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Application of an SPH-DEM Coupled Model for Elastic Fluid–Structure Interaction †

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Abstract: In this work, we present the application of an alternative numerical model for fluid dynamic analyses of structural systems within the Smoothed Particle Hydrodynamics (SPH) framework of DualSPHysics coupled to the Project Chrono library. The Discrete Elements Method-based (DEM) structure model relies on the Euler–Bernoulli theory and utilizes lumped elasticity and rigid body dynamics to reproduce the flexural behavior of two-dimensional beams. The structure and fluid domain are both discretized with SPH particles: the fluid dynamics obey a Weakly Compressible SPH (WCSPH) formulation, whereas the structure particles are assembled into DEM rigid elements, moving according to physically-based, properly developed rotational dynamics. The presented model is of interest for studying complex soil–, solid–, fluid–structure interactions, involving a system that includes all the aforementioned phases in a unitary context—very useful for studying engineered structures under the threat of hazardous natural events. Test cases are presented to validate the SPH-DEM coupled model in both accuracy and stability, starting from an equilibrium test, to the dynamic response, and ending with fluid–structure interaction simulations. This work proves that the developed theory can be used within a Lagrangian framework, using the features provided by a DEM solver, overtaking the intrinsic limitations, and hence applying the results of static theory to complex dynamic problems.

Keywords: SPH; DEM; FSI; Euler–Bernoulli; CFD

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1. Introduction

With the development of powerful computational tools, fluid–structure interaction (FSI) simulations are becoming standard in many application fields [1], as the reliability and flexibility of numerical models give the opportunity to enhance several engineering endeavors where the coupled effects are non-negligible.

The traditional approach to FSI considered mesh-based methods with a partitioned approach, where the Finite Element Method (FEM) treated the structure separately. However, computational costs due to re-meshing processes and convergence issues, led to alternative meshless approaches being considered. The Lagrangian methods are naturally structured to avoid the common drawbacks of grid-based methods, providing a robust computational tool to enhance the fluid phase simulation in FSI problems. Among this methods, the Meshfree Particle Methods (MPMs) gained much interest in many research fields and engineering applications. The MPMs employ a set of a finite number of discrete particles to represent the state of a system and to record its movement: each particle can either be directly associated with one discrete physical object, or be generated to represent a part of the continuum problem domain [2]. These methods provide a robust computational tool to handle fluid phases characterized by relevant deformations, as moving boundaries and complex interfaces are easily tracked by assigning particular physical quantities to each computational point (particle). One of the most used Lagrangian approaches in recent years

has been the Smoothed Particle Hydrodynamics (SPH) method [3], especially regarding coastal engineering applications, but, in general, in all water-related fields [4,5].

Fully Lagrangian approaches have been used for the FSI application, where both fluids and solids are modeled within the same SPH framework: for example, the ISPH-SPH framework by Khayyer et al. (2022) [6] or the δSPH-total Lagrangian SPH solver by Sun et al. (2019) [7]. Other coupled solutions pair an SPH fluid phase with a structural solver [8] to exploit the strengths of both numerical schemes. Along this path, the proposed work showcases the reliability of an alternative coupled framework, developed in Capasso et al. (2022) [9], composed of a Discrete Elements Method (DEM) structure solver within a homogeneous SPH environment. The DEM model, suitable for breakage, rupture and large deformations phenomena [10], is developed starting from the Euler–Bernoulli beam theory [11] by a lumped elasticity approach. It follows a set of rigid elements connected by relative constraints implemented in the DualSPHysics framework that allows the simulation of a wide range of flexible elements.

DualSPHysics is an open-source software based on the SPH method [12]. Its highly parallelized structure harnesses the computing power of graphics processing unit (GPU) cards [13]. The capability of DualSPHysics to address complex multiphysics applications has been enhanced by its coupling with other numerical models [13–15]. In particular, thanks to the coupling to the Project Chrono multiphysics library [16], it can simulate complex mechanical systems and multibody problems. The effectiveness of the DualSPHysics-Chrono framework, including multibody dynamics simulations and mechanical constraints, has been extensively proved [14,15,17,18]. Within the field of renewable energy, many successful applications have been presented over the last years [19–22].

Two FSI test cases for the SPH-DEM model are presented here to showcase the applicability of the proposed scheme. The theoretical fundamentals of the SPH-DEM scheme are briefly introduced in Section 2, while the benchmark tests are shown in Section 3. Final considerations and further possible developments can be found in Section 4.

