

Article

Thermoporoelastoplastic Wellbore Breakout Modeling by Finite Element Method

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Abstract: Drilling a hole into rock results in stress concentration and redistribution close to the hole. When induced stresses exceed the rock strength, wellbore breakouts will happen. Research on wellbore breakout is the fundamental of wellbore stability. A wellbore breakout is a sequence of stress concentrations, rock falling, and stress redistributions, which involve initiation, propagation, and stabilization sequences. Therefore, simulating the process of a breakout is very challenging. Thermoporoelastoplastic models for wellbore breakout analysis are rare due to the high complexity of the problem. In this paper, a fully coupled thermoporoelastoplastic finite element model is built to study the mechanism of wellbore breakouts. The process of wellbore breakouts, the influence of temperature and the comparison between thermoporoelastic and thermoporoelastoplastic models are studied in the paper. For the finite element modeling, the D-P criterion is used to determine whether rock starts to yield or not, and the maximum tensile strain criterion is used to determine whether breakouts have happened.

Keywords: wellbore breakout; finite element method; Drucker-Prager criterion; the maximum tensile strain criterion; thermoporoelasticity; thermoporoelastoplasticity



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1. Introduction

When a hole is drilled into the crust, the rock mass removed from the hole will not support the surrounding rocks, which leads to the stress concentration and redistribution. If the induced stresses exceed the rock strength, parts of rock fall from the wellbore wall, which is called a wellbore breakout [1,2]. Wellbore breakouts result in problematic wellbore stability and affects drilling efficiency, so the research of wellbore breakouts is important in petroleum engineering [3,4].

Numerous studies have been made to explain the mechanism of wellbore breakouts. Zoback found the analytical solution of initial breakout zone by incorporating the Mohr-Coulomb criterion to the Kirsch equation [2]. The studies of wellbore breakouts show that the wellbore breakouts occur by a series of successive spalls that result from shear failure subparallel to the direction of the local minimum principal stress [3,5–9]. In addition, some studies also show that a wellbore breakout can also influence the wellbore breakdown pressure [10]. To consider the effect of pore pressure and obtain effective stress distribution in a porous medium, Biot [11] proposed a linear poroelastic model based on Terzaghi's theory and introduced Biot's coefficient. Later, Fluid seepage and pressure diffusion between the wellbore and the formation were considered for both vertical and inclined wells [12–14]. Recently, numerical methods were also used to analyze the breakout mechanisms [15–19].

Most of these models do not consider the influence of plastic damage of the rock. For the plastic model, different yield criteria are used to analyze the character of rock and soil. The Mohr-Coulomb criterion is the most popular one, which relates the shearing resistance to the contact forces and friction [20]. The Drucker-Prager criterion is an extended version of the Von Mises criterion [21]. Whittle and Kavvas [22] presented the MIT-E3 soil model to describing the behavior of overconsolidated clays that obey normalized behavior and are rate-independent. Akl and Whittle [23] analyzed horizontal wellbore stability in clay shale based on MIT-E3 soil model. Zhang and Yin [24] made a wellbore study using the tangent stiffness matrix method based on the Drucker-Prager criterion.

However, in reality, a wellbore breakout is a complex time-dependent process including initiation, propagation, and stabilization sequences [25]. The breakout initiation and its propagation are the result of stress concentration on the wellbore wall, further stress concentration at the tip of breakout, and the formation of a plastic zone around the tip [26]. Poroelastoplastic analysis of wellbore breakouts is still not well studied due to the complexity of the problem.

With the development of a deep well, consideration of the thermal effects is essential. Coupled thermal-hydraulic-mechanical processes play an important role in the stability of wells in thermal reservoir formations [27]. Lewis [28] used FE simulation to study thermal recovery processes and heat losses problems to surrounding strata. Aboustit et al. [29] used a general variational principle to investigate thermo-elastic consolidation problems, and Vaziri [30] also presented a fully coupled thermo-hydro-mechanical FE model.

In this paper, a fully coupled thermoporoelastoplastic finite element model is built to study the mechanism of wellbore breakouts. The Drucker-Prager criterion is used to determine whether rock starts to yield or not, and the maximum tensile strain criterion is used to determine whether breakouts happened.

2. Model Structure and Methodology

To simulate wellbore breakouts in plastic rock, the Drucker-Prager yield criterion and maximum tensile strain criterion are used to analyze the yield and failure of the rock. The finite element implementation of thermoporoelasticity is based on Biot's theory with the compressible fluid flowing through the saturated non-isothermal deformable porous medium.

2.1. Rock Failure Criteria

In this paper, the Drucker-Prager yield criterion and maximum tensile strain criterion are used to analyze the yield and failure of the rock. Figure 1 shows a simplified sketch for the relationship between stress and strain in the condition of yield and failure of the rock, where σ_{yield} and ε_{yield} are the yield stress and strain of rock, $\varepsilon_{breakout}$ is the maximum allowable tensile strain of rock.

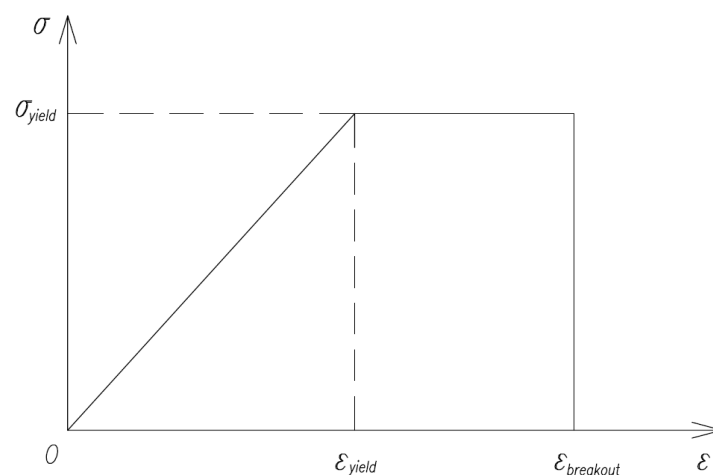


Figure 1. Sketch of the stress-strain relationship.

