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Linguistic Intuitionistic Fuzzy VIKOR Method with the Application of Artificial Neural Network †

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Abstract: This paper proposes Linguistic Intuitionistic Fuzzy (LIF) aggregation operators, LIF-energies, LIF-correlation, and LIF-correlation coefficients. Supporting theorems are also proven for the proposed functions, which are utilized in the Linguistic Intuitionistic Fuzzy–Vlse Kriterijumska Optimizacija Kompromisno Resenje (LIF-VIKOR) method within Decision Support Systems (DSS). Additionally, numerical examples are presented to validate the method. The sensitivity analysis of weighting vectors is conducted, and the consistency of final rankings affirms the robustness of the proposed approaches. Arithmetic operations, specifically subtraction and division, are applied to LIF numbers (LIFNs) within the LIF-VIKOR algorithm. Furthermore, a function called the Linguistic Median Membership (LMM) function is introduced to convert LIFN values into crisp numbers. In the LIF-VIKOR algorithm, the proposed correlation coefficient is used for ranking alternatives, while the entropy method is applied to compute weights. Sensitivity analysis is performed to ensure the consistency of the proposed method. Finally, an Artificial Neural Network (ANN) is integrated into the VIKOR algorithm to enhance computational efficiency, reducing the time and manpower required to solve the model.

Keywords: LIF-energies; LIF-correlation; LIF-correlation coefficients; LIF-VIKOR; DSS; ANN

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1. Introduction

Decision Support Systems (DSS) integrated with Artificial Neural Networks (ANN) are gaining significant attention among researchers worldwide. Various multi-criteria decision-making (MCDM) methodologies have been proposed to address complex decision problems. For instance, a methodology for transforming depressed industrial areas into sustainable urban spaces, prioritizing social, environmental, and economic factors, was developed in [1]. Theoretical foundations, empirical validations, and a framework for interdisciplinary applications in sequential decision-making were explored in [2]. Several studies have introduced novel approaches to handle uncertainty in decision-making. Interval-valued linguistic neutrosophic sets, Z-numbers, and the trapezium cloud model have been effectively combined to address randomness and uncertainty, as presented in the Z-IVLNS-TTC model, which minimizes information loss and distortion while developing a novel multi-objective programming approach for weight calculation. A group decision-making problem and sensitivity analysis demonstrated the model's practicality and superiority over existing methods in [3]. In the field of ranking methodologies, a novel coefficient aiding educators and institutions in selecting effective software solutions for

undergraduate power systems instruction was proposed in [4]. Two-dimensional linguistic intuitionistic fuzzy variables (2DLIFVs) were introduced to model expert cognitive information while ensuring evaluation reliability in [5]. Furthermore, a novel score function, distance measure, and four new aggregation operators were defined to rank and aggregate 2DLIFVs in a complex decision-making process involving multiple conflicting criteria such as cost, safety, and personal preferences in [6]. The VIKOR method, combined with fuzzy logic, has been widely used to provide effective compromise solutions. One study addressed the challenges of selecting appropriate companies for financing in sustainable supply chain finance by proposing a Multi-Attribute Group Decision-Making (MAGDM) method, addressed in [7]. Similarly, a three-way VIKOR method that combines ranking and classification to handle multi-attribute decision-making with conflicting attributes was introduced in [8]. Additionally, a new correlation coefficient (CC) for evaluating relationships between linguistic intuitionistic fuzzy sets (LIFSs) using linguistic intuitionistic fuzzy numbers (LIFNs) was proposed in [9]. Advancements in fuzzy set theory and optimization techniques have also contributed to improving decision-making models. The normal wiggly hesitant fuzzy set was introduced to overcome limitations in traditional hesitant fuzzy sets, capturing both the explicit and implicit preferences of decision-makers in [10]. A hybrid model combining the Gray Wolf Optimization and Archimedes Optimization Algorithms (AOA) with ANN was developed to predict construction and demolition waste quantities more accurately in [11]. Using data from 200 real-life projects in the Gaza Strip, the AOA-ANN model demonstrated superior accuracy compared to other models. Furthermore, machine learning (ML) and deep learning (DL) techniques have been applied in decision-making. Random Forest (RF), Support Vector Machines (SVM), and ANN models were evaluated, with RF showing the highest accuracy in a study on ML-based decision-making in [12]. An ANN model for accurately forecasting sky temperature in Djibouti, a hot and humid climate where existing estimation models fail to perform well, is introduced in [13]. The relationships between microRNA and cytokine profiles of hematopoietic progenitors from cord blood in hematopoiesis regulation were analyzed in [14]. The MAX78000, an ultra-low-power Edge AI microcontroller with a hardware-based convolutional neural network (CNN) accelerator, is evaluated in [15], focusing on its behavior in radiation environments. The RGB images and multispectral data from UAVs used to classify five crops using machine learning techniques are integrated in [16]. The IVN Fuzzy TOPSIS method is applied in [17] to select the optimal location for international education fairs in Türkiye. Two new group decision-making approaches using the linguistic intuitionistic fuzzy Yager weighted arithmetic aggregation operator are proposed in [18]. Linguistic Intuitionistic Fuzzy ANN was introduced in [19]. In this paper, new aggregation operators for linguistic intuitionistic fuzzy sets are proposed, along with a novel correlation coefficient. The proposed methods are applied to select the best alternative using the VIKOR method and ANN-based decision-making approaches.

