

Article

Fault Location on Series-Compensated Transmission Lines Using Incremental Quantities and Emulating a Nonlinear Metal-Oxide Varistor

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Abstract

This paper presents a staged two-terminal fault-location method for transmission lines with series compensation, based on incremental quantities and a least-squares error (LSE) formulation. The fault distance is first established through an analytical estimator with a bounded linear capacitive correction. This distance is subsequently held fixed during subsample time alignment and the emulation of the nonlinear capacitor and metal-oxide varistor (MOV), thereby preventing the later correction stages from shifting the fault location solely to reduce the fitting residual. Among the 18 paired Common Format for Transient Data Exchange (COMTRADE) records, 10 cases corresponded to the reference distance category, including seven instances with series compensation and three instances without compensation. For these 10 records, the proposed procedure achieved a mean absolute error of 2.26 km, with eight cases within 5 km; excluding the record used during method development, it was 2.48 km, with seven of nine within 5 km. For the compensated subset, the mean absolute error decreased from 4.44 km (raw LSE) to 3.04 km (from 3.94 to 3.50 km excluding that record), whereas the uncompensated subset yielded 0.43 km. Sensitivity and stress analyses quantify the influence of the incremental delay and delimit the applicability of the time-alignment and residual-diagnosis stages. The results show the performance of the staged formulation on the available records while indicating the need for broader field and electromagnetic-transient validation before generalization to other compensation levels, MOV characteristics, and fault-resistance conditions.

Keywords: fault location; series compensation; incremental quantities; metal-oxide varistor; COMTRADE; least-squares error



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1. Introduction

1.1. General Context

Transmission lines are used to transfer large amounts of energy from generation to consumption centers. The transfer capability is limited by the inductive reactance of the lines, system stability limits, voltage limits, etc. Series capacitor banks are installed to compensate inductive reactance and increase transfer capacity [1]. However, the presence

of these elements also modifies the response of the transmission line during a fault and complicates the interpretation of the signals used for fault location.

To reduce potential overvoltages, metal-oxide varistors (MOVs) are often connected in parallel with the capacitors in series compensation banks [2–5]. During a fault, the increase in current may raise the capacitor voltage until the MOV conducts and diverts part of the current. As the voltage–current characteristic of the MOV is nonlinear and its conduction changes during the transient, the response of the compensation assembly varies over time [2,4]. Furthermore, the interaction between the line inductance and the series capacitor can produce subsynchronous components, harmonics, and a decaying aperiodic component in the fault signals. These disturbances can affect the estimation of the fundamental phasors and reduce the accuracy of algorithms that represent the signals using sinusoidal components [6–8].

The precise location of a fault helps reduce inspection time, dispatch maintenance crews to the affected section, and speed up service restoration. In long lines, even a moderate percentage error can represent several kilometers. For this reason, fault location in lines with series compensation requires methods that take into account both the electrical configuration of the line and the characteristics of the signals recorded during the fault [1,5].

1.2. Definition of the Problem

Conventional impedance-based fault-location methods assume that transmission line parameters are uniform along its entire length. Using the voltages and currents measured at one terminal, they calculate an apparent impedance and relate it to the distance to the fault. The presence of a series compensation bank introduces a localized impedance discontinuity and modifies the voltage drop observed from the terminal. Consequently, the apparent impedance also depends on the capacitor bank location relative to the fault and on its operating state. If these effects are not properly modeled, the estimated distance may differ from the actual fault point [1,4,9–13].

In methods that use measurements from both ends of the line, it is also necessary for the records to be correctly time-aligned. Even if the relays share a time reference, their acquisition, filtering, and processing stages can introduce relative delays between the signals. Even a phase shift shorter than one sampling period can disrupt the correspondence between the measurements at both terminals and affect the fault-distance calculation [14,15].

Thus, the problem involves the impedance discontinuity caused by the capacitor bank, the variable MOV response, and the possible time offset between the records. These factors can interact during signal fitting. If the distance, the offset, and the MOV voltage are modified simultaneously to minimize a single error metric, an incorrect combination of those values could yield an acceptable numerical fit while also producing an erroneous fault location. Therefore, the problem is to estimate the true distance without having the time-alignment adjustments or the MOV response artificially bias that estimate.

1.3. Literature Review

In the specialized literature, fault location in series-compensated transmission lines has been addressed using time-domain formulations, phasor methods based on impedance, adaptive strategies with synchronized measurements, techniques that use unsynchronized records, traveling waves, and advanced signal processing procedures [6,8]. Despite their differences, these approaches face a common difficulty: the relationship between the voltages, currents, and distance to the fault is altered when the compensation bank is included in the fault circuit. In addition, the nonlinear conduction of the MOV and the interaction between the series capacitance and system inductance introduce transient

components that affect the conventional line models and the estimation of the quantities used in the fault-location techniques.

The first problem addressed was the representation of the capacitor bank and MOV. In [2], a linearized model of a series capacitor bank protected by an MOV was developed to aid its incorporation into fault and stability studies. This formulation represents the behavior of the varistor using an equivalent impedance that depends on the current flowing through the circuit. This approach reduces the computational complexity; however, it does not fully reproduce the instantaneous and nonlinear evolution of the MOV voltage during all conduction intervals.

In [9], the authors proposed a location algorithm based on a distributed line model and time-domain assessment. They used voltage and current measurements acquired at both ends and considered two possible configurations corresponding to faults located before or after the compensation bank. Subsequently, a selection logic determines which solution represents the fault location. This formulation reduces the influence of fault resistance and source equivalents but requires synchronized records, knowledge of the capacitor bank position, and an adequate representation of the compensation elements included between the terminals and fault point.

In [10], the authors sought to reduce the dependence on the explicit MOV model and developed an impedance-based method for locating ground faults in lines with series capacitive compensation using synchronized measurements from both terminals. This formulation avoids the direct use of the nonlinear MOV model and natural fault circuit, thus reducing the errors associated with the inaccurate representation of the varistor. However, the method is conditioned by the quality of the phasor estimation, alignment of the records, and assumptions used to construct the equations for each type of fault.

In [16], an algorithm was proposed based on synchronized measurements from phasor measurement units (PMUs). It estimates the line parameters and Thévenin equivalents at both terminals from sets of pre-fault measurements, where the adaptive estimation of the electrical parameters makes it an alternative for reducing modeling errors. This strategy allows the fault-location technique to be adapted to existing operating conditions and reduces the need to use fixed values for specific system parameters. However, its implementation requires a PMU infrastructure, reliable time reference, and sufficient pre-fault information to perform adaptive estimation.

In a related approach [17], the authors proposed a method that eliminated the need to use previously known line parameters. The formulation uses measurements from both ends and estimates the equivalent impedance of the compensator from the phase currents, considering the nonlinear behavior associated with MOVs. This feature reduces the sensitivity of the system to variations in the electrical parameters of the line. However, the procedure still depends on synchronized measurements and the correct identification of the capacitor bank equivalent response during a fault.

In contrast, Hussain et al. [14] used a time reference proposed as a double-terminal method based on unsynchronized measurements. Using this approach, the information acquired after the fault began was used to estimate the existing time difference between the records before calculating the distance. Eliminating the requirement for Global Positioning System (GPS) synchronization improves the practical applicability of the method but introduces a time offset as a new variable. Therefore, the accuracy of the location depends on both the electrical model used and the ability to distinguish between a time alignment error and an apparent change in the estimated distance.

Traveling wave methods estimate fault locations from the propagation times of transient fronts as an alternative to the impedance-based formulations. In [18], a double-terminal fault-location technique was proposed that uses the first arrival times recorded at

both ends and considers the possible positions of the fault with respect to the compensation bank. Because it depends mainly on the propagation times, this method avoids explicitly representing the nonlinear characteristics of the MOV and reduces the influence of fault resistance and pre-fault conditions. However, it requires a high sampling frequency, reliable detection of wavefronts, and a time resolution that is higher than that available in many conventional protection records.

The challenges posed by series compensation in distance protection were also examined from a broader perspective. Thus, in [19], the authors validated different developments aimed at the protection of lines with capacitive compensation, highlighting that the modification of the apparent impedance, possible inversion of voltage or current, and operation of bank protection devices can affect the reach and directionality of the relays. These effects show that compensation should not be considered solely as a static modification of the line reactance, as its response during a fault depends on the location of the capacitor bank and the conduction state of its protection elements.

Another line of research has focused on improving the estimation of the fundamental component when the signals contain transients produced by series compensations. Fan and Yun [6] employed the Prony algorithm to decompose the signals and separate the fundamental component from the low-frequency damped oscillations before performing the fault location. This processing can reduce the errors caused by subsynchronous components; however, its performance depends on the selection of the model order and can deteriorate in the presence of noise, closely spaced modes, or signals with frequencies different from the nominal values.

In [8], a two-stage particle swarm optimization (PSO) approach was proposed to estimate the current phasor in transmission lines by incorporating series capacitive compensation. This method accounts for the simultaneous presence of decaying direct current (DC) components, subsynchronous oscillations, harmonics, noise, and deviations in the fundamental frequency. The results show an improvement over conventional Fourier-based estimators. Nevertheless, the main contribution of this study is the estimation of the fundamental component and not the direct determination of the distance or physical reconstruction of the capacitor–MOV voltage.

The limitations of conventional methods have been observed in recent studies. This difficulty was observed in [13], where the simple reactance method, Takagi, Eriksson, and a formulation based on superimposed components were evaluated for different fault resistances, loading conditions, and locations. The results showed that when the fault was located beyond the capacitor bank, the discontinuity introduced by the capacitor and the nonlinear operation of the MOV could produce errors of several tens of percent. Therefore, the methods developed for lines with uniformly distributed parameters do not necessarily maintain their accuracy when the compensator is located between the measuring terminal and fault.

The transformation of the sources connected to the system introduces additional difficulties; this is the case in [20], where the authors analyze the distance and directional elements in compensated lines connected to inverter-based resources through analytical studies, electromagnetic simulations, and tests using commercial relays. The results show that the characteristics of the currents supplied by the inverters can invalidate some of the assumptions used in traditional practices for the protection of compensated lines. Although this study does not propose a location algorithm, it demonstrates that the variables observed by the relays depend on the interaction between the capacitor bank, MOV, and dynamic characteristics of the connected sources.

Within the time-domain formulations, [15] proposed a method based on incremental quantities for fault location using multiterminal measurements. The method calculates the

voltage change corresponding to different candidate positions from each end and uses a least-squares error (LSE) estimator to determine the distance that produces the best match between the signals. The procedure works directly with instantaneous samples, does not depend on the previous selection of the fault loop, and allows the use of processing windows with different durations. In addition, it includes a procedure to correct misaligned records by shifting the remote data and searching for the minimum LSE, with interpolation to obtain a time resolution that is smaller than one sampling period.

In [21], the adoption of the Common Format for Transient Data Exchange (COMTRADE) records enabled the application of this method to oscillographic data generated by relays and recorders deployed at both terminals of the transmission line, thereby eliminating the need for additional measurement hardware. However, the standardization of the format does not guarantee that the signals are perfectly aligned, as there may be differences associated with the sampling, filtering, triggering, and group delays of each device used for data acquisition. Additionally, a formulation based on incremental quantities was developed for lines without a nonlinear discontinuity equivalent to that introduced by a conducting capacitor bank and MOV. Although these studies show progress through explicit MOV models, algorithms that avoid directly representing the compensator, adaptive parameter estimation, processing unsynchronized records, traveling wave techniques, and advanced phasor estimation methods are required. These approaches generally treat distance estimation, time misalignment, and the dynamic response of the capacitor bank separately. In the analyzed studies, no method was identified that first computed the distance using an analytical formulation based on incremental quantities, kept this distance fixed during the time correction, and subsequently reconstructed the voltage drop of the capacitor bank and MOV configuration using dynamic equations.

1.4. Literature Gap

The algorithms evaluated in [6,9,10,16–18] were validated using the Electromagnetic Transients Program/Alternative Transients Program (EMTP/ATP) or MATLAB, in which the capacitor–MOV bank, instrument transformers, and alignment between terminals were represented by ideal models. Field oscillography introduces effects that are not reproduced in the simulation, such as different relay group delays at each end, missing transformation ratios in the COMTRADE configuration (.CFG) file, truncated recording windows, and banks whose protective levels and varistor exponents were absent from the line database. Some studies have used simulations to examine the performance of conventional methods under adverse conditions [13,20]. Nevertheless, the degree to which the accuracy reported in these simulation-based studies is maintained when the methods are applied to oscillographic records acquired from protection devices under real-world operating conditions has not yet been quantified.

In the reviewed studies, existing approaches estimated the fault location while fitting a detailed series compensator bank model or avoided explicit compensator modeling by employing features that were largely invariant to its dynamic behavior. Some studies represented capacitor banks using linearized equivalent impedance [2] or time-domain formulations that include compensation elements between the terminal and fault [9,10]. This approach is accurate but depends on poorly documented parameters and simultaneously fits the capacitor bank response and fault distance using the same residual. The optimizer can reduce the error by shifting the fault instead of improving the model. Other studies use quantities that are not very sensitive to the capacitor bank, such as the arrival times of the traveling wave [18] or formulations without line parameters [17]. These studies are more robust; however, they discard the signature of the capacitor bank or, in the case of

those based on traveling waves, require sampling rates on the order of megahertz, which are incompatible with the current relays and protection devices.

The sub-sample time offset between the terminals is usually not treated as an independent phenomenon. Therefore, its effect remains as an unexplained part of the residual error. Some double-terminal techniques estimate and correct the time misalignment between the records [14,15]. Nevertheless, the offset caused by the different group delays of the relays at the sub-sample level was not separated from the nonlinear behavior of the capacitor bank; both contributed a reactive component to the residual and appeared as a single fitting error. Studies that filter the fundamental component in the presence of compensation transients address the problem as a signal estimation task [6,8] without distinguishing between the two effects. Thus, a double-terminal fault location technique only provides a distance and, at most, a fitting error. When this error is high, the analyst does not know whether the estimate is unreliable (because of the data related to faults) or whether the distance is correct but limited by the imperfect MOV model. However, this distinction is crucial for the system operation to decide and act on the reported locations.

1.5. Objective and Novelty

This study aimed to develop and validate a method for locating faults in transmission lines with series capacitive compensation. The input data were obtained directly from COMTRADE files [21] generated by the protection relays installed at both terminals of the transmission line. The proposed method operates exclusively on these records and does not require any additional hardware. The incremental quantity and LSE formulation proposed by Kasztenny and Smelich [15] were adopted as a starting point because the common fault-resistance term cancels algebraically, and they do not depend on the pre-fault load, and allow a time-domain implementation.

The proposed method was evaluated using field measurements acquired from a series-compensated transmission line. Two possible implementations, not incorporated in the original formulation, were identified: (i) nonlinear behavior attributable to the MOV and (ii) sub-sample inter-terminal time misalignment arising from relay-dependent group delays. The present study asserts that accurate fault-location estimation in the presence of series compensation requires the simultaneous satisfaction of three conditions. First, the line distance must be estimated a single time using the analytical LSE estimator and subsequently held constant, rather than iteratively re-optimized to minimize the fitting residual. Second, the sub-sample time offset between the relays must be compensated in a manner that does not alter the fixed distance estimate. Third, the voltage drop across the capacitor bank–MOV branch must be reconstructed by numerically emulating its differential equation, with this compensation applied after the distance has been determined.

The novelty of the work is the combination of multiple signal-processing stages in a two-terminal incremental-quantity framework for fault location on series-compensated transmission lines. The proposed workflow explicitly separates the initial distance-to-fault estimation, subsample-level time alignment, nonlinear emulation of the series capacitor–MOV assembly, and residual-based diagnosis. Specifically, after fixing the fault distance, the following processing stages are constrained such that no correction step can change the estimated fault location to reduce the fitting errors. Both effects are present in the residual mainly as a reactive component, but the subsample timing offset between the two relays is parameterized independently of the capacitor–MOV nonlinearity. The series-compensation bank response is simulated in the time domain numerically from its differential equation, fed by the measured current, and is realized only after the fault distance is determined. The reactive part of the residual is then employed as a diagnostic metric for distinguishing between an unreliable distance estimate and a correct distance with limited achievable

accuracy due to a mismatch in the compensation-bank model. It is implemented only on COMTRADE records of traditional protective relays and does not require GPS synchronization nor megahertz sampling rates.

1.6. Contributions

The main contributions of this study are described as follows:

- A fault-location procedure is proposed for compensated lines. The procedure is organized into six phases, in which the fault-location distance is determined analytically and is fully specified. Subsequently, all further corrections, including sub-sample time alignment and nonlinear bank emulation, are applied while keeping this distance invariant. Consequently, since the distance is no longer treated as a variable within the optimization process, no subsequent correction can modify the estimated fault location to reduce the residual.
- A time-domain emulation of the capacitor bank and MOV is conducted. It is obtained by integrating $C dv/dt = i_{\text{line}} - i_{\text{MOV}}(v)$ using the standard zinc-oxide characteristic, driven by the current measured at the terminal where the capacitor bank is installed. Evidence is provided that its internal signals reproduce the pulse pattern of the varistor and the clipped capacitor current shown by the actual operating oscillograms of the capacitor bank.
- A correction of the sub-sample time offset between terminals caused by the different group delays of the relays was performed. Moreover, this effect can be distinguished from the nonlinearity of the MOV; thus, two previously conflated phenomena, formerly aggregated into a single reactive residual, are treated as separate components. The detailed development case is reported through raw and final values. The controlled alignment analysis further defines the range over which this correction can be interpreted.
- A quality assessment based on the reactivity of the residual is provided. This classifies a remaining fitting error as a physical limit of the compensation bank model rather than as evidence of an incorrect fault location. It is validated on a record whose residual survives the full nonlinear emulation while being 99% reactive, and whose estimated distance nevertheless remains within the expected tolerance.
- Two rejected methodologies and a control experiment are analyzed, demonstrating that minimization of the fitting error does not produce an accurate distance estimate when the fault location is treated as a free parameter during optimization. Instead of yielding consistent or accurate distances, these approaches systematically biased the inferred location to compensate for unmodeled capacitor-bank nonlinearities. These findings indicate that, for the records analyzed, distance estimation should not be based solely on error minimization.
- The method is applied to 18 paired COMTRADE records from three transmission lines. Quantitative distance errors are reported for the 10 records with reference distances, seven from a series-compensated line and three from an uncompensated line, both with and without the record used during method development. The remaining eight records have no reference distance and are used only in the sensitivity analysis and implementation checks.

