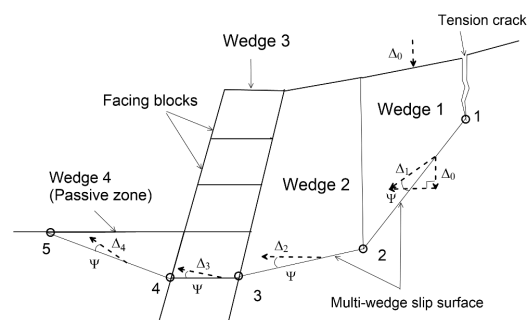


Figure 7. A schematic illustration of the multi-wedge failure mechanism used in the FFDM.

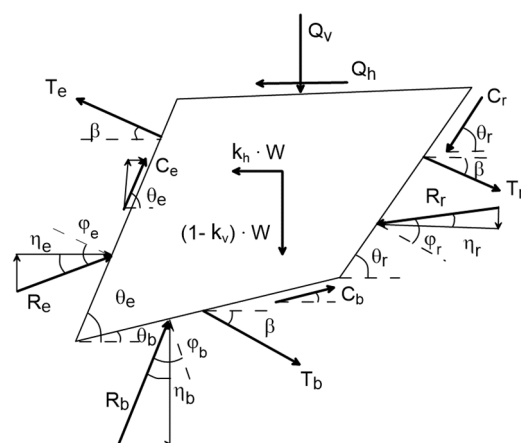


Figure 8. Force equilibrium conditions for a generalized wedge in the multi-wedge system.

By applying force equilibrium in both horizontal (x) and vertical (y) directions, the reaction force on the left side of each wedge can be expressed as:

$$R_e = f(R_r, R_b, T_e, T_r, T_b, C_e, C_r, C_b, \varphi_e, \varphi_r, \varphi_b, k_h, k_v, Q_v, Q_h) \quad (5)$$

$$C_e = c \times l_e / FS_i, C_r = c \times l_r / FS_i, C_b = c \times l_b / FS_i,$$

$$\varphi_e = \tan^{-1}(\tan \varphi / FS_i), \varphi_r = \tan^{-1}(\tan \varphi / FS_i), \varphi_b = \tan^{-1}(\tan \varphi / FS_i)$$

C_e, C_r, C_b : Cohesive resistance along soil–block interfaces.

l_e, l_r, l_b : Lengths of soil–block interfaces.

$\varphi_e, \varphi_r, \varphi_b$: Mobilized internal friction angles at interfaces.

T_e, T_r, T_b : Reinforcement force at interfaces.

R_r, R_b : Reaction forces at interfaces.

To determine the reaction force acting on the left side of each wedge in the conventional F_s analysis, the computation advances sequentially from Wedge 1 to Wedge 3 (or Wedge 4,

if present). A constant safety factor, F_s , is assumed and applied uniformly to all wedges. With this assumption, all terms on the right-hand side of the governing force–equilibrium relationship are known, except for the base reaction force R_b . Because both horizontal and vertical force equilibrium are imposed, the system is statically determinate. The F_s is updated through iterative equilibrium calculations, typically starting with an initial trial value of $F_s = 1.0$, and is adjusted until the virtual external force R_e on the left side of wedge 3 (or wedge 4) becomes negligibly small ($R_e < 0.1$ kN is used in this study).

In contrast, when the same governing relationship is solved within the FFDM framework, the safety factor for each soil block (FS_i) and the mobilized reinforcement force (T_r , T_b , T_e) depend on its displacement (Δ_i). The system remains statically determinate, but the FS_i is updated iteratively according to the displacement field at the block base and along the interfaces. The detailed computational procedure for the FFDM is summarized in Section 2.7, *Algorithmic procedure for FFDM implementation*.

2.4. Post-Peak Stress–Displacement Relationship

The post-peak segment of the shear stress–displacement (τ – Δ) curve is modeled using the Versoria (or Witch of Agnesi) curve [40,41], schematically illustrated in Figure 9. For efficient integration into the overall stress–displacement framework, a normalized local coordinate system (X – Y) is adopted. The X -axis origin ($X = 0$) aligns with the peak stress point (τ_f). The Y -axis origin ($Y = 0$) corresponds to the asymptotic value of the residual stress (τ_r). A distinctive feature of this curve is that the residual state (τ_r) is theoretically reached at an infinite Δ . In practice, however, a finite displacement (Δ_r), observed experimentally at the onset of the residual state (defined at $X = 1$), can be used to approximate τ_r with a negligibly small error (normally less than 1% error). The shear strength in the post-peak regime ($\tau_{post-peak}$) can be expressed as:

$$\tau_{post-peak} = \tau_f - \tau_f \times (t - Y) = \tau_f \times (1 - t + Y) \quad (6)$$

$$Y = t^3 / (t^2 + X^2) \quad (7)$$

$$X = (\Delta - \Delta_f) / (\Delta_r - \Delta_f) \quad (8)$$

$$t = (\tau_f - \tau_r) / \tau_f \quad (9)$$

$$\Delta_r = \Delta_f \times \Delta_{ratio} \quad (10)$$

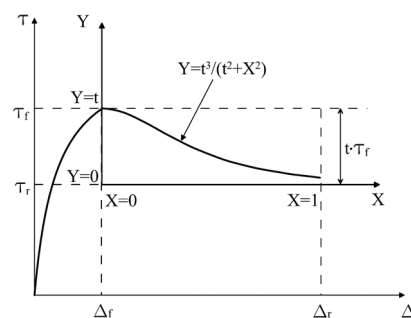


Figure 9. The transformed coordinate system for the “Versoria” curve, representing post-peak stress–displacement behavior.

t : Normalized strength reduction between peak and residual states.

Y : Normalized post-peak shear stress.

X : Normalized post-peak shear displacement.

Δ_f : Shear displacement at peak stress state (obtained from Equation (1) by setting $\tau = \tau_f$).

Δ_r : Shear displacement at the entrance of the residual state.

Δ_{ratio} : Residual-to-peak displacement ratio.

In geotechnical applications of post-peak soil behavior, the generalized and widely accepted term “residual friction angle, φ_{res} ” is preferred over the parameter t used in Equation (6). To maintain consistency with common geotechnical practice and to improve interpretability, t is reformulated based on Equation (9) in terms of φ_{res} as follows:

$$t = 1 - \tau_r/\tau_f = 1 - (\sigma'_n \times \tan \varphi_{res}) / (c + \sigma'_n \times \tan \varphi_{peak}) \quad (11)$$

This formulation is incorporated directly into the FFDM iterative procedure. When post-peak behavior is considered in the slope displacement analysis using SLOPE-ffdm 2.0 [41], Equation (6) is applied to update the available soil strength along the slip surface whenever the condition $FS_i < 1$ is detected during the iterative displacement computations. In this situation, the available shear strength $\tau_{post-peak}$ corresponding to intermediate states between “peak” and “residual” is represented through a “transient friction angle,” denoted as $\varphi_{transient}$:

$$\varphi_{transient} = \tan^{-1} (\tau_{post-peak} / \sigma'_n) \quad (12)$$

The value of $\tau_{post-peak}$ in Equation (12) is obtained from Equation (6), using the normalized displacement X and normalized stress Y defined by Equations (8) and (7), respectively. This ensures that the reduction from peak to residual strength is consistently captured within the displacement-driven FFDM framework.

2.5. Displacement Compatibility

A hodograph (displacement diagram) that satisfies *displacement compatibility*—as schematically illustrated in Figure 10—is derived following the approach in [42]. This formulation maintains displacement compatibility across soil interfaces and forms the basis for kinematic analysis of the sliding block system. The displacement at the soil wedge (or slice) i can be expressed as:

$$\Delta_i = \Delta_0 \times f(\alpha_i) \quad (13)$$

$$f(\alpha_i) = \cos(\alpha_1 - 2\Psi) / [\sin(\alpha_1 - \Psi) \times \cos(2\Psi - \alpha_1)] \quad (14)$$

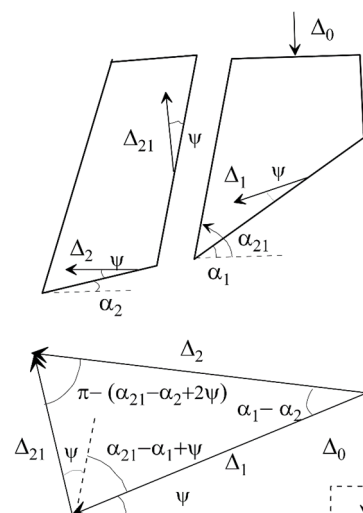


Figure 10. The displacement compatibility conditions between adjacent soil blocks in the multi-wedge failure mechanism.