The Drucker-Prager yield criterion is an extended version of the Von Mises criterion. To obtain a smooth yield surface approximate to Mohr-Coulomb surface, Drucker and Prager [21] put forward the following yield criterion:

$$F = \alpha I_1 + \sqrt{J_2} - k_0 \quad (1)$$

where F is the yield criterion, I_1 is the first stress invariant, J_2 is the second deviatoric stress invariant, and α, k_0 are the material constants.

Two of the most common approximations used are obtained by making the yield surfaces of the Drucker-Prager and Mohr-Coulomb criteria coincident either at the outer or inner edges of the Mohr-Coulomb surface. Coincidence at the outer edges is obtained when

$$\alpha = \frac{2\sin\varphi}{\sqrt{3}(3 - \sin\varphi)}, k_0 = \frac{6c\cos\varphi}{\sqrt{3}(3 - \sin\varphi)} \quad (2)$$

whereas, coincidence at the inner edges is given by the choice

$$\alpha = \frac{2\sin\varphi}{\sqrt{3}(3 + \sin\varphi)}, k_0 = \frac{6c\cos\varphi}{\sqrt{3}(3 + \sin\varphi)} \quad (3)$$

where c is the cohesion of the material; φ is the internal friction angle of the material.

The yield condition of rock can be determined by the following equation.

$$\begin{cases} F \leq 0 & \text{no yield} \\ F > 0 & \text{yield} \end{cases} \quad (4)$$

The failure (*breakout*) condition of rock can be determined by the following equation.

$$\begin{cases} \varepsilon_{\max} \leq \varepsilon_{\text{breakout}} & \text{no breakout} \\ \varepsilon_{\max} > \varepsilon_{\text{breakout}} & \text{breakout} \end{cases} \quad (5)$$

2.2. Finite Element Implementation

The general theory of thermoporoelasticity is based on Biot's theory. With the compressible fluid flowing through the saturated non-isothermal deformable porous medium, in the form of displacements, pressure and temperature as unknowns, the governing equations can be described as follows [31].

$$G\nabla^2\mathbf{u} + (G + \lambda)\nabla\text{div}\mathbf{u} - \left(1 - \frac{K}{K_m}\right)\nabla p - K\beta_s\nabla T + \mathbf{f}^u = 0 \quad (6)$$

$$\left(1 - \frac{K}{K_m}\right)\text{div}\mathbf{u}_t + \left[\frac{1-\phi}{K_m} + \frac{\phi}{K_f} + \frac{1}{(3K_m)^2}i^T D i\right]\mathbf{p}_t + \frac{k}{\mu}\nabla^2 p - \left[\phi\beta_f + (1-\phi)\beta_s + \frac{\beta_s}{9K_m}i^T D i\right]\mathbf{T}_t + \mathbf{f}^p = 0 \quad (7)$$

$$T\left[(1-\phi)\frac{\rho_s c_s}{K_m} + \phi\frac{\rho_f c_f}{K_f}\right]\mathbf{p}_t + \left[(1-\phi)\rho_s c_s + \phi\rho_f c_f - T(1-\phi)\rho_s c_s\beta_s - T\phi\rho_f c_f\beta_f\right]\mathbf{T}_t + \lambda_T\nabla^2 T + \mathbf{f}^T = 0 \quad (8)$$

where K , K_f and K_m are the bulk moduli of the skeleton, fluid and matrix, respectively. k is the permeability of the porous medium, μ is the viscosity of the fluid, G and λ are Lamé constants. $\mathbf{u}$, p and $\mathbf{p}_t$ denote the displacement of the porous medium, the pore pressure and its time derivative, ϕ is the porosity of the porous medium, λ_T is the thermal conductivity matrix of the porous media, T and $\mathbf{T}_t$ are the temperature and time derivative, $\rho_s c_s$ is the heat capacity of the solid phase, $\rho_f c_f$ is the heat capacity of the fluid phase, β_s is the thermal expansion coefficient of the matrix, and β_f is the thermal expansion coefficient of the fluid. Furthermore, $i^T = [1, 1, 1, 0, 0, 0]$, and D is the elastic stiffness matrix, the subscript t denotes time derivative.

Using Galerkin finite element method to approximate the governing equations, the final form of the FEM solution is as follows.

$$\begin{bmatrix} M & -C_{sw} & -C_{sT} \\ 0 & H_{ww} & 0 \\ 0 & 0 & H_{TT} \end{bmatrix} \begin{Bmatrix} u \\ p \\ T \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ C_{sw}^T & R_{ww} & -C_{wT} \\ 0 & C_{Tw} & R_{TT} \end{bmatrix} \begin{Bmatrix} u_t \\ p_t \\ T_t \end{Bmatrix} = \begin{Bmatrix} f^u \\ f^p \\ f^T \end{Bmatrix} \quad (9)$$

where $[u, p, T]^T$, $[u_t, p_t, T_t]^T$ are the vectors of unknown variables and corresponding time derivatives, and $[f^u, f^p, f^T]^T$ is the vector for the nodal loads, flow source and heat sources. The explicit expressions of above matrixes are as follows.

$$M = \int B^T D B d\Omega \quad (10)$$

$$H_{ww} = \frac{k}{\mu} \int (\nabla N)(\nabla N)^T d\Omega \quad (11)$$

$$R_{ww} = \int N \left[\frac{1-\phi}{K_m} + \frac{\phi}{K_f} + \frac{1}{(3K_m)^2} i^T D i \right] N^T d\Omega \quad (12)$$

$$C_{sw} = \int \left(B^T i N - B^T D \frac{i}{3K_m} N \right) d\Omega \quad (13)$$

$$C_{sT} = \int B^T D i \frac{\beta_s}{3} N d\Omega \quad (14)$$

$$C_{wT} = \int N \left[(1-\phi)\beta_s - \phi\beta_f + \frac{\beta_s}{9K_m} i^T D i \right] N^T d\Omega \quad (15)$$

$$C_{Tw} = \int N \left[(1-\phi)\frac{\rho_s c_s}{K_m} - \phi\frac{\rho_f c_f}{K_f} \right] N^T N^T d\Omega \quad (16)$$

$$H_{TT} = \lambda_T \int (\nabla N)(\nabla N)^T d\Omega \quad (17)$$

$$R_{TT} = \int \left[(1-\phi)\rho_s c_s - \phi\rho_f c_f - (1-\phi)\rho_s c_s \beta_s N^T - \phi\rho_f c_f \beta_f N^T \right] N^T d\Omega \quad (18)$$