1.7. Organization

The remainder of this paper is organized as follows. Section 2 presents the background for fault location in series-compensated lines with MOV, covering incremental quantities and replica current, positive-sequence two-phase loops, the LSE estimator and its fitting error, first-order correction of capacitive voltage drop, the origin of subsample terminal time

offsets, and the differential equation emulating the nonlinear capacitor–MOV configuration. Section 3 describes the six-phase procedure for distance calculation, correction, and residual reactivity assessment. Section 4 evaluates 18 paired COMTRADE records from three transmission lines. The quantitative location errors were calculated for the 10 records with reference distances, and one selected record was examined in detail to analyze the residual behavior. Section 5 examines the impact of series compensation and time misalignment on the fault location, interprets the fitting error and its residual, compares the proposed method with previous techniques for compensated lines, and outlines the limitations. Finally, Section 6 presents the conclusions and suggestions for future research.

2. Background

2.1. System Model and Incremental Formulation

The method developed in this study considers a transmission line with measurements available at the local and remote terminals and with a series capacitive compensation bank installed near the remote end, as shown in Figure 1. The line has a total length L , and the fault location is represented by the normalized variable $m \in [0, 1]$, which is measured from the local terminal. Therefore, the distance between this terminal and the fault point is mL , whereas the remaining section toward the remote terminal has a length of $(1 - m)L$. The incremental voltage and current quantities available at both ends are denoted as $(\Delta v_L, \Delta i_L)$ and $(\Delta v_R, \Delta i_R)$, respectively.

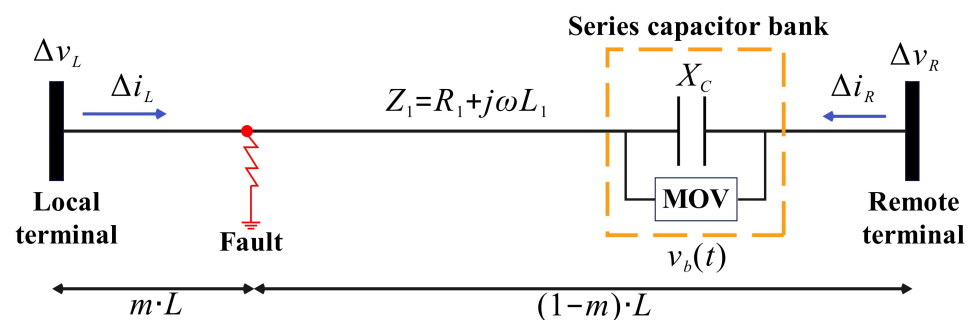


Figure 1. Double-terminal line model used for the formulation of the locator. The impedance Z_1 represents the longitudinal behavior of the line, while the capacitor and MOV constitute an additional lumped element near the remote terminal.

To represent the longitudinal voltage drop between the terminals and fault point, the line was modeled using its positive-sequence impedance, as defined in Equation (1).

$$Z_1 = R_1 + j\omega L_1. \quad (1)$$

Here, Z_1 , R_1 and L_1 correspond to the positive sequence impedance, resistance, and inductance, respectively. The term $\omega = 2\pi f_n$ represents the angular frequency associated with the nominal frequency f_n . This representation characterizes the longitudinal $R\sim L$ behavior of a line. The drop introduced by the compensation bank constitutes an additional lumped element and is therefore not included in Z_1 .

The formulation was developed directly from the signals recorded at both ends. To attenuate the influence of the pre-event operating state and isolate the fault-induced variation, incremental quantities were employed [15]. For a generic signal $x(t)$, these quantities are defined by Equation (2).

$$\Delta x(t) = x(t) - x(t - T). \quad (2)$$

The parameter T corresponds to an integer number of fundamental periods and is selected such that the sample $x(t - T)$ remains within the pre-fault condition throughout the analysis window. When the system remains approximately periodic before the disturbance, the difference between the two signals is small; after the occurrence of the fault, $\Delta x(t)$ mainly preserves the variation produced by the event. In this manner, the incremental voltage and current quantities are calculated and subsequently used to reconstruct the fault location.

The incremental current does not directly represent the longitudinal voltage drop along the line. A replica current was used to incorporate the resistive and inductive components of the model. Its formulation is presented in Equation (3).

$$\Delta i_Z = k_R \Delta i + k_L \frac{d(\Delta i)}{dt}, \quad k_R = \frac{R_1}{|Z_1|}, \quad k_L = \frac{L_1}{|Z_1|}. \quad (3)$$

The coefficients k_R and k_L normalize the resistive and inductive contributions of the line. By multiplying Δi_Z by $|Z_1|$, the drop $R_1 \Delta i + L_1 d(\Delta i)/dt$ is recovered; thus, this variable reproduces the R–L behavior in the time-domain simulation, sample by sample. This function enables the direct use of oscillograms, thereby eliminating the need for prior estimation of the fundamental phasor.

Finally, the previous quantities were arranged in three loops: AB, BC, and CA. The corresponding incremental voltages are defined by Equation (4).

$$\begin{aligned} \Delta v_{AB} &= \Delta v_A - \Delta v_B, \\ \Delta v_{BC} &= \Delta v_B - \Delta v_C, \\ \Delta v_{CA} &= \Delta v_C - \Delta v_A. \end{aligned} \quad (4)$$

These differences were applied analogously to the incremental and replica currents. The use of phase–phase loops eliminates the zero-sequence component common to the phases and allows the locator to be formulated without directly incorporating the zero-sequence impedance Z_0 or the compensation factor required by the phase–ground loops. In addition, the three loops are processed simultaneously; therefore, the execution of the method does not depend on the fault type being identified beforehand. The quantities obtained at this stage constitute the input variables for the reconstruction of the voltage at the fault point and the analytical estimation of its location.

2.2. Analytical Estimation of Distance

Once the incremental quantities and replica currents are established, the voltage at the fault point can be reconstructed independently of both terminals. For a normalized position m , the expressions used to perform these two reconstructions are given by Equation (5).

$$\begin{aligned} \Delta v_{FL} &= \Delta v_L - m|Z_1|\Delta i_{ZL}, \\ \Delta v_{FR} &= \Delta v_R - (1 - m)|Z_1|\Delta i_{ZR}. \end{aligned} \quad (5)$$

The terms Δv_{FL} and Δv_{FR} represent the estimates of the voltage at the same electrical node obtained from the opposite ends of the line. Under ideal conditions, both the reconstructions must coincide. By enforcing this equality and rearranging the terms, each sample in the window can be expressed as a linear relation of the form $A_k = mB_k$.

From this relation, the terms used by the estimator and the analytical LSE solution are defined by Equation (6).

$$\begin{aligned} A_k &= \frac{\Delta v_{L,k} - \Delta v_{R,k}}{|Z_1|} + \Delta i_{ZR,k}, \\ B_k &= \Delta i_{ZL,k} + \Delta i_{ZR,k}, \\ m &= \frac{\sum_k A_k B_k}{\sum_k B_k^2} = \frac{S_N}{S_D}. \end{aligned} \quad (6)$$

The term A_k combines the difference of the incremental terminal voltages with the remote replica current, while B_k brings together the replica currents observed from both ends. Because all the samples within the window share a single unknown m , the problem becomes an overdetermined system. The LSE solution uses $S_N = \sum_k A_k B_k$ as the sum of products between the two series and $S_D = \sum_k B_k^2$ as the quadratic sum of B_k . Their ratio yields the fraction of the line length that provides the best joint match between the measurements at both terminals. Finally, the physical distance from the local terminal is obtained as $d = mL$.

The analytical estimate is used as the initial distance for the subsequent stages. It should not be interpreted as a guarantee of global best result or as an exact elimination of all parameter uncertainty. Errors in the line model, the capacitive approximation, or the measured signals may affect this initial estimate and may remain in the result.

The fault resistance R_f does not appear explicitly in the analytical solution. This property arises from the double-terminal formulation: by equating the two reconstructions of the fault point, the common term associated with the voltage at the node is eliminated algebraically. Therefore, this behavior must be distinguished from the effect of the incremental quantities, whose main function is to reduce the pre-fault electrical conditions.

The estimated position was complemented by an indicator that quantified the correspondence between the local and remote reconstructions. This measure of consistency is defined by Equation (7).

$$\varepsilon = \frac{\|\Delta v_{FL} - \Delta v_{FR}\|_2}{\|\Delta v_{FL}\|_2 + \|\Delta v_{FR}\|_2} \times 100\%. \quad (7)$$

The numerator represents the magnitude of the difference between the two reconstructions, whereas the denominator normalizes this value with respect to the magnitudes of both signals. Consequently, a small value of ε indicates greater consistency in the estimation of fault location. However, this indicator is not directly interpreted as a measure of distance uncertainty because the discrepancy may also include contributions associated with series compensation and time alignment of the records.

2.3. Representation of the Capacitor–MOV Bank

When the compensation bank is electrically located between the remote terminal and fault point, the voltage recorded at that end includes an additional drop that does not belong to the longitudinal model of the line. As a first approximation, this contribution was reconstructed from an incremental remote current. The linear correction applied before evaluating the fault-location technique is defined in Equation (8).

$$\Delta v_R^{\text{corr}}(t) = \Delta v_R(t) - \omega X_c \int \Delta i_R(t) dt. \quad (8)$$

This relation is obtained from $i = C dv/dt$ and $C = 1/(\omega X_c)$. The voltage Δv_R^{corr} later replaces the original remote recording during the reconstruction of the fault point.

Therefore, the correction does not introduce a new locator; it only modifies the voltage that enters the same analytical formulation developed previously.

The reactance used during this correction was restricted to the interval $0 \leq X_c \leq X_{c,\text{rated}}$. The upper limit corresponds to the rated reactance of the capacitor when the MOV does not reduce its contribution. When the varistor starts conducting, a part of the total current is diverted from the capacitive branch, and the effective contribution of the capacitor decreases.

The linear approximation enables the subtraction of the dominant capacitive contribution. However, it does not reproduce the instantaneous distribution of the current between the capacitor and MOV. To represent this condition, the nonlinear characteristics of the zinc-oxide MOV were modeled using Equation (9) [2,3].

$$i_{\text{MOV}}(v) = I_{\text{ref}} \text{sign}(v) \left(\frac{|v|}{V_p} \right)^q. \quad (9)$$

The parameter V_p represents the protective level associated with the reference current I_{ref} , and q determines the slope of the conduction region. For voltages below the protective level, the MOV conducts a small current, and most of the current flows through the capacitor. As the voltage increases, the varistor progressively diverts a larger fraction of the line current away from the series capacitor.

As the exact protective voltage and nonlinear exponent are generally not available in standard line databases, the parameters V_p and q are treated as equivalent model parameters. Their values are obtained through a bounded search intended to reproduce the measured response. This search does not constitute an exact physical identification of the internal MOV structure.

Because the capacitor and MOV were connected in parallel, the current that charged the capacitor corresponded to the difference between the total current and the portion diverted by the varistor. The complete dynamics used to reconstruct the drop across the bank and return it to the incremental formulation are presented in Equation (10).

$$\begin{aligned} C \frac{dv_b(t)}{dt} &= i_{\text{line}}(t) - i_{\text{MOV}}(v_b), & C &= \frac{1}{\omega X_c}, \\ \Delta v_b(t) &= v_b(t) - v_b(t - T), \\ \Delta v_R^{\text{corr}}(t) &= \Delta v_R(t) - \Delta v_b(t). \end{aligned} \quad (10)$$

The first relation reconstructs the instantaneous voltage $v_b(t)$ from the total current measured at the compensated terminal. The second transforms this response into the same incremental representation used by the locator, and the third provides the corrected remote voltage. The latter is used again in the reconstruction of the fault point, so that both the linear correction and nonlinear emulation converge to the same estimator and differ only in the way the bank drop is represented.

2.4. Subsample Time Misalignment

The double-terminal formulation additionally requires that the signals recorded at both ends correspond to the same time. Even when the devices share a common time reference, differences between their acquisition, filtering, and processing chains can introduce small residual shifts [14,15].

Let τ be the shift expressed in samples, and $\delta t = \tau / f_s$ its time-domain equivalent. For small values of δt , the shifted signal can be approximated as $x(t - \delta t) \approx x(t) - \delta t \, dx(t) / dt$. In a sinusoidal signal, the derivative term appears approximately in quadrature with the original signal. Therefore, a small time difference can increase the discrepancy between the reconstructions of the fault point without necessarily producing an equivalent error in

the distance. In compensated lines, this effect is particularly relevant because their reactive nature can be confused with the voltage drop introduced by the capacitor.

Because τ does not necessarily correspond to an integer number of samples, time correction requires estimating the value of the signal at the intermediate positions. The linear interpolation used for this purpose is defined in Equation (11).

$$x_\tau[k] = (1 - \alpha)x[n] + \alpha x[n + 1], \quad n = \lfloor k - \tau \rfloor, \quad \alpha = (k - \tau) - n. \quad (11)$$

The index n identifies the sample immediately preceding the required instant, whereas α determines its relative position within the interval between $x[n]$ and $x[n + 1]$. Thus, the shift can be corrected with a resolution finer than that of the sampling period. This operation is applied after the distance is established to prevent time alignment from using the fault position as an additional degree of freedom.

The controlled nominal alignment window used in this study is limited to ± 2 samples, whereas the field records are processed with $\tau_{\max} = 3$ samples. This distinction is made explicit because neither setting guarantees robustness to larger timing errors or non-uniform relay filtering. The controlled stress analysis evaluates ± 4 and ± 8 samples.

3. Materials and Methods

The method was implemented as a stepwise procedure in the time-domain simulation using the COMTRADE records obtained from both terminals. The sequence was designed to separate the three phenomena that can modify the correspondence between the reconstructions of the fault point: position, time alignment of the measurements, and voltage drop produced by the capacitor bank–MOV combination.

Figure 2 summarizes the complete architecture. Stages A and B prepare the signals required by the estimator, Stage C determines and fixes the distance, and Stage D corrects the time correspondence while maintaining a constant position. Additionally, Stage E replaces the linear approximation of the bank with its dynamic representation, and Stage F interprets the consistency of the result and selects the most appropriate analysis window when both conditions exist. The fault location is set before the time and nonlinear corrections. After the distance-fixing stage, m_0 remains outside the search space, and the subsequent stages modify only the time correspondence and bank representation. The last block corresponds to the reported final result: the distance obtained in the analysis window (fault or bypass) with the lowest ε , together with ε and the residual diagnosis. Additionally, X_c^* represents the reactance, τ^* is the time shift, and d_{w^*} is the distance obtained in the analysis window.

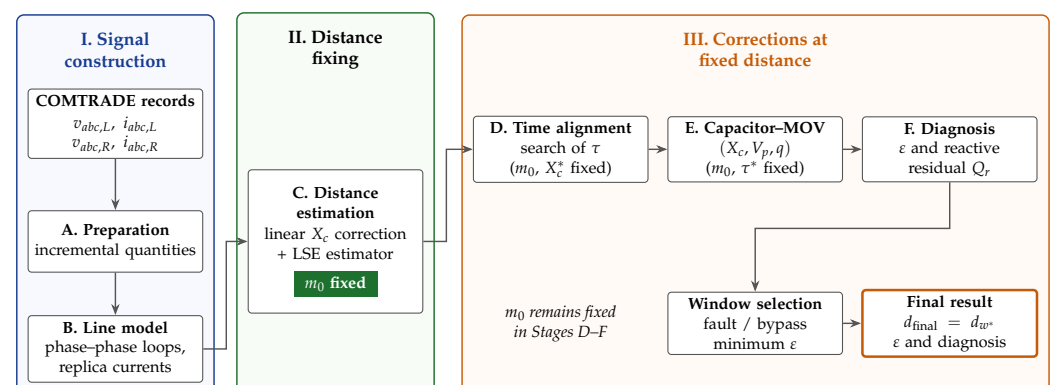


Figure 2. Proposed methodological sequence.

3.1. Preparation and Validation of the Signals

The input data correspond to the COMTRADE records [21] acquired from local and remote terminals. For each event, the three-phase voltages and currents, sampling frequency f_s , nominal frequency f_n , and time information available in both devices were used as inputs for subsequent processing and analysis. The procedure also requires the line length L , parameters R_1 and L_1 , location of the bank, and rated reactance $X_{c,\text{rated}}$.

Before running the locator, it is verified that the channels at both ends are expressed on the same physical base. To this end, the units, polarities, transformation ratios, and declaration of primary or secondary values contained in the .CFG files are checked. This verification is part of the procedure because the COMTRADE standard defines the structure used to exchange the records, but does not guarantee that different devices automatically store their signals using the same electrical scale.

Once the data were validated, stages A and B in Figure 2 were used to construct the incremental quantities, phase-phase loops, and replica currents developed in Section 2.1. The delay T is selected as an integer number of cycles that maintains the incremental reference within the pre-fault condition throughout the analyzed time window. The time derivative required by the replica current was evaluated using central differences. The procedure was implemented using Python 3.14.6 and NumPy 2.5.3.

The records had a single sampling frequency and continuous sample numbering; therefore, no treatment of missing samples was necessary. Second-order central differences were used for the interior samples, and first-order one-sided differences were used only for the two end samples of the processed segment. This segment extends up to six cycles before and after each analysis window, so that these end samples and the start of the numerical integration lie outside the interval over which ε and the distance are evaluated. The resulting signals constitute the input to the estimator developed in Section 2.2. The effect of T was evaluated using six values expressed in cycles: 0.5, 1, 2, 3, 4, and 6. The sensitivity analysis produced 108 configurations based on the available records. The purpose of this analysis was to determine whether the selected delay produced a stable behavior over several events rather than an isolated minimum for a single record.

3.2. Stage C: Distance Fixing

As a reference for the effect introduced by the compensation, each record was initially processed without correcting for the bank drop. This run provided a rough estimate that was preserved solely for later quantification of the improvement produced by the complete procedure.

The fault-location technique uses the linear bank correction developed in Section 2.3. In the implementation, the current integral was computed using the trapezoidal rule to maintain a numerical approximation consistent with that used later during the dynamic reconstruction of the bank.