2.6. Displacement Increment

To evaluate slope displacements resulting from changes in external or internal conditions—such as seismic loading, variations in the water table, or pore water pressure—two displacement values for each slice (Δ_i) are calculated: one representing the state prior to the event and the other representing the state afterward. The displacement increments for slice i , induced by the change in stress conditions, are schematically illustrated in Figure 11 and defined as:

$$\Delta_i = \Delta_i^b - \Delta_i^a \quad (15)$$

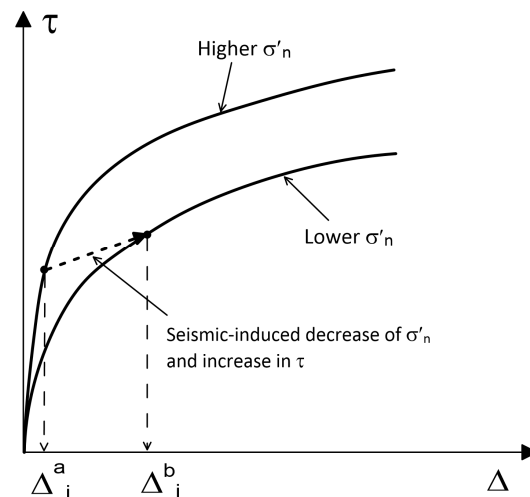


Figure 11. Schematic stress path illustrating shear displacement response to seismic-induced stress changes.

2.7. Algorithmic Procedure for FFDM Implementation

The FFDM analysis implemented in SLOPE-ffdm 2.0 follows the algorithmic sequence below:

- (1) Input slope geometry, soil strength parameters, water table position, and reinforcement or anchor strength data.
- (2) For each trial failure surface, assign an initial vertical displacement at the crest, denoted as Δ_0 (for example, $\Delta_0 = 0.001$ m).
- (3) Compute the displacements of all wedges using the displacement-compatibility functions described in Equation (13).
- (4) Calculate the reinforcement or anchor forces T_r , T_b , and T_e at the pullout displacement corresponding to the local shear displacement Δ_i at the intersection between the reinforcement and the slip surface, following the pullout model in reference [36].
- (5) Calculate the peak shear strength τ_f and the initial stiffness $k_{initial}$ using Equations (2) and (4). Then compute the local safety factors FS_i using Equations (1)–(3). If FS_i is greater than 1, or if post-peak strength is not considered, proceed directly to step (7).
- (6) For wedges where post-peak strength applies, compute the post-peak shear strength $\tau_{post-peak}$ and the transient friction angle $\varphi_{transient}$ using Equations (6) and (12). Reset FS_i to 1.0. The values $\varphi_{transient}$ and $FS_i = 1.0$ are used in Equation (5).
- (7) Starting from wedge 1 at the right-hand crest, use the known boundary value of R_r for wedge 1 as input and compute R_e , using Equation (5) together with the FS_i values obtained above.
- (8) Sequentially compute R_e for Wedges 2 through n , with the value of R_e for wedge i serving as the value of R_r for wedge $i + 1$.

- (9) The value of R_e for wedge n becomes P_{ex} , the virtual external force acting at the left boundary of the wedge at the toe of the slope.
- (10) Adjust Δ_0 and iterate steps (3) through (9) until P_{ex} satisfies the convergence criterion (e.g., within plus or minus 0.1 to 0.01 kN/m).
- (11) Iterate steps (2) through (10) for all trial failure surfaces and identify the critical failure surface associated with the maximum value of Δ_0 .

2.8. Site Explorations

Ground explorations [23] included disturbed and undisturbed soil sampling, standard penetration tests (SPT), and direct shear tests using both small-size specimens (63 mm diameter $\times$ 42 mm) and large-size specimens (200 mm diameter $\times$ 113 mm). The resulting soil properties are summarized in Table 1. Table 1 indicates that Site 1 exhibits slightly higher N-values (8–10) than Site 3 (6–7), which would suggest higher friction angles at Site 1 (25–36°) compared with Site 3 (23–34°). However, the results from large-specimen direct shear tests show the opposite trend: soils at Site 3 exhibit higher friction angles (33.6–34.8°) than those at Site 1 (29.2–30.4°). Overall, the site explorations indicate that soils at Sites 1 and 3 have comparable strength characteristics. Based on these findings, a cohesive intercept of $c_{peak} = 5$ kPa is adopted for soils with an in situ water content of 16.5% in the subsequent analysis. Accordingly, two representative strength conditions are defined: a “low soil strength” case, with $c_{peak} = 0$ and $\varphi_{peak} = 30.4^\circ$, and a “high soil strength” case, with $c_{peak} = 5$ kPa and $\varphi_{peak} = 35^\circ$, consistent with the data summarized in Table 1. The low soil strength case with $c_{peak} = 0$ and $\varphi_{peak} = 30.4^\circ$ is consistent with the $c = 0$ and $\varphi = 30.4^\circ$ used in the post-earthquake analysis [23]. In addition, a strength reduction factor $f_{inter-wedge} = 1.0$ is adopted, indicating that full soil strength is mobilized along the interfaces between soil wedges in the multi-wedge analysis.

Table 1. Soil properties at Sites 1 and 3.

	Site 1	Site 3
Unified Soil Classification System (USCS)	ML, CL	SM, ML
Unit weight (kN/m ³)	21.3	18.9
Water content (%)	16.5	12.1
Cohesion intercept (kPa)	0	0
Internal friction angle, φ	29.2–30.4° ⁽¹⁾	33.6–34.8° ⁽¹⁾
SPT-N value	8–10	6–7
Estimated φ from N-value	25–36° ⁽²⁾	23–34° ⁽²⁾

⁽¹⁾ Obtained from direct shear tests on large specimens (200 mm $\times$ 113 mm). ⁽²⁾ According to Dunham’s empirical equations [23].

2.9. Input Soil and Reinforcement Parameters

The input parameters summarized below were selected based on published experimental databases and site-specific material information. Their uncertainty ranges and influence on computed seismic displacements are further examined through the sensitivity analyses in Appendix A. The sensitivity analysis in Appendix A quantifies the influence of both high-sensitivity and low-sensitivity parameters and shows that the soil-uncertainty-induced variations in the computed seismic displacements are well bounded. The results confirm that, despite the variability of individual input parameters, the overall conclusions drawn in this study remain robust and reliable.

- (1) Peak soil strengths (c_{peak} , φ_{peak}): Three sets of soil strengths are used to represent possible variations in in situ conditions. The low-strength case uses $c_{peak} = 0$ and $\varphi_{peak} = 30.4^\circ$. The high-strength case uses $c_{peak} = 5$ kPa and $\varphi_{peak} = 35^\circ$. The integrated pre- and post-peak soil strength model uses $c_{peak} = 5$ kPa and $\varphi_{peak} = 35^\circ$, and $c_{res} = 0$ kPa and $\varphi_{res} = 31^\circ$. The adopted residual-to-peak friction ratio $\varphi_{res}/\varphi_{peak} = 0.88$ is close to the median of the range 0.82–0.93 reported from a series of direct shear tests on various soil types [43,44].
- (2) Post-peak soil stress–displacement model: A residual displacement ratio $\Delta_{ratio} = 2.0$ is adopted, consistent with the post-peak soil stress–displacement relationships reported in [38,43,44]. For medium to dense sands, reported values of Δ_{ratio} typically range from 2.0 to 6.0.
- (3) Shear stiffness number K : Stiffness values of $K = 200$ and $K = 350$ are adopted based on a database of direct shear tests [45]. These values represent approximately the lower-bound and median stiffnesses for soils with $\varphi = 35^\circ$.
- (4) Pressure dependency exponent n : Approximate median values of $n = -0.1$ and $n = 0.2$, obtained from the same database [45], for sands with $\varphi = 30^\circ$ and 35° are used here.
- (5) Failure ratio R_f : A majority of data on soils with $\varphi \geq 30^\circ$ indicates that R_f values typically fall within the range 0.79 and 0.87 [45]. A median value of $R_f = 0.83$ is used throughout the main analysis.
- (6) Displacement-dependent reinforcement pullout model: The mobilized reinforcement force in the backfill is computed using the hyperbolic reinforcement pullout model described in [36,46]. The peak adhesion at the soil–reinforcement interface is taken as $c_{s-r} = 0$, and the peak interface friction angles are $\varphi_{s-r} = 24^\circ$ and 28° , approximately $0.8 \varphi_{peak}$. The hyperbolic pullout model parameters are the median values for geogrid pullout from silty sands, including the initial pullout stiffness number $K_t = 12$, stress-dependency exponent $n_t = -0.1$, and strength ratio $R_t = 0.7$, as summarized in [46].
- (7) Tie-break strength of reinforcement: In the FFDM hyperbolic pullout model, the tie-break strength $T_{tie-break} = 75$ kN/m is assigned based on the available design and analysis documents for the studied walls.
- (8) Inter-wedge strength ratio: The inter-wedge strength ratio $f_{inter-wedge}$, defined as the ratio of failure shear strength to the shear strength available at the soil block–block interface in the force–equilibrium calculations, is set to 1.0. This choice is consistent with field observations indicating that no permanently open tension cracks developed at the interface between reinforced and unreinforced zones at the investigated sites.