2. New Aggregation Operators for Linguistic Intuitionistic Fuzzy Numbers

Definition 1. The Lin-IFWAA operator: *Let* $\widetilde{\sigma}_1 = < l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ *for j = 1,2,. . .,n be a collection of Linguistic Intuitionistic Fuzzy numbers. The Linguistic Intuitionistic Fuzzy Weighted Arithmetic Averaging (Lin-IFWAA) operator* $Lin - IFWAA : Q^n \to Q$ *is defined as* $Lin - IFWAA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = \left(l_{\sum_{j=1}^{n} \theta_j w_j}, \ 1 - \prod_{j=1}^{n} \left(1 - \alpha_j\right)^{w_j}, \prod_{j=1}^{n} \gamma_j^{w_j}\right)$, *where the weight vector of* $\widetilde{\sigma}_j$, $j = 1, 2, \ldots, n$ *is* $w = \left(w_1, w_2, \ldots, w_n\right)^T$ *and for* $w_j > 0, \sum_{j=1}^{n} w_j = 1$.

Definition 2. The Lin-IFWG operator: *Let* $\widetilde{\sigma}_1 = < l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ *for j = 1, 2, . . ., n be a collection of Linguistic Intuitionistic Fuzzy numbers. The Linguistic Intuitionistic Fuzzy Weighted Geometric (Lin-IFWG) operator* $Lin - IFWG : Q^n \to Q$ *is defined as Lin −*

$IFWG\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = r_{ij} = \left(l_{\sum_{j=1}^{n} \theta_j w_j}, \prod_{j=1}^{n} \alpha_j{}^{w_j}, 1 - \prod_{j=1}^{n} (1-\gamma_j)^{w_j}\right)$, *where the weight vector of* $\widetilde{\sigma}_j, j = 1, 2, \ldots, n$ *is* $w = (w_1, w_2, \ldots, w_n)^T$ *and for* $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$.

Definition 3. The Lin-IFOWG operator: *Let* $\widetilde{\sigma}_1 = < l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ *for j = 1,2,. . .,n be a collection of Linguistic Intuitionistic Fuzzy numbers. The Linguistic Intuitionistic Fuzzy Ordered Weighted Geometric (Lin-IFOWG) operator* $Lin - IFOWG : Q^n \to Q$ *is defined as* $Lin - IFOWG\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = r_{ij} = \left(l_{\sum_{j=1}^{n} \theta_j w_j}, \prod_{j=1}^{n} \alpha_{\sigma(j)}{}^{w_j}, 1 - \prod_{j=1}^{n} \left(1 - \gamma_{\sigma(j)}\right)^{w_j}\right)$, *where the weight vector of* $\widetilde{\sigma}_j, j = 1, 2, \ldots, n$ *is* $w = (w_1, w_2 \ldots, w_n)^T$ *and for* $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$.

Theorem 1. *When Lin-IFWA is used to aggregate a set of Linguistic Intuitionistic Fuzzy numbers* $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$, *the result is also a Linguistic Intuitionistic Fuzzy number.*