To integrate the above equations with respect to time (θ method), the equation becomes:

$$\begin{bmatrix} M & -C_{sw} & -C_{sT} \\ C_{sw}^T & R_{ww} + \Delta t \theta H_{ww} & -C_{wT} \\ 0 & C_{Tw} & R_{TT} + \Delta t \theta H_{TT} \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta p \\ \Delta T \end{Bmatrix} = \begin{Bmatrix} \Delta f^u \\ \Delta t \theta f_1^p + \Delta t(1-\theta)f_0^p - \Delta t H_{ww} p_0 \\ \Delta t \theta f_1^T + \Delta t(1-\theta)f_0^T - \Delta t H_{TT} T_0 \end{Bmatrix} \quad (19)$$

where $\begin{Bmatrix} \Delta u \\ \Delta p \end{Bmatrix}$ is the increment of displacement, pore pressure, $\begin{Bmatrix} \Delta f^u \\ \Delta f^p \end{Bmatrix}$ is the increment vector for the nodal loads and flow source, Δt is the increment of time, θ is the time integration coefficient, f_0^p is the initial vector for the nodal loads in a time step, and p_0 is the initial vector for the pore pressure in a time step.

If the stresses are in the plastic state at a special time t_a based on the yield criterion, Equation (19) becomes Equation (20) at the special time t_a [32].

$$\begin{bmatrix} M & -C_{sw} & -C_{sT} \\ C_{sw}^T & R_{ww} & -C_{wT} \\ 0 & C_{Tw} & R_{TT} \end{bmatrix}_a \begin{Bmatrix} \Delta u \\ \Delta p \\ \Delta T \end{Bmatrix}_a = \begin{Bmatrix} \Psi_n \\ 0 \\ 0 \end{Bmatrix} \quad (20)$$

$$\Psi_n = \Sigma_e \int B^T D^p \Delta \epsilon d\Omega \quad (21)$$

$$D^p = \frac{D \left\{ \frac{\partial Q}{\partial \sigma} \right\} \left\{ \frac{\partial F}{\partial \sigma} \right\}^T D}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T D \left\{ \frac{\partial Q}{\partial \sigma} \right\} - A} \quad (22)$$

where Σ_e is the set of element nodal forces round the node, and Ψ_n is the unbalanced force. For an ideal elastoplastic body, $A = 0$.

In the process of iteration, the convergence condition can be described as:

$$\text{norm}(\Psi_n) < V \quad (23)$$

where $\text{norm}(\Psi_n)$ is the norm of vector Ψ_n , and V is the convergence precision.

For different iterative methods, the global stiffness matrixes are different, which are shown in Figure 2. For the constant stiffness method, the values in Equation (20) can be calculated based on Equations (10)–(18). For the tangent stiffness method, the explicit expressions of the values in Equation (20) are as follows.

$$M|_{t=t_a} = \int B^T D^{ep} B d\Omega \quad (24)$$

$$R_{ww}|_{t=t_a} = \int N \left[\frac{1-\phi}{K_m} + \frac{\phi}{K_f} + \frac{1}{(3K_m)^2} i^T D^{ep} i \right] N^T d\Omega \quad (25)$$

$$R_{TT}|_{t=t_a} = R_{TT} \quad (26)$$

$$C_{sw}|_{t=t_a} = \int \left(B^T i N - B^T D^{ep} \frac{i}{3K_m} N \right) d\Omega \quad (27)$$

$$C_{sT}|_{t=t_a} = \int B^T D^{ep} i \frac{\beta_s}{3} N d\Omega \quad (28)$$

$$C_{wT}|_{t=t_a} = \int N \left[(1-\phi)\beta_s - \phi\beta_f + \frac{\beta_s}{9K_m} i^T D^{ep} i \right] N^T d\Omega \quad (29)$$

$$C_{Tw}|_{t=t_a} = C_{Tw} \quad (30)$$

$$D^{ep} = D - D^p = D - \frac{D \left\{ \frac{\partial Q}{\partial \sigma} \right\} \left\{ \frac{\partial F}{\partial \sigma} \right\}^T D}{\left\{ \frac{\partial F}{\partial \sigma} \right\}^T D \left\{ \frac{\partial Q}{\partial \sigma} \right\} - A} \quad (31)$$

where D^{ep} is elastic-plastic matrix [32,33]. For an ideal elastoplastic body, $A = 0$.

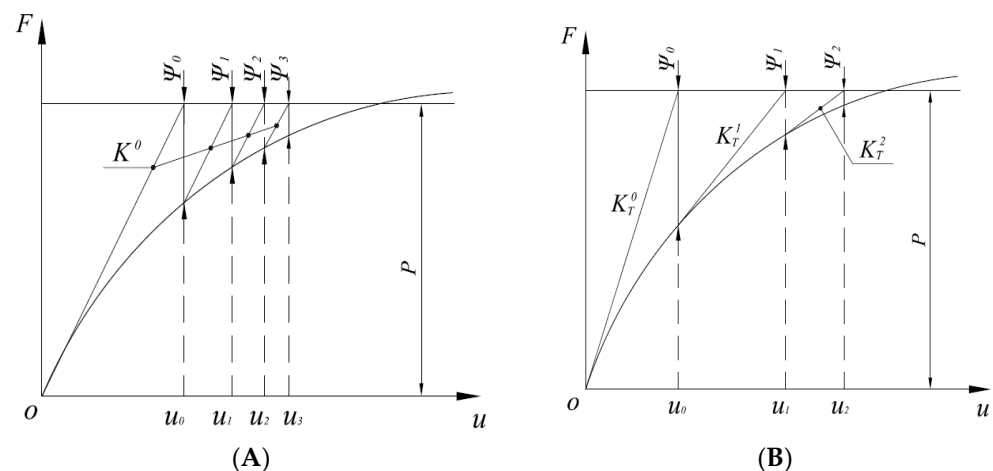


Figure 2. (A) Constant stiffness method, (B) Tangent stiffness method.

