

Article

Influence of Coarse Material on the Yield Strength and Viscosity of Debris Flows

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Abstract: Two properties that are commonly used in the analysis of debris-flow motion and behavior are viscosity and yield strength; however, many of the techniques to measure these properties are tedious, highly theoretical, and use only the finer fraction of debris. The purpose of this study is to develop a practical and consistent method of determining the influence that coarse particles, up to 25.4 mm, have on the viscosity and yield strength of debris flows, using more accessible testing methods. Samples were tested at various sediment concentrations and with increasing maximum grain sizes of particles. Values for viscosity and yield strength of each mixture were measured and compared using four separate, previously derived laboratory tests: an inclined flume box, a slump test, a simple inclined plane, and a rolling sleeve viscometer. The slump test and rolling sleeve viscometer produced the most consistent and reasonable results, particularly as the maximum grain size was increased. In general, the sediment concentration required to produce a given yield strength increased as coarser particles were added to a slurry. While viscosity changes with grain size distribution, its variation can be predicted by sediment concentration alone. Both yield strength and viscosity could be predicted from the finer fraction of sediment, and a proposed method to predict the addition of coarse material is described. Including coarse material, yield strength and viscosity values are expected to be within 25 and 100%, respectively, of values measured by other methods.

Keywords: debris flow; viscosity; yield strength; grain size



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1. Introduction

Debris flows consist of coarse material entrained in relatively small amounts of water and air moving large distances down canyons and slopes. They often occur with very little if any warning and travel at high velocities putting people and structures at significant risk if they are in the potential path of the flow [1–3].

Debris-flow mitigation often requires knowledge of the rheological properties of the debris such as yield strength and viscosity, which are notoriously hard to measure [4–7]. These parameters influence behavioral characteristics that define debris-flow hazards, such as velocity, thickness, runout distance, levee formation, and the ability to entrain large debris such as boulders and trees [2,8].

Yield strength is the finite stress that must be applied to a material to induce flow-like behavior. At stresses below the yield strength, the material will typically deform elastically. With respect to debris flows, yield strength can be used to predict runout distance, channel plugging, levee height, sediment entrainment, and final debris deposit thickness [9–11].

Viscosity is defined as a fluid's resistance to flow. The viscosity of debris flows is not necessarily constant but can depend on the rate of shear, which is governed by its thickness and velocity [7,8,12]. As viscosity increases, more energy—provided by gravity—is required to sustain a given velocity. The ability to predict the velocity of a debris flow is important when designing anything that has a height or runup requirement such as a debris basin or a deflection berm, or that depends on discharge rate, such as channels, culverts, or bridge underpasses. Most debris-flow velocity equations such as the Poiseuille equation include a viscosity parameter [13].

Numerical modeling programs [14–18] and flume modeling [19] also require knowledge of viscosity and yield strength.

It is well known that the solids concentration of a viscous fluid directly influences its rheological parameters of yield strength and viscosity [7,11,20–25]. In the case of viscous debris flows, as the sediment concentration increases so do the yield strength and viscosity. Most debris flows incorporate at least some portion of coarse-grained, non-cohesive material. However, it has yet to be resolved whether the size of the clasts incorporated in a slurry influences its rheological properties in a similar way. This research seeks to address three important issues raised by the presence of coarse material in the debris flows. First, how does the presence of coarse material change yield strength and viscosity values and can the effects of adding each coarse fraction be quantified? Next, is it necessary to collect and test the entire grain size distribution, or is there a way to obtain accurate measurements with small sample sizes of only the finer fraction? Finally, are there testing procedures that do not require prohibitively expensive or complicated equipment, that can be run in a reasonable time, and that can be interpreted without demanding sophisticated technical knowledge?

The purpose of this study is to gain a broader understanding of how the coarse fraction (up to $d_{\max} = 25.4$ mm or 1 inch) of viscous debris flows influences their rheological properties. If it can be determined whether the size of the particles can be treated independently from the sediment concentration when determining the rheological parameters of viscous debris flows, the methods of sample collection and testing of debris-flow material can be better defined and characterized. This has implications for debris-flow hazard assessments where viscous debris flows are known to occur in that velocity, runout, and deposition thickness will be able to be predicted with improved accuracy and confidence.

2. Previous Work

2.1. Debris-Flow Rheology

Debris flows are inherently complex and difficult phenomena to understand and predict. One reason for this is because they are extremely difficult to observe and measure in nature. However, their effects and deposits are readily studied. Based on observations of debris-flow deposits and laboratory experiments, numerous models describe the behavior of debris flows. Ref. [5] provides a review of previously developed models and divides them into two groups: homogeneous and non-homogeneous. Despite their potential for being more theoretically accurate in describing the complex behavior of debris flows, the difficulty of measuring and quantifying the requisite variables has precluded the development of readily applied non-homogeneous models.

Homogeneous models treat debris-flow material as having uniform properties throughout the flow mass. Rheological models fall into this category, and some assume that certain types of debris flows (those with at least 10% of -0.04 mm material) behave as visco-plastic fluids [5,7,12,26,27]. The simplest visco-plastic model is the Bingham model, which Ref. [10] proposed to describe debris flows, where the shear rate and shear stress are linearly correlated, with an intercept representing the yield strength, as shown in Figure 1.

Many authors [12,21–24,27–29] have agreed that most viscous debris flows exhibit shear-thinning behavior or non-constant viscosity and have applied the Herschel–Bulkley model to account for this. The Herschel–Bulkley model allows for the non-linear stress-strain relationships necessary to account for shear-thinning or thickening.

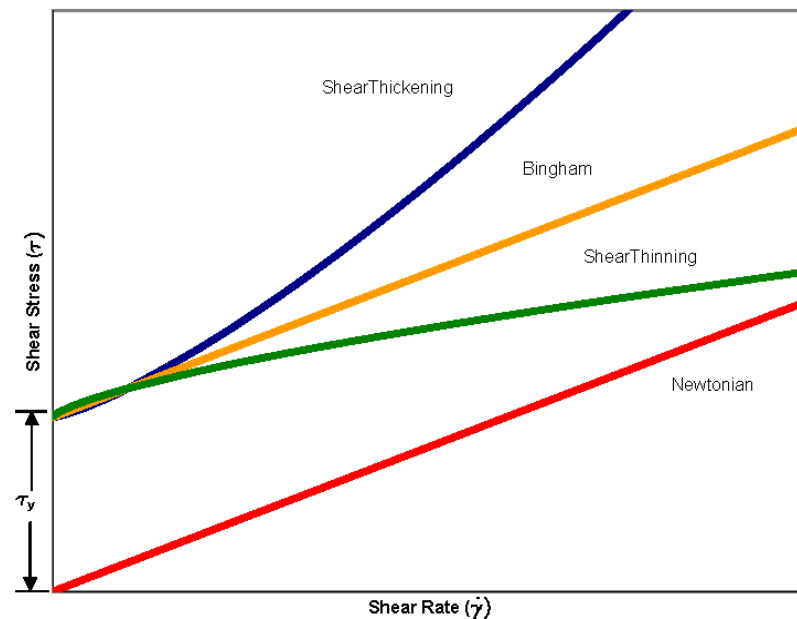


Figure 1. Various types of fluids as modelled by Herschel–Bulkley curves. In these curves, the intercept represents yield strength and the slope represents viscosity. Shear-thickening fluids, such as wet cornstarch, show increases in viscosity with increased shear rates. Bingham fluids, like honey, have constant viscosities. Shear-thinning fluids, like paint and many viscous debris flows, show decreasing viscosity with increased shear rates. Newtonian fluids, like water and oil, have constant viscosities and zero yield strength.

The Bingham model has been applied in early efforts to describe the thickness of deposits on slopes and velocities of material in channels [9,10,30]. More recently, using the Herschel–Bulkley model, other laboratory tests have used viscometers or flume channels to describe debris-flow behavior (e.g., [8,21–23]). Ref. [31] suggested that for some design applications, treating the debris flow as a Newtonian fluid may be sufficient.

While it has been shown that homogeneous models do not exactly predict the behavior of heterogeneous debris flows [32–34] it has been demonstrated that for many applications, the simplicity of the Herschel–Bulkley representation outweighs its lack of predictive precision [10,22,23,31]. Many design methods and procedures involve the use of parameters included in the Herschel–Bulkley model (e.g., [18,31,35–37]).

2.2. Laboratory Experimental Methods

Because debris flows are so short-lived and unpredictable in their occurrence, most work to obtain the relevant properties of the flows and their constituent materials has been carried out in the laboratory. The majority of experiments involve deforming the material using various forms of either rotational viscometers or open-channel flumes. Three other tests that are used in the current study are the slump test, the inclined plane, and the rolling sleeve viscometer.