Proof. Let us use the induction approach to demonstrate this theorem. Take into account $\widetilde{\sigma}_1 \text{and} \widetilde{\sigma}_2$. $\widetilde{\sigma}_1 = \langle l_{\theta_1}, (\alpha_1, \gamma_1) \rangle, \widetilde{\sigma}_2 = \langle l_{\theta_2}, (\alpha_2, \gamma_2) \rangle$. Then, $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2\right) = \widetilde{\sigma}_1 w_1 \oplus \widetilde{\sigma}_2 w_2 = \langle l_{\theta_1 w_1 + \theta_2 w_2}, [1 - ([1-\alpha_1]^{w_1}[1-\alpha_2]^{w_2}), [\gamma_1]^{w_1}[\gamma_2]^{w_2}] \rangle$. Continuing the process with $\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_k$: $Lin - IFWA_w\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_k\right) = \left\langle l_{\sum_{j=1}^{k} \theta_j w_j}, \left[1 - \prod_{j=1}^{k} (1-\alpha_j)^{w_j}, \prod_{j=1}^{k} \gamma_j{}^{w_j}\right] \right\rangle$. Then, when n=k+1, $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_k, \widetilde{\sigma}_{k+1}\right) = \left\langle l_{\sum_{j=1}^{k+1} \theta_j w_j}, \left[1 - \prod_{j=1}^{k+1} (1-\alpha_j)^{w_j}, \prod_{j=1}^{k+1} \gamma_j{}^{w_j}\right] \right\rangle$. Hence, it is clear that for n=k+1, the operator is valid. The operator is thus true for every n according to the induction principle, concluding the proof. Hence, $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_k, \widetilde{\sigma}_{k+1}, \ldots, \widetilde{\sigma}_n\right) = \left\langle l_{\sum_{j=1}^{n} \theta_j w_j}, \left[1 - \prod_{j=1}^{n} (1-\alpha_j)^{w_j}, \prod_{j=1}^{n} \gamma_j{}^{w_j}\right] \right\rangle$, is an LIFN. □

Theorem 2. *Let* $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$ *be a collection of Linguistic Intuitionistic Fuzzy numbers and* $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ *be the weight vector of* $\widetilde{\sigma}_j$, *with* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \omega_j = 1$. *Finally, it can be proven that the Lin-IFWA operator is (i) Idempotent, (ii) Bounded, (iii) Monotonic, (iv) Commutative, and (v) Associative.*

Proof. (i) Idempotency: If all $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$ are equal, that is $\widetilde{\sigma}_j = \widetilde{\sigma}$ for all j, then let us establish that $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = \widetilde{\sigma}$. Since $\widetilde{\sigma}_j = \widetilde{\sigma}$, for all j, $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = \sum_{j=1}^{n} \widetilde{\sigma}\omega_j = \widetilde{\sigma}\sum_{j=1}^{n} \omega_j = \widetilde{\sigma}$. (ii) Boundedness: Let, $\widetilde{\sigma}^- \leq Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) \leq \widetilde{\sigma}^+$. The following representation illustrates this: $\widetilde{\sigma}^- = \left\langle \min_i l_{\theta_i}; \left[\min_i \alpha_{\widetilde{\sigma}_i}\right], \left[\max_i \gamma_{\widetilde{\sigma}_i}\right] \right\rangle$, $\widetilde{\sigma}^+ = \left\langle \max_i l_{\theta_i}; \left[\max_i \alpha_{\widetilde{\sigma}_i}\right], \left[\min_i \gamma_{\widetilde{\sigma}_i}\right] \right\rangle$. (iii) Monotonicity: Let $\widetilde{\sigma}_j{}^*, (j = 1, 2, \ldots, n)$ be a collection of LIFNs. If $\widetilde{\sigma}_j \leq \widetilde{\sigma}_j{}^*$ for all j, then we show that $Lin - IFWA(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) \leq Lin - IFWA(\widetilde{\sigma}_1{}^*, \widetilde{\sigma}_2{}^*, \ldots, \widetilde{\sigma}_n{}^*)$ for all ω. Let $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = \sum_{j=1}^{n} \widetilde{\sigma}_i\omega_i$ and $Lin - IFOWA\left(\widetilde{\sigma}_1{}^*, \widetilde{\sigma}_2{}^*, \ldots, \widetilde{\sigma}_n{}^*\right) = \sum_{j=1}^{n} \widetilde{\sigma}_1{}^*\omega_i$. Since $\widetilde{\sigma}_j \leq \widetilde{\sigma}_j{}^*$ for all j, $Lin - IFWA(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n) \leq Lin - IFWA(\widetilde{\sigma}_1{}^*, \widetilde{\sigma}_2{}^*, \ldots, \widetilde{\sigma}_n{}^*)$, which is possible when we consider the fact that $\widetilde{\sigma}_j = \left\langle l_{\theta_j}, \left[\alpha_{\widetilde{\sigma}_j}\right], \left[\gamma_{\widetilde{\sigma}_j}\right] \right\rangle$ *and* $\widetilde{\sigma}^*{}_j = \left\langle l_{\theta_j{}^*}, \left[\alpha_{\widetilde{\sigma}_j{}^*}\right], \left[\gamma_{\widetilde{\sigma}_j{}^*}\right] \right\rangle$, where $l_{\theta_j} \leq l_{\theta_j{}^*} \text{and} \alpha_{\widetilde{\sigma}_j} \leq \alpha_{\widetilde{\sigma}_j{}^*}; \gamma_{\widetilde{\sigma}_j} \leq \gamma_{\widetilde{\sigma}_j{}^*}, \forall j$. (iv) Commutativity: Let $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$ be a collection of LIFNs. Then we have to prove $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = Lin - IFWA\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)$, $j = 1, 2, \ldots, n$ and for all ω where $\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)$ is any permutation of $\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right)$. Let $Lin - IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = \sum_{j=1}^{n} \widetilde{a}\omega_j = \left\langle \sum_{j=1}^{n} \theta_j\omega_j, \left[\prod_{j=1}^{n} (1-\alpha_j)^{w_j}, \prod_{j=1}^{n} \gamma_j{}^{w_j}\right] \right\rangle = \left\langle l_{\theta_j}, \left[\alpha_{\widetilde{\sigma}_j}\right], \left[\gamma_{\widetilde{\sigma}_j}\right] \right\rangle$. Now, $Lin - IFWA\left(\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)\right) = \sum_{j=1}^{n} \overline{\widetilde{a}}\omega_j = \left\langle \sum_{j=1}^{n} \overline{\theta}_j\omega_{j'}, \left[\prod_{j=1}^{n} (1-\overline{\alpha}_j)^{w_j}, \prod_{j=1}^{n} \overline{\gamma}_j{}^{w_j}\right] \right\rangle = \left\langle l_{\overline{\theta}_j}, \left[\overline{\alpha}_{\widetilde{\sigma}_j}\right], \left[\overline{\gamma}_{\widetilde{\sigma}_j}\right] \right\rangle$. Since $\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)$ is any permutation of $\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right)$, we can have $\overline{\widetilde{\sigma}}_{\sigma(j)} = \widetilde{\sigma}_{\sigma(j)}$, for $j = 1, 2, \ldots, n$