The tangent stiffness method is adopted in this paper.

3. Numerical Experiments

Thermoporoelastoplastic wellbore breakouts are simulated for a vertical well, and there are four parts in this section: the relationship between breakout shape and in situ stresses, verification of wellbore breakout process, influence of drilling fluid temperature, and a comparison between thermoporoelastic and thermoporoelastoplastic modeling.

3.1. Thermoporoelastoplastic Finite Element Modeling of Wellbore Breakouts

For a vertical wellbore shown in Figure 3 that is subjected to horizontal in-situ stresses σ_H and σ_h , the shape of breakouts ϕ_b and r_b can be acquired by finite element modeling, where r_b is the depth of breakouts, and ϕ_b is the width of breakouts (Figure 4).

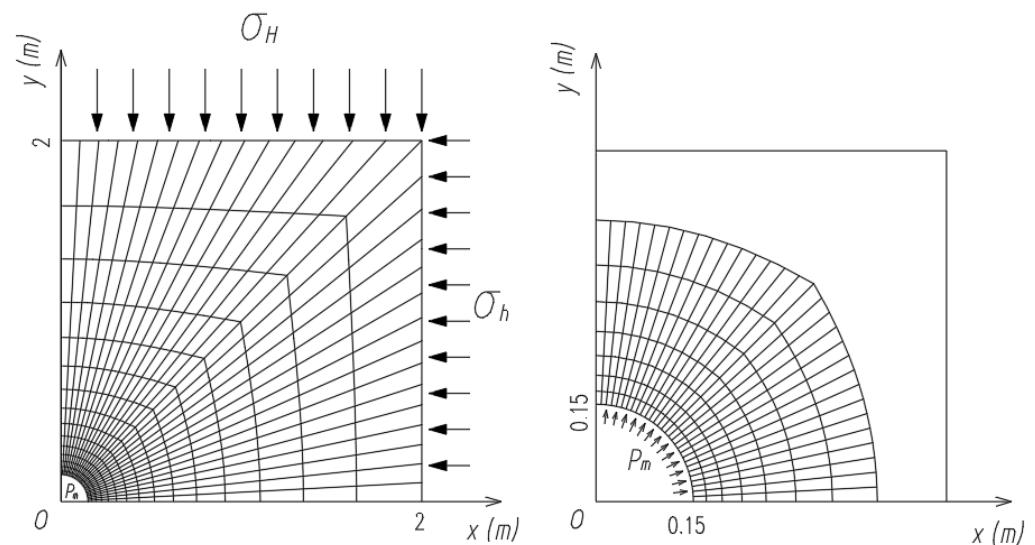


Figure 3. Mesh of finite element model.

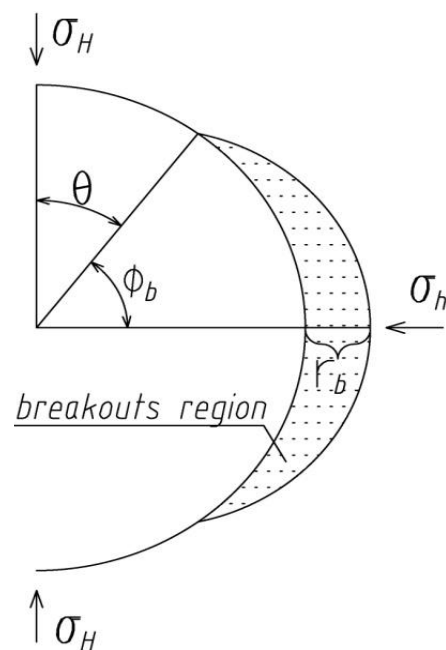


Figure 4. Schematic of a wellbore breakout shape.

The yield condition of rock is determined by Equation (4), and the failure (breakout) condition of rock is determined by Equation (5).

The geometric and mechanical parameters are shown in Table 1.

Table 1. Geometric and mechanical parameters.

Parameter	Value
Young's Modulus, E (MPa)	14,400
Poisson Ratio, ν	0.2
Cohesion, c (MPa)	6
Inner friction angle, φ ($^{\circ}$)	35
Radius of well, R (m)	0.15
Bulk modulus of skeleton, K (MPa)	8000
Bulk modulus of matrix, K_m (MPa)	36,000
Bulk modulus of fluid, K_f (MPa)	2250
Porosity of the porous medium, ϕ	0.19
Permeability of the porous medium, k (μm^2)	0.19
Viscosity of the fluid, μ (MPa s)	10^{-9}
Thermal conductivity, λ_T ($\text{Wm}^{-1}\text{K}^{-1}$)	2.5
Thermal expansion coefficient of solid, β_s (K^{-1})	2.1×10^{-5}
Thermal expansion coefficient of liquid, β_f (K^{-1})	2.0×10^{-4}
Specific heat of solid, c_s ($\text{J kg}^{-1}\text{K}^{-1}$)	800
Specific heat of liquid, c_f ($\text{J kg}^{-1}\text{K}^{-1}$)	4200
Maximum principal stress, σ_H (MPa)	85~92.5
Minimum principal stress, σ_h (MPa)	60~72.5
Vertical stress, σ_V (MPa)	80
Drilling fluid pressure, P_m (MPa)	45
Temperature difference between drilling fluid and surrounding, T (K)	−50
Maximum allowable tensile strain, $\varepsilon_{breakout}$	0.002

The results of the breakout shape are shown in Table 2. From Table 2, Figures 5 and 6, it can be seen that the relationship between breakout shape and in situ stresses is nonlinear. With relatively large σ_H and difference between σ_H and σ_h , as σ_h increases, the breakout width and depth increase. With relatively small σ_H and difference between σ_H and σ_h , as σ_h increases, the difference between σ_H and σ_h is smaller, the in situ stress is closer to symmetric in situ stresses state, and the breakout width and depth decrease.