The numerical integration, selection of the effective reactance, and definition of the distance fixing are summarized in Equation (12).

$$\begin{aligned} \mathcal{I}[k] &= \mathcal{I}[k-1] + \frac{\Delta t}{2} [\Delta i_R[k-1] + \Delta i_R[k]], & \Delta t &= \frac{1}{f_s}, \\ X_c^* &= \arg \min_{0 \leq X_c \leq X_{c,\text{rated}}} \{m(X_c), X_c\}, \\ m_0 &= m(X_c^*), & d_0 &= m_0 L. \end{aligned} \quad (12)$$

The first relation in Equation (12) computes the cumulative integral of the incremental current using the trapezoidal rule. For each candidate value of X_c , this integral is used to reconstruct the capacitive drop and obtain the corrected remote voltage. This signal replaces

the original remote voltage during the reconstruction of the fault point and calculates a unique analytical distance $m(X_c)$.

The second relation selects the reactance that yields the greatest consistency between the two reconstructions within a physically admissible interval. Finally, the third relation determines m_0 and the distance d_0 . Therefore, the search is performed over X_c and not over an independent distance. After determining m_0 , this variable is removed from the search space in all subsequent stages. This requirement is hereafter denoted as the fixed-distance constraint. However, fixing m_0 after the analytical estimation does not provide a universal guarantee that the initial distance is unbiased. This prevents the subsequent alignment and capacitor–MOV stages from compensating for the modeling errors by freely moving the fault location; however, any error introduced during the initial estimation may remain in the result. Therefore, d_0 is treated as a staged estimate whose validity must be evaluated together with the available reference distance and the sensitivity analyses.

3.3. Stage D: Time Alignment

Once the position is established, Stage D corrects the residual time difference between the recordings at both ends of the line. The remote signals were shifted using the previously defined subsample interpolation within a bounded interval around the nominal time. For the field records, the nonlinear procedure used $\tau_{\max} = 3$ samples with 31 mesh points (a step of 0.2 samples). The controlled alignment analysis used $\tau_{\max} = 2, 4$, and 8 samples to test the effect of widening the search interval. As the shift changes the phase relationship between the current and the linear capacitive contribution, the search is performed using the reactance X_c^* obtained in Stage C, and the capacitive contribution is reassessed in Stage E using the nonlinear model of the capacitor bank. The fault position remained fixed at $m = m_0$. Consequently, a reduction in the fitting residual during this stage is interpreted as an improvement in time correspondence and not as evidence that the initial fault position has been independently re-estimated. The search used to determine the time alignment is defined by Equation (13).

$$\tau^* = \arg \min_{-\tau_{\max} \leq \tau \leq \tau_{\max}} \varepsilon(m_0, \tau, X_c^*), \quad \tau_{\max} = 3 \text{ samples (field records)}. \quad (13)$$

The term τ^* represents the time shift that produces the greatest correspondence between both records under the constraint $m = m_0$ and the reactance X_c^* fixed in Stage C. Consequently, any reduction in ε obtained during this stage corresponds to an improvement in the time correspondence and not a new estimate of the fault location. The search interval is an explicit implementation parameter; it does not guarantee the recovery of shifts larger than $\tau_{\max}$ or the nonuniform delays produced by relay filtering. In addition, as m_0 remains fixed during this stage, the alignment cannot correct the effect that a time offset present in the records has had on the distance estimated in Stage C; such an offset must be removed before that stage.

3.4. Stage E: Emulation of the Nonlinear Bank

After determining τ^* , the linear representation used during distance fixing is replaced with the dynamic model of the capacitor–MOV set. For each candidate combination (X_c, V_p, q) , the total measured current is used to reconstruct the instantaneous voltage of the bank and subsequently obtain its incremental contribution to the remote voltage.

The resulting corrected voltage is then used in the same remote reconstruction of the fault point. Thus, the estimator did not change between stages C and E and only changed the model used to represent the compensator drop. The bank dynamics of Equation (10) were integrated with the implicit Euler method using four substeps per sampling interval, $h = 1/(4f_s)$, with the measured current linearly interpolated at the midpoint of each

substep. At each substep, the implicit equation $(v_b^j - v_b^{j-1})/h = \omega X_c [i_{\text{line}}^{j-1/2} - i_{\text{MOV}}(v_b^j)]$ was solved with ten Newton–Raphson iterations. To ensure numerical stability with high exponents, each Newton increment was limited to $\pm 0.3V_p$ and the ratio $|v_b|/V_p$ was capped at 2 in Equation (9). The capacitor voltage is initialized as $v_b(t_0) = \omega X_c Q(t_0)$, where Q is the trapezoidal integral of the measured current after subtracting its mean value over the first cycle; this corresponds to the steady-state voltage of the linear capacitor, for which the MOV does not conduct, and prevents drift in the reconstructed voltage.

During this stage, m_0 and τ^* were fixed. The nonlinear bank model is evaluated conditional on the distance obtained during Stage C. Thus, the nonlinear emulation cannot remove a possible bias introduced during the initial distance estimation by shifting the fault location to reduce ε . The equivalent bank parameters are defined by Equation (14).

$$(X_{c,E}^*, V_p^*, q^*) = \arg \min_{(X_c, V_p, q) \in \mathcal{G}_{\text{MOV}}} \varepsilon(m_0, \tau^*, X_c, V_p, q). \quad (14)$$

The domain $\mathcal{G}_{\text{MOV}}$ contains only the physically admissible combinations of X_c , V_p , and q . Because distance does not belong to this domain, the greater flexibility of the nonlinear representation can improve the reconstruction of the voltage drop of the capacitor bank and reduce the discrepancy between the terminals; however, it cannot modify the previously established location. The optimization problems defined in Equations (12)–(14) are solved via exhaustive evaluation over finite parameter grids. Consequently, no iterative optimization procedure or stopping criterion is required, and the adopted estimate corresponds to the grid point that minimizes ε . In Stage C, X_c is discretized into 20 uniformly spaced values over $[0, X_{c,\text{rated}}]$, corresponding to an increment of approximately 2.6Ω on line A. In Stage D, τ is sampled at 31 uniformly spaced values over the interval $[-3, 3]$. The nonlinear grid $\mathcal{G}_{\text{MOV}}$ is constructed as the Cartesian product of three candidate values of X_c , three protection voltage-stress factors in $[0.12, 1.25]$ (normalized by the peak voltage of the linear capacitor), and $q \in \{11, 23, 45\}$. After fixing m_0 and τ^* , the nonlinear stage therefore evaluates 27 parameter combinations. In the execution environment used for this study, the implementation exhibited per-record runtimes ranging from approximately 0.01 to 3.7 s. These timing results are reported solely to support reproducibility and do not constitute a formal real-time performance guarantee.

3.5. Stage F: Diagnosis and Selection of the Results

Once the corrections have been applied, the local and remote reconstructions are compared again and the indicator ε is calculated. Additionally, the residual $r(t) = \Delta v_{FL}(t) - \Delta v_{FR}(t)$ is analyzed with respect to the total replica current $\Delta i_{ZL}(t) + \Delta i_{ZR}(t)$. The implementation determines the fraction Q_r of the residual associated with a predominantly quadrature component. When $Q_r > 70\%$, the remaining discrepancy is classified as predominantly reactive. This classification indicates an incompatibility between the implemented model and the measured signals, but it does not uniquely identify the MOV as the physical cause of the residual.

As this classification does not modify m_0 , it separates the estimated fault location from the ability of the model to reproduce the measured waveforms. A high value of ε can coexist with an adequate distance when the residual retains an identifiable physical structure after the corrections.

For the events on the compensated line, the fault window and, when available, the post-bypass window were processed independently. The former includes the response of the capacitor–MOV bank, whereas the latter involves short-circuiting the capacitor by its protection, and the line once again approaches the longitudinal model.

Each condition produces an independent solution (d_w, ε_w) . The criterion used to select the result is defined by Equation (15).

$$w^* = \arg \min_{w \in \{\text{fault}, \text{bypass}\}} \varepsilon_w, \quad d_{\text{final}} = d_{w^*}. \quad (15)$$

The term w^* identifies the window that shows the smallest discrepancy between the reconstructions after completing the entire procedure, whereas d_{final} corresponds to the distance calculated for that condition. The comparison is performed only among previously obtained solutions; therefore, it does not constitute a new optimization of m .

For the uncompensated line, only the fault window was considered in this study. In these cases, the stages related to the capacitor–MOV representation were omitted; however, the incremental quantities, replica current, analytical estimator, and time alignment were retained in the model.

4. Results and Analysis

4.1. Case Study

The 11 numbered cases listed in Table 1 correspond to two lines with different characteristics. The validation set considered two lines with different characteristics. The first is line A, which has a length of 194.16 km and includes a series capacitive bank protected by an MOV. The second is line B, which has a length of 84.61 km and does not have series compensation. For these two transmission lines, the recorded events include one-phase, two-phase, two-phase-to-ground, three-phase, and single-phase impedance faults.

Table 1. Numbered cases. Line A with compensation (194.16 km) and line B without compensation (84.61 km). Case 3 could not be matched unambiguously to any available COMTRADE record and is therefore not evaluated.

Case	Line	Fault	Guide (km)	Validation
1	A	1F	149.40	Remote fault with the bus energized; evaluate the procedure under this condition.
2	A	2F	5.42	Fault close to the compensation bank; case used for method development.
3	A	2F	20.86	Recording from the opposite end; record not identified among the available files, not evaluated.
4	A	1F	102.54	Distant single-phase fault; compare the behavior between the fault windows and the bypass.
5	A	1F	135.40	Intermediate distant single-phase fault; both windows yield errors above 5 km.
6	A	1F	186.90	Fault close to the remote terminal; the behavior at one end of the line is evaluated.
7	A	1F	167.77	Distant single-phase fault; the fault window bias is reduced during bypass.
8	A	1F	139.00	Intermediate single-phase fault; presents high accuracy during bypass.
9	B	3F	58.77	Three-phase fault without compensation; it highlights the importance of signal scaling.
10	B	2FT	31.06	Two-phase-to-ground fault; evaluates a fault condition with grounding.
11	B	1F	44.90	Single-phase fault with impedance ($R_f = 20 \Omega$); evaluates the behavior with R_f .

The dataset contains 18 paired COMTRADE records, each including the recordings from both terminals: nine from line A, three from line B, and six from a third line without series compensation (line C, 135.67 km). Table 1 lists 11 numbered cases, eight on line A and three on line B. Ten of these cases (Cases 1, 2, and 4–11) were matched unambiguously to one of the 18 records and form the set used for the location-error metrics. Case 3 could not be matched unambiguously to any available record; therefore, it is listed for completeness but not evaluated. The remaining eight records, two from line A and six from line C, are not numbered and have no reference distance; they are used only in the sensitivity analysis of Section 4.8 and in implementation checks. In addition, Case 2 was used during the development of the method; therefore, the metrics are reported both for the 10 records and for the nine records not used during development.

Table 1 shows that eight of the 11 cases correspond to the series-compensated line, whereas three correspond to the uncompensated line. The listed locations for line A range from the fault close to the compensation bank in Case 2 (5.42 km) to the fault close to the remote terminal in Case 6 (186.90 km). The table also includes single-phase, phase-to-phase, phase-to-phase-to-ground, and three-phase faults. This range documents variation in fault type and location, although the three cases on line B provide a smaller basis for comparison between compensated and uncompensated conditions.

The results were analyzed considering the estimated fault distance d and compared with the reference location available for each event. In addition, the fitting error ε was considered, which is defined as the difference between the two reconstructions of the incremental voltage at the fault point obtained from the local and remote terminals. The first metric assesses the concordance between the estimated location and available reference data, whereas the second metric quantifies the consistency between two independent estimates of the same electrical point.

Two periods were analyzed for the events created in the transmission line with compensation. The first period corresponds to the time when the short-circuit event remains present, and the compensation bank is connected. Under these conditions, the voltage drop across the capacitor and MOV assembly is part of the measured signals and can directly affect the reconstruction of the fault point. The second period is the bypass window, which is the interval following the operation of the capacitor bank protection when the capacitor is bypassed. Under this condition, the line once again approaches the longitudinal model $R_1 + j\omega L_1$ used by the replica current. For line B, as there was no series compensation, only the window corresponding to the fault was considered in the analysis.

The results are presented in the same sequence as defined in the procedure introduced in Section 3. Case 2 was first analyzed, as it was used during the development of the method to observe the contribution of each correction factor. Case 1 was analyzed to study a condition in which the distance was suitable, even though a predominantly reactive residual remained. Finally, the systematic results obtained for the lines with and without compensation are presented, along with a synthesis of the complete set.

4.2. Development Case: Evolution of Fault Location and Fitting Error

Case 2 corresponds to a two-phase fault located 5.42 km from the local terminal of line A, while the compensation bank remained in service. This event was used during the development of the method because it simultaneously encompasses the main difficulties considered, such as the voltage drop associated with the series bank, nonlinear conduction of the MOV, and time discrepancy between the records acquired at both ends.

Figure 3 shows the three-phase voltages and currents recorded during the event. At the fault event, a pronounced reduction in voltages and an increase in currents are observed. These variations constitute the information later isolated by means of the incremental quantities and used by the fault-location technique.

As an initial condition, the LSE estimator was applied directly to the fault window without correcting for the contribution of the capacitor bank and without performing time alignment between the records. Under these conditions, a distance of 12.88 km and a fitting error of $\varepsilon = 30.8\%$ were obtained. The difference with respect to the reference was therefore +7.46 km. This result corresponds to the behavior of the estimator when the replica $R_1 + j\omega L_1$ attempts to reconstruct a line in which the measurement includes an additional voltage drop associated with the compensation bank.

Subsequently, the linear capacitive correction used during the distance-fixing stage was applied to the model. The physical sweep of X_c produced a distance of 4.63 km with $\varepsilon = 20.4\%$. Although the distance was considerably closer to the reference value,

a difference remained between the two fault voltage values. The next stage involved correcting the time shift between the records. By keeping the distance fixed at 4.63 km, the adjustment of τ reduced ε from 20.4% to 9.8%. As the distance was not modified during this stage, the observed reduction in the fitting error can be attributed to a better time correspondence between the signals and not to an additional shift of the fault point.

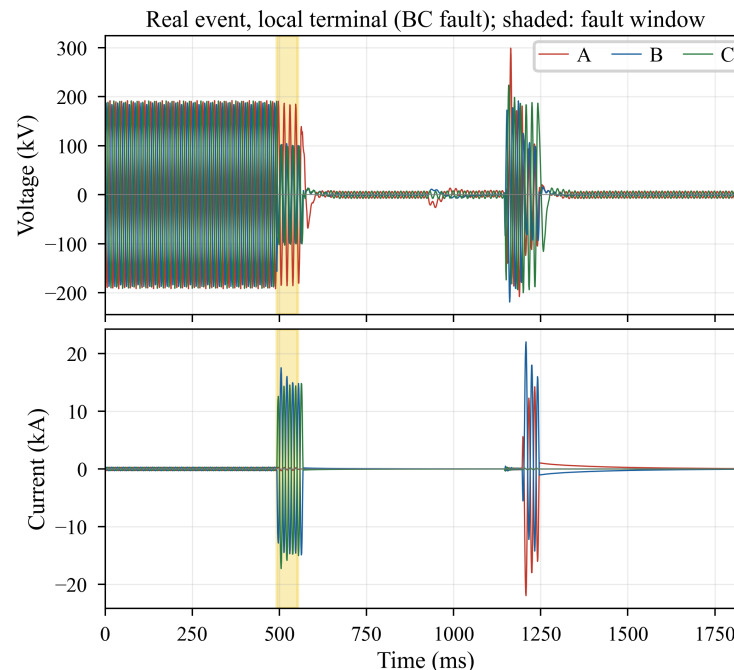


Figure 3. Three-phase voltages and currents recorded at the local terminal during Case 2.

For the implementation of the case under analysis, the capacitive term was recalculated using the trapezoidal integration adopted for the dynamic emulation of the capacitor bank. This update established a corrected distance of 5.05 km. From this value, the nonlinear emulation of the capacitor and MOV set was applied, finally obtaining $\varepsilon = 5.9\%$. The estimated distance was 0.37 km below the reference of 5.42 km. Therefore, the full evolution of Case 2 is from 12.88 km, 30.8% to 4.63 km, 20.4%, then to 4.63 km, 9.8%, and ends at 5.05 km, 5.9%.

Within this sequence, it can be observed that the different stages do not affect the same phenomena. The capacitive correction mainly modifies the initial fault location, time alignment reduces a part of the residual while keeping the distance fixed, and nonlinear emulation reduces the remaining discrepancy associated with the dynamic behavior of the capacitor bank.

The raw LSE estimator was first applied to the fault window without correcting the contribution of the capacitor bank or the time correspondence between the records. A distance of 12.86 km and a fitting error of $\varepsilon = 30.90\%$ were obtained, compared with the reference distance of 5.42 km. The complete staged procedure subsequently applied the prescribed capacitive correction, time alignment, and capacitor–MOV emulation. The final selected distance was 5.11 km, with $\varepsilon = 7.54\%$ and an absolute distance error of 0.31 km. The selected time shift was $\tau = 1.4$ samples. The initial estimate was retained as an auditable result and was not replaced by an unconstrained joint minimization of distance and MOV parameters. Consequently, the change from 12.86 to 5.11 km represents the effect of the prescribed correction stages, whereas the fitting error is evaluated after the distance-estimation stage. This separation reduces the possibility that the distance is shifted merely to obtain a smaller residual.

Figure 4 compares Δv_{FL} (local voltage) and Δv_{FR} (remote voltage) for the raw and final results. In the raw result, the two reconstructed fault voltages showed differences in amplitude and phase. After applying the complete procedure, the signals showed improved agreement. Figure 5 shows the corresponding comparison evaluated at the final selected distance of 5.11 km.