and hence $Lin-IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \ldots, \widetilde{\sigma}_n\right) = Lin-IFWA\left(\overline{\widetilde{\sigma}}_1, \overline{\widetilde{\sigma}}_2, \ldots, \overline{\widetilde{\sigma}}_n\right)$. (v) Associativity: Let $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$ be a collection of Linguistic Intuitionistic Fuzzy numbers. Then, $Lin-IFWA\left(\widetilde{\sigma}_1, \widetilde{\sigma}_2, \widetilde{\sigma}_3\right) = Lin-IFWA\left(\widetilde{\sigma}_1, \left(\widetilde{\sigma}_2, \widetilde{\sigma}_3\right)\right) = Lin-IFWA\left((\widetilde{\sigma}_1, \widetilde{\sigma}_2), \widetilde{\sigma}_3\right)$. □

Theorem 3. *When Lin-IFWG is used to aggregate a set of Linguistic Intuitionistic Fuzzy numbers* $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$*, the result is also a Linguistic Intuitionistic Fuzzy number.*

Theorem 4. *Let* $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$*, be a collection of Linguistic Intuitionistic Fuzzy numbers and* $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ *be the weight vector of* $\widetilde{\sigma}_j$*, with* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \omega_j = 1$*. Finally, it can be proven that the Lin-IFWG operator is (i) Idempotent, (ii) Bounded, (iii) Monotonic, (iv) Commutative and (v) Associative.*

Theorem 5. *When Lin-IFOWG is used to aggregate a set of Linguistic Intuitionistic Fuzzy numbers* $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$ *the result is also a Linguistic Intuitionistic Fuzzy number.*

Theorem 6. *Let* $\widetilde{\sigma}_j, (j = 1, 2, \ldots, n)$ *be a collection of Linguistic Intuitionistic Fuzzy numbers and* $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ *be the weight vector of* $\widetilde{\sigma}_j$*, with* $\omega_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \omega_j = 1$*. Then the Lin-IFOWG operator is (i) Idempotent, (ii) Bounded, (iii) Monotonic, (iv) Commutative, and (v) Associativity.*

All the above theorems can be proven in a similar way to Theorems 1 and 2.