Rotational (e.g., [8,29]) and parallel-plate (e.g., [38]) viscometers are widely used because of their ability to produce very precise values for shear stresses and deformation rates of fluids; however, due to their size and the way in which they shear the fluid, they tend to only be accurate for fluids containing relatively small grain sizes, generally up

to a maximum of medium sand [21]. In an effort to determine the rheology of debris-flow material that contained gravel-sized material, Ref. [7] developed a cone and plate viscometer that was 2.0 m in diameter and could test material with clasts as large as 12 cm. Ref. [27] also constructed a large-scale cylindrical viscometer that could hold up to 0.5 m³ of sample and test a maximum grain size ($d_{\max}$) of 2 cm. Both papers reported that problems with segregation, settlement, and compaction of the material arose during the tests. Ref. [7] reported that the large clasts were heard rattling and banging against the device, raising questions about the accuracy of the data.

Due to the close approximation with actual debris-flow behavior, open-channel flumes are often used to measure various behaviors and parameters of the flow. For example, the flume operated by the U.S. Geological Survey in Oregon USA can release up to 10 m³ of material down the 95 m long, 2 m wide, 1.2 m deep rectangular channel [39]. The slope is fixed at 31° with the runout surface grading to 3°. Such parameters as debris-flow pore water pressure, velocity, thickness, runout distance, deposit morphology, and others, have been measured. Ref. [22] used a 10-meter-long semi-circular flume with a variable slope angle between 10.7° and 15.2°. Measurements were obtained by filming the flows to determine their velocity, the shape of the snout, and the thickness of the material. Ref. [40] conducted experiments using a rectangular flume much in the same way as Ref. [22]. Ref. [41] used a parabolic-shaped flume. Ref. [42] included a lateral element at the base of the flume to study the rheology of deposit lobes and Ref. [43] used a flume to compare gradient, viscosity, and flow depth.

The rheological properties of debris have been measured using an inclined plane test by mounding or dumping material onto a surface with a known inclination and measuring the final, static depth of the material after it flows. The procedure is detailed in Refs. [24,28,40,44,45].

The slump test has long been used for determining the workability of concrete and to provide a rough estimate of the strength of the concrete based on its fluidity [46]. Ref. [47] developed a method to calculate the yield strength of any non-Newtonian fluid by measuring the amount of slump using a slump cone. Ref. [4] found a good relationship between sediment concentration and viscosity for fine-grained debris material. Ref. [6] simplified the test and calculations required by substituting a straight cylinder for the cone. Their results showed that the increase in shear stress along the side of the cylinder when it was removed was very small compared to the shear developed by the weight of the fluid and was therefore negligible.

The viscosity of sediment/water mixtures has been measured using a rolling sleeve viscometer [48,49]. The test consists of filling a thin flexible membrane tube with material to form an ellipse and rolling it down a known slope angle. The dimensions of the ellipse and the velocity of the sleeve are recorded. The authors derived two simple equations from which the shear rate and the shear stress can be determined from this test.

Other research has shown direct and exponential effects on viscosity and yield strength when adding coarse material to the sediment/water mixture [7,11,20–23,38]. However, as described above, the major drawback of these studies has been their inability to successfully incorporate large clasts due to size limitations on the testing equipment.

Ref. [5] performed a series of experiments to determine how the rheological parameters of portions of the parent debris-flow material compared to the field-estimated value of yield strength. The work outlines a procedure in which, through the addition of successively coarser particles, the yield strength of segregated debris-flow material approached the field-estimated yield strength. A sample containing only material <50 mm was obtained from a fresh debris flow with fines content (<0.04 mm) of approximately 10%. They then

visually estimated the percentage of material >50 mm to estimate the solids concentration by mass of the complete debris flow using Equation (1).

$$C_{vDF} = \frac{C_v}{p - C_v(p - 1)} \quad (1)$$

where:

C_{vDF} = the solids concentration by mass of the complete debris-flow material;

C_v = the solids concentration by mass of a given portion of the debris-flow material;

p = the percentage of the represented portion from the complete debris-flow grain size distribution.

Flow curves were developed for five different solids concentrations using rotational viscometers, and yield strength was measured for nine more solids concentrations using the inclined plane test. As the solids concentration increased, so did the yield strength. However, no attempt was made in the work to separate the influence of sediment concentration from the addition of coarse particles. It appears that water was added to the initial test sample such that if the entire grain size distribution of the parent debris-flow material was included, it would equal the solids concentration of the actual debris flow. Progressively coarser material was then added to the finer slurry, which also caused increases in the solids concentration. From this experiment, it cannot be determined whether the coarse clasts, the solids concentration, or both are responsible for the changes in the rheology of the fluid.

Ref. [26] performed some initial experiments measuring the effect of varying grain size distributions on the rheology of clay-water suspensions. The results of the study agree with those of Refs. [12,21] when a single grain size distribution was tested; namely, they observed an exponential increase in yield strength with increasing sediment concentration. They also performed a preliminary test first with sand, then with sand and gravel up to 20 mm. Although they only reported a single data point for the sand and gravel slurry, the result showed that the same yield strength was recorded at a higher sediment concentration for the sand and gravel slurry than for the slurry with sand alone.

3. Methods

3.1. Sample Collection

Two sites were chosen for sample collection of debris-flow deposits, Georgetown and Glenwood Springs, Colorado. The locations of the debris flows are shown in Figure 2.

In all but one case (Glenwood Springs sample number 2), the debris-flow deposits contained clasts that were too large to sample. Photographs of the sampling sites which include an appropriate scaling device such as a sphere of known diameter or tape measure were taken. The photographs were analyzed using Whipfrag (Whipware 2006 version) [50], a proprietary software program that determines the approximate grain size distribution of a sampled area. The program provided the grain size distribution down to a diameter of approximately 2.5 to 5 cm (1 to 2 in). Mechanical sieve analyses were performed on the collected material to determine the distribution of finer-grained material. Details of each site and the sampling procedure are provided in Ref. [51] (available from the authors upon request).

The material passing the #40 sieve was used to measure the Atterberg limits of the sample. This material was then mixed back into the bulk of the material and the entire sample was segregated using the following sieves: 25.4 mm (1-in), 12.7 mm (½-in), #4, #10, and #40. These sieve sizes were chosen to provide a wide range of grain size distributions that were tested for viscosity and yield strength.

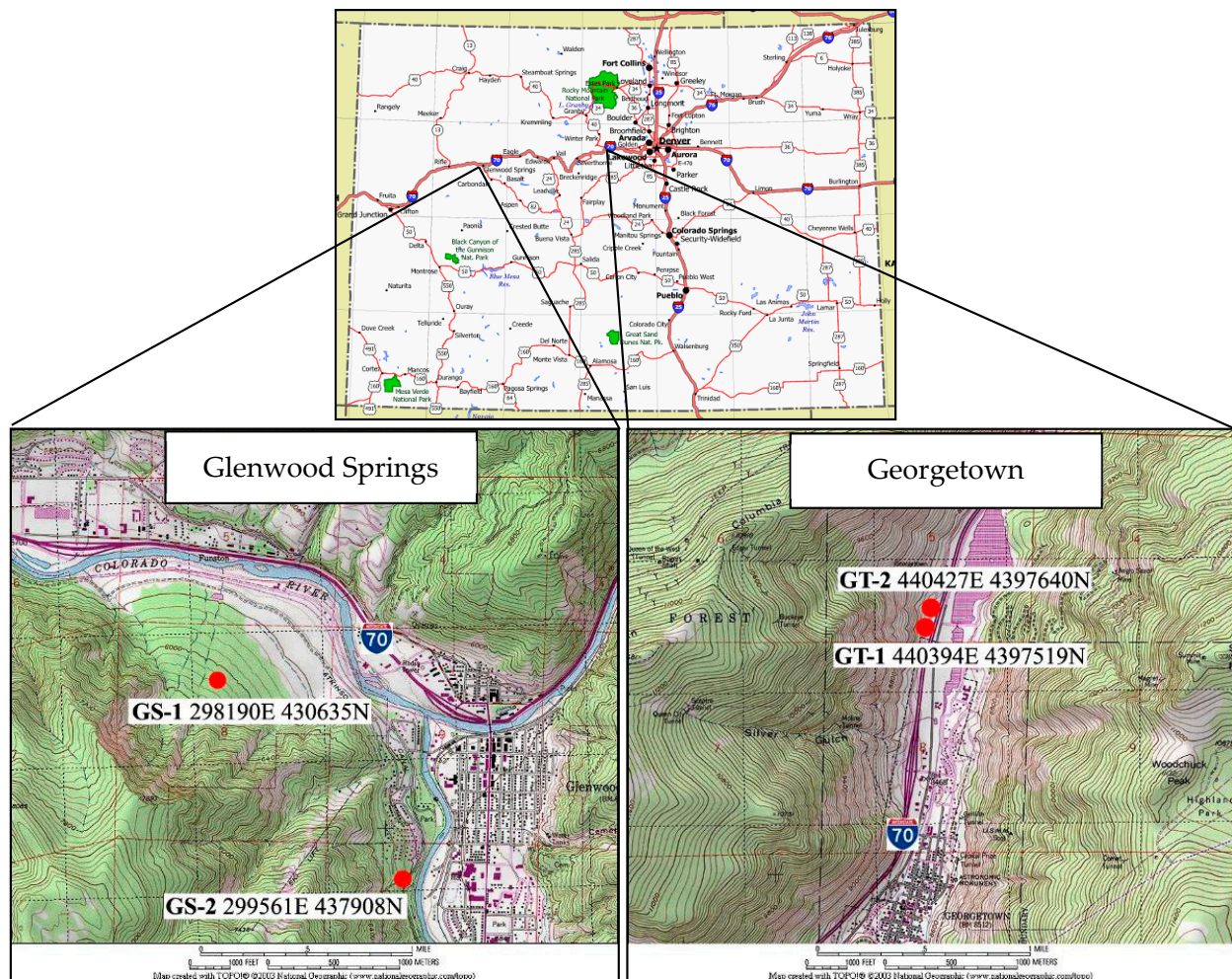


Figure 2. Location Map and UTM coordinates for debris-flow sample collection sites. Top image shows the state of Colorado.