Table 2. Result of the shape of wellbore breakouts r_b and ϕ_b by FEM.

No.	σ_h MPa	σ_H MPa	ϕ_b $^{\circ}$	r_b mm	No.	σ_h MPa	σ_H MPa	ϕ_b $^{\circ}$	r_b mm
1	85.0	60.0	23.3	53.5	13	90.0	67.5	33.2	112.7
2	85.0	62.5	22.9	51.2	14	90.0	70.0	33.3	115.1
3	85.0	65.0	22.1	48.3	15	90.0	72.5	33.7	118.8
4	85.0	67.5	21.2	44.1	16	90.0	60.0	34.3	120.9
5	85.0	70.0	20.4	40.3	17	90.0	62.5	35.0	124.0
6	85.0	72.5	19.3	33.4	18	90.0	65.0	35.8	129.6
7	87.5	60.0	29.7	90.6	19	92.5	67.5	36.2	132.2
8	87.5	62.5	29.8	90.1	20	92.5	70.0	36.4	136.1
9	87.5	65.0	29.9	89.3	21	92.5	72.5	36.9	139.8
10	87.5	67.5	30.0	88.9	22	92.5	60.0	37.5	144.6
11	87.5	70.0	30.2	88.2	23	92.5	62.5	38.2	149.3
12	87.5	72.5	30.4	85.2	24	92.5	65.0	39.3	154.6

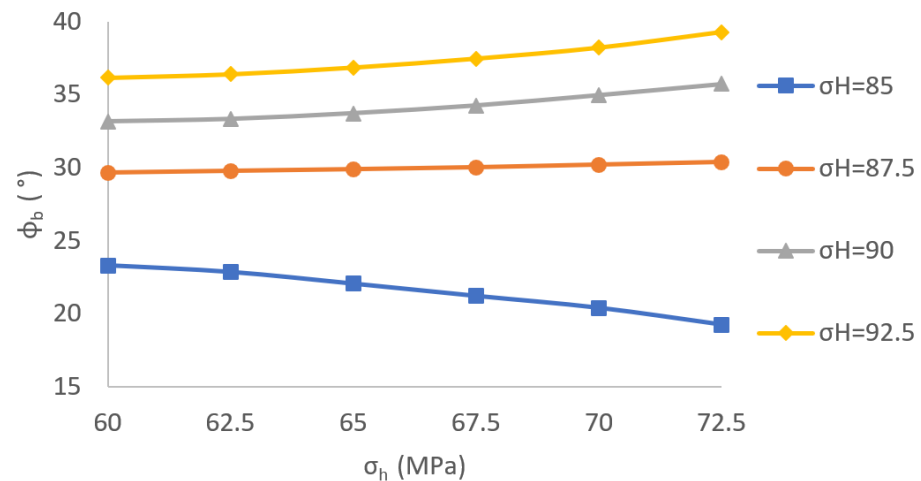


Figure 5. Breakout width for different in situ stresses.

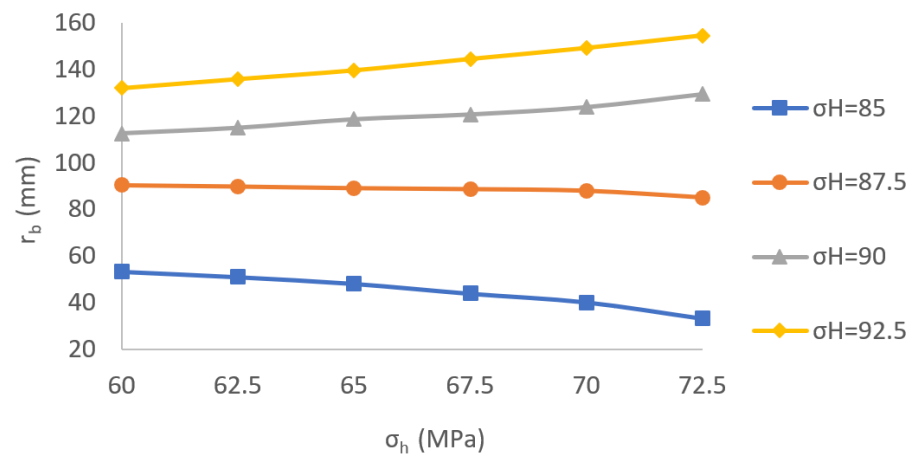


Figure 6. Breakout depth for different in situ stresses.

3.2. Verification of Wellbore Breakout Process

For a vertical well shown in Figure 5, associated with horizontal in situ stresses $\sigma_H = 87.5$ MPa, $\sigma_h = 72.5$ MPa and other parameters are listed in Table 1, the process of well breakout is shown in Table 3 and Figures 7–9.

Table 3. Data of effective principal stresses and breakout region in the process of breakouts.