3. Correlation Coefficient of LIFNs

The following method will be useful for figuring out an LIFN's correlation coefficient excluding hesitance degree: Let $\widetilde{\sigma}_j = < l_{\theta(\nu)}, (\alpha(\sigma_j), \gamma(\sigma_j)) >$. Here, $l_{\theta(\nu)}, \alpha(\sigma_j), \gamma(\sigma_j)$ represent the normalized linguistic degree, membership and non-membership, respectively, where $0 \leq S_{\theta(\nu)} \leq 1$. For LIFN, no correlation measure has been explicitly defined so far in the literature. For LIFN, the correlation measure is defined as follows: Let $G = \left\{ \left\langle l_{\theta_G(\nu)}, \alpha_G(\nu), \gamma_G(\nu)) \right\rangle : \nu \in \wp \right\}$ and $H = \left\{ \left\langle l_{\theta_H(\nu)}, (\alpha_H(\nu), \gamma_H(\nu)) \right\rangle : \nu \in \wp \right\}$ be two LIFSs. Then, for each $G, H \in LIFS(\wp)$, the informational LIF energy of G and H is $E_{LIFS}(G) = \frac{1}{2q}\sum_{i=1}^{q} ({l_{\theta_G}}^2(\nu_i) + {\alpha_G}^2(\nu_i) + {\gamma_G}^2(\nu_i))$, $E_{LIFS}(H) = \frac{1}{2q}\sum_{i=1}^{q} ({l_{\theta_H}}^2(\nu_i) + {\alpha_H}^2(\nu_i) + {\gamma_H}^2(\nu_i))$. The correlation of G and H: $C_{LIFS}(G, H) = \frac{1}{2q}\sum_{i=1}^{q} (l_{\theta_G}(\nu_i) l_{\theta_H}(\nu_i) + \alpha_G(\nu_i)\alpha_H(\nu_i) + \gamma_G(\nu_i)\gamma_H(\nu_i))$. The correlation coefficient between G and H is $K_{LIFS}(G, H) = \frac{C_{LIFS}(G,H)}{\sqrt{E_{LIFS}(G).E_{LIFS}(H)}}$.

Proposition 1. *For any* $G, H \in LIFS(\wp)$*, the propositions are given as follows: (i)* $0 \leq C_{LIFS}(G, H) \leq 1$*; (ii)* $C_{LIFS}(G, H) = C_{LIFS}(H, G)$*; (iii)* $K_{LIFS}(G, H) = K_{LIFS}(H, G)$ *and (iv)* $K_{LIFS}(G, H) = 1$ *if* $G = H$*.*

Proof. Here, $0 \leq \alpha_i, \gamma_i \leq 1$, $0 \leq l_{\theta_i} \leq 1$. Hence $0 \leq C_{LIFS}(G, H) \leq 1$. (i) Clearly, $C_{LIFS}(G, H) = C_{LIFS}(H, G)$, from the definition of $C_{LIFS}(G, H)$. (ii) $K_{LIFS}(G, H) = \frac{C_{LIFS}(G,H)}{\sqrt{E_{LIFS}(G).E_{LIFS}(H)}} = K_{LIFS}(H, G)$, If $G = H$, then $K_{LIFS}(G, G) = \frac{C_{LIFS}(G,G)}{\sqrt{E_{LIFS}(G).E_{LIFS}(G)}} = 1$. □

Theorem 7. *For* $G, H \in LIFS(\wp)$, $0 \leq K_{LIFS}(G, H) \leq 1$.

Proof. Obviously $0 \leq K_{LIFS}(G, H)$, and $\left[\frac{1}{2q}\sum_{i=1}^{q}(l_{\theta_G}(\nu_i) l_{\theta_H}(\nu_i) + \alpha_G(\nu_i)\alpha_H(\nu_i) + \gamma_G(\nu_i)\gamma_H(\nu_i))\right]^2 \leq \left[\frac{1}{2q}\sum_{i=1}^{q}\left({l_{\theta_G}}^2(\nu_i) + {\alpha_G}^2(\nu_i) + {\gamma_G}^2(\nu_i)\right)\right] \times \left[\frac{1}{2q}\sum_{i=1}^{q}\left({l_{\theta_H}}^2(\nu_i) + {\alpha_H}^2(\nu_i) + {\gamma_H}^2(\nu_i)\right)\right]$. Hence, $K_{LIFS}(G, H) = \frac{C_{LIFS}(G,H)}{\sqrt{E_{LIFS}(G).E_{LIFS}(H)}} \leq 1$, where $0 \leq K_{LIFS}(G, H) \leq 1$. □

Theorem 8. *$C_{LIFS}(G,H) = 0$ if fG and H are non-fuzzy sets with the condition $\alpha_G(\nu_i)\alpha_H(\nu_i) = 1$ or $\gamma_G(\nu_i)\gamma_H(\nu_i) = 1$.*

