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A Methodology to Evaluate the Horizontal Earth Pressure Acting on Circular Shafts Based on Its Lateral Displacements

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Abstract: One of the most common solutions to access the underground is through the construction of circular shafts. At a preliminary design stage, analytical expressions are often used to estimate the horizontal earth pressure acting on the lining. However, most of these solutions are based on active state conditions and do not consider the restrictions imposed on the shaft lateral displacements by the rigidity of its lining or by the construction method employed. To overcome this limitation, a new methodology to account for the shaft lateral displacements in the determination of the horizontal earth pressure acting on circular shafts is developed and validated against the results of relevant physical scale models. The obtained results show good agreement with the measured data, adequately reproducing the reduction in the horizontal earth pressure from its initial value to an active state condition with an increase in the shaft lateral displacement. Following this approach, more realistic predictions of the earth pressure acting on the lining of circular shafts can be performed.

Keywords: cylindrical shafts; shaft displacements; earth pressure; analytical methods



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1. Introduction

The construction of shafts to directly access the underground in urban areas is increasing considerably around the world [1–6]. Although some innovative shapes have been employed [7–9], the circular shape is clearly the most commonly adopted due to its structural superiority, which minimizes the lining thickness and the ground deformations due to the development of the arching effect in the perpendicular plane to the excavation [10,11]. As a result of that arching effect, the use of classical earth pressure theories, such as those proposed by Coulomb [12] or Rankine [13] derived for plane strain conditions, are not suitable and lead to over-conservative predictions of the earth pressure [11,14]. Consequently, many authors have developed methods to calculate the axisymmetric earth pressure acting on a shaft based on different frameworks [15–24].

Unfortunately, for some of those methods, there are no analytical solutions available, making them cumbersome to use in practice. Moreover, all methods present limitations, which can have a relevant impact on the predicted horizontal earth pressures (referred to as earth pressures from this point forward). One assumption that is common to all methods, apart from that proposed by Xiong et al. [22], is that the circumferential stress ratio ($\lambda = \sigma_\theta / \sigma_v$) is constant with depth. Often a value of 1 is assumed (the Haar and von Kármán hypothesis), while in other cases, a different value can be selected [23]. According to Cheng et al. [25], λ tends to increase from K_0 to 1 during the shaft construction, reflecting the arching effect, while the radial stress ratio ($K = \sigma_r / \sigma_v$) tends to decrease from K_0 to K_a , as verified in plane strain conditions. However, Xiong et al. [22] opposes the hypothesis of defining a circumferential stress ratio, claiming that this assumption is merely speculation and does not have a deep theoretical basis. This claim is supported to some extent by the results of the numerical analyses performed by Janeiro et al. [11] where it is shown that λ begins equal to K_0 but its value varies with depth as the shaft is excavated. However, the

results also show that for large shaft movements, λ tends to become equal to 1 along the entire shaft depth, as suggested by Cheng et al. [25].

Another limitation common to all methods is that an active limit state is assumed for the evaluation of earth pressures. However, for this condition to be admissible, a significant lateral movement of the shaft is required. Although there are some discrepancies among studies [26], a value of 0.3% of its height ($s_a = 0.3\%H$) is generally accepted as enough to mobilize the active limit state [21]. In the case of circular shafts, that lateral displacement might not be possible to achieve since the movements are often restricted by the high stiffness of the lining ring and are also very dependent on the construction method employed in the excavation. As shown by Faustin et al. [1], the construction method is directly related with the magnitude of the ground movements. In the cases where the support is installed prior to the excavation (SBE—Support Before Excavation), the movement is almost inexistent, and the earth pressures remain almost identical to those in greenfield conditions [27,28]. In contrast, when the excavation is performed in advance and only after the lining is installed (EBS—Excavation Before Support), there is significant soil decompression that leads to large movements towards the opening, with a consequent reduction in the earth pressure. Moreover, even in the cases where the shaft is excavated following the EBS methodology, the field [29,30] and numerical [31,32] results suggest that an active limit state is not reached in typical construction conditions.

This latter limitation is particularly relevant since the active earth pressures are non-conservative and should not be employed in practise unless shaft lateral displacements higher than $0.3\%H$ are expected to occur. One attempt to account for the lateral shaft displacements directly in the evaluation of the earth pressures was made by Liu [21]. The author proposed Equations (1) and (2) for the determination of the mobilised friction angle (ϕ'_{mob}) and of the circumferential stress ratio (λ) as a function of the shaft normalized displacement (s/s_a). According to Liu [21], at initial conditions, ϕ'_{mob} corresponds to the initial angle of internal friction, ϕ'_0 (Equation (3)), and $\lambda = K_0$, and as the shaft lateral displacement (s) increases, both parameters vary, reaching at the active limit state values of $\phi'_{mob} = \phi'$ (soil friction angle) and $\lambda = 1$.

$$\phi'_{mob} = \phi'_0 + \frac{4 \cdot \text{atan}(s/s_a)}{\pi} \cdot (\phi' - \phi'_0), \quad s \leq s_a \quad (1)$$

$$\lambda = \frac{4 \cdot \text{atan}\left(\frac{s}{s_a}\right)}{\pi} \cdot (1 - K_0) + K_0, \quad s \leq s_a \quad (2)$$

$$\phi'_0 = \text{asin}\left(\frac{1 - K_0}{1 + K_0}\right) \quad (3)$$

with K_0 given by Jaky's [33] equation ($K_0 = 1 - \sin\phi'$).

However, as Liu [21] acknowledges these relationships influence significantly the accuracy of the predictions and require further refinement and validation. It should also be pointed that while different ϕ'_{mob} and λ are employed for each shaft lateral displacement, the earth pressure prediction is still performed based on an analytical solution deduced based on an active limit state.

Considering these limitations and the demand for a more global solution, this paper presents a new methodology capable of incorporating shaft lateral displacements in the prediction of earth pressures acting on circular shafts. The method is developed having as reference the results of relevant physical tests, both small-scale and centrifuge, where the shaft excavation was simulated. Based on the measured results, it was possible to define an expression that relates the reduction in the earth pressure with the shaft lateral displacements, varying from the initial stress state (K_0 condition) until an active limit state is reached. The obtained results show good agreement with the measured data and represent an improvement on the previous methods, making the methodology suitable for being used at early stages of design.