Both the hyperbolic curves and the integrated curves combine a pre-peak hyperbolic model with a post-peak strength degradation model, as presented in Figure 12. The conditions of “high soil strength” with $c_{peak} = 5$ kPa, $\varphi_{peak} = 35^\circ$, and $c_{res} = 0$, $\varphi_{res} = 31^\circ$, associated with hyperbolic parameters of $K = 350$, $n = 0.2$, and $R_f = 0.83$, are used. These soil strength parameters are determined from the results of ground exploration and from Table 2, as discussed previously. The use of these soil parameters is further supported by the consistent and physically meaningful trends observed in the sensitivity investigation presented in Appendix A.

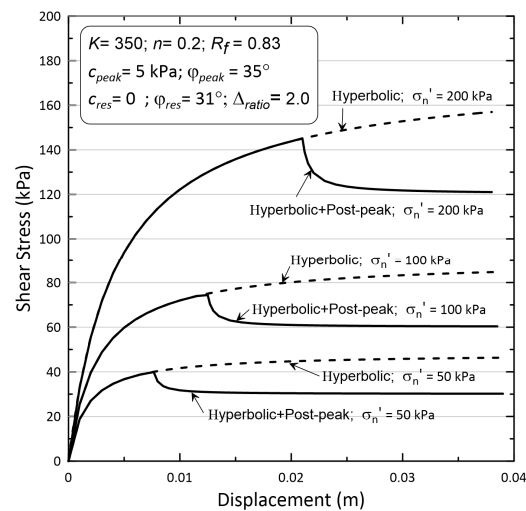


Figure 12. Hyperbolic and composed stress–displacement curves used in the FFDM analyses.

Table 2. Material properties used in the FFDM analysis for the Chi-Chi GRS-MB walls.

Soil Hyperbolic Model		Pullout of Reinforcement		Facing and Connection Strengths	
c	0, 5 kPa	c_{s-r}	0	c_{b-r}	2.5 kPa
φ	30.4°, 35°	φ_{s-r}	24°, 28°	φ_{b-r}	40°
K	200, 350	K_t	12	c_{b-b}	45 kPa
n	−0.1, 0.2	n_t	−0.1	φ_{b-b}	35°
R_f	0.83	R_t	0.7	c_{back}	0 kPa
Ψ	0°, 15°	$T_{tie-break}$	75 kN/m	φ_{back}	30°, 35°
Post-peak model		Post-peak model		c_{base}	0 kPa
c_{peak}	5 kPa	Not available		φ_{base}	30°, 35°
φ_{peak}	35°			Post-peak model	
c_{res}	0			Not available	
φ_{res}	31.0				
Δ_r / Δ_f	2.0				

c_{s-r} and φ_{s-r} : the adhesion and friction angles, respectively, at the soil–reinforcement interface. c_{b-b} and φ_{b-b} : the adhesion and friction angles, respectively, at the facing block–block interface. c_{b-r} and φ_{b-r} : the adhesion and friction angles, respectively, at the facing block–reinforcement interface. c_{back} and φ_{back} : the adhesion and friction angles, respectively, at the back-face of the facing. c_{base} and φ_{base} : the adhesion and friction angles, respectively, at the base of the facing. $T_{tie-break}$: the tie-break strength of reinforcement.

2.10. FFDM Displacement Analysis

In the Type 4 (multi-wedge) FFDM analysis implemented in SLOPE ffdm 2.0, more than 18,000 trial facing surfaces are evaluated to identify the critical failure mechanism, defined by the minimum safety factor (F_s) and the maximum vertical displacement at the slope crest (Δ_0), for a specified pseudo-static seismic coefficient k_{ft} (=HPGA/g). The seismic displacement at the base of the facing (Δ_3) is then computed using Equation (11). The horizontal component of Δ_3 , denoted Δ_{3h} , is used to evaluate the seismic resistance behavior of the wall. To ensure the robustness of the analytical procedure and the hyperbolic stress–displacement model used in the subsequent analyses, a systematic sensitivity investigation is conducted and presented in Appendix A.

The facing and reinforcement properties, except the friction angle-related terms, are kept identical for all cases.

3. Results

3.1. Results of Comparative Study on Sites 1 and 3

Figure 13 presents the F_s – k_h curves obtained from conventional slope stability analyses using SLOPE ffdm 2.0. For both Sites 1 and 3, two soil strength conditions were examined: a low-strength case, with $c = 0$ and $\varphi = 30.4^\circ$, and a high-strength case, with $c = 5$ kPa and $\varphi = 35^\circ$. The two strength conditions produce distinctly different F_s – k_h relationships, with the high-strength case yielding k_{hc} values nearly twice those of the low-strength case. In addition, Site 3 exhibits a markedly different set of F_s – k_h curves compared with Site 1, despite their similar reinforcement, soil, and facing properties. This contrast will be examined further later.

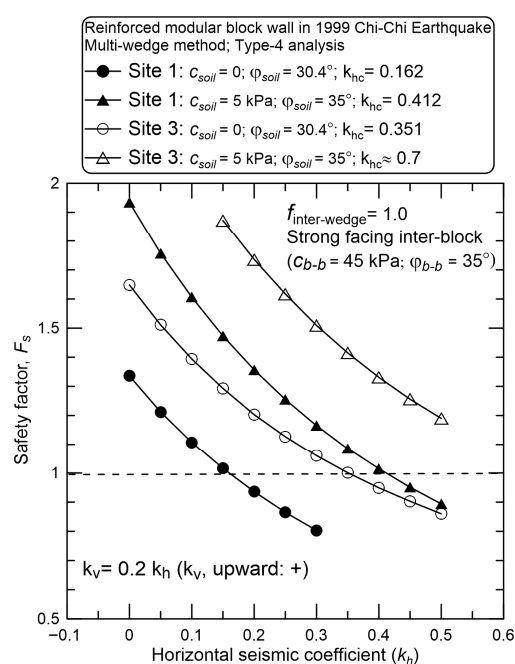


Figure 13. F_s – k_h relationships obtained from conventional LEM slope stability analyses implemented in SLOPE-ffdm 2.0.

Figure 14 summarizes the Δ_{3h} – k_h relationships computed using the FFDM for Site 1, where Δ_{3h} denotes the horizontal displacement at the base of the facing column. Three input conditions were analyzed: low soil strength with a hyperbolic model, high soil strength with a hyperbolic model, and high soil strength with a post-peak model. For each condition, two dilation angles ($\Psi = 0^\circ$ and $\Psi = 15^\circ$), representing large-displacement and peak states, respectively, for the soil with $\varphi = 30^\circ$, were used to evaluate the influence of Ψ on the displacement distribution along the failure surface. The resulting Δ_{3h} – k_h curves show limited sensitivity to Ψ . The low-strength hyperbolic model predicts extremely large Δ_{3h} values at $k_h \approx 0.25$, whereas the high-strength hyperbolic model yields negligible displacements (approximately 0.007–0.008 m) at $k_h = 0.45$. The curve with integrated pre- and post-peak model exhibits yielding behavior beginning around $k_h \approx 0.15$ – 0.20 , and approaches $\Delta_{3h} \approx 0.4$ m at $k_h = 0.45$. This behavior contrasts sharply with the low-strength hyperbolic model, which predicts unrealistically large displacements for $k_h > 0.25$. In this regard, the curve with integrated post-peak provides a more realistic representation of the observed performance than the low-strength hyperbolic model.