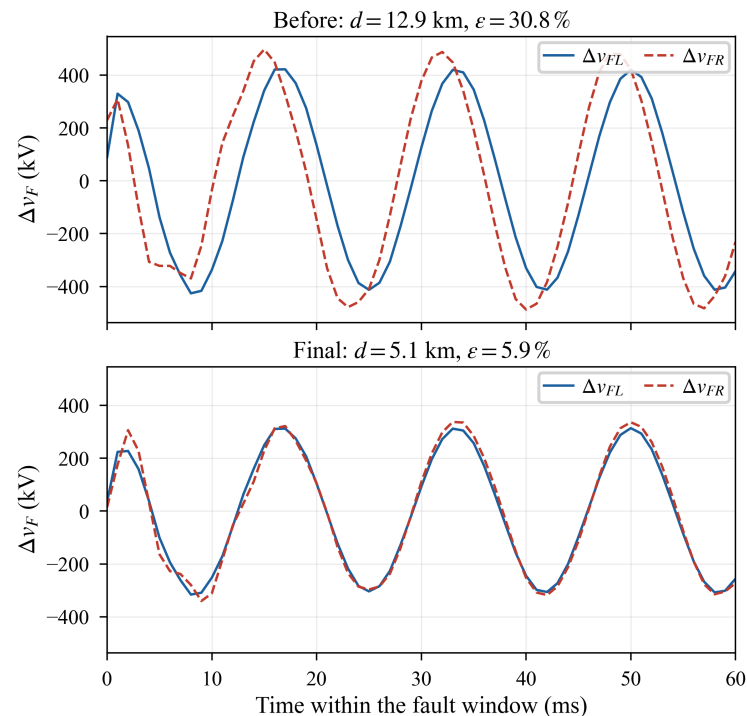


Figure 4. Case 2. Comparison between the raw least-squares error (LSE) result ($d = 12.86$ km, $\varepsilon = 30.90\%$) and the final selected result ($d = 5.11$ km, $\varepsilon = 7.54\%$).

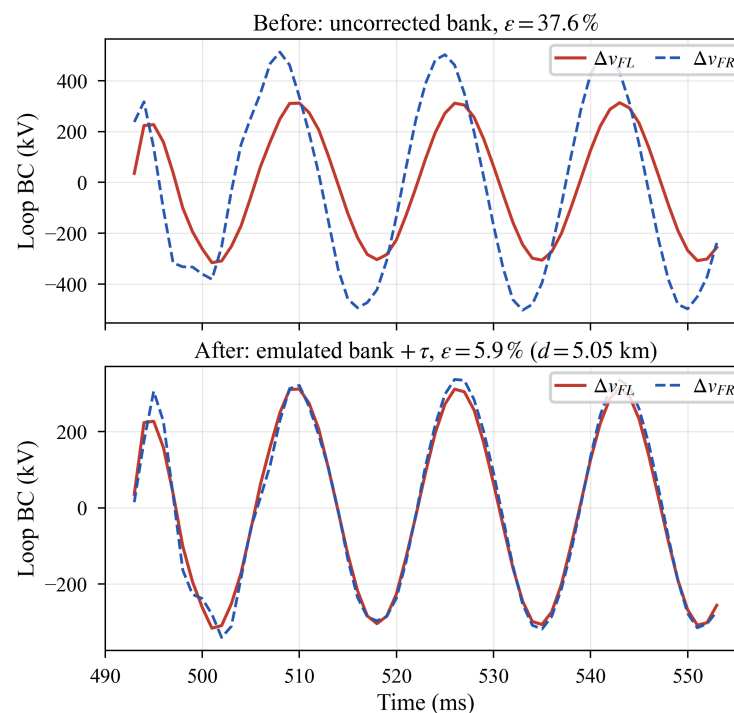


Figure 5. Case 2 evaluated at the final selected distance of 5.11 km before and after applying the final corrections.

As both panels in Figure 5 are evaluated at the same selected distance, the closer agreement between the reconstructed voltages after correction cannot be attributed to a change in fault location. The figure therefore illustrates the combined effect of the final time-alignment and capacitor-bank corrections at a fixed distance. It does not separate the contribution of each correction individually.

In contrast, another important result corresponds to the internal variables reconstructed through the physical emulation of the capacitor bank. In this case, the selected combination was $X_c = 39.6 \Omega$, $V_p = 98 \text{ kV}$, and $q = 11$. Figure 6 shows the calculated voltage for the capacitor bank and the current shared between the capacitor and MOV branches. The shaded area in red represents the fault windows of the Case 2. During the first peaks following a fault event, the capacitor voltage reached the conduction region of the varistor, and current pulses appeared through the MOV. Simultaneously, the current corresponding to the capacitive branch decreased because part of the total current was diverted through the varistor. As the fault current decreased, the conduction of the MOV also decreased, and most of the current flowed through the capacitor. Therefore, this pattern is qualitatively consistent with the behavior observed in the operating oscillograms of the actual bank. Thus, in addition to reducing the fitting error, emulation generates internal signals that are consistent with the expected physical mechanism of a capacitor protected by an MOV.

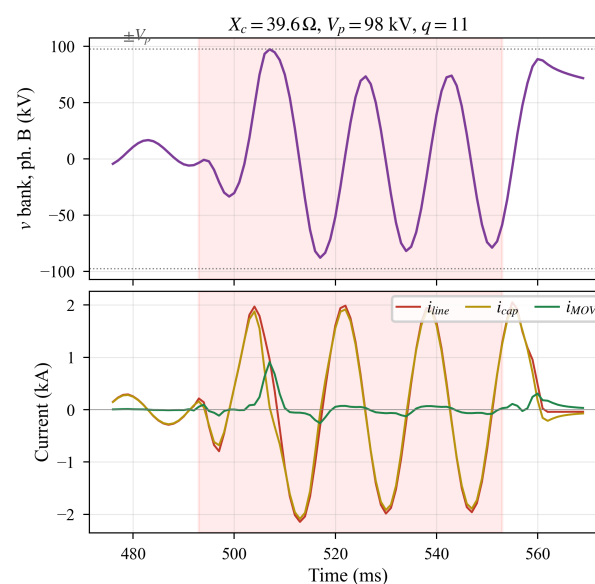


Figure 6. Voltage in the capacitor bank and currents between the capacitor and the MOV for Case 2.

For Case 2, the equivalent fit selected $X_c = 33.0 \Omega$, $V_p = 73.38 \text{ kV}$, and $q = 45$. These values were obtained through a constrained search and should be interpreted as equivalent parameters that reproduce part of the recorded response, rather than as an exact identification of the internal MOV characteristics. The controlled MOV and compensation analysis is presented separately in Section 4.10.

4.3. Reference Record and Residual Behavior

Case 1 corresponds to the field record employed to evaluate scenarios in which the estimated distance closely matches the reference value, while a substantial residual error persists. The reference distance is 149.40 km. The raw LSE estimator yielded 158.60 km with $\varepsilon = 31.61\%$, whereas the final selected distance was 148.55 km, with an absolute error of 0.85 km and a final fitting error of $\varepsilon = 20.37\%$.

Figure 7 shows two reconstructions of the fault voltage for the obtained distance and the evolution of the residual. The reactivity analysis indicated that approximately 99% of the difference $\Delta v_{FL} - \Delta v_{FR}$ was reactive in nature; that is, it was predominantly in quadrature with the fault current. This indicates that the value of ϵ cannot be directly interpreted as a measure of distance uncertainty. In this case, a discrepancy remains between the two reconstructions, even though the computed distance is close to the reference. The predominantly reactive nature of the residual indicates that the remaining difference is consistent with a voltage component that the implemented model of the capacitor bank does not fully reproduce.

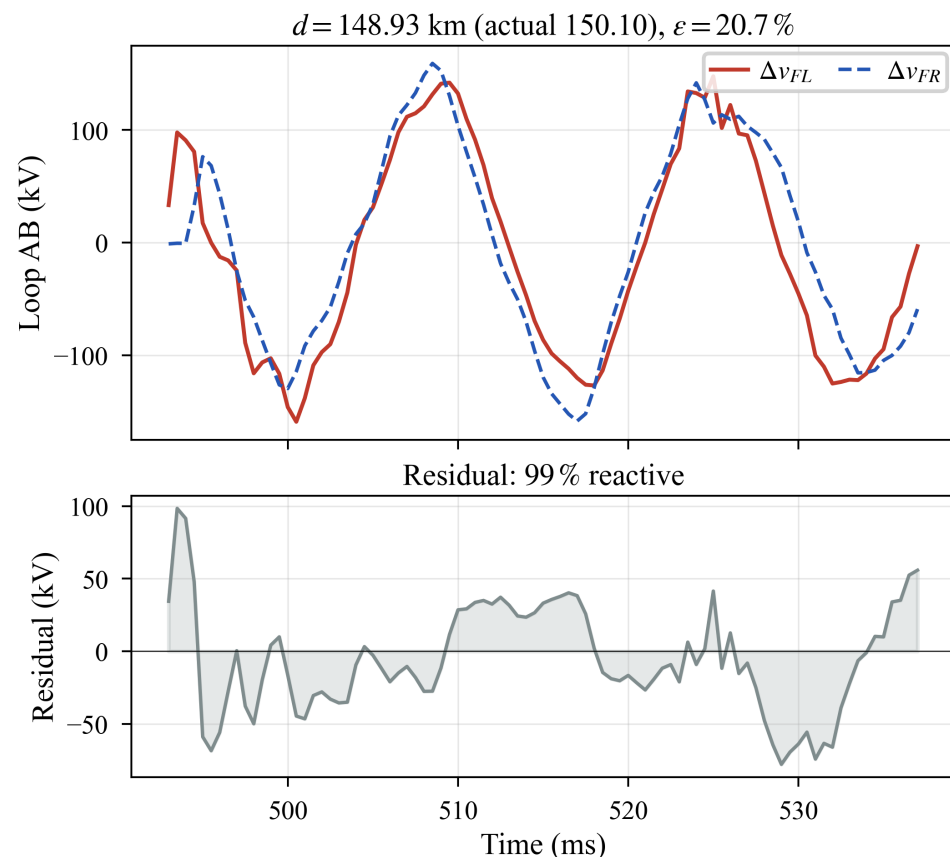


Figure 7. Case 1 evaluated at the final selected distance of 148.55 km. The remaining residual presents a predominantly reactive component.

Therefore, the quality assessment uses both the value of ϵ and the nature of the residuals. When the discrepancy remains after the corrections and shows a dominant reactive component, the result is identified as being limited by the representation of the MOV, and the distance is no longer adjusted to artificially reduce the fitting error.

Therefore, the quality assessment uses both the value of ϵ and the nature of the residual. In Case 1, approximately 99% of the residual was reactive, while the distance remained close to the reference. This result is used as a non-identifying diagnostic and not as proof that the MOV is the unique cause of the residual. In particular, the available records do not allow the effects of the MOV, unmodeled shunt capacitance, line-parameter variations, and measurement-chain uncertainties to be separated independently. Consequently, the distance is not modified solely to obtain a lower fitting error.

4.4. Line with Series Compensation

Once the two cases used to study the main stages of the procedure individually were analyzed, the behavior was evaluated over the eight cases corresponding to line A. Table 2

shows the distance and fitting error obtained using the LSE without corrections, the result during the fault window, and, when available, the result corresponding to the bypass window.

Table 2. Fault location results on line A with series compensation. Distances are expressed in km and ϵ in percent.

Case	Fault	Ref.	LSE Without Correction		Fault Window		Bypass	
			d	ϵ	d	ϵ	d	ϵ
1	1F	149.40	158.60	31.6	148.90	20.7	152.70	21.6
2	2F	5.42	12.88	30.8	5.05	5.9	—	—
3	2F	20.86	17.10	5.9	15.50	5.2	12.90	10.2
4	1F	102.54	94.60	32.9	79.30	16.9	95.60	9.7
5	1F	135.40	165.30	51.0	130.30	10.6	135.50	8.7
6	1F	186.90	188.30	34.2	188.30	29.3	187.50	10.0
7	1F	167.77	171.20	44.6	155.80	27.4	165.80	19.0
8	1F	139.00	160.50	43.4	145.50	20.8	139.50	4.2

The initial behavior showed that the presence of the capacitor bank affected the consistency of the two reconstructions. In seven of the eight cases, the LSE without corrections showed ϵ values between 30.8% and 51.0%. Case 3 was an exception, with an initial error of 5.9%. Considering all eight events, the average value of ϵ was 34.3%. After applying the corrections during the fault window, ϵ decreased in all cases, and its average was reduced to 17.1%. The reduction was particularly observed in Cases 2 and 5. In Case 2, the fitting error decreased from 30.8% to 5.9%, whereas in Case 5, it decreased from 51.0% to 10.6%. Cases 6 and 7 retained higher fitting errors of 29.3% and 27.4%, respectively. This result shows that the magnitude of the residual after the corrections depends on the specific conditions of the event and that the model used for the capacitor bank does not fully reproduce all the signals recorded during the fault window.

Figure 8 shows the comparison of the fitting error before and after the corrections for the compensated line cases. This distance behavior was also not uniform for all events. During the fault window, Cases 1 and 2 showed differences of only -0.50 and -0.37 km with respect to their references. In contrast, Cases 4 and 7 maintained considerably larger deviations. In Case 4, a value of 79.30 km was obtained compared to a reference of 102.54 km, while Case 7 produced 155.80 km compared to 167.77 km.

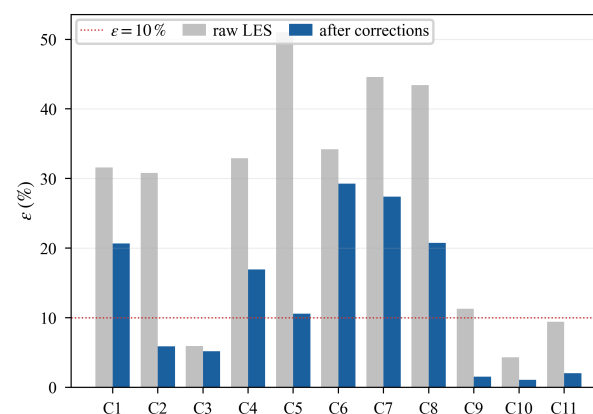


Figure 8. Fitting error ϵ before and after applying the corrections during the fault window.

For the events where a time interval after the capacitor bank bypass was available, this second window was evaluated as well. Figure 9 compares the values of ϵ obtained during the fault and bypass windows. Additionally, the bypass window showed a lower fitting error in five of the seven cases for which both windows were available: Cases 4, 5,

6, 7, and 8. In contrast, Cases 1 and 3 showed a lower value of ε during the fault window. Therefore, the results do not indicate a systematic superiority of the bypass for all events.

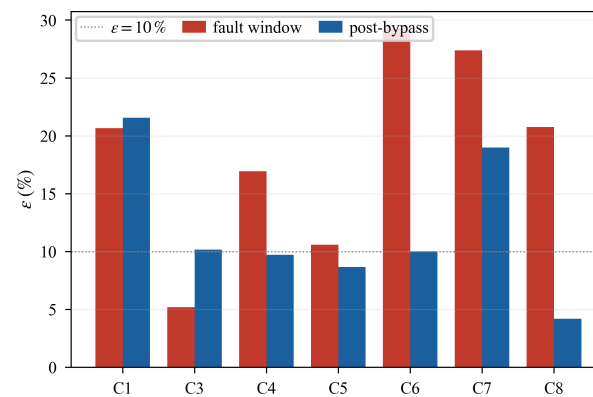


Figure 9. Fitting error ε during fault and bypass windows for the cases of the compensated line.

The same trend was observed when comparing the distances between the two groups. In Case 4, the result changed from 79.30 km during the fault window to 95.60 km in the bypass, reducing the absolute error from 23.24 to 6.94 km. For Case 7, the deviation decreased from 11.97 to 1.97 km. Cases 5, 6, and 8 reached differences of only 0.10, 0.60, and 0.50 km, respectively, during the bypass. In Case 3, the opposite behavior was observed. The fault window produced a distance of 15.50 km with $\varepsilon = 5.2\%$, whereas the bypass window produced 12.90 km with $\varepsilon = 10.2\%$, against a reference of 20.86 km. Case 1 also showed a greater agreement during the fault window.

A relevant result from the set is that in the seven cases where both windows could be compared, the window with the smallest ε was also the one with the smallest absolute distance error with respect to the reference window. This behavior distinguishes between the two uses of the fitting error ε and indicates that it should not be used to freely shift the distance during the fault location process. However, once each solution is obtained through the structured procedure, it can be used as an internal indicator to select the available time windows.

The selected distances for the eight adjusted cases were 148.90, 5.05, 15.50, 95.60, 135.50, 187.50, 165.80, and 139.50 km. The corresponding errors were -0.50 , -0.37 , -5.36 , -6.94 , $+0.10$, $+0.60$, -1.97 , and $+0.50$ km. Six of the eight adjusted cases had an absolute error less than or equal to 5 km, whereas all eight were within approximately 7 km of their respective reference values. The mean absolute error for the set was 2.04 km, and the median absolute error was 0.55 km. When the LSE was used directly without corrections, the mean absolute error for the eight cases was 10.57 km. Therefore, the complete procedure reduced the mean absolute error by approximately 80.7% relative to the initial estimate.

The systematic analysis of the series-compensated line was performed using the seven records with reference distances. Case 3 was not included in the quantitative statistics because its correspondence with the published reference could not be established unambiguously.

For the seven series-compensated records, the mean absolute error decreased from 4.44 km for the raw LSE result to 3.04 km after the complete procedure. Five of the seven records remained within 5 km of their reference distances, and the largest absolute error was 7.14 km. The median absolute error of the results was 2.28 km.

These results show that the compensation-related correction improves the aggregate location performance for the available records, but the improvement is not uniform across all events. The residual fitting error varies among records and should therefore be inter-

puted together with the distance result rather than as an independent measure of location uncertainty.

The case-by-case comparison in Table 3 shows that the aggregate improvement is not shared equally across the records. The raw absolute error decreased from 9.20 to 0.85 km in Case 1 and from 7.44 to 0.31 km in Case 2. In Case 4, however, it increased from 1.45 to 7.14 km; in Case 5, it remained at 7.05 km. The raw and final distances are identical for Cases 6–8. These individual changes explain why the mean absolute error decreases overall while some events show little improvement or a larger final error.

Table 3. Fault location for the series-compensated line.