2. Materials and Methods

2.1. Comparison of the Analytical Methods

As mentioned previously, the classical earth pressure theories derived for plane strain conditions are not applicable in circular shafts, where axisymmetric conditions prevail. As a result, several authors developed and proposed analytical methods for evaluating the active earth pressure acting on circular shafts [15–24]. These methods can be divided into three principal groups according to their frameworks. (1) The limit equilibrium theories proposed by Terzaghi [15] and Prater [17], where a suitable failure surface is adopted and a minimum pressure is determined in order to ensure the static equilibrium of the soil wedge. (2) A different approach was followed by Berezantzev [16] and further extended by Cheng et al. [18], Liu et al. [19], Liu et al. [20], Liu [21] and Xiong et al. [22], where the slip line method was employed to derive the partial differential equations governing the plastic equilibrium of a conical surface. (3) Wong and Kaiser [24] proposed a methodology based on the convergence–confinement method usually applied for tunnels under isotropic stress conditions. However, this latter approach is usually not considered since it does not take into account the important effect of gravity forces. A detailed description of all these methods and derived equations is out of the scope of this work and can be found in the references provided.

The general expression for evaluating the active earth pressure acting on a circular shaft is similar to that proposed for plane strain conditions by the classical theories (Equation (4)), with the difference being the earth pressure coefficients ($K_{a\gamma}$, K_{ac} and K_{aq}), which were deduced for axisymmetric conditions.

$$p_a = K_{a\gamma} \cdot \gamma \cdot h + K_{ac} \cdot c' + K_{aq} \cdot q \quad (4)$$

In this expression, γ represents the unit weight of the soil, h the depth, c' the cohesion and q a surcharge applied at ground surface. A list of all input parameters required for each analytical solution is presented in Table 1. All methods use Mohr–Coulomb as failure criterion (although in some cases just for cohesionless soils), except for the method proposed by Xiong et al. [22], where the Drucker–Prager failure criterion was adopted, so that it could describe the axisymmetric limit problem without the need for defining a constant circumferential stress ratio. It should also be mentioned that Xiong et al. [22] also consider in their expression the soil dilation effect through the plastic potential parameter, α_ψ . The most employed analytical method was proposed by Berezantzev [16], where the Haar and von Kármán hypothesis ($\lambda = 1$) is adopted. Having as reference this method, Cheng et al. [18] and then Liu et al. [20] proposed an extension to allow for the definition of a different circumferential stress ratio, λ . Liu et al. [19] also derived a solution to include as parameters the inclination of the shaft (β), the backfill inclination (ϵ) and the soil–shaft friction angle (δ), while keeping $\lambda = 1$. As mentioned previously, the latest development made by Liu [21] was to include in the Liu et al. [20] formulation the adoption of a normalized lateral displacement (s/s_a) as an input parameter, which, through Equations (1) and (2), adjusts the ϕ'_{mob} and λ values.

In order to compare the active earth pressures estimated by the different analytical solutions, a parametric study was performed. For this purpose, a soil with a unit weight of 20 kN/m³, a friction angle of 40°, a cohesion of 0 kPa and no-dilatant ($\psi = 0^\circ$) was adopted. A levelled ground surface, without the presence of any surcharge, and a vertical shaft with a depth of 30 m and a radius of 3 m, were also assumed. For the solutions proposed by Prater [17], Cheng et al. [18] and Liu et al. [20], the active earth pressures were determined considering λ is equal to K_0 and 1, in order to assess the influence of this parameter. For the methodology proposed by Liu [21], shaft lateral displacements of 0 (corresponding to a K_0 condition), 0.05% H , 0.10% H , 0.15% H and 0.3% H (corresponding to an active limit state) were considered.

Table 1. Input parameters of the analytical methods.

Method	Shaft Geometry			Soil				Soil–Wall Friction	Surcharge	Backfill Inclination	Circumferential Stress Ratio— σ_θ/σ_v	Shaft Lateral Displacement
	Height	Radius	Inclination	Weight	Cohesion	Friction Angle	Dilation					
Terzaghi [15]	H	a	—	γ	—	ϕ'	—	—	—	—	1	—
Berezantzev [16]	H	a	—	γ	c'	ϕ'	—	—	q	—	1	—
Prater [17]	H	a	—	γ	—	ϕ'	—	—	—	—	λ	—
Cheng et al. [18]	H	a	—	γ	c'	ϕ'	—	—	q	—	λ	—
Liu et al. [19]	H	a	β	γ	c'	ϕ'	—	δ	q	ε	1	—
Liu et al. [20]	H	a	—	γ	c'	ϕ'	—	—	q	—	λ	—
Liu [21]	H	a	—	γ	—	ϕ'	—	—	q	—	—	s/s_a ¹
Xiong et al. [22]	H	a	—	γ	c'	ϕ' ²	α_ψ ³	—	q	—	—	—

¹ s_a corresponds to the shaft lateral displacement in active state condition. Liu [21] recommends the use of $0.3\%H$.

² It is also possible to define a distinct Drucker–Prager friction angle, ϕ'_e . ³ α_ψ corresponds to the Drucker–Prager model parameter that controls the plastic potential function and is directly related to the soil dilation angle, ψ .

The evolution of the normalised earth pressure ($P/\gamma a$) with the normalised depth (H/a) is presented in Figure 1. Superimposed in the figure are the represented active and at-rest earth pressures determined for the plane strain conditions using the Rankine [13] and Jaky [33] proposals. The results confirm that all axisymmetric methods estimate an active earth pressure smaller than that obtained by the plane strain theory (magenta solid line), with the difference being particularly relevant as depth increases, since in the axisymmetric case, the earth pressure tends to stabilize with depth as a result of the arching effect, while in plane strain conditions, a typical linear increase with depth is observed.

As expected, the active earth pressures calculated by Berezantzev [16], Cheng et al. [18], Liu et al. [19] and Liu et al. [20] methods (black solid line) give the exact same results when λ is equal to 1. For the same circumferential stress ratio ($\lambda = 1$), it is possible to observe that the Terzaghi [15] solution (red solid line) estimates a higher active earth pressure distribution, though with a similar trend with depth. The active earth pressures predicted by Prater [17] for $\lambda = 1$ (blue solid line) are smaller and present a different trend, initially increasing with depth and then decreasing to a value of zero. As discussed by Prater [17], the decrease observed in the active earth pressure is a consequence of the assumptions made in the derivation of the analytical solution and is considered to be unrealistic. Prater [17] suggests that the maximum value estimated for the active earth pressure should be adopted from that depth further, resulting in a trend similar to that observed in the other analytical solutions.