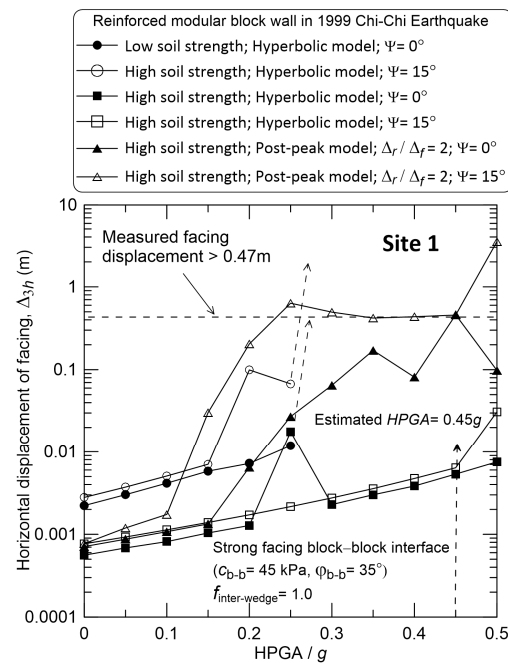


Figure 14. The calculated Δ_{3h} vs. k_h curves for Site 1 obtained from the FFDM analysis.

Figure 15 presents the corresponding results for Site 3. Compared with Site 1, the Δ_{3h} – k_h curves for Site 3 exhibit more numerically stable behavior. All three input conditions show consistent trends. The low-strength hyperbolic model predicts $\Delta_{3h} = 0.07$ – 0.08 m at $k_h = 0.45$, comparable to the displacement predicted by the integrated post-peak model ($\Delta_{3h} = 0.06$ – 0.07 m). In contrast, the high-strength hyperbolic model produces much smaller displacements (0.003–0.005 m) at the same seismic coefficient, indicating a highly stable response. Overall, regardless of the assumed soil strength model, all curves for Site 3 indicate a stable condition, with wall movements that would go unnoticed in the field unless a displacement monitoring system were used.

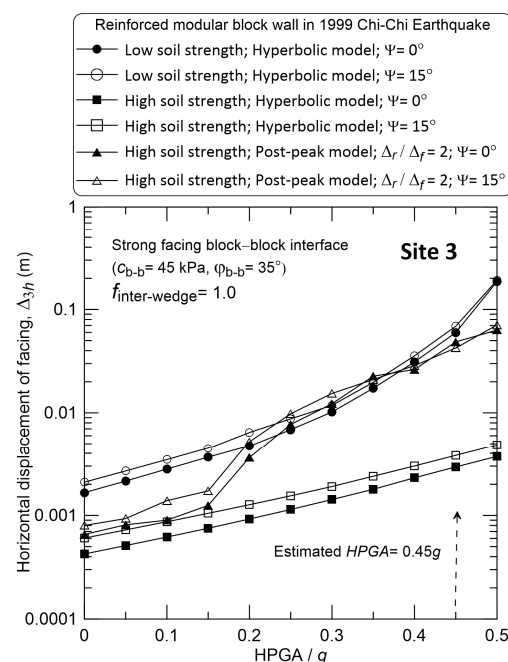


Figure 15. The calculated Δ_{3h} vs. k_h curves for Site 3 obtained from the FFDM analysis.

3.2. Factors Influencing the Failure Mechanism of Site 3

To investigate the primary cause of the higher stability of the GRS-MB at Site 3 compared with Site 1, the gravity concrete wall located behind the GRS-MB at Site 3 was removed from the slope profile, and is referred to as “Site 3, no gravity wall.” In contrast, the actual configuration is referred to as “Site 3, with gravity wall”. Figure 16 shows that the F_s values obtained from conventional stability analyses for the no-gravity-wall case are significantly smaller than those for the with-gravity-wall case. This difference arises because the no-gravity-wall condition allows the critical failure surface to extend freely into the backfill, whereas the presence of the gravity wall constrains the failure mechanism by forcing the critical surface to pass outside the wall (as shown in Figure 17). The resulting difference in k_{hc} between the two cases exceeds 50%. For reference, two dotted curves representing Site 1 are also included in Figure 16. The difference in F_s between these dotted curves and the curves for the no-gravity-wall case reflects factors other than the presence of the gravity wall, including reinforcement configuration and the slight difference in wall height. These factors account for differences in k_{hc} on the order of 13–38%, which are comparatively smaller than the influence of the gravity concrete wall.

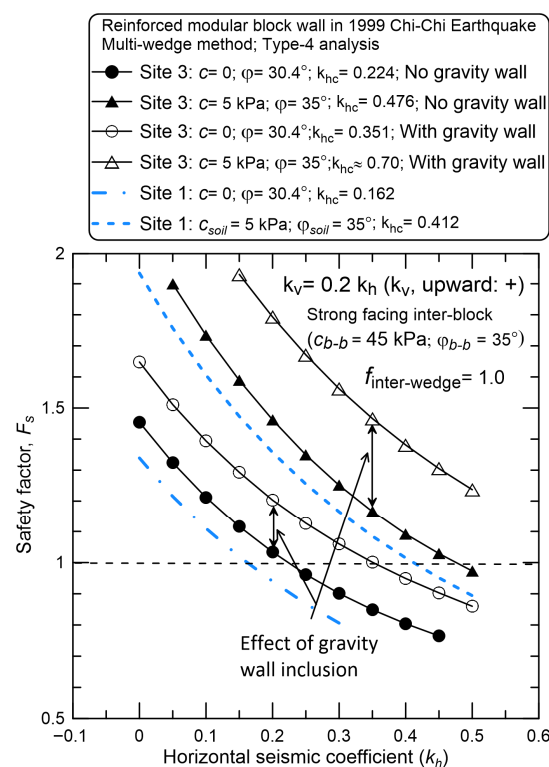


Figure 16. F_s – k_h relationships for Site 3, comparing cases with and without the gravity wall.

From the perspective of the FFDM, differences in seismic displacement behavior are also evident. Figure 18 shows that, for a given value of k_h , the resulting Δ_{3h} values follow the sequence Site 1 > Site 3 (no gravity wall) > Site 3 (with gravity wall). This ordering is the reverse of the F_s – k_h relationships shown in Figure 16. Figure 19 presents results similar to those in Figure 18 but based on the soil model integrating pre- and post-peak soil strengths. The sequence of Δ_{3h} among the three cases remains the same as in Figure 18, with a minor exception at $k_h = 0.25$ – 0.30 , where Δ_{3h} for Site 1 becomes slightly smaller than that for Site 3 (no gravity wall).

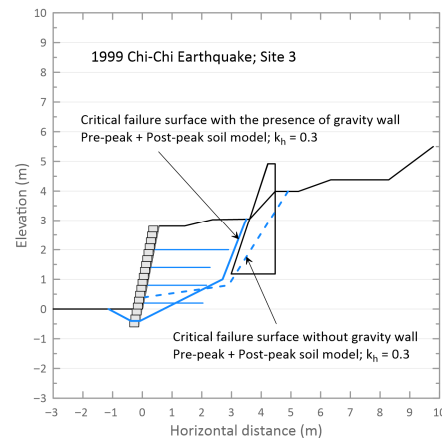


Figure 17. Examples of critical failure surfaces obtained from the FFDM analyses with and without the gravity wall.

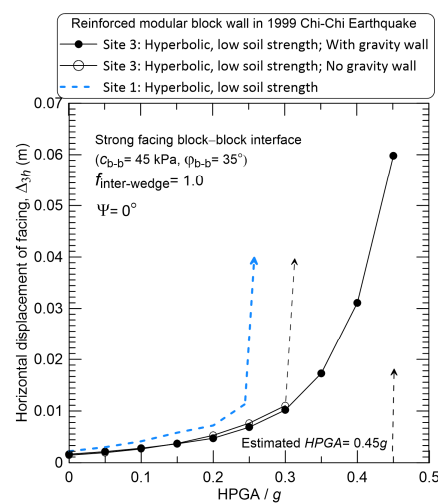


Figure 18. Δ_{3H} - k_{Ht} relationships for Site 3 with and without the gravity wall (using the hyperbolic soil model).