Case	Window	Ref.	Raw	Final	$ \Delta d $ (km)	Final ϵ (%)
1	Fault	149.40	158.60	148.55	0.85	20.37
2	Fault	5.42	12.86	5.11	0.31	7.54
4	Bypass	102.54	101.09	95.40	7.14	8.34
5	Bypass	135.40	142.45	142.45	7.05	11.44
6	Bypass	186.90	187.07	187.07	0.17	10.36
7	Bypass	167.77	165.49	165.49	2.28	18.91
8	Bypass	139.00	135.49	135.49	3.51	4.16

4.5. Line Without Compensation

Cases 9–11 were used to analyze the behavior of the same formulation in the absence of a capacitor bank and MOV. Before implementing the fault-location technique, a scaling difference was identified between the measurements stored by the relays at both terminals. One of the devices recorded the voltages in secondary values, with amplitudes close to 133 V, whereas the device at the opposite terminal stored the voltages in primary values, with amplitudes close to 132 kV. The transformation ratio corresponding to the first terminal was not correctly specified in the .CFG file.

Table 4 summarizes this condition, in which a difference of approximately three orders of magnitude directly alters the equations for the fault location. The term PS is used to represent primary (P) or secondary (S) values. Under these conditions, the term associated with the drop was reconstructed from the replica current and artificially dominated the incremental terminal voltage, thereby producing an incorrect fault location. Therefore, the signals were first brought to the same physical base using the available transformation information for every line.

Table 4. Assessment of signal scaling on line B.

Channel	PS	Measured Peak	Condition
Voltage, terminal 1	S	≈ 133 V	Secondary value; potential-transformer ratio not available in the .CFG file.
Voltage, terminal 2	P	≈ 132 kV	Primary value; correct scale.
Currents	—	≈ 4 kA	Recorded values in primary scale.

After correcting the scaling, the same estimator based on incremental quantities was applied without incorporating any series-compensation model. Table 5 presents the obtained results.

Table 5. Fault location on line B without series compensation: initial LSE estimate.

Case	Fault	Ref. (km)	LSE (km)	Error (km)	Initial ε	Final ε
9	3F	58.77	58.94	+0.17	11.3%	1.6%
10	2FT	31.06	31.71	+0.65	4.3%	1.1%
11	1F	44.90	44.10	−0.80	9.4%	2.0%

The three events showed fault location errors of less than or equal to 0.80 km. Case 9 had the smallest difference, with an estimated distance of 58.94 km compared with a reference distance of 58.77 km. For Case 10, the result was 31.71 km versus 31.06 km, whereas Case 11 yielded 44.10 km compared with a reference value of 44.90 km. The mean absolute error for the three events was 0.54 km, and the median absolute error was 0.65 km. These values are considerably smaller than those initially obtained on the compensated line and provide a point of comparison for examining the behavior of the formulation when the nonlinear drop in the capacitor bank is absent.

Time correction also produced a clear improvement in the consistency of the reconstruction. The average of ε decreased from 8.33% to 1.57%, whereas the distances remained practically unchanged. This reproduces the behavior observed in the alignment stage of Case 2, where the time shift mainly affects the correspondence between the waveforms and can be corrected without modifying the fault location.

The final procedure produced the results summarized in Table 6. The final distances were 58.82, 31.51, and 44.10 km for Cases 9–11, respectively. The corresponding absolute errors were 0.05, 0.45, and 0.80 km, yielding a mean absolute error of 0.43 km and a median absolute error of 0.45 km.

Table 6. Fault location on line B without series compensation: final result.

Case	Fault	Ref. (km)	Final (km)	$ \Delta d $ (km)	Final ε	τ (Samples)
9	3F	58.77	58.82	0.05	1.74%	−0.6
10	2FT	31.06	31.51	0.45	1.06%	0.2
11	1F	44.90	44.10	0.80	2.09%	−0.8

The average fitting error for Cases 9–11 decreased from 8.36% before time alignment to 1.63% after alignment, while the final distances remained close to the reference values. This result provides a direct control condition: the time correction improves the consistency of the reconstructions even when no series compensation model is required.

Figure 10 shows the result corresponding to Case 9. Before the time correction, the two reconstructions of the fault voltage showed a small phase difference. After performing the alignment, the signals practically overlapped, and ε decreased to a final value of approximately 1.74%. In Case 11, presented in Figure 11, it is additionally possible to analyze the behavior in the presence of a single-phase impedance fault event. For $R_f = 20 \Omega$, the estimated distance was 44.10 km compared to a reference of 44.90 km, corresponding to a difference of −0.80 km. After performing the time alignment, the final fitting error was approximately 2.09%.

This result is consistent with the algebraic cancellation of the term associated with R_f in the incremental formulation of the double-terminal fault-location technique. However, because the available set contained only one impedance fault event, the result should be interpreted as a verification of the behavior for this particular case and not as a statistical characterization of different fault resistance values.

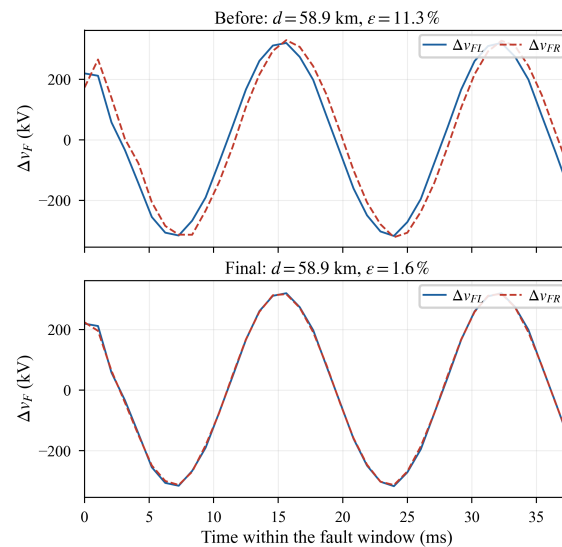


Figure 10. Case 9, three-phase fault in line B before and after performing the time alignment.

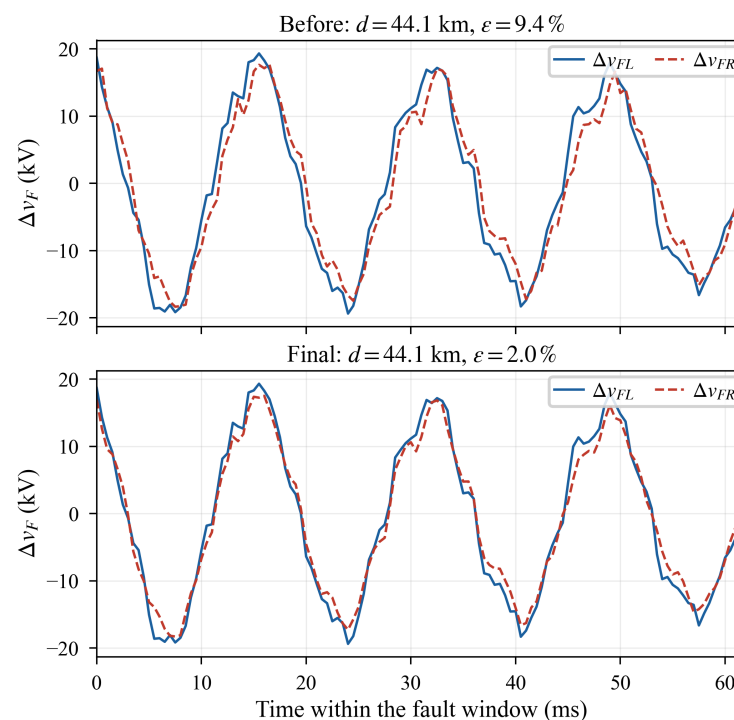


Figure 11. Case 11, single-phase impedance fault on line B.

4.6. Validation of Results

The complete dataset contains 18 paired COMTRADE records; the remaining eight records are retained in the dataset inventory but are excluded from the quantitative error metrics because no reference distance is available. For the seven series-compensated records, when available, the window that showed the smallest value of ε after applying the complete procedure was used. For the three uncompensated records, only the available window was used. The complete comparison is shown in Figures 12 and 13, and the individual values are listed in Table 7.

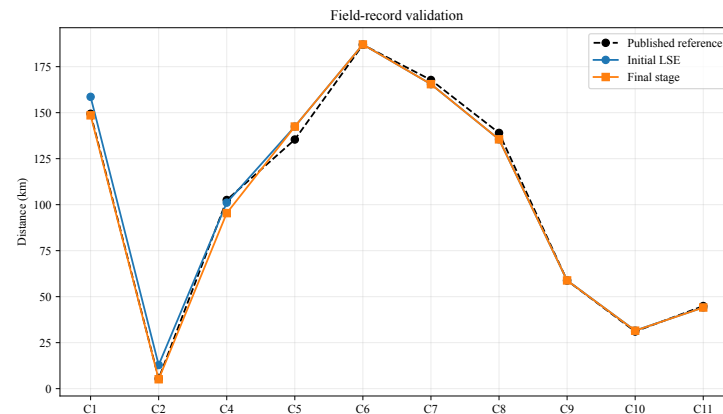


Figure 12. Reference, raw LSE, and final fault-location distances for the 10 records with reference distances. The other eight paired COMTRADE records are not shown because no independent reference distance is available.

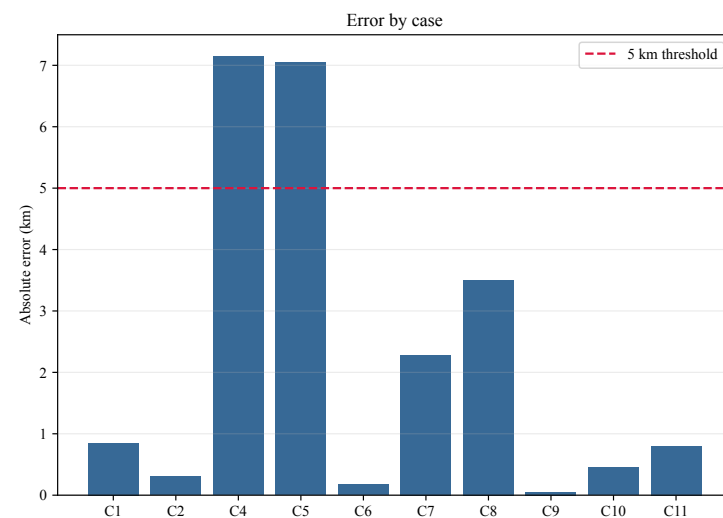


Figure 13. Absolute distance errors for the 10 records with reference distances; the dashed horizontal line marks the 5 km threshold.

Table 7. Individual validation results for records with reference distances.

Case	Line	Window	Reference (km)	Raw (km)	Final (km)	$ \Delta d $ (km)	Final ε (%)	τ (Samples)
1	A	Fault	149.40	158.60	148.55	0.85	20.37	1.4
2	A	Fault	5.42	12.86	5.11	0.31	7.54	1.4
4	A	Bypass	102.54	101.09	95.40	7.14	8.34	3.0
5	A	Bypass	135.40	142.45	142.45	7.05	11.44	1.8
6	A	Bypass	186.90	187.07	187.07	0.17	10.36	−3.0
7	A	Bypass	167.77	165.49	165.49	2.28	18.91	−3.0
8	A	Bypass	139.00	135.49	135.49	3.51	4.16	0.4
9	B	Fault	58.77	58.82	58.82	0.05	1.73	−0.6
10	B	Fault	31.06	31.58	31.51	0.45	1.06	0.2
11	B	Fault	44.90	44.10	44.10	0.80	2.09	−0.8

Figure 12 compares the reference, raw LSE, and final distances for each record. The final procedure reduces the absolute error in Cases 1, 2, and 10, leaves it unchanged in Cases 5–9 and 11, and increases it in Case 4. Figure 13 shows that Cases 4 and 5 are the only records above the dashed 5 km threshold; the remaining eight of the 10 records remain below it, consistent with the aggregate results reported in the following paragraph.

Eight of the 10 cases had an absolute error equal to or less than 5 km. The two events that exceeded this range were Case 4, with an absolute error of approximately 7.14 km,

and Case 5, with an absolute error of approximately 7.05 km. Consequently, the largest absolute error in the evaluated set was approximately 7.14 km. Considering the 10 records with reference distances, the mean absolute error was 2.26 km, and the median absolute error was 0.82 km. The difference between these two metrics was mainly influenced by Cases 4 and 5, whereas the remaining estimates exhibited tighter clustering around their corresponding reference values. As Case 2 was employed during method development, the performance metrics were additionally evaluated on the nine records excluded from the development set, the mean absolute error was 2.48 km, the median absolute error was 0.85 km, and seven of the nine records were found within 5 km. For the 10 records, the root mean square error (RMSE) is 3.46 km (3.65 km excluding Case 2), the sample standard deviation is 2.76 km, and the mean and maximum absolute errors relative to the line length are 1.25 % and 3.68 %, respectively.

To complement the comparison, Table 7 lists the individual results, and Table 8 summarizes the aggregated metrics for both lines.

Table 8. Fault location for records with reference distances. Rows excluding Case 2 omit the record used during the development of the method.

Description	Cases	MAE (km)	Median (km)	$ \Delta d \leq 5$ km (Cases)
Series-compensated line, raw LSE	7	4.44	3.51	4/7
Series-compensated line, final result	7	3.04	2.28	5/7
Uncompensated line	3	0.43	0.45	3/3
Complete dataset	10	2.26	0.82	8/10
Series-compensated line excluding Case 2, raw LSE	6	3.94	2.90	4/6
Series-compensated line excluding Case 2, final result	6	3.50	2.90	4/6
All records with reference distances excluding Case 2	9	2.48	0.85	7/9

Table 7 shows that five of the seven series-compensated records have final absolute errors below 5 km. Cases 4 and 5 are the exceptions, with errors of 7.14 and 7.05 km, respectively. A comparison of the raw and final distance columns indicates that the error reduction is primarily concentrated in Cases 1 and 2, with errors decreasing from 9.20 to 0.85 km and from 7.44 to 0.31 km, respectively. In Case 4, the error increases from 1.45 to 7.14 km, while it remains unchanged in Case 5. The raw and final distances are also unchanged in Cases 6–9 and 11; in Case 10, the error decreases slightly, from 0.52 to 0.45 km. Thus, the overall results do not represent a uniform improvement across all records. This table also shows that neither the final fitting error nor the selected time shift can be used by itself to rank location accuracy. Case 1 has a final fitting error of 20.37% but a distance error of only 0.85 km, whereas Case 8 has a lower fitting error of 4.16% and a larger distance error of 3.51 km. The selected shifts range from -3.0 to $+3.0$ samples; for example, Cases 4 and 6 use shifts at opposite limits of this range but have final distance errors of 7.14 and 0.17 km. These results support evaluating the distance error, fitting error, and time shift as distinct quantities.

In the series-compensated line, the mean absolute error decreased from 4.44 km when directly using the raw LSE result to 3.04 km after applying the complete procedure and selecting the window with the best consistency. This corresponds to an approximate reduction of 31.5%. Excluding Case 2, which was retained for development purposes, the mean absolute error across the remaining six compensated records decreased from 3.94 to 3.50 km (approximately 11.2%), while the number of records with an error below 5 km remained unchanged at 4/6.

Table 8 shows the results of fault location considering reference distances. The term MAE represents the mean absolute error. This table shows that the change in the compensated subset also affects the number of records within the 5 km threshold: this count increases from 4/7 for the raw LSE to 5/7 after the complete procedure, while the median absolute error decreases from 3.51 to 2.28 km. For the uncompensated line, all three records remained within 5 km, with a mean absolute error of 0.43 km and a median of 0.45 km. These values show that the larger errors in the combined set are concentrated in the compensated cases, particularly Cases 4 and 5.

A comparison between the two systems also revealed three relevant behaviors. First, in the absence of series compensation, the formulation yields errors of at most 0.80 km for the three evaluated events, whereas time alignment reduces ε without appreciably modifying the distance.

Second, in the compensated line, the fault window can provide accurate results when the corrections sufficiently represent the recorded signals, as in Cases 1 and 2. However, in other events, a larger discrepancy remains, although the capacitor bank is connected. In five of the seven cases with information after bypass (Cases 4–8), this second window simultaneously produced both a lower fitting error and a location closer to the reference.

Third, the combined behavior of the cases shows that the fitting and distance errors represent different aspects of the results. Case 2 illustrates that a reduction in ε can be obtained by applying corrections with the distance kept under control; Case 1 shows that a residual may remain even with a distance close to the reference; and Cases 9–11 show that time alignment can reduce ε without practically modifying the calculated location. Therefore, within the set analyzed, ε is useful as an indicator of consistency between the two reconstructions of the fault point and as a criterion for comparing windows once the corresponding solutions are obtained. However, this should not be interpreted as a measure of distance uncertainty.

Finally, the above metrics must be analyzed considering the nature of the available references. The positions used as reference correspond to the approximate locations associated with the system events and not to the distances verified by an independent field measurement. For this reason, the reported errors characterize the performance observed on the available set, but do not yet constitute a statistical distribution of the uncertainty for the fault location.

4.7. Quantitative Two-Terminal Benchmark

A quantitative benchmark was evaluated using the same records with reference distances. For each event, the estimated distance, reference distance, and absolute error were calculated using the selected two-terminal reference formulation. For the 10 records with references, the benchmark produced a mean absolute error of approximately 10.33 km, with two of the ten cases within 5 km. The proposed procedure produced a mean absolute error of approximately 2.26 km, with eight of ten cases within 5 km. For this benchmark, the reference phasor formulation was evaluated as in Equation (16):

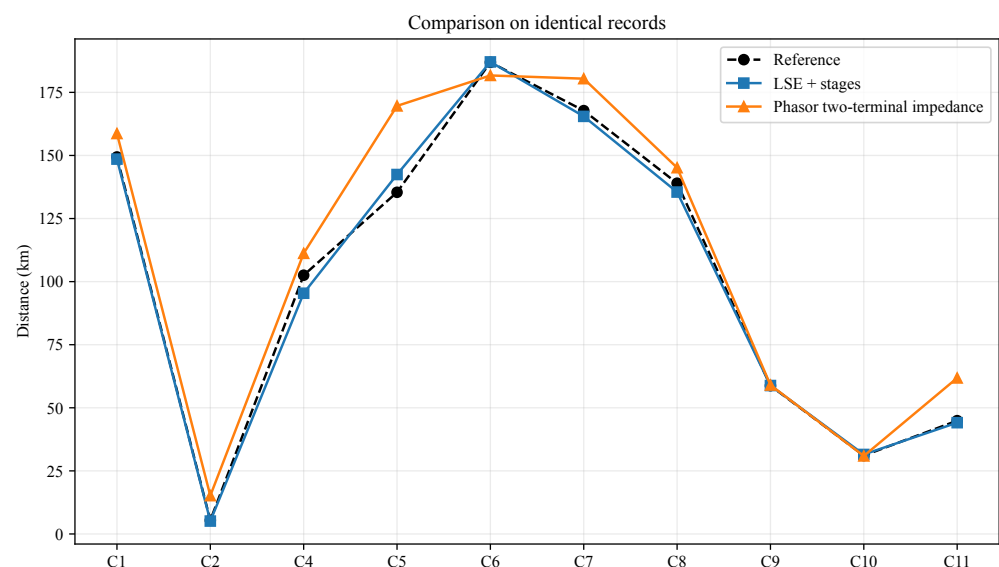
$$m_{\text{ph}} = \frac{V_L - V_R + Z_1 I_R}{Z_1 (I_L + I_R)}, \quad (16)$$

where V_L and V_R are the terminal voltages, I_L and I_R are the terminal currents with the adopted sign convention, and Z_1 is the positive-sequence impedance of the complete line, as defined in Equation (1). The resulting distances are listed in Table 9.