From the obtained results, it is possible to conclude that the stress ratio has an important impact on the prediction of the active earth pressures. For a value equal to K_0 , a significant increase in the active earth pressures was observed for both Prater [17] (blue dashed line), Cheng et al. [18] and Liu et al. [20] methods (black dashed line), with the last two predicting equal active earth pressures, as both methods have the same general formulation. These results highlight the importance of adopting an adequate circumferential stress ratio in the prediction of active earth pressures when using these analytical solutions. In contrast, in the method proposed by Xiong et al. [22] (orange solid line), no circumferential stress ratio is imposed, with the active earth pressure distribution prediction being between the results obtained for $\lambda = 1$ and $\lambda = K_0$, slightly higher than the Terzaghi [15] proposal.

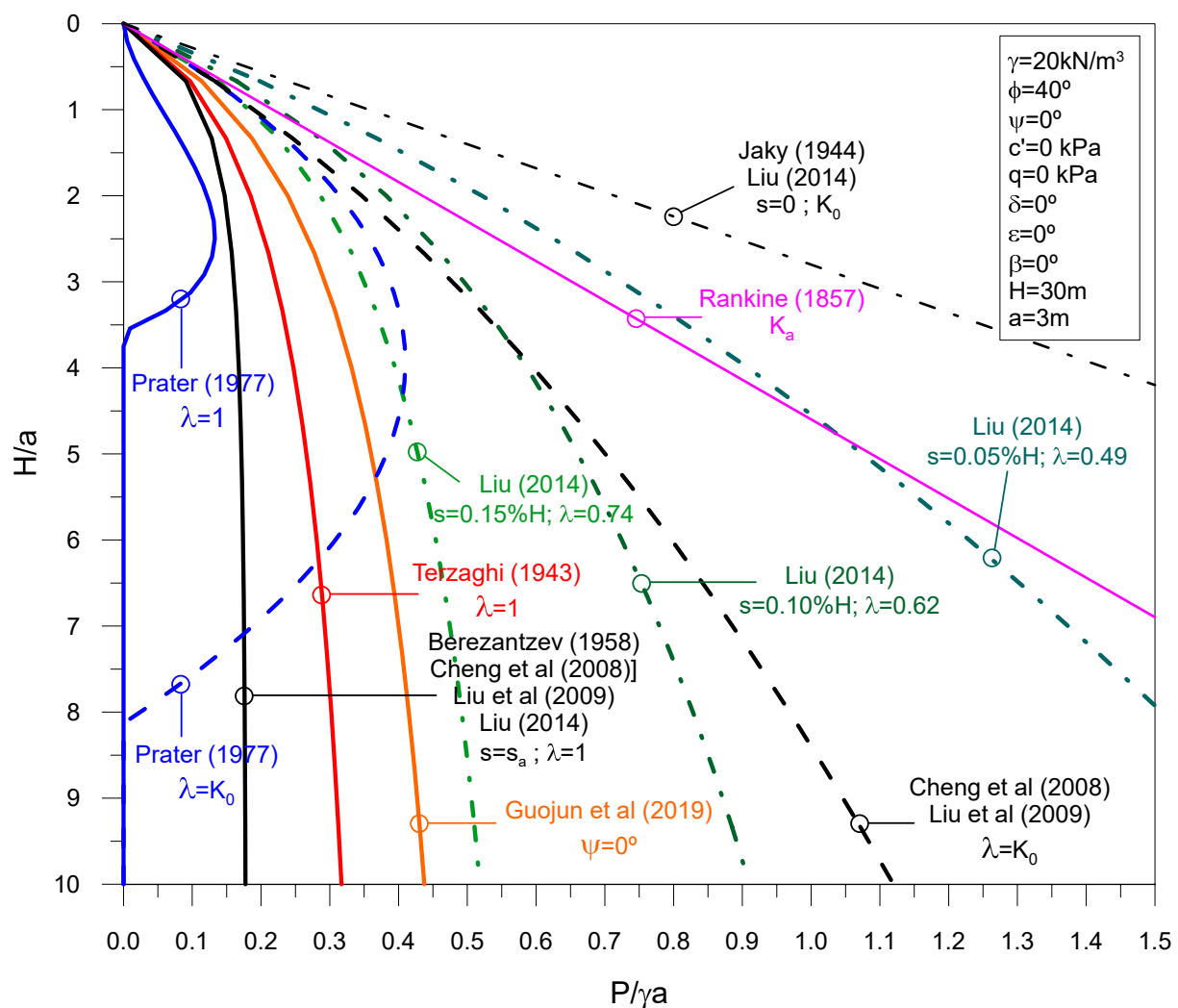


Figure 1. Comparison of the different analytical methods [13,15–22,29].

Finally, in the analyses considering different shaft lateral displacements, it is possible to observe the reduction in the earth pressure with the increase in the lateral displacement (green dashed–dot lines). For the initial condition ($s = 0$ with $\lambda = K_0$), Liu's [21] method exactly predicts the at-rest earth pressures, while for a lateral displacement equal to $0.3\%H$ ($s = s_a$), the prediction is equal to that obtained by the different slip line methods which consider $\lambda = 1$. Based on the obtained results, it is possible to verify that Liu's [21] method can effectively take into account the shaft lateral displacements in the earth pressure prediction.

2.2. Shaft Excavation Physical Models

The excavation of circular shafts has also been investigated using physical models, such as small-scale models or more elaborate centrifuge tests. These experiments primarily focused on evaluating the earth pressure distribution with depth, although in some cases, the ground deformations and associated failure mechanism were also analysed. One of the first small-scale models reported was performed by Walz [34] and involved the sinking of a circular shaft in a box filled with sand, reproducing the jacked caisson technique. By modifying the size of the cutting edge, Walz [34] created a recess between the soil and the lining and observed that the earth pressure decreased considerably with the increase in the recess, which was justified by the larger movements induced in the soil.

