# Supplementary Materials

Text in these Supplementary Materials provides further technical details.

## Problem statement

Mathematically, the goal is to calculate the variance of the total seabird bycatch $T = r^T E$, where the covariance matrices for the vector of seabird bycatch rates *r* and fishing efforts E are $\Sigma_r$ and $\Sigma_E$, respectively. Another popular measure of uncertainty is the coefficient of variation (CV), calculated as the ratio of standard error and the estimated bycatch rate. When the total fishing effort is known from each source, for example, based on fishery logbooks, the variance of the total bycatch T is $var(T) = E^T \Sigma_r E$. Note that only the diagonal elements of the covariance matrix $\Sigma_r$ are known based on individual assessments, and the off-diagonal elements are generally unknown, since the bycatch rates are assessed separately locally in each source. Denote the standard deviations of the bycatch rate from source *i* as $\sigma_i$. Then, the covariance matrix $\Sigma_r$ can be written as $diag(\sigma_1, \dots, \sigma_i, \dots, \sigma_n) \cdot C \cdot diag(\sigma_1, \dots, \sigma_i, \dots, \sigma_n)$, where *C* is the correlation matrix of the bycatch rates from all n sources. Next, we consider the case when the total fishing effort is also estimated. We can reasonably assume that the covariance matrix $\Sigma_E$ is diagonal, because the uncertainty of the estimate of total fishing effort from different sources can be assumed to be independent of each other, and the independence of the vector of the bycatch rate and the vector of the total fishing effort. Further denote the standard deviation of the total fishing effort from the *i*th source as $\sigma_{Ei}$. After some algebraic manipulation, it can be shown that $var(T) = E^T \Sigma_r E + \sum_{i=1}^{n} \sigma_{Ei}^2 (\mu_i^2 + \sigma_i^2)$, where $\mu_i$ and $\sigma_i$ are the mean and standard deviation of the bycatch rate from the *i*th source. From this formula, we see that, when the vector of the total fishing effort is known, the variance reduces to the previous formula, and when the vector of the total fishing effort *E* is also estimated, the variance of *T* includes an additional term due to the uncertainty in $E$.

## Three special cases of correlation structure

There are three special cases where additional knowledge of the correlation matrix is not needed for the calculation of the uncertainty estimate of the total bycatch. The first one is the completely synchronized case, where the bycatch rates from any two sources are perfectly correlated, i.e., the off-diagonal elements of $C$ are all ones. In this case, the variance of the total bycatch is simply $var(T) = (\Sigma_{i=1}^{n} E_i \sigma_i)^2$. In the second case, the bycatch rates from all of the sources are assumed to be independent from each other, i.e., the off-diagonal elements of $C$ are all zeros, and in this case, $var(T) = \Sigma_{i=1}^{n} (E_i \sigma_i)^2$. In the third case, the variation of the bycatch rates is completely counter-balanced, such that $E^T \Sigma_r E = 0$, and the total bycatch is completely determined. Note that there might be multiple covariance matrices satisfying the counter-balanced condition. The completely synchronized variation and the counter-balanced cases are two limiting cases that, respectively, marks the upper and the lower bound on the size of the uncertainty estimate of the total bycatch, and the independent case stays between those two.

## Compound symmetry correlation structure

In the compound symmetry case, the correlation coefficient between the bycatch rates $r_i$ and $r_j$ for $i \neq j$ is assumed to be the same $corr(r_i, r_j) = \rho$. The validity of the correlation matrix requires that $-\frac{1}{n-1} \leq \rho \leq 1$, where $n$ is the total number of regions, and in this study $-\frac{1}{3} \leq \rho \leq 1$ for $n = 4$. The upper bound on $\rho$ corresponds to the limiting case of a completely synchronized variation, the case of $\rho = 0$ corresponds to the independent case, and the lower bound on $\rho$ corresponds to a counter-balanced case with a uniform distribution of fishing effort and equal variance for all of the regions.

## Additional results

In the following, we show that the synchronized variation drives a much faster growth of variability in the estimate of the total bycatch than the independent case, and the effect is more pronounced as we have more individual sources. The analytical results can be obtained by assuming that each area has the same fishing effort ($y$ thousand hooks annually), and the seabird bycatch rate in each area has a common standard deviation σ. In the completely synchronized case, the standard deviation of the total bycatch $\sigma_T$ is $\sigma_T^1 = y\sigma n$, and $\sigma_T^0 = y\sigma\sqrt{n}$ in the independent case. The difference between the two estimates grows steadily with the number of sources $n$. For example, with $n = 10$, $\sigma_T^1$ is more than twice the size of $\sigma_T^0$, and with $n = 40$, $\sigma_T^1$ is more than five times the size of $\sigma_T^0$. By assuming the compound symmetry correlation structure, we can also obtain an analytical result for the cases between the completely synchronized case and the independent case. With $-\frac{1}{n-1} \leq \rho \leq 1$, the standard deviation of the total bycatch is $\sigma_T^\rho = y\sigma\sqrt{(\rho n^2 + (1-\rho)n)}$, of which the perfect synchronization case and the independent case are special cases with $\rho = 1$ and $\rho = 0$, respectively. Here, $y$ and σ are the scaling factors and they do not change the relative size of these functions. With $y =$ 1 million hooks and $\sigma = 0.01$ captures per 1000 hooks, we can plot these functions and visually compare their growth with the number of sources. For reference, the last global review of the seabird bycatch in longline fisheries compiled estimates from 68 individual sources, and still with missing assessments from some of the regions and fleets. In terms of CV, the level of uncertainty of the total seabird bycatch remains constant with respect to the number of sources in the completely synchronized case; for the compound symmetry correlation with an intermediate positive correlation, the CV quickly reaches a finite asymptote at $\sqrt{\rho} \cdot \mathrm{CV}_i$, where $\mathrm{CV}_i$ is the CV of seabird bycatch rate from an individual source; for the independent case, the CV vanishes when the number of sources tends to infinity.