Table 9. Quantitative comparison with the classical two-terminal phasor benchmark on the same records.

Case	Reference (km)	Benchmark (km)	Benchmark Error (km)	Proposed (km)	Proposed Error (km)
1	149.40	158.68	9.28	148.55	0.85
2	5.42	15.18	9.76	5.11	0.31
4	102.54	111.21	8.67	95.40	7.14
5	135.40	169.63	34.23	142.45	7.05
6	186.90	181.70	5.20	187.07	0.17
7	167.77	180.43	12.66	165.49	2.28
8	139.00	145.15	6.15	135.49	3.51
9	58.77	59.01	0.24	58.82	0.05
10	31.06	30.92	0.14	31.51	0.45
11	44.90	61.82	16.92	44.10	0.80
Mean absolute error	—	—	10.33	—	2.26

Across the same 10 records, the proposed procedure reduced the mean absolute error from 10.33 to 2.26 km, a reduction of approximately 78.1%. It also placed eight records within 5 km of their references, compared with two for the phasor benchmark. The proposed procedure had a lower absolute error in nine of the 10 cases; Case 10 was the exception, for which the benchmark error was 0.14 km and the proposed error was 0.45 km. Figure 14 displays these case-by-case differences, while Table 9 provides their numerical values. Excluding Case 2, the mean absolute errors were 10.39 km for the benchmark and 2.48 km for the proposed procedure, with two and seven of the nine records within 5 km, respectively.

**Figure 14.** Quantitative comparison between the classical two-terminal phasor benchmark and the proposed procedure for the 10 records with reference distances.

This comparison is intended as a reproducible reference under explicit assumptions and not as an exhaustive reproduction of all the published fault-location algorithms.

4.8. Sensitivity to the Delay Parameter

The sensitivity to the delay parameter T was evaluated using 108 configurations, obtained by applying six values of T to the 18 available paired records. The tested values of T were expressed in cycles and applied to all records using the same signal-processing procedure.

The results show that T affects both the incremental quantities and the estimated distance. In practical applications, T should be selected by considering the sampling frequency, transient duration, and filtering characteristics of the recording devices. The selected value should provide a stable error behavior over several records rather than an isolated minimum for a single event.

Table 10 summarizes the mean and maximum fitting errors obtained for each tested value of T .

Table 10. Sensitivity of the raw incremental formulation to the delay parameter T .

T (Cycles)	Records	Mean ε (%)	Maximum ε (%)
0.5	18	16.32	99.74
1.0	18	33.89	94.71
2.0	18	28.53	72.63
3.0	18	26.39	68.91
4.0	18	26.39	71.66
6.0	18	47.89	97.10

The values in Table 10 reveal a non-monotonic dependence of the fitting error on T . The lowest mean error was obtained for $T = 0.5$ cycles, with 16.32%; however, this value also produced the highest maximum error, equal to 99.74%, indicating a highly dispersed response among the records. The mean error decreased to 26.39% for both $T = 3$ and $T = 4$ cycles, whereas the corresponding maximum errors decreased to 68.91% and 71.66%, respectively. In contrast, $T = 6$ cycles increased the mean and maximum errors to 47.89% and 97.10%. Therefore, the lowest mean error alone does not identify the most stable value of T ; the mean error, maximum error, and record-to-record dispersion must be considered.

The corresponding error and distance responses are shown in Figures 15 and 16, respectively. In both figures, the gray markers represent individual records, the blue boxes summarize the concentration of the results, the whiskers indicate the minimum and maximum values, the solid black line represents the mean, and the dashed red line represents the median. Figure 15 shows the effect of T on the fitting error for the 18 records used in this study. Although $T = 0.5$ provides the lowest mean error, equal to 16.32%, it also produces the widest dispersion and the highest maximum error, which is equal to 99.74%. For $T = 1$, the mean error increases to 33.89%. It then decreases to 28.53%, 26.39%, and 26.39% for $T = 2$, $T = 3$, and $T = 4$, respectively. For $T = 6$, the mean error increases again to 47.89%. The values $T = 3$ and $T = 4$ also provide lower maximum errors than $T = 0.5$, $T = 1$, and $T = 6$, although record-to-record variability remains.

Figure 16 presents the corresponding response of the raw LSE distance data. For $T = 0.5$, the individual estimates showed a very wide dispersion, ranging from approximately -217.11 km to 188.39 km. The negative values were not interpreted as physical fault distances; they indicated instability in the raw estimator for this delay value. For $T = 1$, $T = 2$, $T = 3$, and $T = 4$, the mean distances remained between 115.39 km and 125.32 km, while the median values remained close to 120 km. However, the individual records still show considerable dispersion. For $T = 6$, the mean increased to 144.21 km and the median to 158.84 km, indicating a displacement of the central distance estimates from the original location.

Table 10 and Figures 15 and 16 show a trade-off rather than a single optimum. Although $T = 0.5$ yields the lowest mean fitting error (16.32%), its maximum error is the largest (99.74%). Values $T = 3$ and $T = 4$ have the same mean fitting error (26.39%), while $T = 3$ gives a slightly lower maximum (68.91% versus 71.66%). Thus, $T = 3$ is a candidate for record-by-record verification within this dataset, not a universally validated

setting. Because the distance plot combines records with different reference locations, its across-record mean and median describe the aggregate data and do not measure location accuracy. This evaluation requires analyzing the distance response and, when available, its associated reference error.

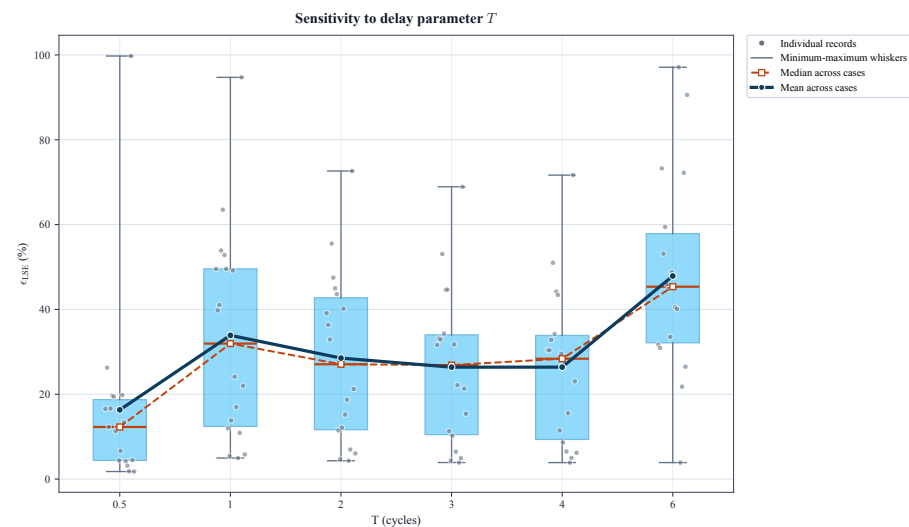


Figure 15. Sensitivity of the fitting error to the delay parameter T over the 18 available paired records.

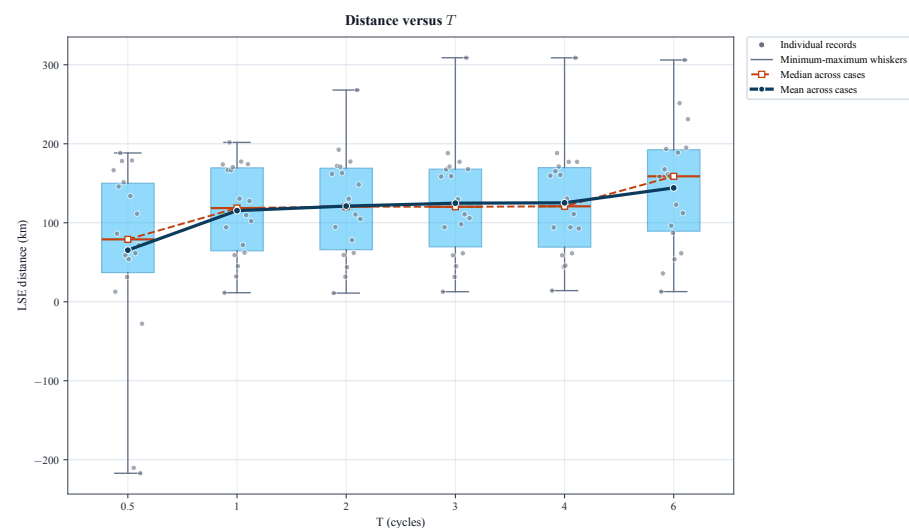


Figure 16. Sensitivity of the raw LSE distance to the delay parameter T over the 18 available paired records.

These results show that T should be regarded as part of the signal-processing configuration rather than merely as a numerical tuning parameter. Because T modifies the construction of the incremental quantities, it affects both the fitting residual and the raw LSE distance. For reproducibility, the selected value of T should be reported together with the sampling frequency, nominal system frequency, filtering characteristics, and analysis window used in the fault location procedure.

4.9. Controlled Fault Resistance Analysis

The effect of fault resistance was evaluated with a controlled synthetic sweep at $R_f = 0, 20, 50, 100, 200,$ and 500Ω . The idealized two-terminal formulation returned the same distance, 67.956 km, for all tested values, with a numerical error below 10^{-12} km. The fitting residual was high only for the zero-resistance condition because the corresponding synthetic signals generated a nonzero residual under the selected construction. These

results support the algebraic cancellation of the fault resistance term in the ideal model, but they do not establish immunity under field conditions with noise, saturation, parameter errors, or unmodeled elements.

Figure 17 shows a horizontal distance estimate at 67.956 km across the tested values of R_f , consistent with the distances listed in Table 11. The table also shows that the fitting residual is 81.41% for $R_f = 0$ and below $10^{-12}\%$ for the other tested values. This controlled result illustrates the distance invariance of the idealized formulation under the tested construction; it does not establish performance for high-resistance field faults with reduced incremental-signal magnitudes, measurement noise, or other non-ideal effects.

Table 11. Controlled synthetic sweep of the fault resistance.

R_f (Ω)	Final Distance (km)	$ \Delta d $ (km)	ε (%)
0	67.956	$<10^{-12}$	81.41
20	67.956	$<10^{-12}$	$<10^{-12}$
50	67.956	$<10^{-12}$	$<10^{-12}$
100	67.956	$<10^{-12}$	$<10^{-12}$
200	67.956	$<10^{-12}$	$<10^{-12}$
500	67.956	$<10^{-12}$	$<10^{-12}$

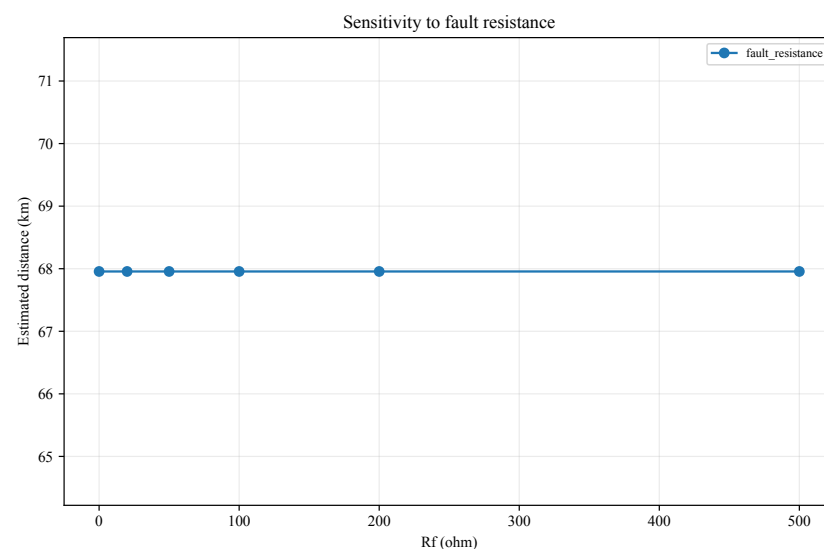


Figure 17. Controlled synthetic fault resistance sweep.

4.10. Controlled MOV and Compensation Analysis

A separate controlled analysis evaluated four synthetic compensation values, $X_c = 0, 20, 49.5$, and 70Ω , combined with four normalized MOV perturbation levels, 0, 0.25, 0.50, and 0.75. The 16 combinations are summarized in Table 12 and Figure 18. The analysis is used to show the response of the implementation to controlled changes in the bank representation; because the scenarios are synthetic and do not include independently measured MOV parameters, they do not constitute identification of the physical varistor or EMTP validation.

Table 12 and Figure 18 show that the fitting error increases with the normalized MOV perturbation, from 0% at level 0 to approximately 34–35% at level 0.75. The distance error, however, does not increase monotonically with either perturbation level or X_c : at level 0.75, it ranges from 0.053 to 1.269 km across the tested compensation values. Thus, a larger residual in this controlled sweep does not necessarily correspond to a larger location error.

These results describe the response of the synthetic implementation and do not identify the physical MOV parameters of the line.

Table 12. Controlled sensitivity analysis for compensation and MOV perturbation. The reference distance was 67.956 km.

X_c (Ω)	MOV Level	Final Distance (km)	$ \Delta d $ (km)	Final ε (%)
0	0.00	67.956	0.000	0.00
0	0.25	68.379	0.423	12.11
0	0.50	68.802	0.846	23.40
0	0.75	69.225	1.269	34.51
20	0.00	67.956	0.000	0.00
20	0.25	68.379	0.423	12.11
20	0.50	68.194	0.238	23.30
20	0.75	68.009	0.053	34.28
49.5	0.00	67.956	0.000	0.00
49.5	0.25	68.379	0.423	12.11
49.5	0.50	68.802	0.846	23.40
49.5	0.75	67.720	0.236	34.48
70	0.00	67.956	0.000	0.00
70	0.25	68.379	0.423	12.11
70	0.50	68.802	0.846	23.40
70	0.75	67.097	0.859	35.20

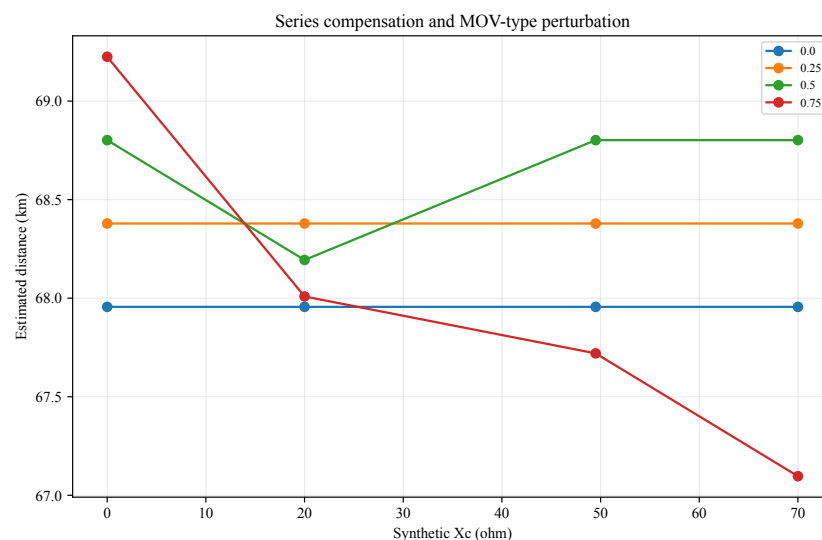


Figure 18. Controlled sweep of compensation and MOV perturbation levels.

4.11. Stress Analysis of the Time-Alignment Window

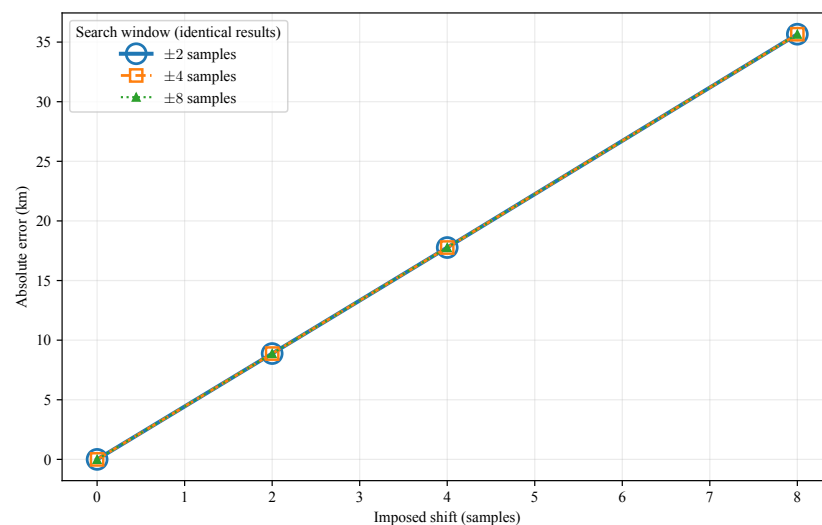
The alignment procedure was tested using imposed integer-sample shifts of 0, 2, 4, and 8 samples. For each condition, the search was repeated with windows of ± 2 , ± 4 , and ± 8 samples. Table 13 reports whether the imposed shift was recovered and the best absolute location error among the tested windows, as well as the range of the selected $|\tau^*|$ over the three windows.

Table 13. Recovery of imposed time-alignment shifts and associated location errors for the tested search windows.