2. SPH-DEM Fundamentals

2.1. Fluid Phase

The strategy in SPH is to discretize the physical domain (fluid and/or solid objects) into a set of particles, where the physical quantities are obtained as an interpolation of the corresponding quantities of the surrounding particles. The contribution of those particles is weighted using a kernel function, with an area of influence that is defined using a characteristic smoothing length [2].

The integral representation of a generic spatial function $f(\boldsymbol{r})$ within an integral volume $\boldsymbol{\Omega}$, is given by

$$\langle f(\boldsymbol{r})\rangle = \int_{\boldsymbol{\Omega}} f(\boldsymbol{r}')W(\boldsymbol{r}-\boldsymbol{r}',h)d\boldsymbol{r}' \tag{1}$$

where W is the smoothing kernel function or kernel, whereas $\boldsymbol{r}$ is the position vector. In the smoothing function, h is the smoothing length defining the influence area of W.

Within a discrete space, the function is approximated by summing up the values of the nearest neighbor particles that possess fixed mass m_b and finite volume V_b. The discrete form of Equation (1) for a particle a, being $b = 1 \dots N_p$ part of its support domain

$$\langle f(\boldsymbol{r}_a)\rangle = \sum_{b=1}^{N_p} \frac{m_b}{\rho_b} f(\boldsymbol{r}_b)W_{ab} \tag{2}$$

The kernel function, hence, plays a fundamental role in the SPH method. In DualSPHysics the Wendland quintic kernel function [23] has been utilized.

2.1.1. Governing Equations

The fluid domain behaves according to the Navier–Stokes (NS) equations

$$\frac{D\rho}{Dt} = -\rho\nabla \cdot \boldsymbol{u} \tag{3}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla p + \boldsymbol{g} + \nabla \cdot \boldsymbol{\Gamma} \tag{4}$$

where ρ is the density, $\boldsymbol{u}$ is the velocity vector, $\boldsymbol{g}$ the gravitational acceleration vector, p is the pressure and $\boldsymbol{\Gamma}$ is the deviatoric stress tensor. An extensive treatise of the Weakly Compressible SPH (WCSPH) discretization of the NS equations can be found in [9].

2.1.2. Boundary Treatment

Dynamic Boundary Conditions (DBC) [24] for solid surfaces are standard in DualSPHysics [25]. Solids are represented by fixed particles whose pressure is dependent on the same state equation as the fluid particles.

To enhance the DBC formulation, the modified Dynamic Boundary Conditions (mDBC) [26] has been developed and validated [27,28]. The particle arrangement remains the same as in the DBC, while a mirroring technique [29] helps compute smoother pressure fields by using ghost nodes located in the fluid domain, where the properties are found through a first-order consistent SPH spatial interpolation [30] over the surrounding fluid particles only. In this way the fixed boundaries are presented as a part of a fluid mass, avoiding any non-physical separation between particles.

2.1.3. Rigid Body Dynamics and Coupling Strategy

The rigid bodies in the DualSPHysics environment are discretized with a different set of particles, which obey Newton's law for the rigid body dynamics [14,31]. The force per unit of mass experienced by a boundary particle k is

$$\boldsymbol{f}_k = \sum_{b \in fluid} \boldsymbol{f}_{kb} \tag{5}$$

where $\boldsymbol{f}_{kb}$ is the unitary force exerted by the fluid particle b on the boundary particle k.

The interaction between bodies is handled by the Project Chrono module embedded in the original framework of DualSPHysics: arbitrarily defined structures with dynamic- and kinematic-type restrictions can be simulated by the Chrono library integrating active and reactive forces. DualSPHysics calculates the forces exerted on the bodies by the fluid particles, while Project Chrono updates their position $\boldsymbol{R}$, angular $\boldsymbol{\Omega}$ and linear velocities $\boldsymbol{V}$ [9].

2.2. Structure Model

The DEM model formulation presented in Capasso et al. (2022) [9,32] relies on the assumptions of the Euler–Bernoulli (EB) beam theory, which lead to ordinary differential equations able to describe the static beam deformation through linear elasticity relations called constitutive bonds. If the flexural behavior is considered, as a matter of course in many engineering applications, the bending moment, M, governs the deflection of the beam with a rotation rate per cross-section defined by

$$\frac{d\varphi(z)}{dz} = \frac{M}{EI} \tag{6}$$

with E being the Young's modulus, and I the second moment of inertia of the cross-section.