3.2. Laboratory Tests of Yield Strength and Viscosity

Four tests were run on the samples, namely open channel flume box test, rolling sleeve viscometer test, inclined plane test, and slump test. During the testing phase, inconsistencies in results were noted for the flume box and inclined plane tests, so only results for the rolling sleeve viscometer and slump tests are reported here. Full procedures and results for all of the testing methods may be found in Soule [51] (available from the authors upon request).

The tested samples represent a large range of grain-size distributions. Sample GS-2, at the finer end, has 43% of material finer than #200 sieve (clay and silt fraction), with 10% of the material larger than 29 mm. Sample GT-1 is much coarser, with 4% finer than #200, and 10% larger than 300 mm. Samples GT-2 and GS-1 represent a mid-range, with 10 and 19% finer than #200, and 10% larger than 58 and 114 mm, respectively. These also fall within the range of debris tested by previous researchers [7,11,20–24].

Testing started with a sample sieved to use only the fraction finer than $d_{\max} = 0.425$ mm (#40 sieve). Tests were run for three to five different moisture contents so that curves relating sediment concentration to viscosity and yield strength could be developed. The range of moisture contents, and, therefore, sediment concentrations, was limited by the ability to produce flow movement. At moisture contents that were too low, flow was arrested as the moisture content approached the liquid limit. At moisture contents that were too high, segregation and flood-like behavior were observed. Next, the material up to $d_{\max} = 2$ mm

(#10 sieve) from the original sample was added to the base slurry, and the tests were run again with the new sediment concentrations. The process was repeated until the coarsest material ($d_{\max} = 25.4$ mm, or 1 inch) was mixed in and the original sample was reformed. In this way, any differences and variations in the behavior of the slurries could be compared.

3.3. Rolling Sleeve Viscometer Methodology

The rolling sleeve viscometer was developed by the authors of [48,49]. In [48], the authors describe that, according to finite-strain theory, an ellipse transforms into another ellipse of the same size and shape if the deformation is homogeneous. If a mass of material moves down a slope in the form of an ellipse without significantly changing shape like a “caterpillar-tractor tread”, it will be uniformly sheared.

Based on the experiments by the authors of [48], a slurry was placed in a thin latex or plastic tube and tied at both ends. Care was taken to remove as much air as possible from the tube. The short (c) and long (a) axes of the ellipse were then measured with the use of calipers. According to Ref. [48], the ratio of the axes should be between 0.2 and 0.4 for best results, however, ratios up to 0.5 were tested for this study with no detrimental effects.

The sleeve was then placed on a slope with a known inclination and the time it took to roll a specific distance was measured. Time was started when the sleeve had reached terminal velocity. Typically, this occurred within 20 to 30 cm of travel for the smaller sleeves and 50 to 60 cm for larger sleeves. The deformation was varied by changing the inclination of the slope. The sleeve was rolled down the plane at each angle at least five times. The travel times were then averaged into a single data point for each slope angle. A typical test included five different slope inclinations.

Experiments were conducted using a latex sleeve for $d_{\max} = 0.425$ and 2 mm (#40 and #10 sieves) material. For slurries containing $d_{\max} = 4.75$ and 12.7 mm (#4 and ½-inch sieves), a 6-mil plastic sleeve with a circumference of 30.5 cm (12 inches) was used. The coarsest material tested was $d_{\max} = 25.4$ mm (1 inch) and a 4-mil plastic sleeve 71.1 cm (28 inches) in circumference was used.

Ref. [48] provides the necessary equation for calculating the shear stress (Equation (2)), which would be correct if the sleeve provided no resistance. However, the sleeve has some strength of its own, which inhibits it from rolling down the plane. Therefore, Ref. [48] recommends that each type of sleeve be filled with water (a fluid with zero strength and very low viscosity) and the lowest angle at which the sleeve rolls is recorded. This angle relates to the shear stress provided by the sleeve and must be subtracted from the shear stress generated by the slurry. In addition, a viscosity correction must be made by determining the slope of a flow curve for each sleeve filled with water. This apparent viscosity is multiplied by the shear rate and subtracted from the shear stress. This provides the correct slope or viscosity without the effect of the sleeve. Equation (3) shows the shear stress on the slurry with the effect of the sleeve removed and Equation (4) shows how to calculate the shear rate. Table 1 gives the shear stress and viscosity correction coefficients for the three sleeves used.

$$\tau = \frac{\pi \rho g c \sin i}{4.75(1 - \frac{c}{a}) - 0.75(1 - \frac{c}{a})^5} \quad (2)$$

$$\tau = \frac{\pi \rho g c (\sin i - \sin i_s)}{4.75(1 - \frac{c}{a}) - 0.75(1 - \frac{c}{a})^5} - (\mu_s \dot{\gamma}) \quad (3)$$

where: τ = shear stress on the fluid;

c = 1/2 the short axis of the ellipse;

a = 1/2 the long axis of the ellipse;

i = the inclination of the plane;

i_s = the minimum angle at which the sleeves roll when filled with water;
 μ_s = the apparent viscosity of the sleeve;
 $\dot{\gamma}$ = shear rate of the fluid;
 ρ = fluid density;
 g = gravitational acceleration.

$$\dot{\gamma} = \left(\frac{V}{2c} \right) \left[1 - \left(\frac{c}{a} \right)^2 \right] \quad (4)$$

where V = velocity of the rolling sleeve.

Table 1. Correction coefficients for sleeves used for the rolling sleeve viscometer.

	Latex Sleeve	6-mil Plastic Sleeve 31.1 cm Circumference	4-mil Plastic Sleeve 71.1 cm Circumference
Shear stress correction ($^\circ$) (i_s)	1	5	2.5
Viscosity correction (Pa-s) (μ_s)	0.24	2.57	5.40

The viscosity of the slurry is found by the slope of the line on a graph of shear stress versus the shear rate. A limitation of the test is that there is no direct way to measure the yield strength of the material (shear stress at a shear rate of zero) since the sleeve must be in motion to determine a shear stress. However, regression of the data points to the y-axis is used to extrapolate the yield strength.

3.4. Slump Test Methodology

The slump test is used extensively in geotechnical applications, most commonly to determine the workability of concrete. It has been shown that the slump test can be used to determine the yield strength of any fluid, even slurries with coarse particles [6,28,47]. The current study employed two different sizes of slump cylinders. For slurries with $d_{\max} = 0.425$, 2, and 4.75 mm (#40, #10, and #4 sieves) a small plastic cylinder measuring 6.36 cm in diameter and 7.0 cm high was used. For slurries with $d_{\max} = 12.7$ and 25.4 mm ($\frac{1}{2}$ and 1 in) wax-lined cardboard cylindrical tubes approximately 25.4 cm in diameter and the same height were used so that the apparatus was at least 10 times the size of the largest clast.

After filling the cylinder with debris, the height of the slurry was measured, and the cylinder was removed with a vertical motion and the final height of the slumped slurry was measured. While this test can measure yield strength, viscosity cannot be measured because of the lack of velocity data.

The method for calculating the yield strength of the fluid as given by ref. [6] is shown in Equation (5).

$$\frac{s}{h} = 1 - 2 \left(\frac{\tau_y}{\rho g h} \right) \left[1 - \ln \left(2 \frac{\tau_y}{\rho g h} \right) \right] \quad (5)$$

where: τ_y = the yield strength;
 s = the final slump height;
 h = the initial height of the fluid;
 (other variables defined previously).

4. Results

The number of tests performed in the general testing program for each sample is shown in Table 2. For each test, the value used was typically obtained by repeating each test three to five times and averaging the result.

Table 2. Summary of tests performed in general testing program.

Test Type	Sample				Total
	Georgetown-1 (GT-1)	Georgetown-2 (GT-2)	Glenwood-1 (GS-1)	Glenwood-2 (GS-2)	
Slump Test	16	25	25	25	91
Rolling Sleeve	10	24	22	24	80
Total	26	49	47	49	171

Rheological tests were performed on the samples to determine how the sediment concentration and addition of coarse material would affect the yield strength and viscosity of the samples. A typical series of results are shown in Figures 3 and 4 from sample GT-2 (full results are presented in Ref. [51], available from the authors upon request). Best-fit regression curves were determined using exponential terms in the form of Equation (6), as recommended by Refs. [12,21,26]. The R^2 value is included to provide the user with an assessment of the variance accounted for with the predicted equation and a general sense of the quality of the trend indicated by the fitted curve. Terms for Equation (6) are summarized in Tables 3 and 4.