Elem	Iteration1				Iteration2				Iteration3			
	$\sigma_{1'}$ (MPa)	$\sigma_{2'}$ (MPa)	$\sigma_{3'}$ (MPa)	ε_{max}	$\sigma_{1'}$ (MPa)	$\sigma_{2'}$ (MPa)	$\sigma_{3'}$ (MPa)	ε_{max}	$\sigma_{1'}$ (MPa)	$\sigma_{2'}$ (MPa)	$\sigma_{3'}$ (MPa)	ε_{max}
1	−19.64	−87.86	−54.31	0.00294	−20.56	−91.03	−55.97	0.00315	−28.26	−113.46	−66.27	0.00397
2	−19.86	−87.44	−54.21	0.00292	−20.79	−90.46	−55.86	0.00314	−28.98	−111.17	−65.55	0.00387
3	−20.29	−86.59	−54.00	0.00288	−21.28	−89.41	−55.64	0.00311	−30.35	−106.72	−64.05	0.00362
4	−20.93	−85.35	−53.70	0.00282	−21.97	−87.94	−55.30	0.00305	−31.98	−99.88	−61.76	0.00328
5	−21.73	−83.74	−53.31	0.00275	−23.08	−86.13	−54.89	0.00295	−33.25	−90.66	−58.60	0.00290
6	−22.67	−81.78	−52.84	0.00266	−24.60	−84.05	−54.28	0.00277	−33.25	−79.86	−54.76	0.00236
7	−23.71	−79.54	−52.31	0.00257	−26.56	−81.64	−53.33	0.00248	−29.37	−64.73	−48.08	0.00168
8	−24.85	−77.08	−51.74	0.00246	−27.60	−78.51	−51.49	0.00211	−25.69	−53.71	−43.49	0.00133
9	−26.13	−74.51	−51.08	0.00231	−27.75	−73.95	−50.08	0.00191	−23.57	−49.53	−40.89	0.00094
10	−27.67	−71.91	−50.22	0.00205	−27.14	−66.88	−48.84	0.00199	−23.10	−47.39	−40.83	0.00091

Table 3. Cont.

Elem	Iteration4				Iteration5				Iteration6			
	$\sigma_{1'}$ (MPa)	$\sigma_{2'}$ (MPa)	$\sigma_{3'}$ (MPa)	ε_{max}	$\sigma_{1'}$ (MPa)	$\sigma_{2'}$ (MPa)	$\sigma_{3'}$ (MPa)	ε_{max}	$\sigma_{1'}$ (MPa)	$\sigma_{2'}$ (MPa)	$\sigma_{3'}$ (MPa)	ε_{max}
1	−44.48	−142.16	−85.00	0.00501	−73.96	−174.26	−101.91	0.00270	−89.52	−181.38	−111.19	0.00196
2	−44.71	−128.38	−76.66	0.00523	−64.49	−137.95	−83.14	0.00305	−58.62	−116.89	−72.32	0.00194
3	−42.72	−108.77	−70.07	0.00433	−39.34	−69.62	−54.19	0.00185	−25.22	−50.52	−42.48	0.00083
4	−40.12	−87.33	−61.17	0.00323	−17.79	−38.71	−37.73	0.00080	−15.15	−29.09	−35.71	−0.00004
5	−32.66	−62.74	−49.07	0.00195	−17.15	−29.80	−36.13	−0.00007	−15.68	−26.52	−35.48	−0.00014
6	−23.86	−45.66	−40.42	0.00097	−16.17	−27.53	−35.79	−0.00006	−15.10	−24.65	−34.91	−0.00012
7	−20.75	−38.00	−37.25	0.00014	−15.95	−25.85	−35.30	−0.00008	−15.20	−23.19	−34.55	−0.00014
8	−19.36	−36.17	−37.26	0.00020	−16.14	−25.73	−35.25	−0.00006	−15.57	−23.52	−34.70	−0.00013
9	−19.50	−36.24	−37.52	0.00019	−16.71	−26.65	−35.51	−0.00006	−16.02	−24.40	−34.95	−0.00012
10	−19.25	−36.35	−37.60	0.00020	−16.87	−27.24	−35.63	−0.00004	−16.25	−25.08	−35.11	−0.00010

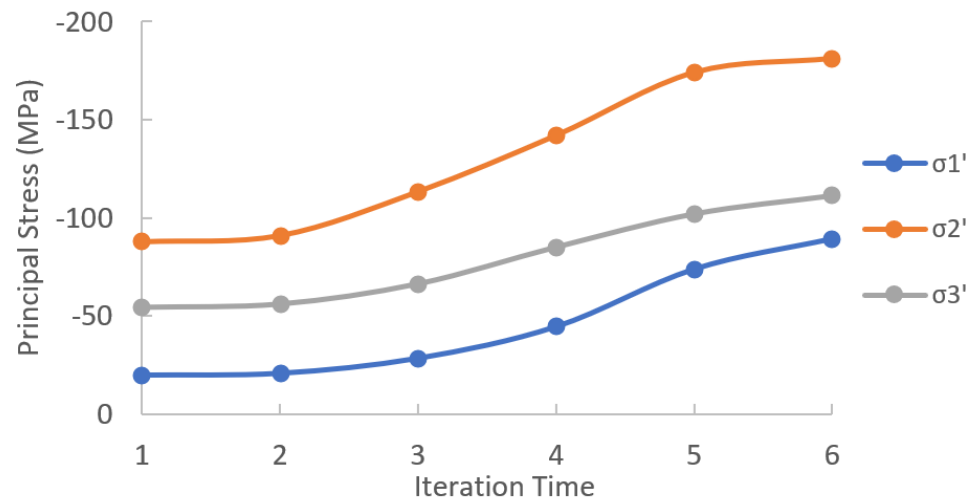


Figure 7. Principal stresses for the element in the breakout tip in the process of breakouts.

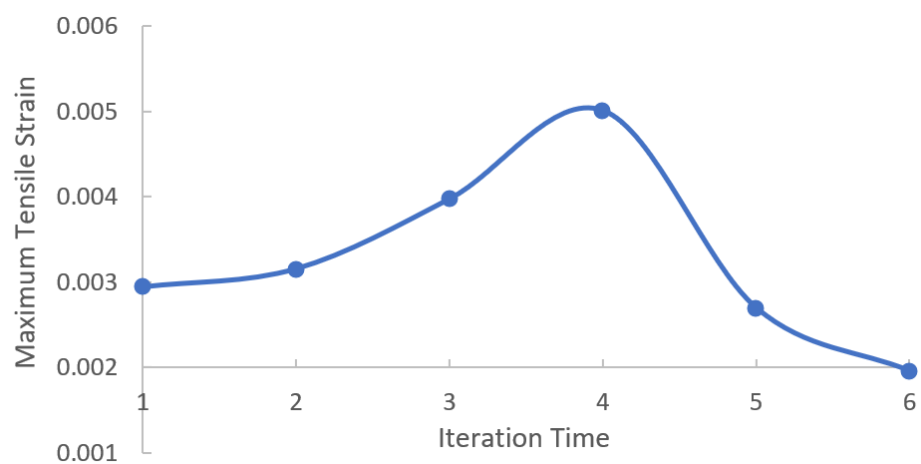


Figure 8. Maximum strain in the breakout tip in the process of breakouts.