Imposed Shift (Samples)	Recovered	Best $ \Delta d $ (km)	Selected $ \tau^* $ (Samples)	Search Windows Tested
0	Yes	0.00	0.00	$\pm 2, \pm 4, \pm 8$
2	No	8.88	0.00–0.07	$\pm 2, \pm 4, \pm 8$
4	No	17.77	0.20–0.27	$\pm 2, \pm 4, \pm 8$
8	No	35.66	0.77–0.80	$\pm 2, \pm 4, \pm 8$

Table 13 and Figure 19 show that only the zero-shift case was recovered. The imposed shifts of 2, 4, and 8 samples were not recovered; the best absolute distance errors were 8.88, 17.77, and 35.66 km, respectively. Increasing the search window from ± 2 to ± 8 samples did not recover any of these nonzero shifts. Thus, the tested procedure did not demonstrate recovery of nonzero integer-sample offsets under these conditions, and widening the search window alone was insufficient.

The intermediate results of this test explain why the imposed shifts were not recovered. As the shift is applied to the remote record before Stage C and the distance is fixed in that stage prior to any time correction, the analytical estimator absorbs the time mismatch as an error in the fault position. In this test, this error increased by approximately 4.4 km for each sample of imposed shift. Once m_0 is fixed at that shifted position, the minimum of ε in Stage D is located near zero shift: the selected $|\tau^*|$ value did not exceed 0.80 samples in any test, even though recovery required a magnitude equal to the imposed shift, as presented in Table 13.

**Figure 19.** Stress analysis of the time-alignment search window.

The three search windows produced the same distance, and widening the window did not alter the result. The final fitting error remained between 1.5 and 8.9%, a range comparable to that obtained in field records with small location errors, confirming that a low value of ε alone does not guarantee distance accuracy. Consequently, under the fixed-distance constraint, Stage D reduces the residual inconsistency between terminals, but it cannot correct a location error caused by a time mismatch present since the determination of m_0 .

4.12. Reactive-Residual Stress Analysis

The interpretation of the reactive residual was also tested under controlled shunt perturbations of 0, 0.25, 0.50, and 1.00 in normalized units. The final distance changed by

less than 0.01 km across these cases, while the reactive fraction remained close to one. This behavior suggests that a predominantly quadrature residual is not sufficient to identify the MOV as its unique physical source. It can also be compatible with unmodeled shunt capacitance, line-parameter variation, filtering, or measurement-chain effects.

Table 14 shows that, as the normalized perturbation increases from 0 to 1, the estimated distance changes only from 67.956 to 67.961 km, while ε increases from 0 to 0.157% and the reactive fraction reaches 1.000. Figure 20 displays the small distance change; the residual fraction and fitting error are reported in the table. Together, these results show that the reactive fraction alone does not identify the physical source of the residual and do not validate a universal attribution threshold for the MOV.

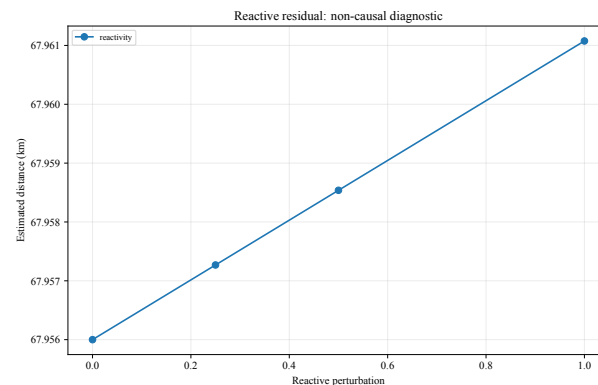


Figure 20. Reactive-residual stress analysis and its non-identifying interpretation.

Table 14. Reactive-residual stress test under controlled shunt perturbations.

Perturbation	Final Distance (km)	ε (%)	Reactive Fraction	Interpretation
0.00	67.956	0.000	0.960	Multiple sources possible
0.25	67.957	0.039	1.000	Multiple sources possible
0.50	67.959	0.078	1.000	Multiple sources possible
1.00	67.961	0.157	1.000	Multiple sources possible

5. Discussion

The results presented in the previous section show that the fault location on lines with series capacitive compensation does not depend solely on the estimator used to calculate the distance. The presence of the capacitor bank and MOV introduces a voltage drop that does not belong to the longitudinal model $R_1 + j\omega L_1$ of the line, whereas the time differences between the recordings at both terminals generate an additional discrepancy between the signals used by the fault-location technique. Therefore, the differences observed between Δv_{FL} and Δv_{FR} should not be interpreted directly as an error in the fault position, because they may simultaneously contain information associated with distance, time alignment, and the response of the capacitor bank.

The results also show that the fitting error ε and the location error represent different criteria. In Case 2, the stepwise procedure reduced the raw estimate from 12.86 km with $\varepsilon = 30.9\%$ to a final estimate of 5.11 km with $\varepsilon = 7.54\%$, against a reference of 5.42 km. However, during the study, it was observed that an additional reduction in ε could be obtained by artificially shifting the distance when it remained as an optimization variable. Complementarily, Case 1 showed that a distance close to the reference can coexist with a fitting error near 21%. These two situations indicate that the consistency between the waveforms and the accuracy of the distance must be analyzed jointly, but should not be regarded as equivalent.

5.1. Effect of Series Compensation on Fault Location

Double-terminal fault-location techniques have been used previously to reduce the limitations associated with conventional impedance-based methods. In [9], a formulation was developed for series-compensated lines using measurements from both ends and considering the possible positions of the fault concerning the capacitor bank. Similarly, in [10], synchronized measurements from both terminals were used to reduce the dependence on an explicit representation of the MOV. These studies show that having information at both ends improves the observability of the problem; however, the presence of the compensator continues to alter the relationship between the voltages and currents used during the location. This difficulty has been observed in recent studies. In [13], different single-terminal impedance-based methods showed large errors when the fault was located after the compensation bank. The discontinuity introduced by the capacitor and the operation of the MOV modified the apparent impedance observed from the relay, directly affecting the estimates obtained using the simple reactance and the superimposed components.

These results are consistent with the behavior observed in the present work, where the raw LSE result showed a mean absolute error of 4.44 km for the seven records of the series-compensated line. After applying the proposed method, the mean absolute error was reduced to 3.04 km. Therefore, the difference between both conditions does not indicate that the formulation based on incremental quantities completely loses the ability to locate the fault when there is compensation, but rather that the voltage drop associated with the capacitor bank must be separated from that corresponding to the longitudinal segment of the line. This behavior was observed in Case 2. The LSE estimator without corrections produced 12.86 km for a fault whose reference was 5.42 km, while the complete procedure reduced the estimate to 5.11 km.

The results obtained during development showed that a more detailed representation of the capacitor bank does not guarantee a better distance. When the location and the parameters used to represent the compensator were adjusted simultaneously, the fitting error could be reduced while the estimated distance moved away from the reference value. Similarly, allowing the complete nonlinear parameter set (X_c, V_p, q) to participate directly in the distance calculation introduced additional degrees of freedom without necessarily improving the location. This result introduces an important difference with respect to a strategy based exclusively on minimizing the error between the two fault point reconstructions. When the distance remains free, the capacitor bank parameters and the fault position can compensate each other, producing a reduction in ε without necessarily representing the best fault location. Therefore, in the proposed procedure, the distance and subsequent corrections are not treated as equivalent variables within a single optimization process.

5.2. Effect of Time Alignment and the Nonlinear Response of the MOV

The use of measurements from both terminals introduces a second requirement associated with the time correspondence between the signals. In [14], this problem was addressed using a method designed to operate with unsynchronized measurement data. Likewise, the formulation based on incremental quantities in [15] considers the time alignment between records when there is a difference between the signals at the ends of the line. The alignment stage is documented through the selected shift for each record and through the controlled stress analysis in Section 4.11. For Case 2, the raw result was 12.86 km with $\varepsilon = 30.90\%$, whereas the final selected result was 5.11 km with $\varepsilon = 7.54\%$ and $\tau = 1.4$ samples. The distance was not re-optimized as part of the time correction.

Similar behavior was observed on the line without compensation. In Cases 9–11, the average value of ε decreased from 8.36% to 1.63% after applying time alignment, whereas

the final distances remained close to the reference values. This control condition supports the interpretation that time mismatch can contribute to the residual independently of the capacitor bank. A small time shift can generate a component approximately in quadrature with the original signal and may therefore be confused with a reactive voltage drop.

However, the signals recorded during faults on the compensated lines may contain components that hinder the fault processing. This limitation was reported in [6], where the authors used the Prony algorithm to separate the fundamental component from the transient oscillations before performing fault location. A similar situation occurred in [8], where the estimation of the current phasor was analyzed in the presence of a decaying DC component, subsynchronous oscillations, harmonics, noise, and frequency deviation. In both cases, additional processing was performed to improve the quality of the data used by the fault-location technique. In this study, a different strategy was used. The formulation remains in the time-domain simulation and does not require the fundamental phasor to be obtained in advance. However, this does not eliminate the need to ensure that the samples from both terminals correspond to the same time. Once this correction was made, the remaining discrepancy could be examined in terms of the capacitor–MOV representation.

The nonlinear capacitor–MOV emulation is reported through the final selected result. For Case 2, the result was 5.11 km with $\varepsilon = 7.54\%$, using the equivalent fit $X_c = 33.0 \Omega$, $V_p = 73.38$ kV, and $q = 45$. The controlled sweep in Section 4.10 evaluates the response to synthetic compensation and MOV perturbations; it does not establish a unique physical identification of the bank.

The representation of the MOV has been studied in works such as [2], where equivalent models were developed for series capacitors protected by varistors. In this study, the dynamic representation is based on the differential equation of the capacitor–MOV set and the exponential characteristic of the varistor. For Case 2, the equivalent fit selected $X_c = 33.0 \Omega$, $V_p = 73.38$ kV, and $q = 45$. Because these parameters were obtained through a bounded search and the internal nameplate data were unavailable, they should not be interpreted as an exact identification of the physical MOV.

5.3. Fitting and Residual Errors

The results show that the fitting error ε and the fault-location error are not equivalent quantities. In Case 2, allowing the distance to vary jointly with the parameters used to represent the capacitor bank reduced the fitting residual but produced locations farther from the reference. In contrast, Case 1 produced a distance close to the reference while retaining a fitting error of approximately 21%. In this case, approximately 99% of the residual $\Delta v_{FL} - \Delta v_{FR}$ was associated with a reactive component. These results indicate that an additional reduction in ε obtained by freely changing the distance may reduce the residual at the expense of introducing a location bias.

Accordingly, the proposed procedure uses ε only after the distance has been established by the analytical estimator. The time-alignment and capacitor–MOV stages are therefore evaluated conditionally on $m = m_0$ and cannot modify the location solely to reduce the residual. This structure reduces the risk of residual-driven overfitting, but it does not constitute a formal guarantee that the initial estimate is unbiased. A remaining residual may be related to the capacitor–MOV representation, unmodeled shunt capacitance, line-parameter variations, filtering, time alignment errors, or other measurement effects. Therefore, the reactive-residual analysis is reported as a non-identifying diagnostic rather than as proof of a unique physical cause.

This interpretation should be treated with caution, because the behavior observed in Case 1 may reflect a bank topology that is more complex than that captured by a single equivalent parameter set (V_p, q) . Potential contributing factors include multiple MOV

subgroups, additional resistive components, unmodeled shunt capacitances, variations in line parameters, filtering effects, time alignment errors, and other sources of uncertainty with measurement.

In particular, the available records do not allow the effects of the MOV, unmodeled shunt capacitance, line-parameter variations, and measurement-chain uncertainties to be separated independently. Therefore, the result allows us to state that the implemented model does not fully reproduce the observed reactive component, but it does not allow us to uniquely determine which phenomenon is responsible for the residuals.

5.4. Comparison with Other Fault-Location Techniques Applied to Lines with Series Compensation

The methodologies reported in the literature present different trade-offs among accuracy, required information, acquisition infrastructure, and prior knowledge of the system parameters. Therefore, a comparison with the results of the present study must be performed by considering these conditions simultaneously and not only the final percentage error. Therefore, the double-terminal fault-location technique based on electrical magnitudes, such as those presented in [9,10], can achieve reduced errors by using information available at both ends of the line. This feature makes them conceptually similar to the proposed method in this study. However, the formulations differ in their handling of compensators. Some methods explicitly incorporate the location of the capacitor bank or formulate equations that avoid direct dependence on the MOV. However, the present approach maintains an incremental formulation in the time-domain simulation and separates the distance calculation from the subsequent corrections applied to the signals.

Other researchers have sought to increase the adaptability of the fault-location technique [16], where PMU measurements were used to estimate the Thévenin equivalents and line parameters from pre-fault online information. This strategy reduces the dependence on fixed values and allows the fault location technique to adapt to the operating conditions. However, it requires the availability of synchrophasors and an adequate alignment infrastructure at both terminals.

The procedure developed in this study is based on a distinct infrastructure framework. The input data comprised the COMTRADE records generated by the installed protection equipment. This choice enables the exploitation of information already embedded in the operational data stream of a transmission line. However, it also introduces practical challenges that are not necessarily present in an ideal PMU infrastructure, such as heterogeneous delays among devices, incorrectly specified transformation ratios, and constraints related to limited sampling frequencies.

In [17], parameter uncertainty was addressed using a formulation that reduced the need to use previously known information about the line. This feature is advantageous when the system is incompletely characterized. In the proposed method, a different decision is adopted: the positive-sequence model $R_1 + j\omega L_1$ is preserved, and the search associated with the capacitor bank is restricted to values that are physically acceptable. This formulation preserves a direct relationship between the model used and the physical elements of the line, although it depends on the quality of the positive-sequence parameters.

Traveling wave techniques offer another tradeoff. In [18], a method was developed that mainly uses the arrival times of the transient fronts and avoids explicitly representing the nonlinear characteristics of MOV. This strategy can achieve high accuracy because the estimation depends mainly on the propagation time and not on the apparent impedance of the capacitor bank. However, it requires a time resolution that is considerably higher than that of many conventional protection records. Therefore, the traveling waves and proposed method do not necessarily compete under the same acquisition conditions. When high-frequency instrumentation and proper alignment are available, a solution based on

arrival times can offer advantages. In contrast, the procedure developed in this work was designed to use conventional oscillograms stored after events, without requiring additional sensors or recorders.

Methods based on advanced signal processing show similar performance differences. Strategies based on [6] or optimized phasor estimators, such as those in [8], seek to reduce the influence of transient components and improve the quality of the magnitude used for the fault location. The proposed method avoids phasor estimation and works directly with instantaneous samples; however, it must explicitly handle time alignment and capacitor bank bypass to guarantee consistency between the two terminals.

From a quantitative standpoint, the proposed procedure achieved a mean absolute error of 3.04 km in the seven records of the series-compensated line. Five of the seven events presented errors equal to or less than 5 km, and the largest absolute error was 7.14 km. For the line without compensation, the mean absolute error was 0.43 km, and the maximum error observed was 0.80 km. For the complete set of 10 records with reference distances, the proposed procedure obtained a mean absolute error of 2.26 km, with eight of the ten cases within 5 km (2.48 km and seven of nine when Case 2, used during development, is excluded).

For the same 10 records with reference distances, the classical synchronized two-terminal benchmark obtained a mean absolute error of 10.33 km, with only two of ten cases within 5 km. In comparison, the proposed procedure obtained a mean absolute error of 2.26 km, with eight of ten cases within 5 km. This comparison is used as an auditable reference under the same records and does not represent a reproduction of a specific published implementation.

In [6,9,10,18], lower errors were reported for various validation conditions. However, these values should not be directly compared without considering the differences in the type of signals, sampling frequency, knowledge of parameters, alignment, and definition of the distance used as a reference. In this study, the field records included time differences between relays, non-uniform electrical scales, and incomplete bank parameters, whereas the available positions for several faults corresponded to approximate operational references. Therefore, the contribution of the method is not the lowest nominal error in the literature, but a robust location performance on oscillograms from existing protection systems when the main discrepancies in the records are explicitly addressed. This characteristic positions the method as an alternative oriented toward operational applications with COMTRADE, complementary to strategies that require PMUs or high frequency acquisition.

5.5. Selection of the Analysis Window and Operational Application

Both candidate windows were evaluated for the seven series-compensated records. In all seven, the window with the lower fitting error was also the window with the smaller distance error. The fault window was selected for Cases 1 and 2, with absolute errors of 0.85 and 0.31 km, compared with 3.15 and 9.17 km in the post-bypass window. The post-bypass window was selected for Cases 4–8, with absolute errors of 7.14, 7.05, 0.17, 2.28, and 3.51 km, compared with 22.32, 7.56, 1.39, 13.70, and 7.14 km in the fault window. Consequently, the present results do not support a general claim that bypass is systematically superior. The window must be selected case by case after the structured procedure has been applied.

The fitting error has a second, limited use after the candidate solutions have been obtained with the distance constrained by the staged procedure. It can be used to compare the internal consistency of different electrical windows without using the reference distance, but this criterion must be accompanied by the selected window and the alternative candidate values. It must not be used to re-optimize the fault distance.

This interpretation must be made within the context of the size and structure of the evaluated set. The available records are insufficient to establish that the smallest ε universally corresponds to the smallest distance error. Its validation requires a larger set of faults, different compensation levels, other bank positions, independently verified references, and records obtained using different relay models.

The operational use of COMTRADE records introduces an additional condition related to data quality. On line B, it was identified that one terminal stored the voltages in secondary values, whereas the other stored the voltages in primary values, without the correct transformation ratio being declared in the .CFG file. This difference initially produced an incorrect location. After correcting the scale, all three cases showed errors of at most 0.80 km. This result shows that the COMTRADE standard enables information exchange, but does not automatically guarantee that signals from different devices can be used directly within a formulation of the double-terminal fault-location technique. Therefore, verification of units, scales, polarities, and time alignment constitutes a necessary condition before running the fault-location technique.

5.6. Behavior in the Absence of Compensation and Fault Resistance

The results for line B allow for analyzing how the formulation behaves when the nonlinear drop introduced by the capacitor bank is eliminated. After correcting the scaling of the signals, Cases 9–11 showed errors of approximately +0.05, +0.45, and −0.80 km, respectively, with a mean absolute error of 0.43 km.