Chun and Shin [35] and Tobar and Meguid [36] (additional data of these tests are reported in Tran et al. [37]) also used small-scale models to assess the evolution of the earth

pressures with the increase in the shaft lateral displacement. Both studies used dense sand in the tests and a similar shaft apparatus, capable of imposing uniform lateral displacements along the shaft perimeter and depth by contracting its lining. To measure the earth pressure, several stress transducers were installed along the shaft depth. The main properties of the models are presented in Table 2, and the principal results are shown in Figures 2 and 3. The results of both studies clearly show the impact on the earth pressure caused by the increase in the shaft lateral displacement. For the initial condition ($s = 0.00\%H$), the earth pressures are well estimated in both cases by Jaky's [33] K_0 proposal.

Table 2. Physical model properties.

Reference	Chun and Shin [35]	Tobar and Meguid [36]	Hagiwara et al. [38]	Imamura et al. [39]
Type of test	Small-scale	Small-scale	Centrifuge	Centrifuge
Soil	Dense	Dense	Dense	Dense
γ (kN/m ³)	16.4	14.7	15.2	15.2
ϕ' (°)	41.6	41.0	42.0	42.0
Acceleration (g)	—	—	60	100
Model/Prototype	H (m)	H (m)	H (m)	H (m)
	0.75	1.00	30.00	50.00
	a (m)	a (m)	a (m)	a (m)
	0.175	0.750	3.600	6.000
Maximum shaft displacement	0.25% H	0.40% H	0.60% H	0.20% H

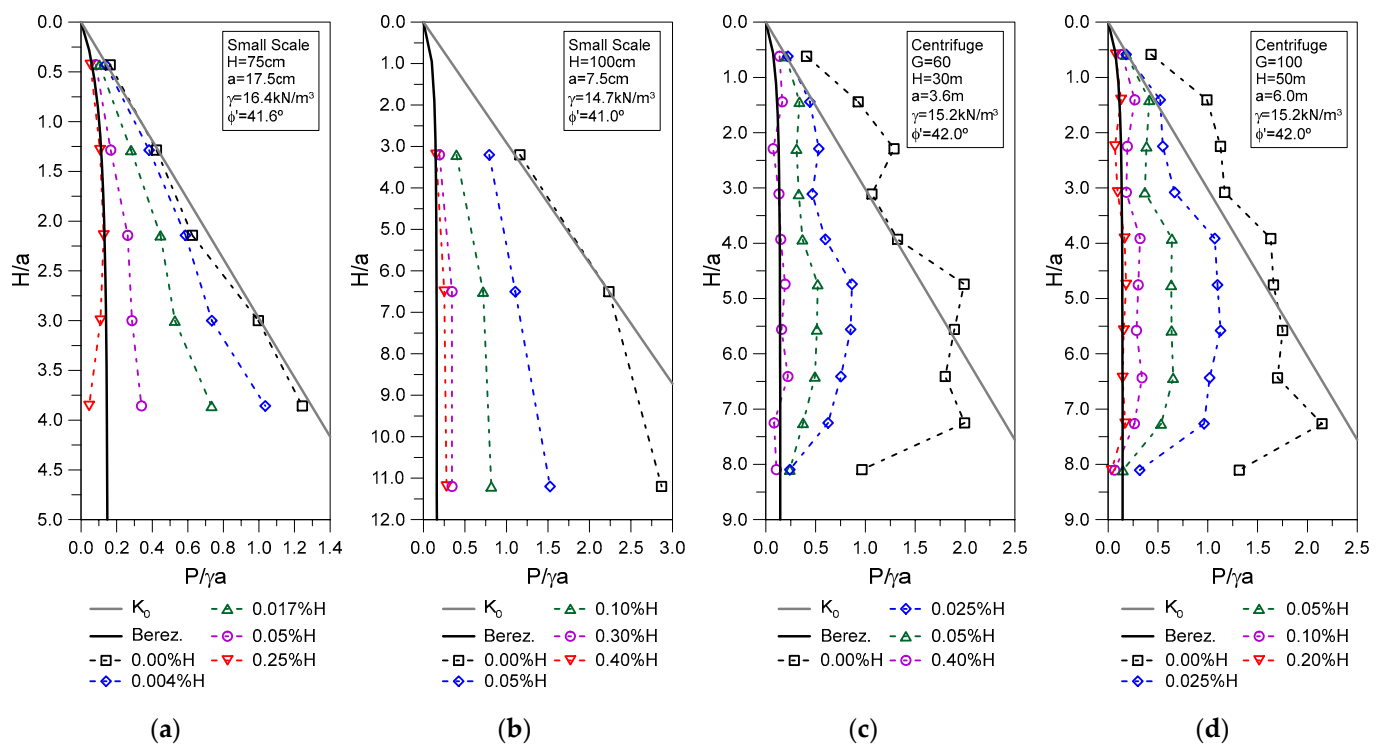


Figure 2. Earth pressure distribution with depth measured in the physical models: (a) Chun and Shin [35]; (b) Tobar and Meguid [36]; (c) Hagiwara et al. [38]; and (d) Imamura et al. [39].