Proof. For all $\nu_i \in \wp, l_{\theta_G}(\nu_i)l_{\theta_H}(\nu_i) + \alpha_G(\nu_i)\alpha_H(\nu_i) + \gamma_G(\nu_i)\gamma_H(\nu_i) \geq 0$. If $C_{LIFS}(G,H) = 0 \forall \nu_i \in \wp$, then $\alpha_G(\nu_i)\alpha_H(\nu_i) = 0, \gamma_G(\nu_i)\gamma_H(\nu_i) = 0$. If$\alpha_G(\nu_i) = 1$, then $\gamma_G(\nu_i) = 0$and $\alpha_H(\nu_i) = 0, \Rightarrow \alpha_G(\nu_i) + \alpha_H(\nu_i) = 1$. If $\alpha_H(\nu_i) = 1$then $\gamma_H(\nu_i) = 0$and$\alpha_G(\nu_i) = 0, \Rightarrow \alpha_G(\nu_i) + \alpha_H(\nu_i) = 1$. Similarly, it can be proven for $\gamma(\nu_i)$. □

4. The Linguistic Intuitionistic Fuzzy VIKOR Method with Sensitivity Analysis Using Correlation Coefficients

Algorithm for LIF-VIKOR Method

The working rule of the VIKOR method using a correlation coefficient is given as follows:

Step 1. Make the integrated matrix;

Step 2. Determine the best attributes q_j^+ and the worst attributes q_j^- involved in the problem;

Step 3. Compute the values of A_i and B_i given as

$$A_i = \sum_{j=1}^{M} w_j\left(\left[q_j^+ - q_{ij}\right] / \left[q_j^+ - q_j^-\right]\right); B_i = \underset{j}{Max}\left\{w_j\left(\left[q_j^+ - q_{ij}\right] / \left[q_j^+ - q_j^-\right]\right)\right\}, j = 1,2,\ldots M;$$

Step 4. Compute $C_i = v\frac{(A_i - A^*)}{(A^- - A^*)} + (1-v)\frac{(B_i - B^*)}{(B^- - B^*)}$, where $A^* = \underset{i}{min} A_i; A^- = \underset{i}{max} A_i$. A^- is the maximum value of A_i, and A^* is the minimum value of A_i; B^- is the maximum value of B_i, B^* is the minimum value of B_i, and v is the weight of the strategy of 'the majority of attributes'. The value of v lies in the range of 0 to 1. Values of C are in the form of LIFN;

Step 5: Three ranking lists should be prepared based on A_i, B_i and C_i. The one with the minimum value of C_i is chosen as the best alternative;

Step 6. Rank the alternatives according to the values of $K_{LIFS}(C_{\min}, C_i)$. The theorem given below discusses the changes in the weights of attributes.

Theorem 2. If the weight of the pth attributes changes to Δp, then the weight of the other attributes changes by $\Delta_j = \frac{(\Delta_p . w_j)}{(w_p - 1)}; j = 1,2,\ldots .k, j \neq p$.

5. The Entropy Method to Determine the Weight of Each Indicator

The method of finding weights of the attributes by the entropy function is given as follows:

Step 1. Calculate $D_{ij} = \frac{g_{ij}}{\sum_{j=1}^{m} g_{ij}}$, where r_{ij} is the ith scheme's jth indicators value;

Step 2. Calculate the entropy value e_j, $e_j = -k\sum_{i=1}^{m} D_{ij}\ln(D_{ij})$, $k = \frac{1}{lnm}$, where m is the number of schemes;

Step 3. Calculate $w_j = \frac{(1-e_j)}{\sum_{j=1}^{n}(1-e_j)}$ and $0 \leq w_j \leq 1, \sum_{j=1}^{n} w_j = 1$, where n is the number of indicators.

6. Numerical Illustration: LIF-VIKOR with Entropy, Sensitivity and ANN

Assume there are four industries (alternatives) $\{L_1, L_2, L_3, L_4\}$ to be weighed against certain criteria. Evaluate industries in terms of their technological innovation capability, evaluating 'factors' such as resource ability for digitalization (C_1), organizational innovation (C_2), Innovation Centers (C_3), and Innovative products (C_4). Consider a group of experts whose weights are given as $\lambda = (0.4, 0.32, 0.28)$. The experts' assessments of the four industries are listed in the following tables.