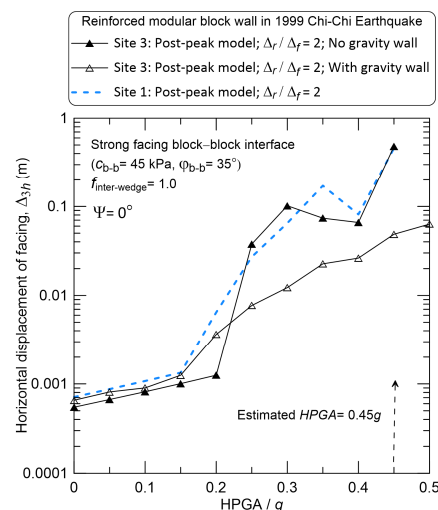


Figure 19. Δ_{3H} - k_{Ht} relationships for Site 3 with and without the gravity wall (using post-peak soil model).

3.3. Comparisons Between Newmark's Chart and the FFDM

To compare the effectiveness of the FFDM with Newmark's method, a Newmark chart compiled from [3,25,47–49] is presented in Figure 20. In this chart, the seismic

displacement of slopes is normalized by v_{max} (maximum velocity) and HPGA of selected seismographs, and the critical seismic coefficient k_{hc} is normalized by k_m (design value of seismic coefficient, or seismic coefficient corresponding to an anticipated ground shaking). Among these curves, those associated with the El Centro and Chi-Chi earthquakes reported in [49] are selected for comparison. These two curves approximately represent the lower and upper bounds of the reported range and are used in the following comparative study.

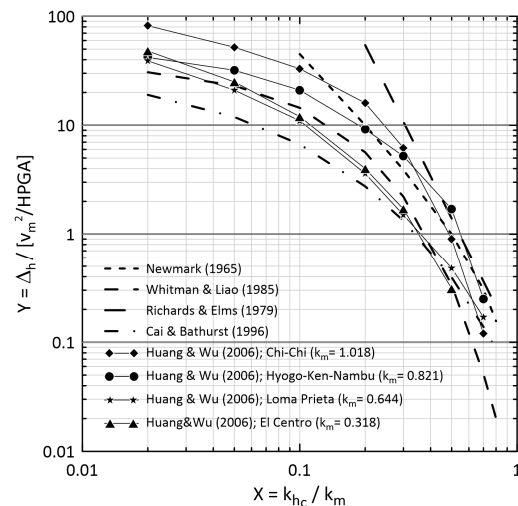


Figure 20. Normalized Newmark's seismic displacement chart [3,25,47–49].

Table 3 compares the predicted Δ_{3h} for Site 1 under a ground shaking intensity of $k_m = 0.45$ using Newmark's chart and the FFDM. Newmark's chart estimates Δ_{3h} values ranging from 0.044 to 0.575 m for the Chi-Chi and El Centro earthquakes, respectively. This range spans more than a factor of 13, covering conditions from lightly damaged to severely damaged. In contrast, the FFDM predicts $\Delta_{3h} = 0.456$ – 0.460 m for $k_m = 0.45$ (Figure 14). The FFDM-predicted displacement of $\Delta_{3h} = 0.456$ – 0.460 m is consistent with the observed displacement of 0.47 m at Site 1, where part of the facing column collapsed.

Table 3. Comparisons of Δ_{3h} for Site 1 ($k_{hc} = 0.162$, $k_m = 0.45$ for Newmark's chart; with post-peak model, $K = 350$, $n = 0.2$, $R_f = 0.83$, $\Psi = 0$ – 15° , and $k_m = 0.45$ in the FFDM analysis).

Curve and Earthquake	X	Y	v_{max} (m/s)	HPGA (g)	Newmark's Δ_{3h} (m)	FFDM's Δ_{3h} (m)
Chi-Chi [49]	0.356	3.5	1.28	1.018	0.575	0.456–0.460
El Centro [49]	0.356	1.0	0.369	0.318	0.044	

For Site 3, the FFDM predicts seismic displacements of $\Delta_{3h} = 0.042$ – 0.048 m under $k_m = 0.45$ (Figure 15). This displacement range is consistent with the misalignment observed at the top of the facing column during the post-earthquake investigation, as shown in Figure 5. When applying Newmark's chart to Site 3, the parameter $X = 0.361/0.45 = 0.802$ falls beyond the upper limit of the charted range, even when the lowest possible soil strength is used. This illustrates the difficulty of applying Newmark's chart to reinforced soil structures such as GRS-MBs, which typically exhibit higher k_{hc} values than conventional soil slopes.

4. Discussion

- (1) The distinctive responses of the two GRS-MB walls (Sites 1 and 3), despite their similar geological and geotechnical conditions, are attributed primarily to the presence of the

gravity concrete wall behind the reinforced zone (Figure 4). The concrete gravity wall impeded the development of the slip surface and strongly influenced the critical failure mechanism in both force-based F_s analyses and displacement-based FFDM analyses. Secondary factors include the smaller vertical reinforcement spacing ($S_v = 0.6$ m) used at Site 3 and its smaller wall height (2.6 m) compared with Site 1 (3.2 m).

- (2) For a unified interpretation of the seismic resistance of the two sites, the FFDM response curves, based on the soil model integrating pre- and post-peak strengths, provide a consistent explanation. These curves indicate $\Delta_{3h} = 0.456\text{--}0.460$ m at $k_h = 0.45$ for Site 1 (Figure 14), and a much smaller $\Delta_{3h} \approx 0.042\text{--}0.048$ m for Site 3 (Figure 15), consistent with the observed performance.
- (3) The use of high soil strength with a hyperbolic model may underestimate seismic displacement. This is evident for Sites 1 and 3, as shown in Figures 14 and 15, where Δ_{3h} values on the order of 10^{-3} m are predicted under $k_h = 0.45$. These results arise from the inherent approximation of the hyperbolic model, which does not capture soil behavior as accurately as the model incorporating both pre- and post-peak strengths. The discrepancy, therefore, reflects limitations of the hyperbolic curve itself rather than uncertainty in the selected soil parameters.
- (4) For the two GRS-MB walls examined, the FFDM does not require a pre-determined critical acceleration (k_{hc}). Instead, seismic displacements are computed directly through a displacement-driven formulation that enforces compatibility and equilibrium at each iteration. In these case histories, this feature allowed the FFDM to capture deformation patterns that were not represented in conventional approaches, such as Newmark's method, which relies on a pre-determined critical acceleration and does not explicitly model the evolution of soil deformation. The constitutive model adopted in the FFDM enabled a more detailed representation of pre- and post-peak soil behavior than was possible with the operational strength parameters used in limit-equilibrium-based methods. These observations suggest potential advantages of the FFDM for reinforced soil structures similar to those studied here.
- (5) For the GRS-MB walls analyzed in this study, the critical seismic coefficient k_{hc} , derived from limit-equilibrium analyses, was relatively high, making direct application of Newmark's chart difficult. For example, at Site 3, k_{hc} remained as high as 0.361 even under the lowest plausible soil strength. Given the recorded HPGA of 0.45 g at station TCU052, the resulting k_{hc}/k_m ratio exceeded the upper limit of Newmark's chart. This illustrates a practical challenge when applying Newmark's method to reinforced soil structures with relatively high stability margins, at least for the two cases examined here. In these specific situations, the FFDM provided an alternative means of estimating seismic displacement by explicitly incorporating interaction effects and pre- and post-peak soil behavior.
- (6) In these case histories, Newmark's chart provided only order-of-magnitude displacement estimates and did not resolve the small but engineering-significant movements (several 10^{-3} to 10^{-2} m) observed in the field. In contrast, the FFDM is capable of generating displacements in the 10^{-3} to 10^{-2} m range in response to small to intense seismic inertial forces. For the wall systems examined here, this capability was essential for interpreting performance where millimeter- to centimeter-scale movements governed serviceability.
- (7) For the two GRS-MB walls studied, the FFDM served as a useful complement to Newmark's sliding-block method. The two approaches differ fundamentally in how they treat soil behavior and seismic loading: Newmark's method relies on operational strength parameters and double integration of ground accelerations, whereas the FFDM incorporates additional deformation-related soil parameters to represent pro-

gressive strength and stiffness changes. In these case histories, the simplified inertial loading used in the FFDM, combined with its mechanistic soil modeling, provided displacement estimates that aligned with observed performance. Nevertheless, uncertainties in deformation-related soil parameters and variability in ground-motion characteristics remain important considerations, underscoring the need for continued calibration of the FFDM against additional well-documented case histories before broader generalization.