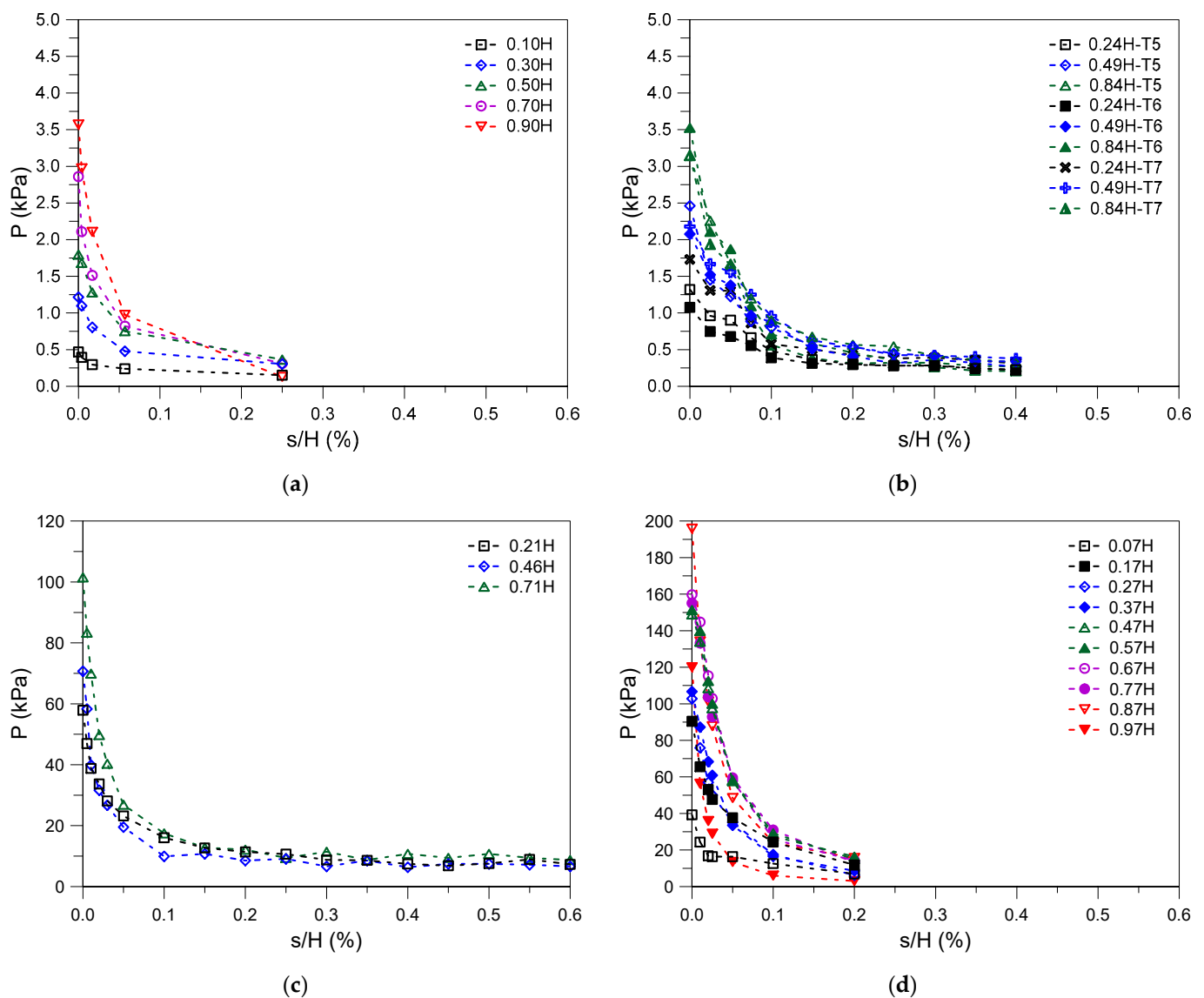


Figure 3. Reduction in the earth pressure at different depths measured in the physical models: (a) Chun and Shin [35]; (b) Tobar and Meguid [36]; (c) Hagiwara et al. [38]; and (d) Imamura et al. [39].

In Chun and Shin's [35] model, a shaft lateral displacement of just $0.017\%H$ caused a reduction in the earth pressure of about 35%. Figure 3a shows that the reduction in earth pressure is highly nonlinear, decreasing sharply until a shaft lateral displacement of $0.05\%H$ and then more gently until the imposed maximum lateral displacement of $0.25\%H$. For the maximum lateral displacement imposed, it is possible to observe in Figure 2a that the earth pressures measured are very similar to those proposed by the Berezantzev [16] solution where an active limit state is considered, implying that this condition has probably been reached in the test. In Tobar and Meguid's [36] tests, similar behaviour is observed, although for a maximum shaft lateral displacement of $0.4\%H$, the earth pressures measured do not appear to have completely stabilized and are still slightly above those estimated using the Berezantzev [16] solution, which can be considered a strange result as for that shaft lateral displacement, an active limit state should already have been reached. Nevertheless, a similar earth pressure reduction is observed in all tests (T5; T6; T7), as can be seen in Figure 3b.

The first centrifuge tests to evaluate the earth pressure induced by shaft excavation were performed by Lade et al. [40]. The study confirmed that the earth pressure reduces

with the increase in the shaft lateral displacement. However, Lade et al.'s [40] results were influenced by the mechanism used in the centrifuge to simulate the excavation and, consequently, are not considered to be entirely reliable [26]. In the 90's, several centrifuge tests to evaluate the behaviour of shaft excavation were carried out at different Japanese facilities. In all cases, the tests were performed in sand and a mechanical system similar to that employed by Chun and Shin [35] and Tobar and Meguid [36] was used to apply a contraction in the shaft lining, reducing its radius. Fujii et al. [41] observed that the increase in roughness between the soil and lining slightly decreased the earth pressure acting on the shaft. The effects of the lining stiffness and of the radius of the shaft were investigated by Ueno et al. [42]. This study observed that the earth pressure increased when stiffer linings were adopted. In contrast, the earth pressure remained almost unchanged for the different shaft radius tested, although it should be mentioned that a very stiff lining was employed in that set of tests, which might have influenced the results. The results of both studies are in agreement with the typical behaviour observed in the shafts and retaining walls. The soil–wall roughness decreases the horizontal component of the earth pressure and contributes to an increase in the overall stability, while a stiffer lining reduces the shaft lateral displacement and consequently higher pressures are expected. The centrifuge tests performed by Hagiwara et al. [38] and Imamura et al. [39] are particularly relevant since they presented high-quality results of the evolution of the earth pressure with the shaft lateral displacement for several points located along the shaft height. The characteristics of the centrifuge tests performed in both of those studies are presented in Table 2 and the principal results are plotted in Figures 2 and 3. The evolution of the earth pressure with the shaft lateral displacement is very similar in both tests and is in close agreement with the results of the small-scale models. The initial earth pressure is similar to that estimated by Jaky's [33] K_0 proposal, while the minimum earth pressure measured in the tests is in very good agreement with the Berezantzev [16] prediction, although the magnitude of the lateral displacements required for reaching the active condition might be slightly different in each study. The earth pressure reduction (Figure 3) observed in both studies is also in agreement with the results of the small-scale models, with the tests performed by Hagiwara et al. [38] confirming that an active limit state is reached around $0.3\%H$.