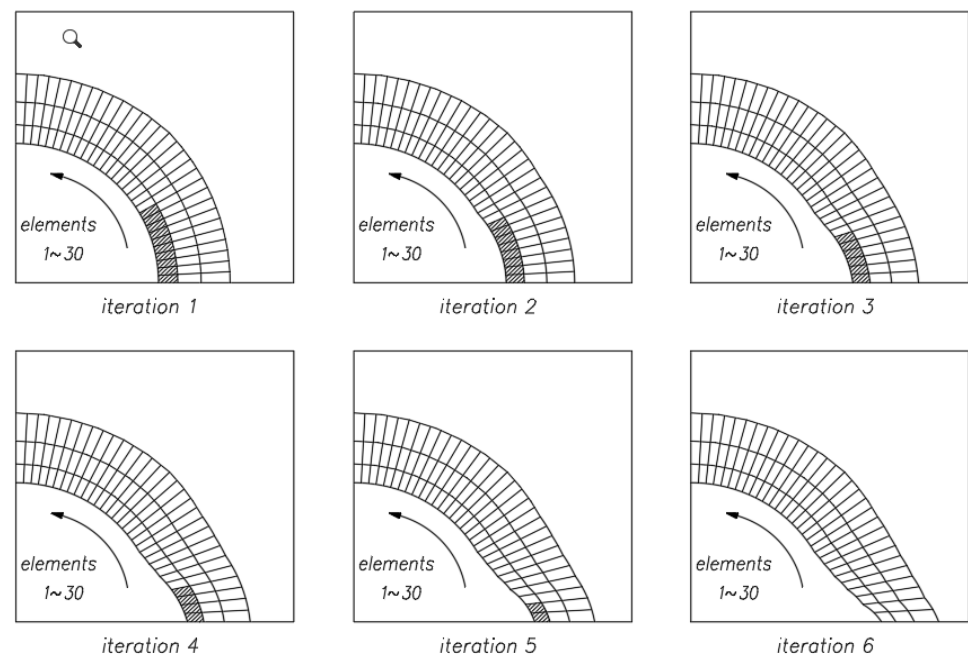


Figure 9. Changing of breakout region in the process of breakouts.

From Table 3 and Figures 7–9, the principal stresses of elements close to the tip of wellbore breakouts are increasing in the process of breakouts, but the maximum tensile strain changes to less than $\varepsilon_{breakout}$. As a breakout develops, the breakouts depth increases and the breakout width decreases, which means the depth of breakouts increases till a stable state, but the width of breakouts remains unchanged [3].

3.3. Influence of Drilling Fluid Temperature

For a vertical well shown in Figure 3, associated with horizontal in situ stresses $\sigma_H = 85$ MPa, $\sigma_h = 72.5$ MPa, and the temperature difference between drilling fluid and surrounding rock $\Delta T = -50 \sim 50$ K, and other parameters are listed in Table 1, the breakout shape for different drilling fluid temperature is shown in Figure 10.

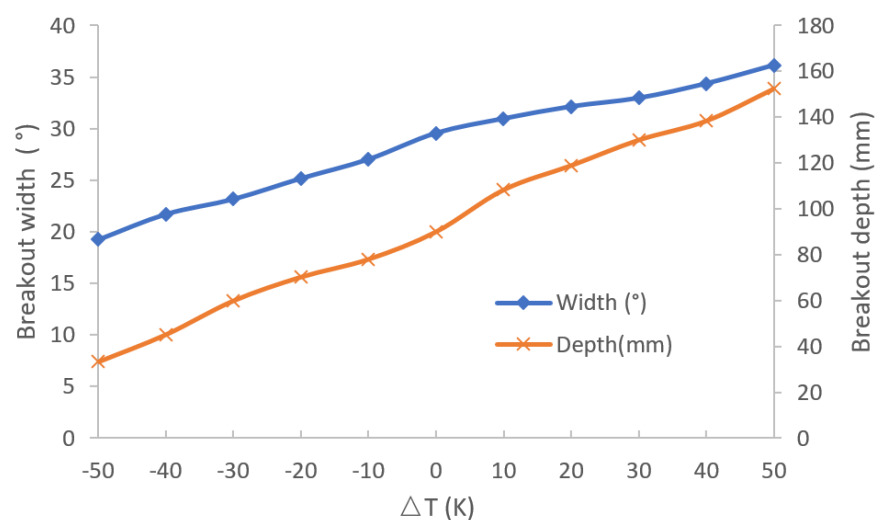


Figure 10. Breakout width and depth for different drilling fluid temperature.

Based on Figure 10, it can be seen that as temperature of drilling fluid increases, the breakout width and depth will increase.

3.4. Comparison between Thermoporoelastic and Thermoporoelastoplastic Modeling

For a vertical well, shown in Figure 3, which is subjected to horizontal in-situ stresses σ_H and σ_h , thermoporoelastic and thermoporoelastoplastic modeling are compared in this section, and parameters are listed in Table 1.

For the thermoporoelastoplastic model, the yield condition of rock is determined by Equation (4), and the failure (breakout) condition of rock is determined by Equation (5). For the thermoporoelastic model, the failure (breakout) condition of rock can be determined by Equation (4).

Figures 11 and 12 show different breakout depths and widths corresponding to different in situ stresses by thermoporoelastic and thermoporoelastoplastic modeling.