Considering the rotation finite ($d\varphi \rightarrow \Delta\varphi$) costs an error proportional to the dimension of the interval ($dz \rightarrow \Delta z$). The rotational stiffness of the whole trunk, hence, is lumped in a single point, namely the rotational hinge: in this way, the deformation function is represented by a physical set of linear interpolating polynomials (trunks) that rotate

according to the geometrical and mechanic characteristics of the beam. If the beam is divided in N trunks of dimension $\Delta z = L/N$, then the $j-th$ element rotation with respect to the $i-th$ hinge is given by

$$\overline{\varphi}_i \frac{EI}{\Delta z_{j=i}} = M_i \tag{7}$$

where $\overline{\varphi}_i$ is the local rotation, and M_i is the total reactive moment. The constant quantities in Equation (7) are referred to as rotational stiffness

$$K_{\varphi,i} = \frac{EI}{\Delta z_{j=i}} \tag{8}$$

As a matter of fact, this numerical solution allows a straightforward implementation within the DualSPHysics + Chrono framework: each trunk is modeled as a rigid floating object, while the rotational constraint reactions that reproduce the elasticity of the beam are computed by the Chrono library. Being governed by the rigid body dynamics, the co-rotating elements, modeled upon static hypotheses, can reproduce the flexural behavior of thin beams in complex dynamics and multiphysics phenomena.

3. Applications

3.1. A Breaking Water-Column with an Elastic Gate

This FSI benchmark was first presented in Antoci et al. (2007) [33], in which the experimental setup and results in a two-dimensional fashion are provided; a schematic of the setup is given in Figure 1. The water-column, $\rho = 1000$ kg/m^3, is confined between two rigid walls, one of these with an elastic gate at its end. The latter is modeled with the proposed SPH-DEM scheme to reproduce the elastic deformation induced by the fluid pressure. The experimental reference given in [33] is compared to the displacement of the free-end of the gate in the first instants of the simulation.

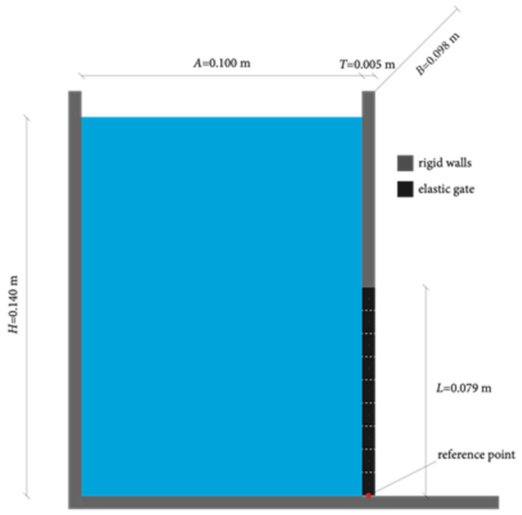


Figure 1. Setup of the benchmark after Antoci et al. (2007) [33] in DualSPHysics.

The results displayed in Figure 2, together with the snapshot comparison in Figure 3, describe an overall acceptable agreement, especially between the presented model and the iSPH-SPH model proposed by Khayyer et al. (2018) [6]. The latter solver is based on a coupled model in which the fluid is treated as incompressible, thus ensuring great stability by the way of a semi-implicit projection-based particle method (iSPH), and an SPH-based structure model. Differently from the explicit WCSPH implementation available in DualSPHysics, the iSPH solver admits larger time-steps and globally solves the fluid–structure interaction, as the computation of pressure follows from the Poisson's equation instead of the very stiff Tate's equation [9].

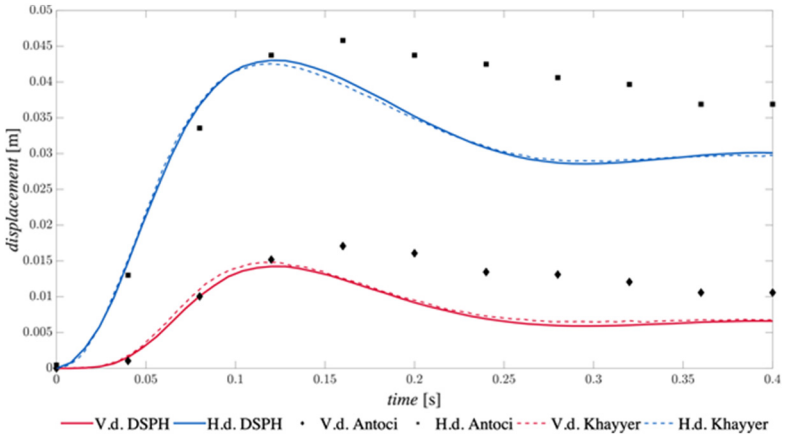


Figure 2. Horizontal and vertical free-end displacements time history with $N = 10$, paired with experimental results from [33] and iSPH simulations from [6].