5. Conclusions

The contrasting seismic responses of the two geologically and geotechnically similar GRS-MB walls (Sites 1 and 3) during the 1999 Chi-Chi earthquake can be mechanistically explained through force-based F_s analysis and displacement-based FFDM. Although the walls shared comparable backfill soils and reinforcement materials, the FFDM revealed the primary factor governing their divergent behaviors: the presence of a concrete gravity retaining wall behind the GRS-MB at Site 3, which restricted the development of the critical slip surface and fundamentally altered the failure mechanism. Secondary influences included the smaller reinforcement spacing at Site 3, which enhanced internal stability, and its slightly smaller wall height. These findings illustrate how site-specific structural constraints can dominate seismic performance even when soil and reinforcement conditions are nominally similar.

Using these site-specific conditions, the FFDM successfully reproduced the observed seismic displacements when the high in situ soil strength was combined with a post-peak strength model. In contrast, lower-bound and upper-bound hyperbolic soil models tended to overestimate or underestimate the observed displacements. This highlights the importance of incorporating post-peak soil behavior and mechanistic soil parameters (c , ϕ , K , n , and R_f) when modeling reinforced soil structures under strong shaking, and underscores the need for a more comprehensive database of deformation-related soil parameters—particularly those governing peak and post-peak response.

For these two case histories, Newmark's displacement method produced a wide range of predicted displacements due to its reliance on operational soil strength parameters and its sensitivity to ground-motion variability. These characteristics limited its ability to resolve the small but engineering-significant movements observed in the field. For the GRS-MB walls examined, the FFDM provided a more consistent interpretation of seismic displacement by combining simplified inertial loading with deformation-related soil parameters. While these findings demonstrate the usefulness of the FFDM for the specific wall systems studied, broader validation across additional case histories is needed to fully establish its general applicability.

As demonstrated in both the main analysis and the sensitivity study, the FFDM remains applicable at strong shaking levels (e.g., $k_h \geq 0.3$) that would cause conventional limit-equilibrium calculations to yield $F_s < 1.0$ —a non-existent equilibrium state implying mobilized shear resistance greater than the soil's ultimate strength. This capability is fundamental to the FFDM's usefulness for displacement-based seismic assessment. The sensitivity investigation further shows that most database-derived soil, reinforcement, and facing parameters influenced predicted displacements only on the order of a few 10^{-3} m, while the parameters exerting the greatest influence, such as the internal friction angles of soils and the friction angle at the base of facing, were well constrained by site exploration. These results confirm that the main conclusions are robust with respect to reasonable uncertainties in input properties.

For the studied walls—and, potentially, for similar reinforced soil systems—the FFDM offers a practical balance between simplified inertial loading and deformation-related

soil behavior, enabling displacement estimates that align with observed field performance across a wide range of shaking intensities. Because the FFDM produces displacement-based seismic resistance curves during the design phase, it may serve as a useful component of seismic resilience planning for reinforced soil structures, particularly those resembling the systems analyzed here. Despite these advantages, several limitations remain. In the short history of reinforced-soil seismic case studies, the lack of a comprehensive database for deformation-related soil parameters, the need for engineering judgment when selecting input soil properties, and the reliance on prescribed failure–surface patterns all constrain current modeling accuracy. These limitations also indicate that the present findings should be viewed as case-specific until additional well-documented seismic case histories become available.

Looking forward, automated back-analysis systems, AI-assisted estimation of soil parameters, AI-based seismic displacement prediction frameworks, and self-updating slope displacement monitoring systems functioning as digital twin models represent promising directions for improving the reliability, adaptability, and resilience of displacement-based seismic design.

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Appendix A

This appendix presents a systematic sensitivity investigation of the input parameters used in the hyperbolic stress–displacement model. The purpose of this analysis is twofold: (1) to verify that the model responds to parameter variations in a physically consistent manner, and (2) to demonstrate the robustness of the FFDM-based displacement framework under a wide range of soil conditions. These checks provide additional confidence that the main text results are not artifacts of a particular parameter choice but instead reflect stable and interpretable model behavior. In this parametric study, all parameters fall within practically meaningful ranges based on preliminary investigations reported in [43–46]. The database used to establish these ranges encompasses soils ranging from coarse granular materials to fine-grained cohesive deposits, ensuring that the selected values are representative of backfill conditions commonly encountered in practice.

Site 3, which exhibited significant seismic stability during the intense ground shaking of the Chi-Chi earthquake, is adopted as the reference case. Throughout the analysis, an intensive shaking level of HPGA = 0.3 g ($k_h = 0.3$) is used. It is worth noting that such a high value of k_h is generally not permissible in conventional limit–equilibrium-based seismic stability analyses because the resulting safety factor F_s would often fall below 1.0. An F_s value less than 1.0 implies that the slope has already reached a failed or non-existent equilibrium state, and that the soil would need to mobilize shear resistance exceeding its ultimate strength—an outcome that is physically impossible. For this reason, engineers avoid $F_s < 1.0$ scenarios in traditional analyses. In contrast, the FFDM-based displacement method does not rely on the existence of a statically admissible equilibrium (i.e., $F_s > 1.0$) as a prerequisite for meaningful results, allowing it to remain applicable even under strong shaking levels that may be physically impermissible in conventional F_s -based approaches. Consequently,

this parametric analysis also highlights the capability of the FFDM framework to transcend limitations inherent in traditional limit-equilibrium methods.

Figure A1 shows that the horizontal displacement at the base of the facing (Δ_{3h}) decreases with increasing K ; increases with increasing R_f and Ψ ; and remains essentially unchanged with variations in n . These trends are consistent with the physical meaning of the parameters. A higher initial stiffness K reduces Δ_{3h} . A larger failure ratio R_f increases Δ_{3h} because a higher R_f produces a flatter stress–displacement segment beyond the peak state, which in turn generates larger displacements than a soil with a lower R_f . A positive dilatancy angle Ψ corresponds to volumetric expansion during shearing, which magnifies the displacement field toward the slope toe due to displacement compatibility requirements among soil wedges. Although $\Psi > 0$ represents dilation, assigning a negative Ψ is also possible within the model to simulate contractive soil behavior when needed. The insensitivity of Δ_{3h} to the stress-dependency exponent n arises because the confining stress within the backfill of the 2.6 m high wall at Site 3 is not sufficiently large to induce notable changes in soil shear stiffness.

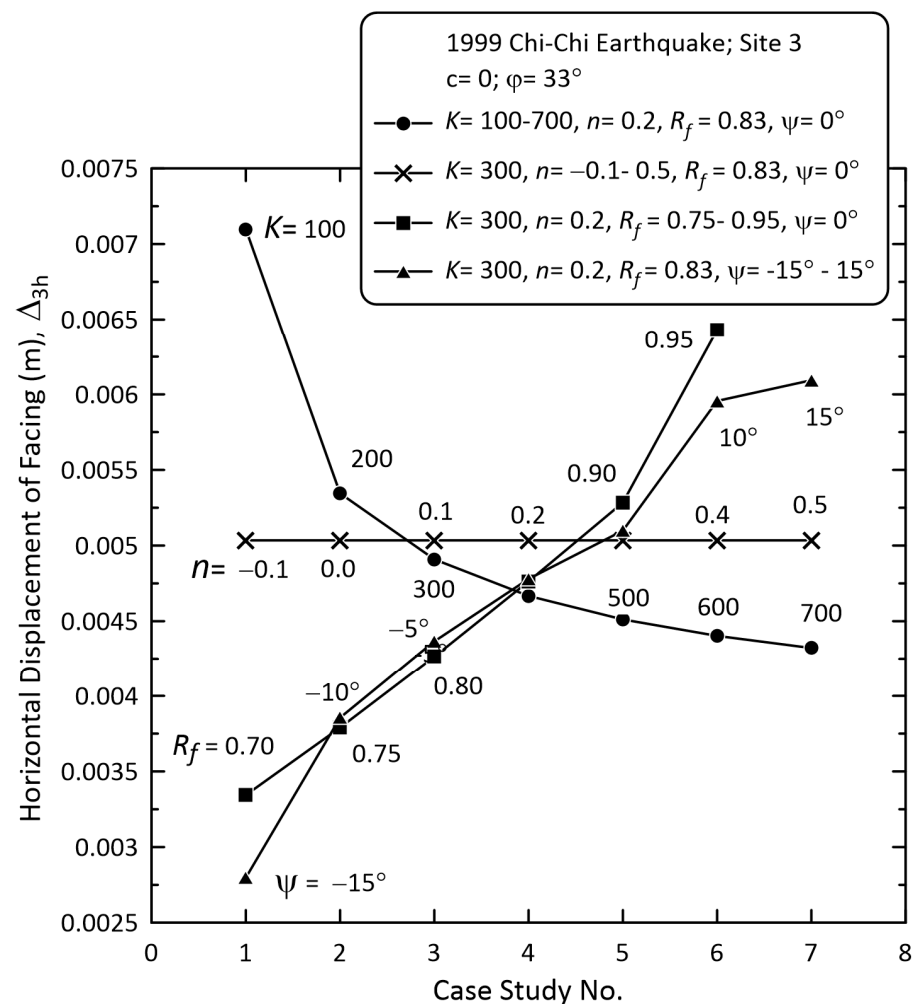


Figure A1. Sensitivities of K , n , R_f , and Ψ on the Δ_{3h} of Site 3.