3. Results and Discussion

To establish a reliable method to evaluate the earth pressure as a function of the shaft settlement, it is necessary to define a credible relationship between both variables. As shown in Figure 3, a common behaviour is observed in both small-scale and centrifuge tests with the earth pressure decreasing sharply for small shaft movements and tending to become constant for large shaft lateral displacements. In Figure 4, the points depicted in Figure 3 for all four studies are replotted, normalized by the initial pressure (P_0), and grouped by depth to investigate if the earth pressure–displacement relationship was significantly impacted by the shaft depth. While a trend is visible in Figure 4, with an increase in the earth pressure reduction with the depth of the shaft, the effect can overall be considered limited, and no clear relationship can be established. To strengthen the simplicity of the proposed method, it was decided to disregard any shaft depth effect. Aspects such as the slenderness of the shaft and the stiffness and roughness of the lining were also not considered given the absence of data available to properly evaluate those effects. Naturally, these limitations should be revised in future work if additional data are obtained, exhibiting a clear dependency on any of the abovementioned factors.

Based on the results plotted in Figure 4, an expression to calculate the reduction factor of the normalized earth pressure (R_f) as a function of the normalized shaft lateral displacement ($(s/H(\%))$) was fitted. Given the data distribution, an initial sharp reduction in the earth pressure and its asymptotic value for large shaft lateral displacements, it was decided to select an exponential expression with decreasing form (Equation (5)).

$$R_f = 100 - A \cdot \left(1 - e^{-k \cdot (s/H(\%))}\right) \quad (5)$$

The proposed equation depends on two parameters, A , which is the asymptotic value of the normalized earth pressure reduction, and k , which controls the decay rate. The gathered data values of $A = 90$ and $k = 18$ can be considered adequate, as shown in Figure 4 by the black solid line. Additionally, in the figure, a conservative curve is represented by a black dashed line, with parameters $A = 80$ and $k = 12$, which can be adopted to calculate a maximum reduction factor and, consequently, estimate a conservative value of the earth pressure. With the proposed curve, a maximum earth pressure reduction of 90% is reached while the active limit state is achieved for approximately $0.25\%H$.

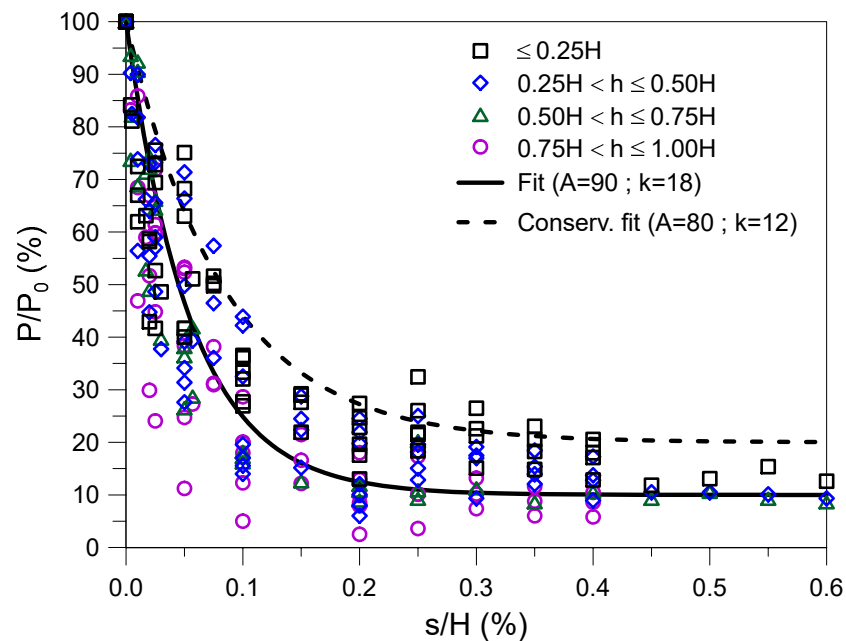


Figure 4. Reduction in the normalized earth pressure.

An established relationship between the normalized earth pressure and the normalized shaft lateral displacement of the earth pressure distribution can be determined. As seen in Figure 2, the initial stresses are usually well reproduced by Jaky's [33] K_0 proposal and the earth pressure at an active limit state is also adequately predicted by Berezantzev's [16] analytical solution. Consequently, Equation (6) is proposed to calculate the earth pressure having upper and lower limits, respectively.

$$P [kPa] = P_{a,Berez} + (P_{K_0} - P_{a,Berez}) \cdot \left(1 + \frac{R_f - 100}{A} \right)^m \quad (6)$$

In the equation, $P_{a,Berez}$ and P_{K_0} correspond to the earth pressure determined by Berezantzev [16] and Jaky's [33] proposals, respectively, which can be determined using the equations presented in Appendix A. R_f is the reduction factor calculated using Equation (5), A is the asymptotic parameter (same as adopted in Equation (5)) and m is a fitting coefficient, which is recommended to be equal to 2.

Validation of the proposed methodology was performed by comparing the predicted values against the data shown in Figure 2 for all four studies. In Table 3, the R_f calculated for all normalized shaft lateral displacements are presented. For comparison, the values of λ and ϕ_{mob} determined using Equations (1) and (2) proposed by Liu [21] are also shown.