$$\widetilde{R}_1 = \begin{bmatrix} \langle l_5,(0.2,0.7)\rangle & \langle l_2,(0.4,0.6)\rangle & \langle l_5,(0.5,0.5)\rangle & \langle l_3,(0.2,0.6)\rangle \\ \langle l_4,(0.4,0.6)\rangle & \langle l_5,(0.4,0.5)\rangle & \langle l_3,(0.1,0.8)\rangle & \langle l_4,(0.5,0.5)\rangle \\ \langle l_3,(0.2,0.7)\rangle & \langle l_4,(0.2,0.7)\rangle & \langle l_4,(0.3,0.7)\rangle & \langle l_5,(0.2,0.7)\rangle \\ \langle l_6,(0.5,0.4)\rangle & \langle l_2,(0.2,0.8)\rangle & \langle l_3,(0.2,0.6)\rangle & \langle l_3,(0.3,0.6)\rangle \end{bmatrix}$$

$$\widetilde{R}_2 = \begin{bmatrix} \langle l_4,(0.1,0.7)\rangle & \langle l_3,(0.2,0.7)\rangle & \langle l_3,(0.2,0.8)\rangle & \langle l_6,(0.4,0.5)\rangle \\ \langle l_5,(0.4,0.5)\rangle & \langle l_3,(0.3,0.6)\rangle & \langle l_4,(0.2,0.6)\rangle & \langle l_3,(0.2,0.7)\rangle \\ \langle l_4,(0.2,0.6)\rangle & \langle l_4,(0.2,0.7)\rangle & \langle l_2,(0.4,0.6)\rangle & \langle l_3,(0.3,0.7)\rangle \\ \langle l_5,(0.3,0.6)\rangle & \langle l_4,(0.4,0.5)\rangle & \langle l_2,(0.3,0.6)\rangle & \langle l_4,(0.2,0.6)\rangle \end{bmatrix}$$

$$\widetilde{R}_3 = \begin{bmatrix} \langle l_5,(0.2,0.6)\rangle & \langle l_3,(0.3,0.7)\rangle & \langle l_4,(0.4,0.5)\rangle & \langle l_4,(0.2,0.7)\rangle \\ \langle l_4,(0.3,0.7)\rangle & \langle l_5,(0.3,0.6)\rangle & \langle l_2,(0.1,0.8)\rangle & \langle l_3,(0.4,0.6)\rangle \\ \langle l_4,(0.2,0.7)\rangle & \langle l_5,(0.3,0.6)\rangle & \langle l_1,(0.1,0.8)\rangle & \langle l_4,(0.2,0.7)\rangle \\ \langle l_3,(0.2,0.7)\rangle & \langle l_3,(0.1,0.7)\rangle & \langle l_4,(0.3,0.6)\rangle & \langle l_5,(0.4,0.5)\rangle \end{bmatrix}$$

Make the integrated matrix, as mentioned in the algorithm.

$$\widetilde{R} = \begin{bmatrix} \langle l_{0.780},(0.169,0.670)\rangle & \langle l_{0.433},(0.313,0.658)\rangle & \langle l_{0.680},(0.388,0.581)\rangle & \langle l_{0.707},(0.270,0.591)\rangle \\ \langle l_{0.720},(0.374,0.591)\rangle & \langle l_{0.727},(0.342,0.558)\rangle & \langle l_{0.507},(0.133,0.730)\rangle & \langle l_{0.567},(0.388,0.586)\rangle \\ \langle l_{0.600},(0.200,0.666)\rangle & \langle l_{0.713},(0.229,0.670)\rangle & \langle l_{0.420},(0.285,0.692)\rangle & \langle l_{0.680},(0.233,0.700)\rangle \\ \langle l_{0.807},(0.365,0.533)\rangle & \langle l_{0.487},(0.246,0.663)\rangle & \langle l_{0.493},(0.262,0.600)\rangle & \langle l_{0.647},(0.300,0.570)\rangle \end{bmatrix}.$$

Computation 1: VIKOR method with known weights

Assign the known weights $w = (0.2383, 0.2714, 0.3294, 0.1609)$ to each indicator and use the cost type indicators $V_{ij} = \left(\frac{minx_{ij}}{x_{ij}}\right)$ and the benefit type indicators $V_{ij} = \left(\frac{x_{ij}}{maxx_{ij}}\right)$ to derive the following decision matrix:

$$v = \begin{pmatrix} \langle l_{0.967},(0.436,0.294)\rangle & \langle l_{0.537},(0,1)\rangle & \langle l_{0.843},(0,1)\rangle & \langle l_{0.876},(0.696,0.125)\rangle \\ \langle l_{0.893},(0,1)\rangle & \langle l_{0.901},(0.880,0.054)\rangle & \langle l_{0.628},(0.343,0.421)\rangle & \langle l_{0.702},(0,1)\rangle \\ \langle l_{0.744},(0.515,0.286)\rangle & \langle l_{0.884},(0.591,0.295)\rangle & \langle l_{0.521},(0,1)\rangle & \langle l_{0.843},(0.601,0.358)\rangle \\ \langle l_1,(0.939,0)\rangle & \langle l_{0.603},(0.633,0.279)\rangle & \langle l_{0.612},(0.674,0.144)\rangle & \langle l_{0.802},(0.773,0.080)\rangle \end{pmatrix}.$$