Figure A2 shows the sensitivity of the internal friction angle φ on the Δ_{3h} – K curves. The consistent trends observed under different combinations of φ and K further demonstrate the precision and robustness of both the hyperbolic soil model and the computational algorithm.

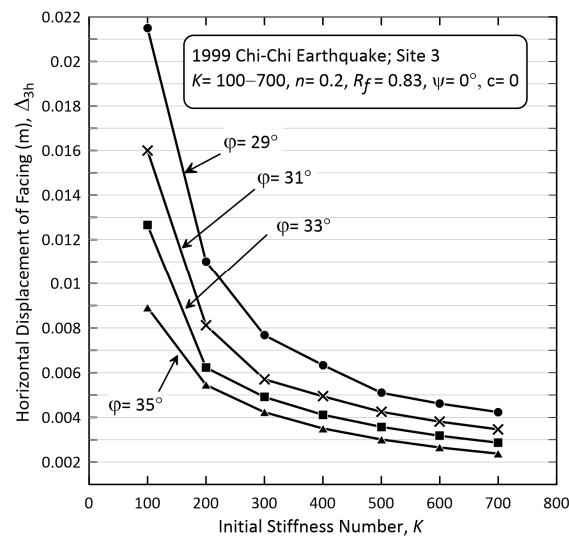


Figure A2. Sensitivities of ϕ on the Δ_{3h} of Site 3.

Figure A3 presents the results of the sensitivity analysis for the two post-peak model parameters, ϕ_{res} and Δ_{ratio} , using peak soil strengths of $c_{peak} = 5$ kPa and $\phi_{peak} = 35^\circ$, which were validated earlier. In this analysis, $K = 300$, $n = 0.2$, and $R_f = 0.83$ are adopted because they represent median values for soils with ϕ_{peak} in the range of $30-35^\circ$. The results show clear trends of decreasing Δ_{3h} with increasing ϕ_{res} and Δ_{ratio} . The physical meaning of the ϕ_{res} curve is straightforward. For the Δ_{ratio} curve, a larger Δ_{ratio} indicates a longer transitional stage between the peak and residual states because Δ_{ratio} represents the ratio of shear displacements between these two states. A longer transition implies a slower rate of strength deterioration, which in turn leads to a smaller Δ_{3h} .

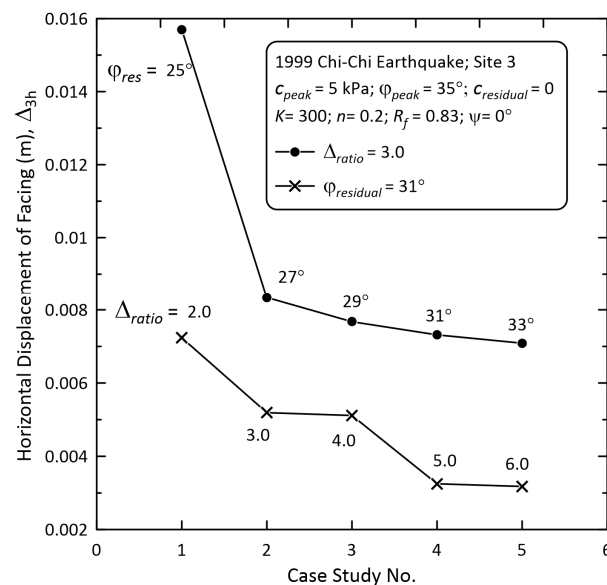


Figure A3. Sensitivities of ϕ_{res} and Δ_{ratio} on Δ_{3h} .

Figure A4 shows the sensitivity of the hyperbolic reinforcement pullout parameters—including the initial pullout stiffness K_t , the stress-dependency exponent n_t , the failure ratio R_t , the tie-break strength T_f , and the soil–reinforcement interface friction angle ϕ_{s-r} —on the horizontal displacement of the facing, Δ_{3h} . Ranges for these parameters were determined based on the reinforcement pullout database reported in [46]. Among these parameters, K_t , n_t , and R_t were assigned approximately median values from the database to represent typical reinforcement behavior. In contrast, T_f was selected based on field exploration

results at the studied sites, and the soil–reinforcement interface friction angle φ_{s-r} was taken from pullout tests conducted using a geogrid similar to that employed in the field.

Among the parameters investigated, n_t exhibits a negligible influence on Δ_{3h} . The reason is similar to that discussed for the soil parameter n in Figure A1: the confining stress levels in the reinforced zone are not sufficiently high to activate meaningful stress-dependent stiffness changes. A subtle but consistent trend of increasing Δ_{3h} with increasing R_t is observed. The underlying mechanism is analogous to that described in Figure A1, although the influence of R_t on Δ_{3h} is much smaller than the influence of R_f on Δ_{3h} . This difference arises because, in the present case, the shear resistance of the soil contributes to wall stability more directly and significantly than the reinforcement pullout resistance.

The influence of T_f on Δ_{3h} is noticeable only when T_f is as small as 25 kN/m. For $T_f \geq 50$ kN/m, further increases in T_f have no measurable effect on reducing Δ_{3h} . Among the parameters examined, K_t has the most pronounced influence on Δ_{3h} . When K_t increases fivefold from 5 to 25 kN/m, Δ_{3h} decreases from 0.0068 m to 0.0038 m, a reduction of approximately 0.003 m. This investigation confirms the accuracy and consistency of the computational algorithms governing hyperbolic reinforcement pullout behavior in the FFDM-based displacement analysis.

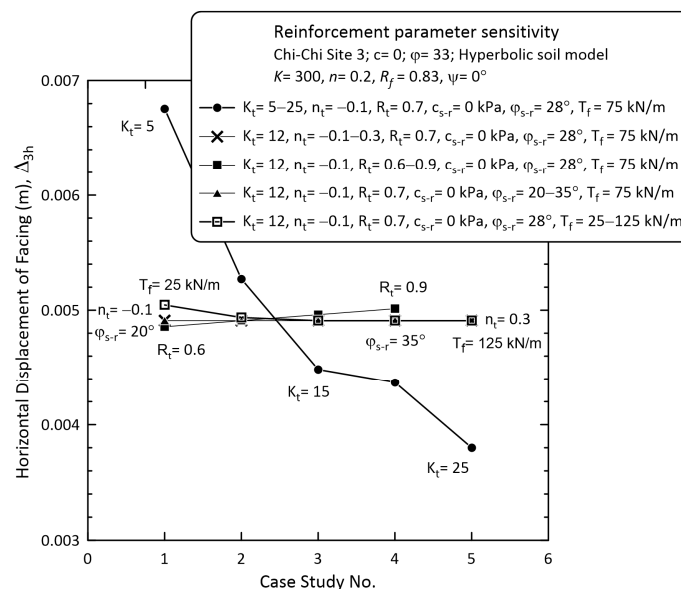


Figure A4. Sensitivities of reinforcement parameters on Δ_{3h} .