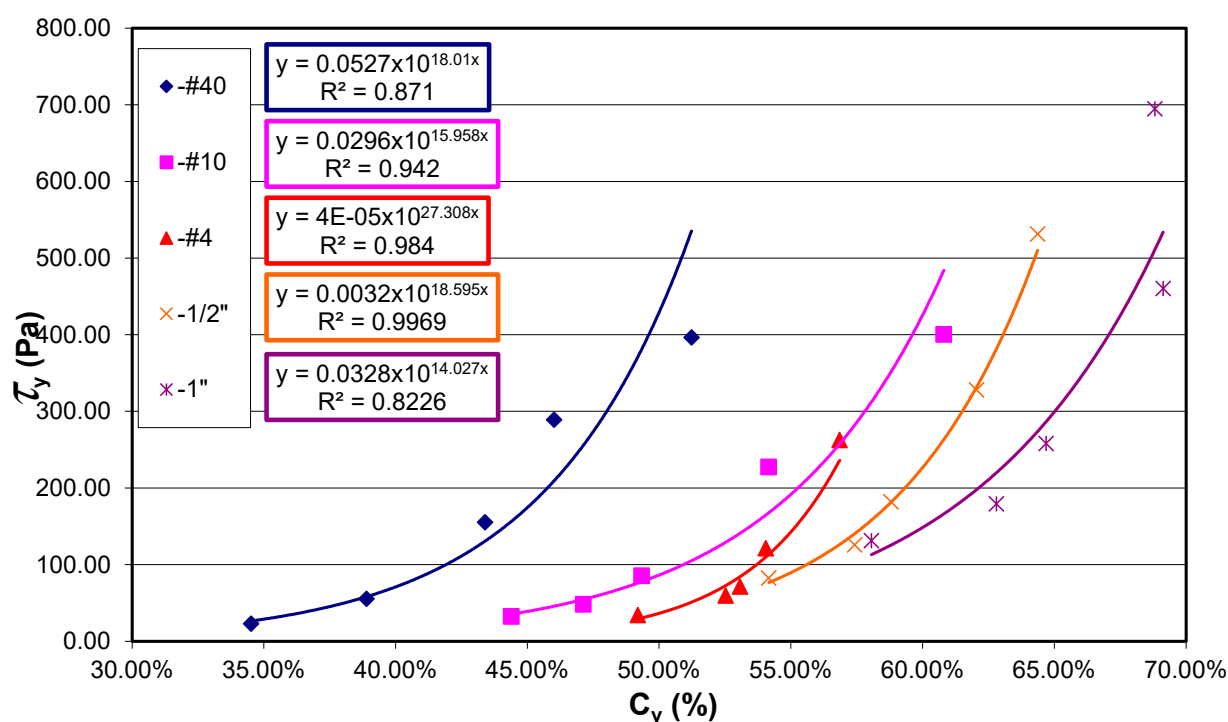
$$\tau_y = \alpha_1 e^{\beta_1 C_v}; \mu = \alpha_2 e^{\beta_2 C_v} \quad (6)$$

where: τ_y = yield strength;

μ = viscosity;

C_v = sediment concentration by mass;

α and β = fitting coefficients determined by testing.

**Figure 3.** Cont.

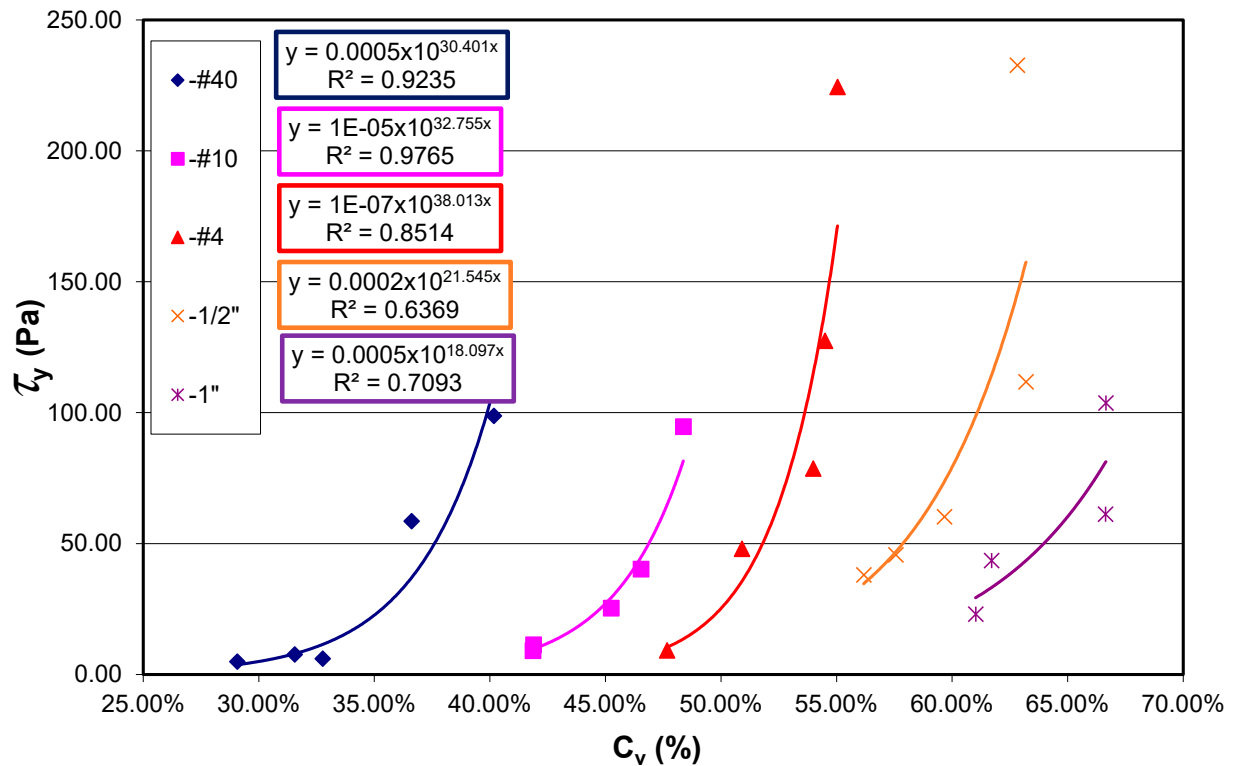


Figure 3. Yield Strength vs. Sediment Concentration by volume for sample GT-2 measured using the rolling sleeve viscometer (top) and slump test (bottom). Sieve numbers represent samples containing only the fraction of original debris less than 0.43 mm (#40 sieve), 2.00 mm (#10), 4.75 mm (#4), 12.70 mm (1/2"), and 25.40 mm (1").

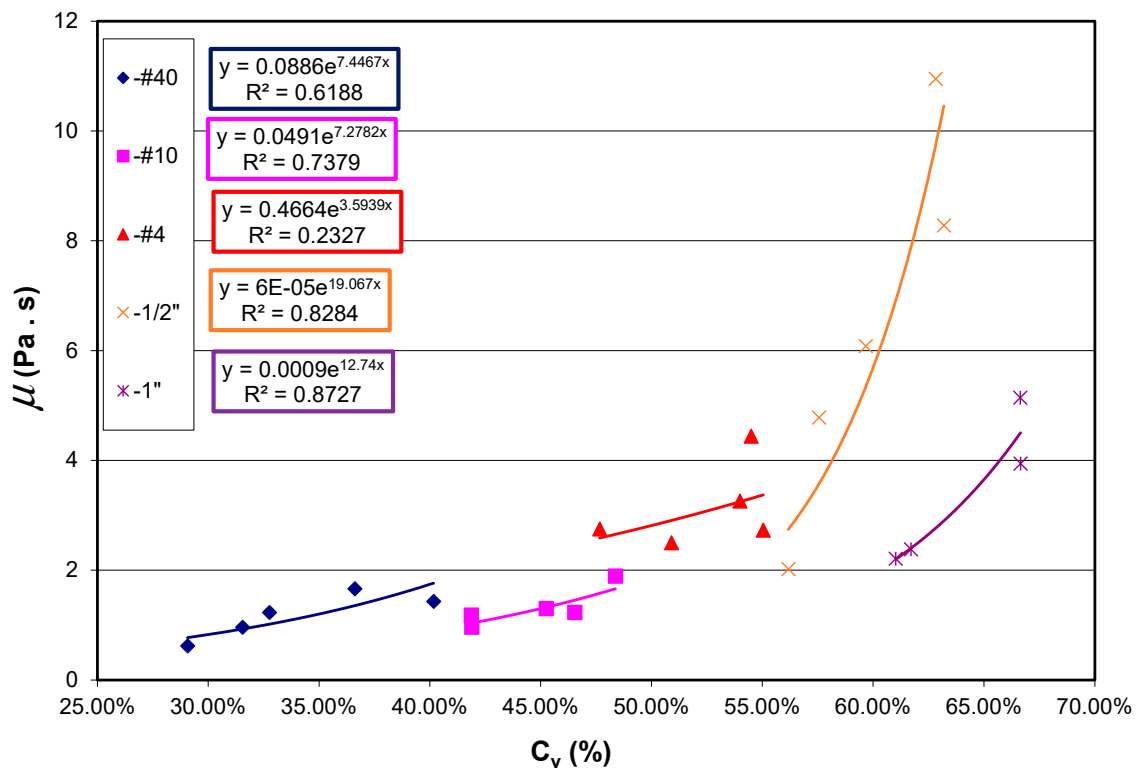


Figure 4. Viscosity vs. Sediment Concentration by volume for sample GT-2 measured using the rolling sleeve viscometer. Sieve numbers represent samples containing only the fraction of original debris less than 0.43 mm (#40 sieve), 2.00 mm (#10), 4.75 mm (#4), 12.70 mm (1/2"), and 25.40 mm (1").