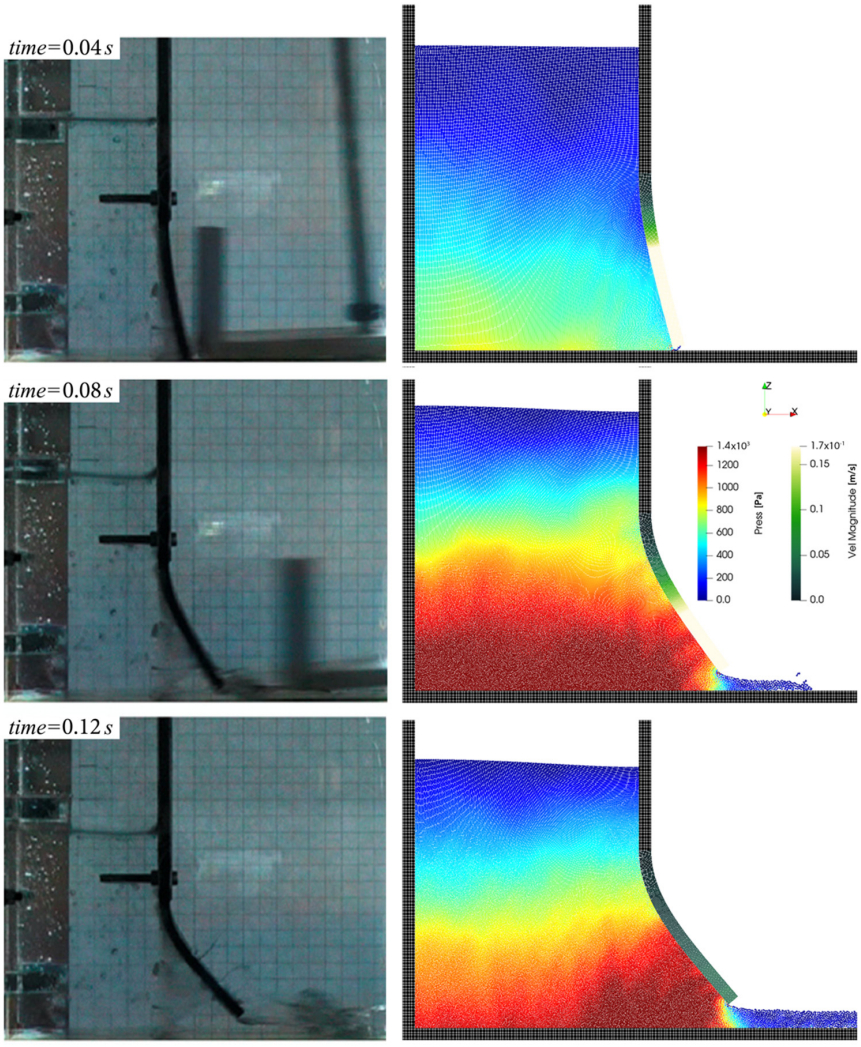


Figure 3. Breaking water-column numerical test: comparison between photos from experiment [33] (**left**) and SPH-DEM results (**right**) at different time instants.

In the first instance of the simulation, the SPH-DEM scheme properly follows the experimental data, while, after the peak of deformation, it appears to be excessively rigid. This behavior is probably due to the absence of damping in the beam, which will require further investigation. The visual comparison (Figure 3) highlights a smooth pressure field,

and confirms the good reproduction of the first instance of the simulation. At time $t = 0.12$ s, the velocity magnitude of the elastic gate is approximately null, in correspondence with the peak of deformation, when the resistance of the structure equalizes the dynamic fluid load.