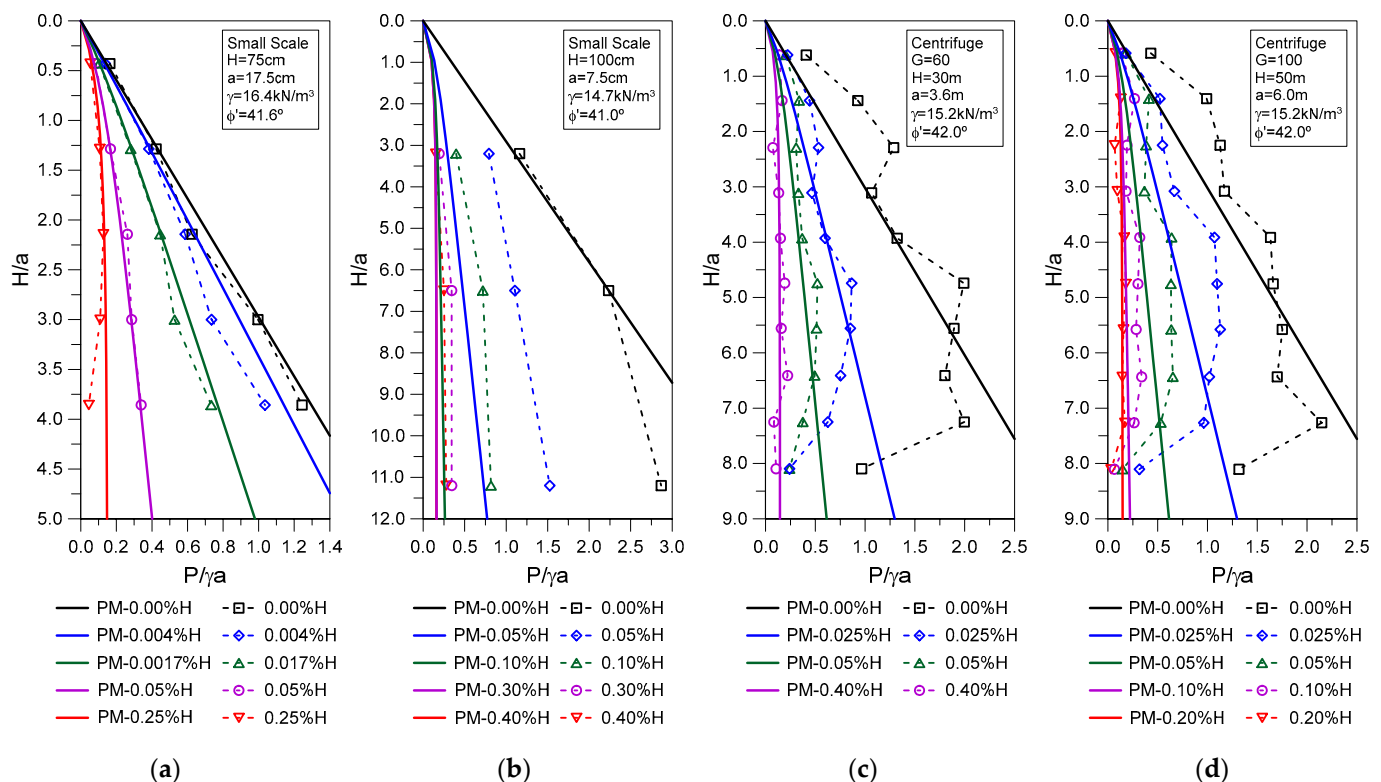
Table 3. Parameters determined for the shaft lateral displacements imposed on the physical models.

Reference		Chun and Shin [35]				Tobar and Meguid [36]				Hagiwara et al. [38]			Imamura et al. [39]			
	s/H (%)	0.004	0.017	0.05	0.25	0.05	0.1	0.3	0.4	0.025	0.05	0.4	0.025	0.05	0.1	0.2
Proposed	R_f	93.8	76.3	46.6.1	11.0	46.6	24.9	10.4	10.1	67.4	46.6	10.1	67.4	46.6	24.9	12.5
Liu [21]	λ	0.35	0.38	0.48	0.92	0.48	0.61	1.0	1.0	0.40	0.47	1.00	0.40	0.47	0.60	0.83
	ϕ'_{mob}	30.0	30.7	32.3	40.2	31.7	34.1	41.0	41.0	31.4	32.7	42.0	31.4	32.7	35.0	39.0

The predictions using the proposed methodology and Liu's [21] analytical solution are shown in Figures 5 and 6, respectively, against the results of the relevant physical models. Overall, both methods can effectively predict with reasonable agreement the evolution of the earth pressure distribution with depth for the different shaft settlements in all studies. From Figure 4, it is clear that Liu's [21] analytical solution tends to over-estimate the earth pressures, while the proposed methodology, apart from the Tobar and Meguid [36] case, where a strange behaviour was observed as discussed previously, presents a better agreement. This is confirmed by the results presented in Table 4, where the root mean squared error (RMSE) is calculated for each case and shaft settlement using the following equation:

$$RMSE [kPa] = \sqrt{\frac{\sum_{i=1}^{N_{obs}} (P_{predicted,i} - P_{observed,i})^2}{N_{obs}}} \quad (7)$$

where N_{obs} is the number of datapoints reported for each shaft lateral displacement, while $P_{predicted,i}$ and $P_{observed,i}$ are, respectively, the earth pressures determined by the analytical solutions and those observed in the physical models. Clearly, smaller values of RMSE, corresponding to a closer fit between the prediction and the observed data, are obtained for the proposed methodology, making it more suitable for predicting the earth pressure acting on circular shafts as a function of the shaft normalized lateral displacement.

**Figure 5.** Comparison of the results of the physical models with the proposed methodology: (a) Chun and Shin [35]; (b) Tobar and Meguid [36]; (c) Hagiwara et al. [38]; and (d) Imamura et al. [39].