Table 3. Regression coefficients and R^2 values as determined by the rolling sleeve viscometer. ND = constants not determinable. Diameter $d_{\max}$ indicates samples containing only the fraction of original debris less than 0.43 mm (#40 sieve), 2.00 mm (#10), 4.75 mm (#4), 12.70 mm (1/2"), and 25.40 mm (1").

Yield Strength (Pa)					Viscosity (Pa.s)		
$\tau_y = \alpha_1 e^{\beta_1 C_v}$					$\mu = \alpha_2 e^{\beta_2 C_v}$		
Sample	d _{max} (mm)	α ₁	β ₁	R ²	α ₂	β ₂	R ²
GT-1	0.43	ND	ND		ND	ND	
	2.00	ND	ND		ND	ND	
	4.75	ND	ND		ND	ND	
	12.70	ND	ND		ND	ND	
	25.40	ND	ND		ND	ND	
GT-2	0.43	5.00×10^{-4}	30.40	0.91	8.86×10^{-2}	7.45	0.71
	2.00	1.00×10^{-5}	32.76	0.98	4.91×10^{-2}	7.28	0.72
	4.75	1.00×10^{-7}	38.01	0.94	6.00×10^{-5}	19.07	0.84
	12.70	2.00×10^{-4}	21.55	0.82	4.66×10^{-1}	3.59	0.24
	25.40	2.00×10^{-4}	18.10	0.77	9.00×10^{-4}	12.74	0.93
GS-1	0.43	2.79×10^{-2}	21.97	0.82	1.15×10^{-1}	5.86	0.40
	2.00	1.69×10^{-2}	20.29	0.92	1.73×10^{-2}	10.35	0.78
	4.75	1.78×10^{-2}	19.71	0.91	2.59×10	−0.91	0.01
	12.70	7.31×10^{-1}	10.00	0.86	3.20×10^{-1}	5.08	0.75
	25.40	2.00×10^{-10}	50.14	0.99	5.03×10	−1.08	0.04
GS-2	0.43	1.72×10	16.01	0.98	1.46×10^{-1}	8.12	0.69
	2.00	1.38×10^{-1}	23.38	0.99	4.57×10^{-1}	2.61	0.18
	4.75	5.97×10^{-1}	17.02	1.00	4.21×10	−2.54	0.32
	12.70	5.42×10	7.75	0.94	2.52×10	0.11	0.00
	25.40	7.00×10^{-3}	22.73	0.98	4.34×10	1.78	0.19

Table 4. Regression coefficients and R^2 values as determined by the slump test. NR = test not run. Diameter $d_{\max}$ indicates samples containing only the fraction of original debris less than 0.43 mm (#40 sieve), 2.00 mm (#10), 4.75 mm (#4), 12.70 mm (1/2"), and 25.40 mm (1").

Yield Strength (Pa)				
$\tau_y = \alpha_1 e^{\beta_1 C_v}$				
Sample	d _{max} (mm)	α ₁	β ₁	R ²
GT-1	0.43	3.57 × 10	8.15	0.94
	2.00	2.84 × 10	7.38	0.93
	4.75	1.19 × 10 ¹	5.79	1.00
	12.70	NR	NR	
	25.40	NR	NR	
GT-2	0.43	5.27 × 10 ^{−2}	18.01	0.96
	2.00	2.96 × 10 ^{−2}	15.96	0.96
	4.75	4.00 × 10 ^{−5}	27.31	0.95
	12.70	3.20 × 10 ^{−3}	19.60	0.99
	25.40	3.28 × 10 ^{−2}	14.03	0.90
GS-1	0.43	1.52 × 10 ^{−1}	17.56	1.00
	2.00	5.50 × 10 ^{−2}	17.84	0.97
	4.75	8.40 × 10 ^{−3}	21.28	1.00
	12.70	5.20 × 10 ^{−3}	21.53	0.91
	25.40	3.83 × 10 ^{−2}	15.31	0.78
GS-2	0.43	3.01 × 10	13.97	1.00
	2.00	2.08 × 10	13.51	0.99
	4.75	2.99 × 10	11.11	0.99
	12.70	1.35 × 10 ^{−1}	17.28	0.92
	25.40	4.60 × 10 ^{−3}	22.80	0.84

Each sample follows the exponential increase in yield strength as the sediment concentration increases with α_1 and β_1 coefficients determinable in most cases. The R^2 values in most cases decrease as the maximum grain size ($d_{\max}$) increases. Therefore, the precision of the data decreases as larger clasts are introduced into the slurry, and there is more variability in the value of the yield strength at higher sediment concentrations. While

there is overlap in the tested range of sediment concentration, the rolling sleeve viscometer generally measured lower sediment concentrations than did the slump test, as can be observed in Figure 3. However, in the zone of overlap, the yield strength values calculated by the two methods are reasonably similar.

The viscosity of the slurries is determined by the slope of the flow curve as in the example in Figure 5. In all but a very few cases, the slurries behaved as Bingham fluids with linear regressions of the data points providing the best fit. Therefore, across the range of shear rates tested, the materials had constant viscosities. Viscosity results in Figure 4 show a dependence on both grain size distribution and the related sediment concentration. In general, viscosities increase with increasing sediment concentration; however, large variations were observed, as seen in Figure 6. These results are typical of the data generated from other samples as well.

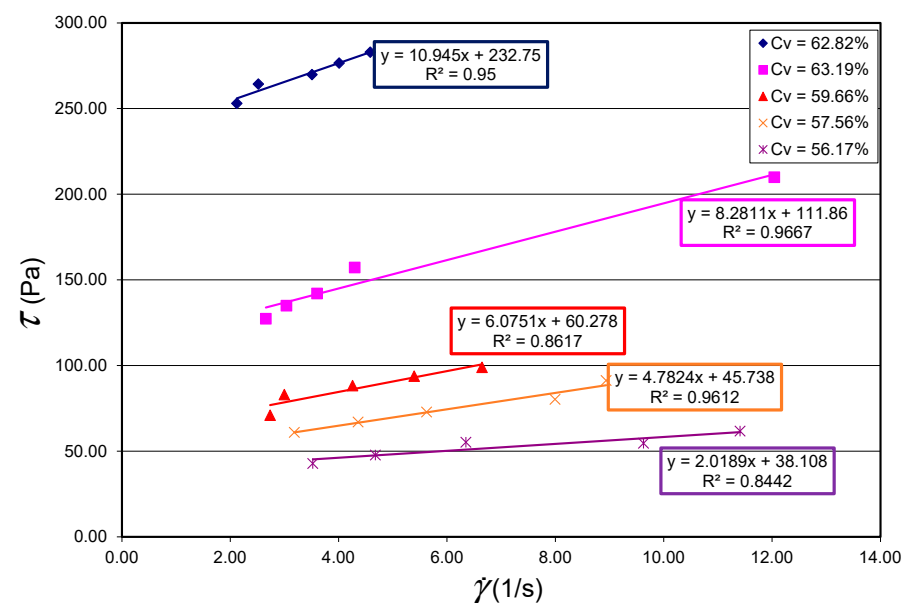


Figure 5. Flow curves from sample GT-2 with $d_{\max} = 12.7$ mm (1/2 in) measured using the rolling sleeve viscometer.