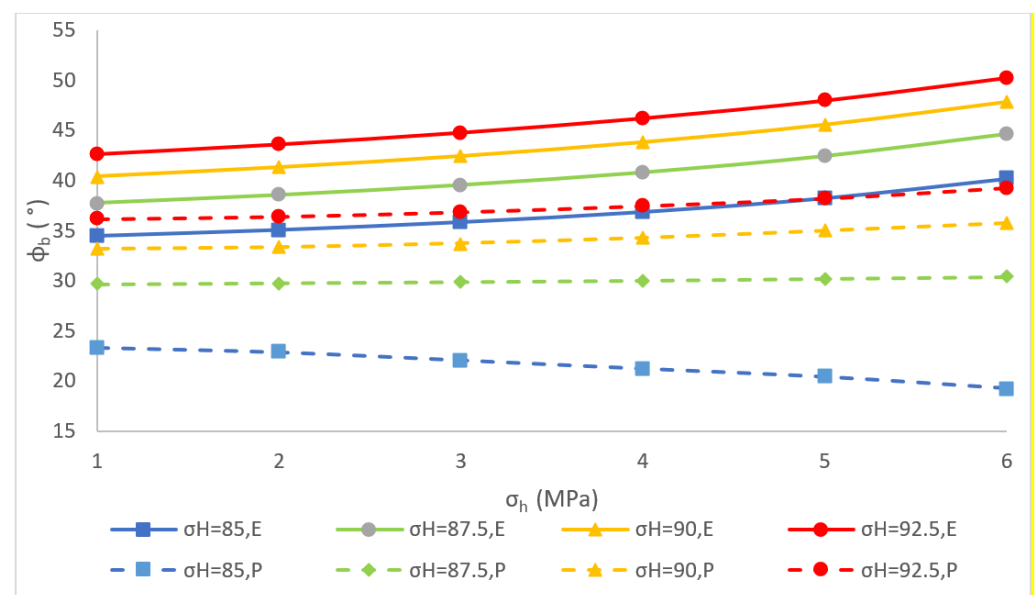


Figure 11. Comparison of breakout width between thermoporoelastic and thermoporoelastoplastic model.

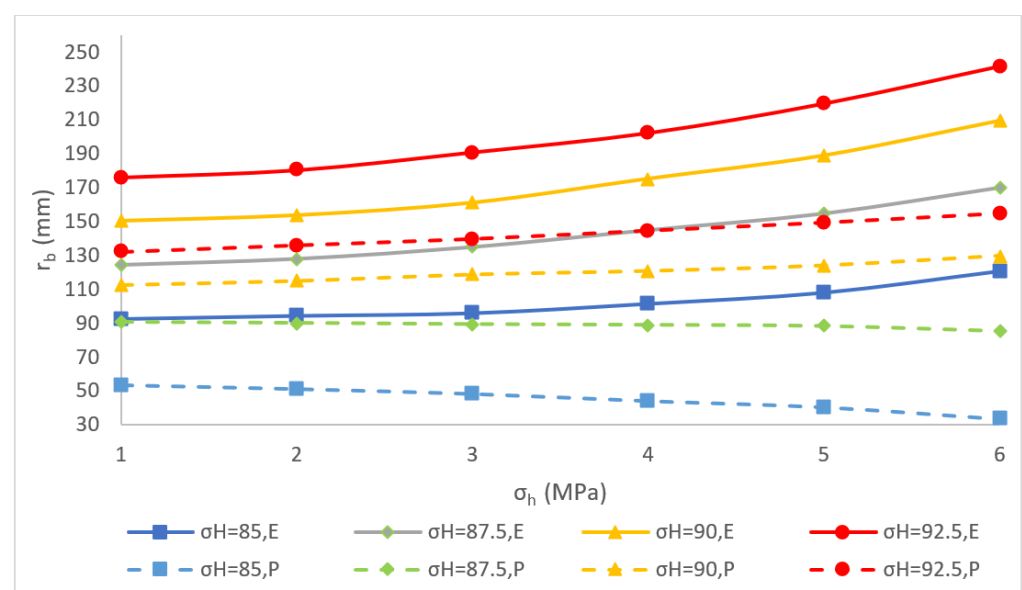


Figure 12. Comparison of breakout depth between thermoporoelastic and thermoporoelastoplastic model.

From Figures 11 and 12, it can be found that for different I-situ stresses, the width and depth of wellbore breakouts for thermoporoelastoplastic model are smaller than those for thermoporoelastic model. For the plastic model, the yield condition of rock is determined by Equation (4), and the failure (breakout) condition of rock is determined by Equation (5). For the elastic model, the failure (breakout) condition of rock can be determined by Equation (4). Therefore, in elastic model, when $F > 0$ in Equation (4), a breakout happens, but in plastic model, rocks just enter plastic state and no breakouts happened. Therefore, the breakout widths and depths are greater for the elastic model compared to the plastic model.

4. Discussion

In this paper, wellbore breakout process is simulated by the finite element method. Plasticity and temperature influence are considered, and results are compared with elastic model. There are some limits for this study. First, mud cake is assumed to be perfect, and there is no liquid transfer across the mud cake. Second, ideal elastictoplastic stress-strain relationship is used. Third, results are not verified by experimental data and field data. Future work is expected to address this in the next step.

5. Conclusions

In this paper, the finite element method is employed to simulate wellbore breakouts based on the thermoporoelastoplastic model. Numerical experiments on finite element modeling of wellbore breakouts show the contrasting tendency of breakouts shape with different in situ stress, and some conclusions can be made as follows:

1. The relationship between breakout shape and in situ stresses is nonlinear. If the σ_h is constant, as σ_H increases, the breakout width and depth become greater. If the σ_H is kept constant while the difference between σ_h and σ_H is relatively large, as σ_h increases, the breakout width and depth increase greater. However, if the σ_H is kept constant while the difference between σ_h and σ_H is relatively small, as σ_h increases, the breakout width and depth decrease instead.
2. In the process of wellbore breakouts, the breakout depth increases till a stable state, but the breakout width remains unchanged.
3. As temperature of drilling fluid increases, the breakout width and depth will increase.
4. For different in situ stresses, the width and depth of wellbore breakouts for thermoporoelastoplastic model are smaller than those for thermoporoelastic model.

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