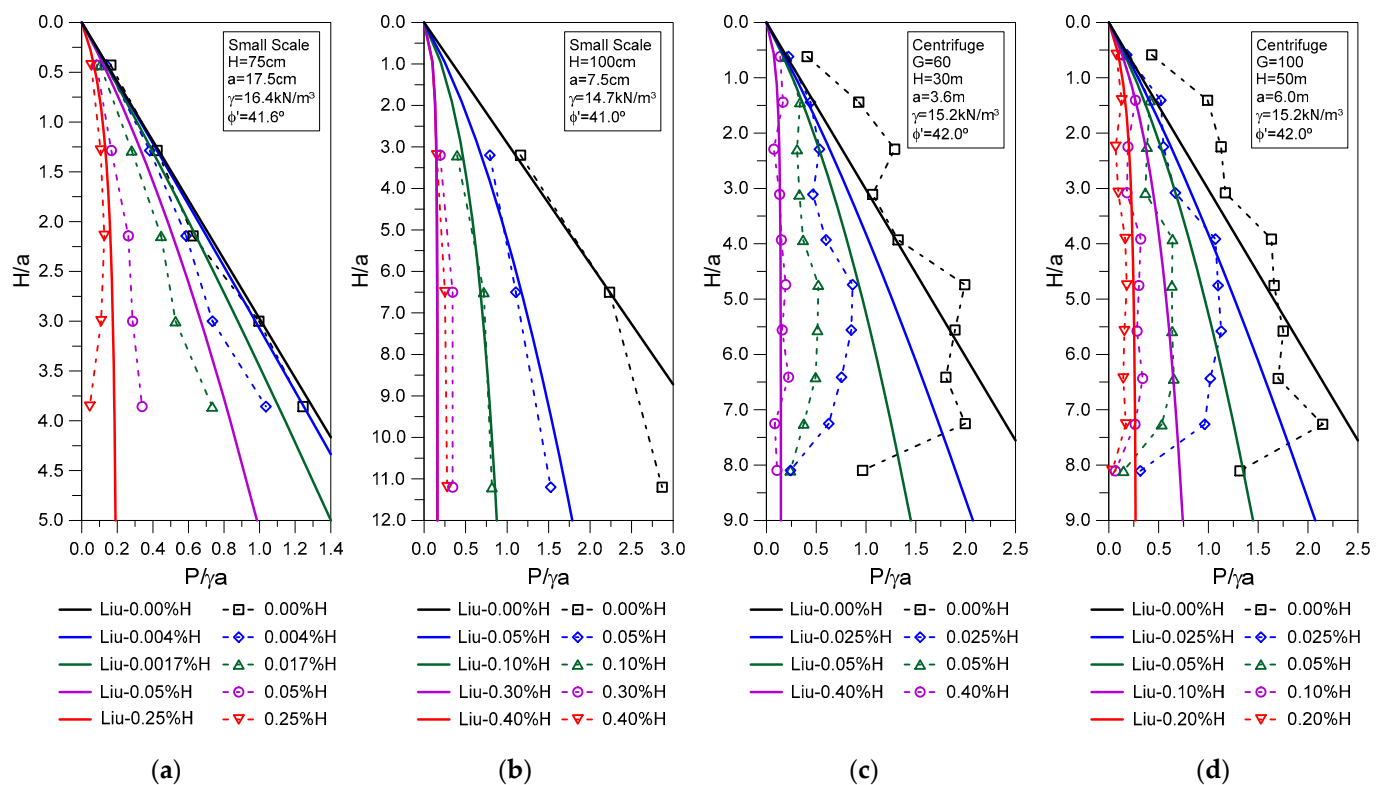


Figure 6. Comparison of the results of the physical models with the methodology proposed by Liu [21]: (a) Chun and Shin [35]; (b) Tobar and Meguid [36]; (c) Hagiwara et al. [38]; and (d) Imamura et al. [39].

Table 4. RMSE [kPa] associated with the use of the analytical methods to predict earth pressures.

Reference	Chun and Shin [35]				Tobar and Meguid [36]				Hagiwara et al. [38]			Imamura et al. [39]			
s/H (%)	0.004	0.017	0.05	0.25	0.05	0.1	0.3	0.4	0.025	0.05	0.4	0.025	0.05	0.1	0.2
Proposed	0.25	0.12	0.04	0.14	0.69	0.50	0.17	0.09	18.8	7.7	2.7	33.2	20.6	9.9	4.2
Liu [21]	0.44	0.72	0.87	0.21	0.19	0.07	0.17	0.09	40.0	31.1	2.7	54.1	46.6	28.8	10.5

4. Conclusions

The prediction of the earth pressures acting on circular shafts is a relevant topic, as shown by the significant number of studies published on the topic. However, most analytical expressions proposed in the literature present limitations, with the most relevant being the assumption of an active limit state, which requires large shaft lateral displacements to occur. As shown by field cases and numerical studies, this condition is often not verified, since the excavation methods employed, and the rigidity of the lining, considerably restrict the ground movements. As a result, the use of those analytical solutions leads to non-conservative predictions of the earth pressures, which might result in safety hazards.

In this article, a new methodology to estimate the earth pressure acting on circular shafts as a function of the lateral displacement is proposed, having as reference the results of relevant physical tests. In the method, the upper limit of the earth pressure is estimated using Jaky's [33] K_0 proposal, while the lower limit is set by adopting Berezantzev's [16] analytical solution, with the transition being made by an exponential decay equation fitted to the physical test results. The validation against the physical test results shows a good agreement with the measured data and provides a better prediction than the method proposed by Liu [21]. However, it should be noted that the proposed methodology was validated against the results of physical tests that were performed in sandy materials and where a uniform lateral deformation was imposed along the shaft height, which might be

unrealistic in real cases, particularly when SBE excavation methodology with very stiff deep linings is adopted. Nevertheless, and despite these limitations, it is believed that with the methodology proposed, it is possible to more adequately predict the earth pressure acting on circular shafts for a pre-specified shaft displacement and regardless of the shaft excavation method adopted, with the obtained results being acceptable for preliminary design stages.

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Appendix A

The active earth pressure acting on a cylindrical shaft according to Berezantzev's [16] analytical solution ($P_{a,Berez}$) can be determined using Equation (1). The earth pressure coefficients for self-weight ($K_{a\gamma}$), cohesion (K_{ac}) and surcharge (K_{aq}) can be calculated using Equations (A1)–(A3), respectively.

$$K_{a\gamma} = \frac{\tan\left(45 - \frac{\phi'}{2}\right)}{\eta - 1} \cdot \left(\frac{a}{h} - \frac{a}{h \cdot r_b^{(\eta-1)}}\right) \quad (\text{A1})$$

$$K_{ac} = \left(1 - \frac{1}{r_b^\eta} \cdot \tan^2\left(45 - \frac{\phi'}{2}\right)\right) \cdot \frac{1}{\tan(\phi')} \quad (\text{A2})$$

$$K_{aq} = \frac{1}{r_b^\eta} \cdot \tan^2\left(45 - \frac{\phi'}{2}\right) \quad (\text{A3})$$

where η and r_b are given by

$$\eta = \tan^2(45 + \phi'/2) - 1 \quad (\text{A4})$$

$$r_b = 1 + \frac{h}{a} \cdot \tan\left(45 - \frac{\phi'}{2}\right) \quad (\text{A5})$$

The earth pressure at-rest (P_{K_0}) at a depth of h can be determined using Equation (A6), where K_0 is calculated using Jaky's [33] equation ($K_0 = 1 - \sin\phi'$).

$$P_{K_0} = \gamma \cdot h \cdot K_0 \quad (\text{A6})$$

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