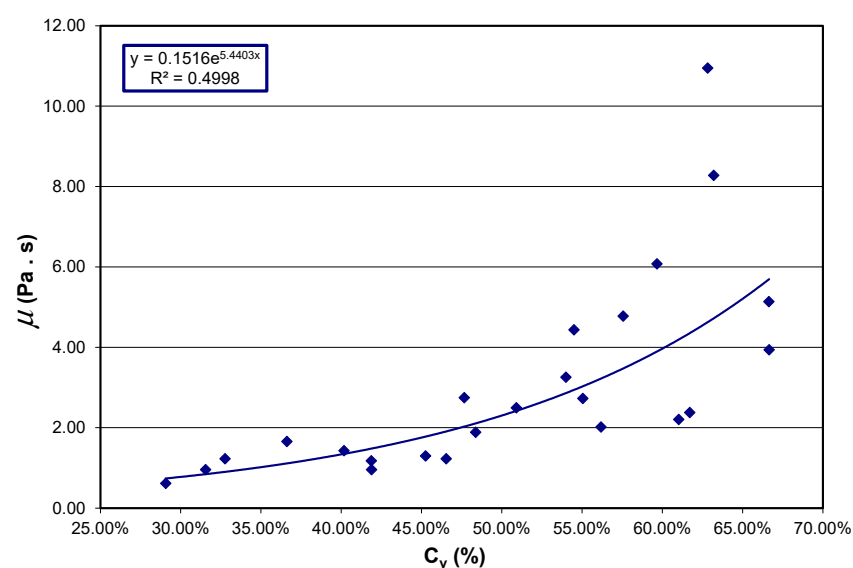


Figure 6. Viscosity vs. sediment concentration for sample GT-2 measured using the rolling sleeve viscometer. Grain size is not considered. R^2 ranges from 0.45 to 0.70 for the four sets of test results.

5. Discussion

5.1. Test Data

The most significant result of the research is the effect of the expanded grain size distribution (through the addition of coarser material) on the yield strength and viscosity of debris-flow material. The addition of coarser particles has the clear effect of increasing the sediment concentration required to produce a given yield stress. Viscosity, however, does not appear to be affected by a wider grain size distribution, as the data generated from all tests on a sample appear to fall along a single curve.

For the tests where flow curves could be determined—the flume box and rolling sleeve viscometer—the data points were typically best fit by a linear relation that defines a Bingham material. This is in contrast to the results from other work [12,21,22,28]. It is likely that the reason the fluids generally behaved as Bingham fluids is because of the relatively small range of shear rates tested. The other researchers noted above have generally tested across a wide range of shear rates—up to 100 to 200 s^{-1} , an order of magnitude greater than those evaluated in this study. This extended range of shear rates allowed the full behavior of the fluid to be observed. However, Ref. [21] found that for most natural debris-flow material shear rates are on the order of 10 s^{-1} or less. Therefore, while the data obtained in this study provides a limited range of shear rates, it is believed to be relevant to conditions in actual debris flows.

Figures 3 and 4 above show yield strength as a function of sediment concentration curves over a limited range of sediment concentrations. This is related to the range in which debris-flow behavior was exhibited by the slurries. At sediment concentrations higher than the extent of the curves, flow typically did not occur because the moisture content approached the liquid limit. At sediment concentrations below the extent of the curves, segregation and flood-like behavior were observed. Therefore, the sediment concentrations tested show a general range in which debris-flow behavior occurred.

5.2. Rolling Sleeve Viscometer

The rolling sleeve viscometer is a particularly attractive test because it is clean, easy to run, requires a relatively small volume of material (for smaller maximum grain sizes), and produces flow curves from which yield stress and viscosity can be determined.

A limitation of the test is that it does not measure yield strength directly. Therefore, flow curves must be extrapolated back to the y-axis to obtain a yield stress value. However, in most cases, shear rates less than 5 s^{-1} were measured, with many less than 2 s^{-1} . Because such low shear rates could be produced, the extrapolation distance of the flow curve to the y-axis was minimal so over-estimation of the yield stress was also minimal. It is assumed that the yield stress would be over-estimated using this method because it has been shown that most debris-flow material exhibits an increase in the slope of the flow curve as shear rates approach zero, i.e., they are shear-thinning fluids [12,21,22,28].

The rolling sleeve viscometer is the only test of the four that did not show a substantially decreasing level of precision for yield strength values as the maximum grain size was increased; however, the accuracy of the results is difficult to judge because the true value is not known.

The use of the latex sleeve for the material with $d_{\text{max}} = 2 \text{ mm}$ or less is ideal. Above this size, polyethylene sleeves were used. For these sleeves, the thicknesses required to resist puncture necessitate the strength of the sleeve that, although corrected for, may be higher than the strength of the material being tested.

5.3. Slump Test

Of the four tests evaluated in this study, the slump test was the simplest and most convenient to run and allowed for the highest sediment concentrations to be tested. The primary constraint recognized was that if the slurry was too dry, it could adhere to the sides of the cylinder and either not slide when the cylinder was lifted or deform significantly along the sides. There also appears to be a practical limitation to the wet end of slurries that can be reliably tested. At these lower sediment concentrations, the slurry may exit the cylinder so rapidly that turbulent motion ensues, allowing it to reach resting thicknesses below the critical thickness dictated by the yield strength.

For this study, only the final resting thickness of the material was measured to determine the yield strength. Ref. [28] evaluated a different method involving the measurement of the profile of the deposit. The method is applicable for deposits where the lateral extension is much larger than the height, i.e., for low-concentration slurries. The method allows a theoretical determination of the profile of the deposit based on the yield strength. Ref. [28] reported good results and fit of their data to the theoretical calculations. It appears that this method may be more reliable than simply measuring the central thickness of the deposit when testing low sediment concentrations; however, Ref. [28] did not compare the results from the two methods. Based on the data obtained from the current study, it appears that, over the range of concentrations tested, measurement of only the central thickness provides good results.

The slump test is a test that most engineers are familiar with for testing the workability of concrete. Of the four tests evaluated, it is most likely that the slump test could be utilized by professionals working toward mitigative efforts because it is easy to run, provides repeatable results, and is in wide use already for concrete testing. Also, if large particles are desired to be tested, the slump test is the easiest of the four tests to be scaled up with the use of larger-diameter cylinders. Ref. [28] reported a similar conclusion and recommended using the slump test for the coarsest grain size distributions.

5.4. Comparison of Test Results to Published Values

Recognizing the large range of viscosity and yield strength values and their dependence on shear rate, maximum grain size, and sediment concentration, it is nevertheless important to compare the range of the values measured in this study with examples of those published elsewhere. Not only will this serve as a confirmation of the general range of values but it will also allow interpretation of the effects of adding coarse sand and gravel, which is not reported elsewhere. Table 5 summarizes these results.

Table 5. Comparison of literature values to this study. “NM” indicates “not measured”.

Reference	Maximum Grain Size (mm)	Shear Rate (s ⁻¹)	Viscosity (Pa·s)	Yield Strength (Pa)	Measurement Method
This study	25.4	5–40	0.5–9	5–115	Rolling sleeve viscometer
This study	25.4	NM	NM	5–700	Slump tube
Ref. [7]	<35	3–16	17–32	5–1000	Cone rheometer
Ref. [24]	0.074 (#200)	Various but 5–40 range reported here	1–60	30–200	Lab viscometer
Ref. [4]	2	Various but 5–40 range reported here	4–60	NM	Slump cone

Table 5. Cont.

Reference	Maximum Grain Size (mm)	Shear Rate (s^{-1})	Viscosity (Pa·s)	Yield Strength (Pa)	Measurement Method
Ref. [52]	2 (sand)	1–100	0.2–10 *	10–800 *	Lab viscometer
Ref. [39]	100+	2	800	200	Field estimate
Ref. [39]	0.5	100	0.01–10	0.1–200	Parallel plate rheometer

* (for C_v range 35–65%, comparable to this study).

The maximum grain size for the materials tested in this study, as designed, is much larger than the maximum for this selection of published results, with the exception of the field estimated values for Ref. [38]. The shear rates for the testing carried out for this study are comparable, yet the viscosity values calculated using the shear rates are generally smaller, with a smaller range, than the published values. We attribute this difference, at least in part, to the narrow range of materials we tested. However, we cannot rule out the possibility that our testing methods may underestimate viscosity. The range of yield strength values for this study is comparable to the published results.

5.5. Proposed Method for Yield Stress Curve Prediction

It appears that the change in yield stress versus sediment concentration for a given grain size distribution can be predicted if the properties of a reduced portion of that distribution are known. The following procedure was conducted from slump test data because it provided the most consistent results of the four tests; however, the method was also used for the other three tests with positive results. The following steps outline the method:

Step 1—Determine yield strength as a function of sediment concentration curves

The first step is to determine yield stress versus sediment concentration curves (such as those shown in Figure 3) for a base grain size distribution and for at least one addition of coarser material. For these tests, a base sample with $d_{\max} = 0.425$ mm (#40 sieve) was used. Added coarse material could be any logical increase such as the addition of $d_{\max} = 2$ mm (#10 sieve) particles. All the material between $d_{\max} = 0.425$ mm and 2 mm should be added to avoid the creation of an artificial gap grading.

Step 2—Calculate sediment concentrations for a given yield strength

This step will determine how much the sediment concentration at a given yield strength is affected by the addition of coarser material. From the yield strength versus sediment concentration curves, choose a yield strength close to the upper limit of the data and calculate the sediment concentration required to achieve that yield strength as seen in Figure 7. It is recommended that the largest yield strength for which data exists for all grain size distributions be used because yield strength versus sediment concentration curves are more parallel at larger yield stresses and will provide better results.