Figure A5 shows the sensitivity of the facing-block-related parameters on Δ_{3h} (investigation-1). Δ_{3h} consistently decreases with increasing block–reinforcement connection adhesion, c_{b-r} , and friction angle φ_{b-r} . The mechanism is straightforward: increasing c_{b-r} and φ_{b-r} enhances the pullout resistance of the reinforcement from the facing block, thereby improving the stability of the facing column. Increasing the block–block interface adhesion, c_{b-b} , from 0 to 3 kPa produces a small reduction in Δ_{3h} of approximately 2×10^{-4} m. For c_{b-b} values greater than 3 kPa, no further reduction in Δ_{3h} is observed. This behavior occurs because the reduction in Δ_{3h} is associated with cases where the failure surface passes through the block–block interface. When c_{b-b} exceeds 3 kPa, the facing column becomes effectively impenetrable, and a passing-through-base failure mechanism becomes dominant. The block–block interface friction angle φ_{b-b} exhibits negligible influence on Δ_{3h} throughout the investigated range of 25–45°. This is because a large value of $c_{b-b} = 45$ kPa is used in the main analysis for Sites 1 and 3, based on site exploration findings that reported FRP rods were used to connect the facing blocks. The value $c_{b-b} = 45$ kPa corresponds to the

converted shear strength at the block–block interface derived from the FRP rod strength. Due to this large c_{b-b} value, Δ_{3h} remains essentially unchanged with variations in φ_{b-b} .

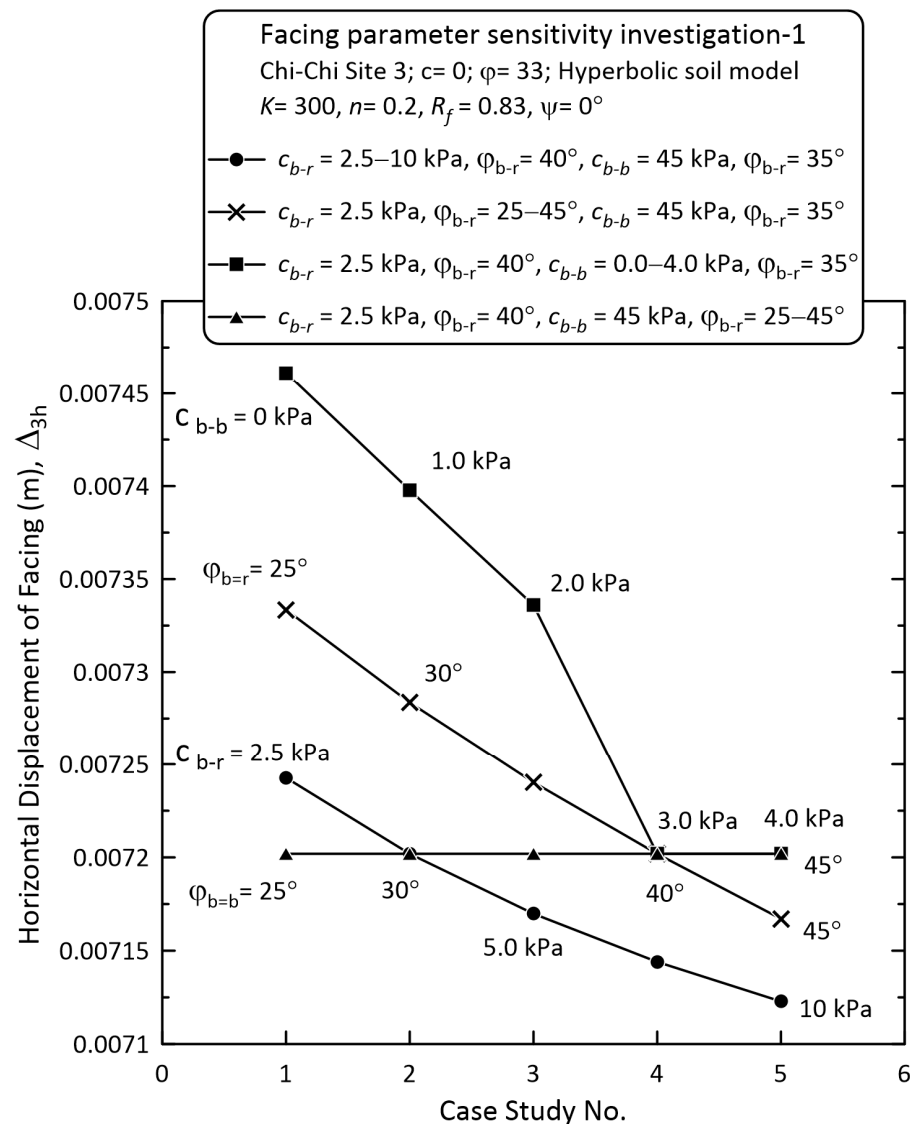


Figure A5. Sensitivities of modular-block-related properties on Δ_{3h} .

Figure A6 presents a similar sensitivity investigation to that in Figure A5, but with a broader set of parameters (investigation-2). In addition to the parameters in Figure A5, Figure A6 includes the soil–facing interface friction angle at the back of the facing column, φ_{back} , and the interface friction angle at the base of the facing column, φ_{base} . The influence of φ_{base} and φ_{back} on Δ_{3h} differs markedly from the trends shown in Figure A5, particularly for φ_{base} . The range of Δ_{3h} variations associated with the parameters in Figure A5 now appears as a negligibly small zone in Figure A6. Increasing φ_{base} from 20 to 40 degrees reduces Δ_{3h} by approximately 0.01 m, an amount comparable to the influence of the internal friction angle φ discussed in Figure A2. The physically and computationally consistent results revealed in this parameter sensitivity analysis confirm the accuracy of the FFDM as a displacement-based analytical tool for movement-sensitive and interaction-rich soil structures, such as geosynthetic-reinforced soil retaining walls with modular block facing.

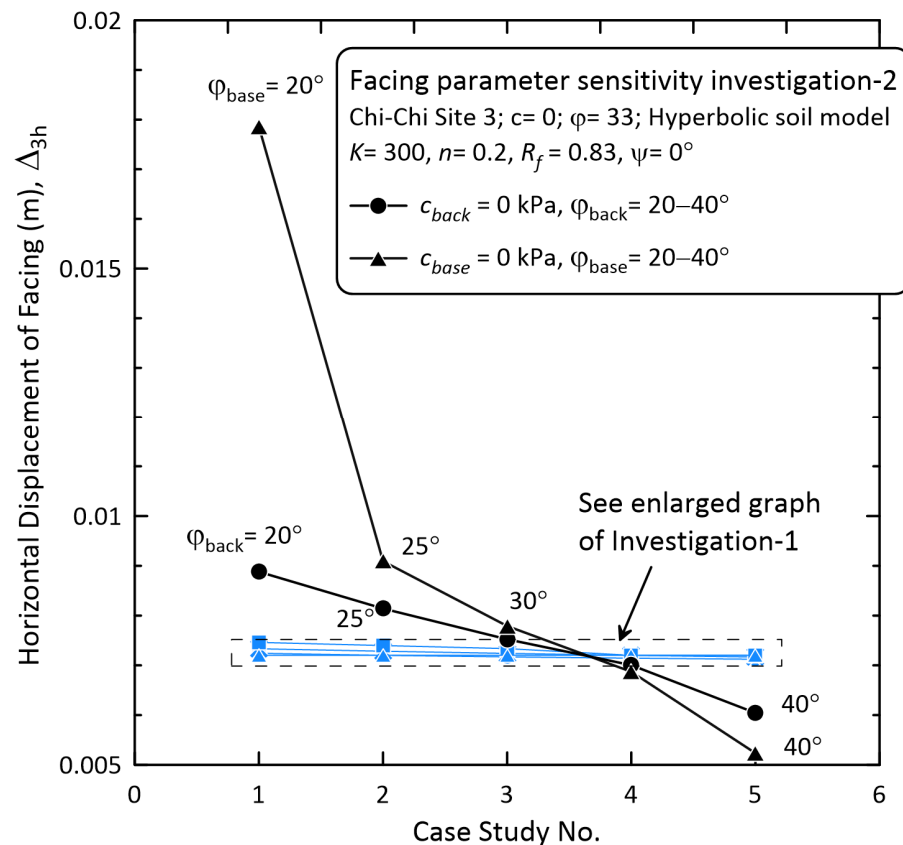


Figure A6. Sensitivities of facing–soil friction angles on Δ_{3h} .

In summary, the sensitivity investigation shows that variations in database-derived soil, reinforcement, and facing parameters typically influence Δ_{3h} only on the order of several 10^{-3} m, whereas the parameters exerting the greatest influence—such as φ_{peak} , φ_{res} , φ_{base} , and φ_{back} —were determined with confidence from site exploration. Taken together, these sensitivity results confirm that the FFDM-based displacement framework responds to parameter variations in a physically consistent manner and that the conclusions of the main analysis remain robust with respect to reasonable uncertainties in soil, reinforcement, and facing properties.

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