3.2. Hydroelastic Slamming of an Aluminium Wedge

The following benchmark consists of a high-speed impact of a deformable aluminum beam with the undisturbed water surface. A semi-analytic solution was made available by [34], while numerical simulations are performed in [6]; both of the references were obtained under incompressible fluid hypothesis. The wedge is composed of two symmetrical wings of length $L_b = 0.6$ m and thickness $h_b = 0.04$ m, tilted at an angle of $\alpha_b = 10^\circ$ with respect to the water surface. The Young's modulus of the beam is $E_b = 67.5$ GPa, while the Poisson's coefficient is $\nu_b = 0.34$. The elastic structure impacts the water at a fixed velocity $v = 30$ m/s, with the central section, as well as the extremities, restrained to a vertical motion without any other degree of freedom. Consistently with this hypothesis formulated by [34], the DualSPHysics setup is built according to Figure 4, with the rigid trunks disposed perpendicularly to the slope and fitted at the far ends.

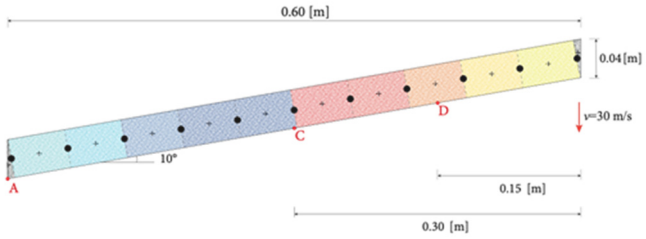


Figure 4. Wedge wing Chrono scheme, with the trunks' (in different colors) and hinges' disposition visible; in red the reference points for pressure and displacement computation.

In the following chart an SPH resolution of $dp = h/20$, with dp being the initial interparticle distance, is used.

The numerical speed of sound for the fluid phase is set at $c_0 = 1500$ m/s to properly simulate the highly dynamics phenomenon, and therefore very small time-steps are required.

In Figure 5 the displacement of the point C is presented: results with two different DEM resolutions, depending on the parameter N, are paired with the semi-analytical and numerical references, showing good agreement. As the number of trunks N increases, the behavior of the SPH-DEM model tends to resemble the semi-analytical one: the inflection point, around $t = 1.3 \times 10^{-3}$ s, when the elastic response of the structure begins to counter the fluid action, is better reproduced as expected.

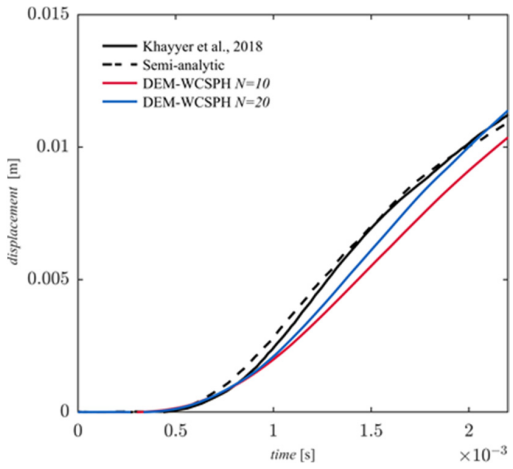


Figure 5. Time history of the deflection at point C, with two different DEM resolutions N.

4. Outlook

The proposed approach makes the most out of the fluid–solid interaction computation capability of the DualSPHysics solver, exploiting the potentiality of the solid body motion solver of the embedded Chrono module to handle complex hydroelastic phenomena, in which the mutual exchange of forces, pressure and viscous stress between the fluid and structure plays a fundamental role.

The presented model relies on the EB hypotheses, but has a broader application field: the validations proposed account for deformations clearly out of the limits imposed by the beam theory but are, anyway, properly reproduced. Furthermore, the inherent nature of the SPH environment where the model is developed allows the direct tracking of interfaces, efficient body force computation and extreme displacements reproduction.

The straightforward implementation procedure of the SPH-DEM model provides a reliable tool, which relies on the well-known capabilities of Smoothed Particle Hydrodynamics, to approach the study of the elastic response of structures through a Discrete Element model. The presented model, hence, can be applied to several real-world engineering problems, for example:

- flow–structure interactions involving coastal vegetation, i.e., bioshields;
- swinging of wind turbine columns subject to waves or seismic action;
- modeling of vibrations of one-dimensional structures interacting with Newtonian or non-Newtonian fluids.

All these fields involve non-negligible coupled effects that need similar numerical frameworks to be correctly assessed, as the optimization of the resources, especially among the emerging renewable technologies, is probably the cornerstone to fully exploit the energy potential stored in our environment. Advanced numerical models, such as the one proposed, indeed, are developed to further enhance the design of systems that require an in-depth analysis of wave– and, in general, fluid–structure interaction.

Further improvements could enhance the performances and widen the application possibilities: including the deformation effects due to the shear and normal stress, or even model elasto-plastic behavior.

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