Step 3—Plot the cumulative mass versus sediment concentration line

Using the sediment concentrations calculated in Step 2 and the mass of added coarse particles, plot cumulative mass versus sediment concentration and calculate the linear regression equation as shown in Figure 8.

Step 4—Predict the yield strength step-over for the desired grain size distribution

Using the linear regression equation found in Step 3, one can calculate the sediment concentration at which the chosen yield strength (for this example 80 Pa) will occur (as shown in Figure 9). For example, say we want to know the behavior of the sample with $d_{\max} = 25.4$ mm (1 in). For sample GS-2, the mass of the particles finer than 25.4 mm

and greater than 0.425 mm (the base sample) is 7446 g. From the regression equation, the sediment concentration would be 40.3%. Note that this is very close to the actual sediment concentration measured during the experiment for $d_{\max} = 25.4$ mm (1 inch) of 42.8%. Predictive curves for the other samples were similarly close to measured values.

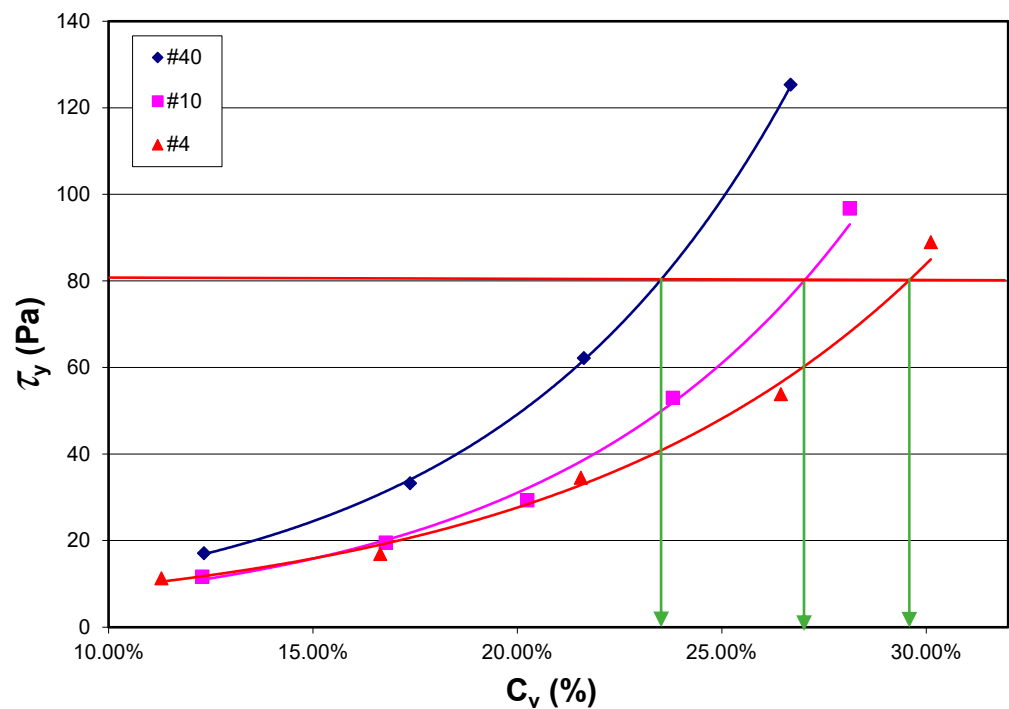


Figure 7. Method of selecting C_v points for plotting to extrapolate to coarser materials. A yield strength should be selected near the highest point where data exists for each curve, for this example for GS-2, the red line corresponds to a yield strength of 80 Pa. Sediment concentration can be read for the intersection points shown by the green lines. While three curves are shown for this example, two will work at a minimum, although accuracy is improved if more are used.

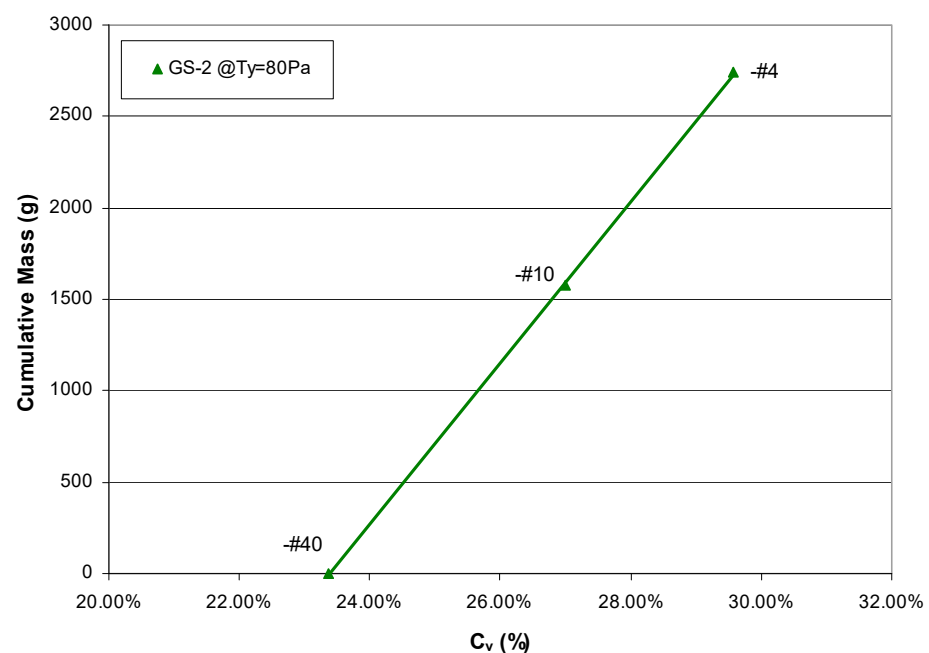


Figure 8. Mass of the sample as a function of the sediment concentration for a yield strength of 80 Pa for sample GS-2. For this data, the linear regression equation is Cum. Mass = $44,177C_v - 10,334$.

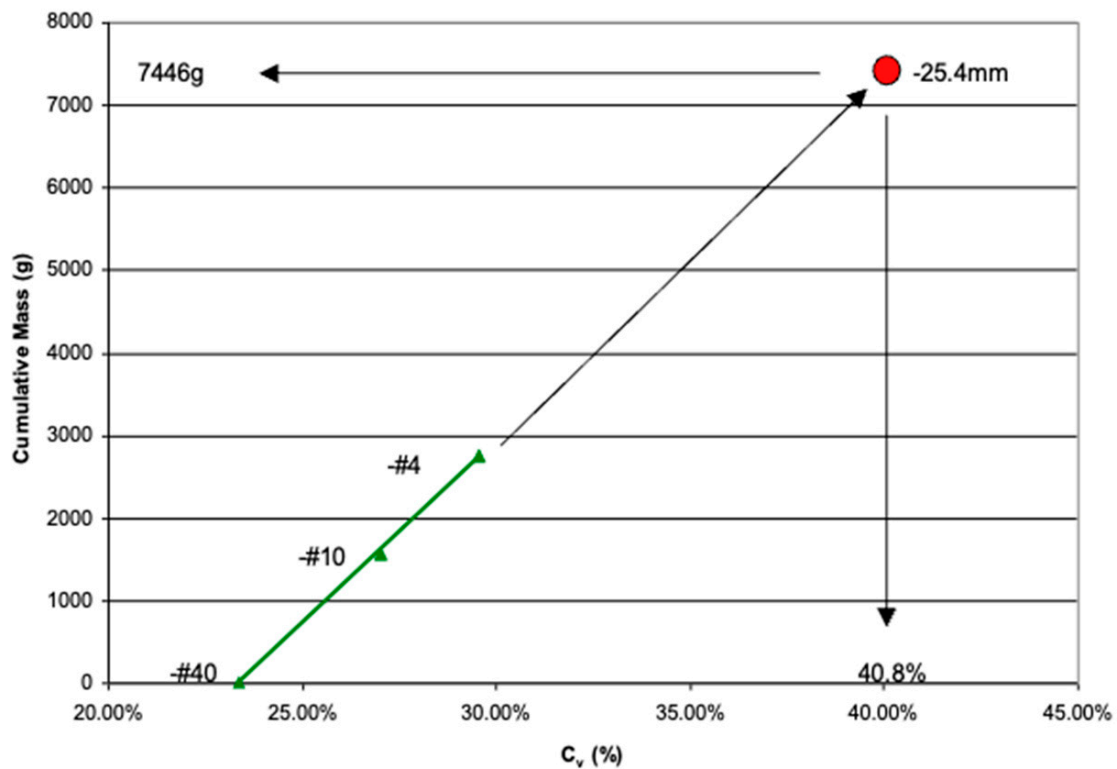


Figure 9. Extrapolation of sediment concentration to $d_{\max} = 25.4$ mm (1 inch). The green line indicates the measured data shown in Figure 8.