The difference with respect to the series-compensated line was considerable. While the raw LSE result showed a mean absolute error of 4.44 km in the seven records of line A, the same formulation on the line without compensation yielded errors below 0.80 km. This contrast indicates that a part of the deterioration observed on the compensated line is associated with elements that are not part of the longitudinal model $R_1 + j\omega L_1$.

The time alignment also produced a reduction in the fitting error from an average of 8.36% to 1.63% without substantially changing the distances. This result indicates that the time mismatch exists whether or not the capacitor bank is present and reinforces the need to separate both effects before interpreting the residual.

Case 11 provides the available field assessment of a single-phase fault with $R_f = 20 \Omega$. The estimated distance was 44.10 km, compared with a reference of 44.90 km, with $\varepsilon = 2.1\%$. This behavior is consistent with the algebraic cancellation of the term associated with fault resistance in the formulation of the double-terminal fault-location technique. This feature represents an advantage over conventional impedance methods, in which R_f can modify the apparent impedance and produce errors in the estimated distance. However, the available field dataset contained only one impedance event. Therefore, the result verified the expected behavior for Case 11 but did not allow us to experimentally establish that the accuracy was maintained for any value of fault resistance.

A complementary controlled sweep was performed for $R_f = 0, 20, 50, 100, 200$, and 500Ω using a synthetic record with known distance. Under the idealized conditions of this controlled experiment, the estimated distance remained equal to the reference, with numerical errors close to zero. This result supports the expected algebraic cancellation of the common fault resistance term, but it does not demonstrate immunity to noise, saturation, unmodeled line effects, or measurement errors. The claim is therefore restricted to the model assumptions and to the controlled sweep.

5.7. Limitations

This study has some limitations that must be considered when interpreting the results. The available dataset contains 18 paired COMTRADE records, but a reference distance is available for only 10 records, including seven series-compensated cases and three uncompensated cases; one of these records (Case 2) was also used during the development of the method. Consequently, the quantitative error statistics cannot be interpreted as a statistical characterization of all possible operating conditions. In particular, the field validation contains only one impedance-fault event with $R_f = 20 \Omega$. In addition, the positions used as references correspond to approximate locations available in the operational information and not to distances verified through independent measurements. Therefore, the reported error incorporates both the uncertainty of the method and that associated with the reference.

Another limitation is related to the representation of the capacitor bank and MOV. The equivalent model uses the parameters X_c , V_p , and q ; however, the exact values of V_p and q were not available and were obtained through a constrained search aimed at reproducing the observed waveforms. Consequently, these parameters represent an equivalent approximation rather than an exact identification of the varistor characteristics. This simplification may partially explain the residual observed in some cases, given that actual configurations may include multiple MOV groups, additional resistive elements, or other features not considered. Likewise, the absence of direct measurements of the current, conduction time, or energy absorbed by the MOV prevents establishing a quantitative relationship between the severity of its conduction and the location error.

The method also depends on certain simplifications and input conditions. The line is represented using lumped parameters and its positive-sequence impedance, without explicitly considering distributed shunt capacitance, non-homogeneous sections, parameter variations along the line, or couplings with parallel circuits. In addition, records from both ends of the line are required, consistently scaled and with sufficiently synchronized timing information. The available records do not allow the effects of the MOV, unmodeled shunt capacitance, line-parameter variations, filtering, alignment, and other measurement-chain uncertainties to be separated independently.

The controlled nominal alignment procedure searches within a window of ± 2 samples, while the field records are processed with $\tau_{\max} = 3$ samples. Controlled stress tests considered imposed shifts of 0, 2, 4, and 8 samples with search windows of ± 2 , ± 4 , and ± 8 samples, but these tests do not establish robustness for arbitrary timing offsets or non-uniform relay filtering delays. Because the distance is fixed before time correction, time offsets present in the records must be removed using independently verified coarse alignment before Stage C; as shown in Section 4.11, widening the search window alone is not sufficient.

Finally, the results show sensitivity to the analysis window, the delay parameter T , and the information available about the compensation bank. The sensitivity analysis considered $T = 0.5, 1, 2, 3, 4$, and 6 cycles. In practical applications, T should be selected as the smallest value that provides a valid pre-fault reference interval and sufficient separation between the delayed and present signals. Likewise, ε should not be interpreted in isolation as a direct measure of location uncertainty. These results require further validation with a larger number of events, different compensation configurations, different fault resistances, and operating conditions.

The controlled analyses do not replace additional field records or electromagnetic-transient simulations. In particular, the available evidence is insufficient to establish generalization across independent MOV protection levels, compensation degrees, relay filtering characteristics, or fault resistance distributions. These conditions are therefore identified as priorities for future validation.

6. Conclusions and Future Work

6.1. Summary

This study presented a staged two-terminal method for locating faults on series-compensated transmission lines, based on incremental quantities and an LSE estimator. The method operates directly on COMTRADE records from protection devices installed at both terminals and does not require phasor estimation, PMU infrastructure, or traveling-wave acquisition. Its main feature is the separation of the effects that jointly affect the agreement between the fault-point reconstructions obtained from both terminals: the fault position, time misalignment between records, and the voltage drop across the capacitor bank, including nonlinear MOV conduction.

6.2. Conclusions

The main conclusion is that these effects should not be fitted simultaneously using a single error function, because the fitting residual can then be reduced by shifting the estimated fault location. For this reason, the proposed procedure fixes the distance before time alignment and capacitor–MOV emulation, so that these stages improve consistency between terminals without changing the location. This constraint limits overfitting driven by the residual but does not guarantee that the initial estimate is unbiased; an error introduced when fixing the distance remains in the final result. For the same reason, the fitting error is a useful consistency indicator for comparing solutions already obtained, such as those from the fault and post-bypass windows, but it is not a direct measure of location uncertainty. Reactive residual analysis complements this indicator as a non-identifying diagnostic: it reveals a discrepancy between the model and the measured signals but does not, by itself, identify its physical cause.

In the evaluation using field records, the staged procedure reduced the location error on the series-compensated line relative to the uncorrected LSE estimate and achieved a mean absolute error of 2.26 km across the 10 records with reference distances (2.48 km when excluding the record used during method development), compared with 10.33 km for the classical two-terminal phasor benchmark applied to the same records. The improvement was not uniform, and two records from the compensated line retained errors of approximately 7 km. On the uncompensated line, the same formulation yielded errors below one kilometer once the scaling of the signals at both terminals had been verified. This contrast indicates that a substantial portion of the error on the compensated line is associated with elements not included in the longitudinal line model, and that verifying units, transformation ratios, and polarities in the COMTRADE files is a necessary step before applying the fault locator.

These conclusions are limited by the available evidence. The reference distances correspond to approximate operational locations rather than independently verified measurements, the number of records with reference distances is small, and the field data contain a single fault event involving fault impedance. The result for this event and the idealized synthetic sweep are consistent with the algebraic cancellation of the fault-resistance term but do not demonstrate that fault location is unaffected by fault resistance under practical conditions involving noise, saturation, parameter errors, or unmodeled elements. The MOV parameters obtained by the method are equivalent values that reproduce the recorded response and do not identify the physical characteristics of the varistor. In addition, the stress analysis showed that the alignment stage cannot eliminate the location error caused by a time offset already present when the distance is fixed; therefore, these offsets must be corrected before estimating the distance.

6.3. Future Work

Future work will extend the validation to a larger set of field events with independently documented fault distances, compensation levels, MOV characteristics, and fault resistances, complemented by electromagnetic-transient simulations of conditions not represented in the available records. A coarse time-alignment stage applied before distance estimation will be developed and validated, and the equivalent MOV parameters will be constrained using manufacturer data when available. The method will also be compared with representative two-terminal algorithms developed specifically for series-compensated lines.

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Abbreviations and Symbols

The following abbreviations and symbols are used in this manuscript:

Abbreviations

.CFG	COMTRADE configuration file
COMTRADE	Common Format for Transient Data Exchange
DC	Direct current
EMTP/ATP	Electromagnetic Transients Program/ Alternative Transients Program
GPS	Global Positioning System
LSE	Least-squares error
MAE	Mean absolute error
MOV	Metal-oxide varistor
PMU	Phasor measurement unit
PS	Primary (P) or secondary (S) value
PSO	Particle swarm optimization
RMSE	Root mean square error

Symbols

A_k, B_k	Terms of the linear relation $A_k = mB_k$ for sample k
C	Equivalent capacitance of the series bank, $C = 1/(\omega X_c)$
d	Fault distance from the local terminal, $d = mL$

d_0	Distance fixed in Stage C, $d_0 = m_0 L$
d_w, d_{final}	Distance obtained in window w and reported distance, $d_{\text{final}} = d_{w^*}$
$ \Delta d $	Absolute location error with respect to the reference distance
f_s, f_n	Sampling frequency and nominal system frequency
h	Integration substep of the bank model, $h = 1/(4f_s)$
I_L, I_R	Local and remote terminal currents in the phasor benchmark
I_{ref}	MOV reference current
$\mathcal{I}[k]$	Cumulative trapezoidal integral of Δi_R
i_{line}	Total line current through the series bank
i_{MOV}	MOV current
j	Imaginary unit
k_R, k_L	Normalized resistive and inductive coefficients of the replica current
L	Total line length
m	Normalized fault distance from the local terminal, $m \in [0, 1]$
m_0	Normalized distance fixed in Stage C
m_{ph}	Normalized distance estimated by the phasor benchmark
n, α	Index of the preceding sample and fractional position in the interpolation
Q	Trapezoidal integral of the measured current used to initialize v_b
Q_r	Fraction of the residual associated with a predominantly quadrature component
q	Nonlinear exponent of the MOV characteristic
R_1, L_1	Positive-sequence resistance and inductance of the line
R_f	Fault resistance
$r(t)$	Residual between reconstructions, $\Delta v_{FL}(t) - \Delta v_{FR}(t)$
S_N, S_D	Numerator and denominator sums of the LSE solution
T	Delay used to construct the incremental quantities
$t, \Delta t$	Time and sampling interval, $\Delta t = 1/f_s$
V_L, V_R	Local and remote terminal voltages in the phasor benchmark
V_p	Equivalent MOV protective voltage
$v_b, \Delta v_b$	Voltage across the capacitor–MOV bank and its incremental value
$\Delta v, \Delta i$	Incremental voltage and current; subscripts L and R denote the local and remote terminals
$\Delta v_{AB}, \Delta v_{BC}, \Delta v_{CA}$	Incremental phase–phase loop voltages
$\Delta v_{FL}, \Delta v_{FR}$	Fault-point voltages reconstructed from the local and remote terminals
Δv_R^{corr}	Remote incremental voltage corrected for the bank voltage drop
$\Delta i_Z, \Delta i_{ZL}, \Delta i_{ZR}$	Replica current and its local and remote values
w, w^*	Analysis window (fault or bypass) and selected window
X_c	Capacitive reactance of the series bank
$X_{c,\text{rated}}$	Rated reactance of the series capacitor
X_c^*	Reactance selected in Stage C
$X_{c,E}^*, V_p^*, q^*$	Equivalent bank parameters selected in Stage E
$x(t), x[n], x_\tau[k]$	Generic signal, its samples, and its value interpolated with shift τ
Z_1, Z_0	Positive- and zero-sequence impedances of the line
δt	Time-domain equivalent of τ , $\delta t = \tau/f_s$
$\varepsilon, \varepsilon_w$	Normalized fitting error between the reconstructed fault-point voltages, and its value in window w
τ, τ^*	Time shift between terminal records, in samples, and selected shift
τ_{max}	Limit of the time-shift search interval
ω	Angular frequency, $\omega = 2\pi f_n$
$\mathcal{G}_{\text{MOV}}$	Set of admissible combinations of (X_c, V_p, q)

References

1. Di Santo, S.G.; Albertini, A.D.R.; Tiferes, R.R. Optimization-based fault location algorithm for series-compensated power transmission lines. *IEEE Access* **2022**, *10*, 46864–46877. <https://doi.org/10.1109/ACCESS.2022.3170041>.

2. Goldsworthy, D.L. A linearized model for MOV-protected series capacitors. *IEEE Trans. Power Syst.* **1987**, *2*, 953–957. <https://doi.org/10.1109/TPWRS.1987.4335284>.
3. *IEEE Std C37.116-2018 (Revision of IEEE Std C37.116-2007)*; IEEE Guide for Protective Relay Applications to Transmission-Line Series Capacitor Banks. IEEE Power & Energy Society: Piscataway, NJ, USA, 2018. <https://doi.org/10.1109/IEEESTD.2018.8497106>.
4. Taheri, R.; Eslami, M.; Damchi, Y. A current-based algorithm for one-end fault location in series capacitor compensated double-circuit transmission lines. *Comput. Electr. Eng.* **2023**, *106*, 108618. <https://doi.org/10.1016/j.compeleceng.2023.108618>.
5. Reis, R.L.A.; Lopes, F.V.; Ribeiro, E.P.; Moraes, C.M.; Silva, K.M.; Britto, A.M.; Agostinho, R.L.; Rodrigues, M.A.M. Traveling wave-based fault locators: Performance analysis in series-compensated transmission lines. *Electr. Power Syst. Res.* **2023**, *223*, 109567. <https://doi.org/10.1016/j.epsr.2023.109567>.
6. Fan, S.; Yun, Z. Fault location algorithm for series-compensated transmission lines using Prony algorithm. In *Proceedings of the 2023 International Conference on Power System Technology (PowerCon), Jinan, China, 21–22 September 2023*; IEEE: Piscataway, NJ, USA, 2023; pp. 1–5. <https://doi.org/10.1109/POWERCON58120.2023.10331384>.
7. Noori Aljweir, H.; Abdoos, A.A.; Ebadi, A. A new least squares error based method for accurate phasor estimation of fault currents in series-compensated transmission lines. *Comput. Electr. Eng.* **2024**, *118*, 109360. <https://doi.org/10.1016/j.compeleceng.2024.109360>. Erratum in *Comput. Electr. Eng.* **2025**, *121*, 109899. <https://doi.org/10.1016/j.compeleceng.2024.109899>.
8. Abed Shiadeh, S.M.; Abdoos, A.A.; Ebadi, A. Accurate fault current phasor estimation in series-compensated transmission lines using improved particle swarm optimisation. *IET Gener. Transm. Distrib.* **2026**, *20*, e70351. <https://doi.org/10.1049/gtd2.70351>.
9. Sadeh, J.; Hadsaid, N.; Ranjbar, A.M.; Feuillet, R. Accurate fault location algorithm for series compensated transmission lines. *IEEE Trans. Power Deliv.* **2000**, *15*, 1027–1033. <https://doi.org/10.1109/61.871370>.
10. Bains, T.P.S.; Sidhu, T.S.; Xu, Z.; Voloh, I.; Dadash Zadeh, M.R. Impedance-based fault location algorithm for ground faults in series-capacitor-compensated transmission lines. *IEEE Trans. Power Deliv.* **2018**, *33*, 189–199. <https://doi.org/10.1109/TPWRD.2017.2711358>.
11. Tabari, M.; Sadeh, J. Fault location in series-compensated transmission lines using adaptive network-based fuzzy inference system. *Electr. Power Syst. Res.* **2022**, *208*, 107800. <https://doi.org/10.1016/j.epsr.2022.107800>.
12. Rithisha, P.; Naidu, O.D.; Srikanth, N.V. Fault location for series compensated lines eliminating parameters as a setting. In *Proceedings of the 2024 IEEE PES Innovative Smart Grid Technologies–Asia (ISGT Asia), Bengaluru, India, 10–13 November 2024*; IEEE: Piscataway, NJ, USA, 2024; pp. 1–6. <https://doi.org/10.1109/ISGTAsia61245.2024.10876364>.
13. Akhavan, M.; Yazdaninejadi, A.; Talavat, V. Performance evaluation of single-ended impedance-based fault location methods in series-compensated transmission lines. In *Proceedings of the 2026 International Conference on Protection and Automation of Power Systems (IPAPS), Shiraz, Iran, 6–7 January 2026*; IEEE: Piscataway, NJ, USA, 2026; pp. 1–6. <https://doi.org/10.1109/IPAPS69408.2026.11499636>.
14. Hussain, S.; Osman, A. Fault location on series-compensated lines using unsynchronized measurements. In *Proceedings of the IEEE Power & Energy Society General Meeting, Vancouver, BC, Canada, 21–25 July 2013*; pp. 1–8. <https://doi.org/10.1109/PESMG.2013.6672958>.
15. Kasztenny, B.; Smelich, G. *Locating Faults by Using Incremental Quantities: Introduction, Application Considerations, and Performance*; Technical Report; Schweitzer Engineering Laboratories (SEL): Pullman, WA, USA, 2025.
16. Al-Mohammed, A.H.; Abido, M.A. A fully adaptive PMU-based fault location algorithm for series-compensated lines. *IEEE Trans. Power Syst.* **2014**, *29*, 2129–2137. <https://doi.org/10.1109/TPWRS.2014.2303207>.
17. Padmanabhan, S.; Terzija, V. Line parameter-free fault location algorithm for series compensated transmission lines. In *Proceedings of the 2013 IEEE Power & Energy Society General Meeting, Vancouver, BC, Canada, 21–25 July 2013*; IEEE: Piscataway, NJ, USA, 2013; pp. 1–5. <https://doi.org/10.1109/PESMG.2013.6672833>.
18. Naidu, O.D.; Pradhan, A.K. Model free traveling wave based fault location method for series compensated transmission line. *IEEE Access* **2020**, *8*, 193128–193137. <https://doi.org/10.1109/ACCESS.2020.3032458>.
19. Purohit, A.M.; Gohokar, V.N. Recent developments in distance protection of compensated transmission lines. In *Proceedings of the 2015 International Conference on Pervasive Computing (ICPC), Pune, India, 8–10 January 2015*; IEEE: Piscataway, NJ, USA, 2015; pp. 1–6. <https://doi.org/10.1109/PERVASIVE.2015.7087055>.
20. Dadashzade, A.; Nasr, M.-A.; Hooshyar, A.; Sharma, P. Protection of series-compensated transmission lines connected to inverter-based resources. *IEEE Trans. Power Deliv.* **2026**, *41*, 2523–2536. <https://doi.org/10.1109/TPWRD.2026.3698958>.
21. *IEEE/IEC Std C37.111-2013 (IEC 60255-24:2013)*; Measuring Relays and Protection Equipment—Part 24: Common Format for Transient Data Exchange (COMTRADE) for Power Systems. IEEE: New York, NY, USA, 2013.

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