Step 5—Plot the predicted yield strength versus sediment concentration curve

It is not very useful to be restricted to a fixed yield strength. It is more desirable to know how the yield strength of a sample will vary with sediment concentration. Earlier results showed that β coefficients of the exponential regressions are essentially equal for a given sample regardless of the grain size distribution. Therefore, the yield strength versus sediment concentration curve for the base sample can be used as an estimate for expanded grain size distributions once the sediment concentration step-over is determined. From the calculation in Step 4, it is known that the sediment concentration that will produce a yield strength of 80 Pa for $d_{\max} = 25.4$ mm (1 inch) is 40.8%. Using the same β coefficient as the base sample (#40 in Figure 7) and forcing the curve through the point (80 Pa, 40.8% C_v) the relationship of yield strength to sediment concentration can be predicted as seen in Figure 10. Plots of measured versus predictive curves are very close for all of the samples, as shown in the example in Figure 11 for sample GT-2.

The proposed method has been demonstrated to predict the yield stress of a sample at a desired sediment concentration from a reduced grain size distribution. The predicted curves fall within about 25% of the original regression curve, which is within the level of precision of the test, especially for the larger $d_{\max}$ samples. The tests were only performed on samples up to a $d_{\max} = 25.4$ mm (1 inch). The assumption can be made that the method is viable across any grain size distribution or maximum particle size, but further investigation is necessary to show whether the method is applicable to debris flows containing boulders or very large clasts. It is also important to note that the methodology predicts laboratory-derived values, which may differ from actual field-scale values.

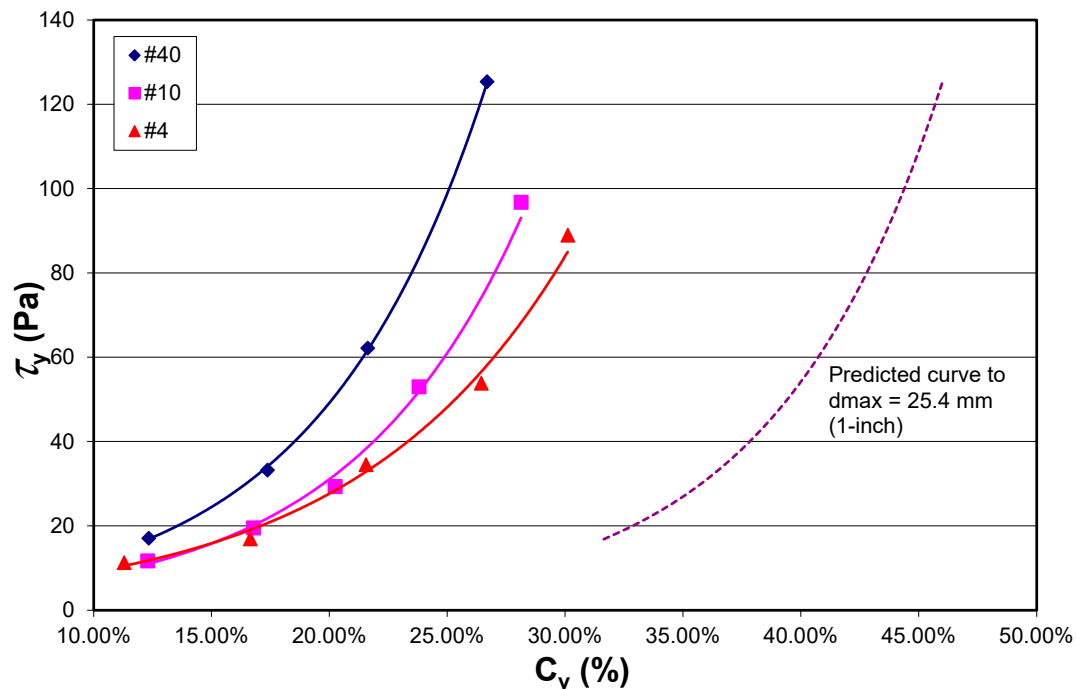


Figure 10. Predicted curve for $d_{\max} = 25.4$ mm (1 inch). The shape of the curve is drawn to mimic the base curves plotted as solid lines, with the predicted data for 80 Pa used as a fixed point from which to start the curve ($d_{\max} = 0.425$ mm, #40 sieve).

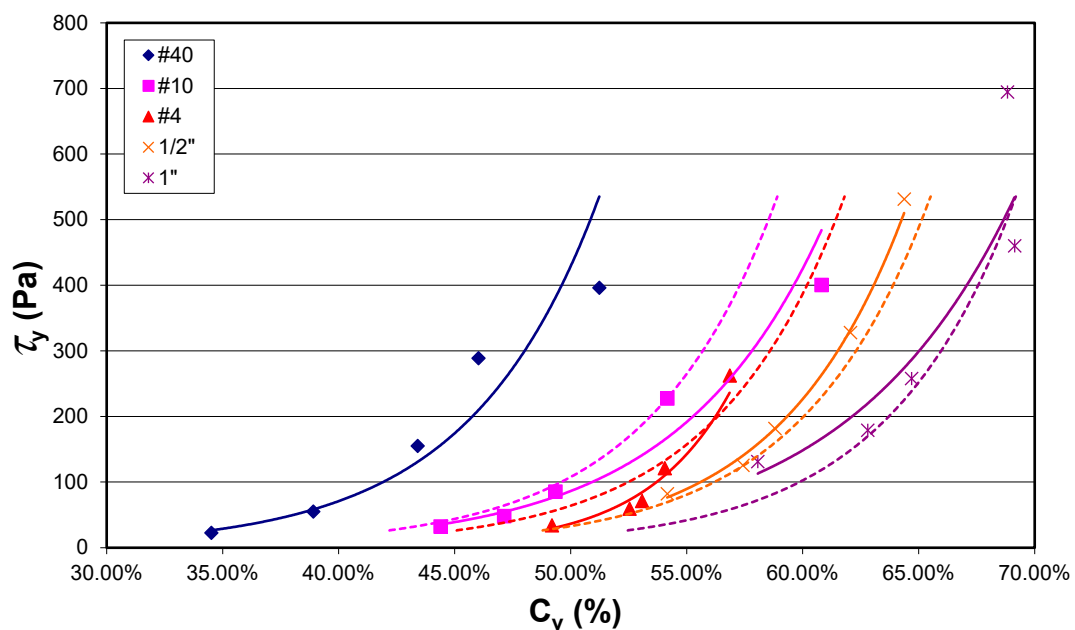


Figure 11. Predicted and actual regression curves for yield strength vs. sediment concentration for sample GT-2 from the slump test. Solid lines = original curves, dashed lines = predicted curves.

5.6. Viscosity Prediction

Prediction of viscosity from a reduced grain size distribution appears to be rather straightforward. The data show that the addition of coarser particles does not produce separate curves, but rather, they all fall along a continuous trend described by exponential regression. Therefore, if viscosity versus sediment concentration can be determined for a portion of the complete grain size distribution, the curve can be used to predict the viscosity at any given concentration or grain size distribution of the sample. From the methods used

in this study, the measurement of viscosity is accurate to about $\pm 100\%$. In a predictive scenario, the accuracy would become less as the data is further extrapolated.

6. Conclusions

This study evaluated the effect of the inclusion of large particles on the yield strength, viscosity, and sediment concentration of debris-flow samples. Four samples were collected and subjected to a set of rheological tests from which the following conclusions can be made:

1. The study confirms that both the yield strength and viscosity plotted as functions of sediment concentration of a slurry follow an exponential regression trend.
2. The addition of progressively larger particles increases the sediment concentration at which a given yield strength is achieved. The addition creates a step-over in the yield strength but the shape of the regression curve remains essentially unchanged.
3. It appears that the addition of coarse particles affects the viscosity of the material, through the change in sediment concentration. The variation in viscosity can be predicted by sediment concentration alone without knowing the exact grain size distribution.
4. Four laboratory tests were evaluated. The flume box and inclined plane tests were found to produce either highly scattered data, results that were largely inconsistent with other tests, or both. The slump test was recommended as the best of the four tests for determining yield strength when large particles ($d_{\max} > 12.7$ mm) are present in the sample. Test results for viscosity are on the low end of the range reported in a selection of published values in the technical literature, but yield strength values are comparable.
5. Methods for the prediction of both yield strength from a reduced portion of a complete grain size distribution were proposed. The method for yield strength as a function of sediment concentration prediction provides satisfactory results, accurate to within about 25% of the original data, which is within the margin of error for the tests.
6. Prediction of viscosity is more straightforward than for yield strength because the grain size distribution is not a controlling factor. Prediction of viscosity is dependent on the test and equipment used, and for this study is accurate to within about 100% at the largest particle sizes tested.

The results for this study represent only a few samples from two locations located near each other, with grain diameters up to 25.4 mm (1 inch), so broader applicability will necessitate testing of a wider spectrum of samples. Also, the results were not correlated to field measurements of yield strength and viscosity, so the true accuracy should be more closely validated.

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