By using the Linguistic Median Membership function, the above matrix is converted into a crisp matrix.

$$v = \begin{pmatrix} 1.054 & \underline{0.269} & 0.421 & 1.224 \\ \underline{0.446} & \underline{\underline{1.364}} & 0.775 & \underline{0.351} \\ 0.986 & 1.090 & \underline{0.260} & 1.043 \\ \underline{\underline{1.470}} & 0.979 & \underline{\underline{1.070}} & \underline{\underline{1.247}} \end{pmatrix}.$$

In the above matrix, negative ideal solutions are highlighted by single lines and positive ideal solutions are highlighted by double lines.

The positive ideal solution and the negative ideal solution are

$$q^* = (\langle l,(0.939,0)\rangle, \langle l_{0.901},(0.880,0.054)\rangle, \langle l_{0.612},(0.673,0.144)\rangle, \langle l_{0.843},(0.601,0.358)\rangle).$$

$$q^- = (\langle l_{0.893},(0,1)\rangle,\ \langle l_{0.537},(0,1)\rangle,\ \langle l_{0.521},(0,1)\rangle,\ \langle l_{0.702},(0,1)\rangle).$$

Calculate the *A*, *B*, and *C* values for all the alternatives in the decision problem, as in Section 4:

$$A_i = \sum_{i=1}^{n} \left(w_i\left(q_i{}^* - q_{ij}\right) / \left(q_i{}^* - q_i{}^-\right)\right),\ B_i = max\left[w_i\left(q_i^* - q_{ij}\right) / \left(q_i^* - q_i^-\right)\right].$$

$$A_1 = \langle l_{1.221}, (1,0)\rangle, A_2 = \langle l_{0.459}, (1,0)\rangle, A_3 = \langle l_{0.910}, (0.357, 0.582)\rangle, A_4 = \langle l_{0.269}, (0,1)\rangle.$$

$$B_1 = \langle l_{0.271}, (1,0)\rangle, B_2 = \langle l_{0.161}, (1,0)\rangle, B_3 = \langle l_{0.012}, (0.357, 0.581)\rangle, B_4 = \langle l_{0.222}, (0,1)\rangle.$$

Then, $C_j = \nu\left(A_i - A^*\right)/\left(A^- - A^*\right) + (1-\nu)\left(B_j - B^*\right)/\left(B^- - B^*\right)$, where $A^* = \min A_j$; $A^- = \max A_j$; $B^* = \min B_j$; $B^- = \max B_j$,

$$A^- = \langle l_{1.221}, (1,0)\rangle,\ A^* = \langle l_{0.269}, (0,1)\rangle, B^- = \langle l_{0.271}, (1,0)\rangle,\ B^* = \langle l_{0.222}, (0,1)\rangle.$$

$$C_1 = \langle l_1, (1,0)\rangle, C_2 = \langle l_{0.719}, (1,0)\rangle, C_3 = \langle l_{2.462}, (0.357, 0.582)\rangle, C_4 = \langle l_0, (0,1)\rangle.$$

C_4 is the lowest value, and hence L_4 is the best alternative.

By using the proposed Correlation coefficient, $K_{LIFS}(C_4, C_1) = 0$, $K_{LIFS}(C_4, C_2) = 0$, and $K_{LIFS}(C_4, C_3) = 0.228$. Hence the ranking of the alternatives is $L_4 \succ L_3 \succ L_1 = L_2$.

Computation 2: VIKOR method with weights from entropy method

Calculate $D_{ij} = \frac{g_{ij}}{\sum_{j=1}^{m} g_{ij}}$, where g_{ij} is computed from the integrated matrix.

$$D_{ij} = \begin{pmatrix} 0.355 & 0.090 & 0.142 & 0.412 \\ 0.253 & 0.311 & 0.174 & 0.263 \\ 0.193 & 0.249 & 0.254 & 0.304 \\ 0.423 & 0.143 & 0.214 & 0.220 \end{pmatrix}.$$

Calculate the entropy value $e_j = -k\sum_{i=1}^{m} D_{ij}\ln(D_{ij}), k = \frac{1}{ln(m)}$, where m = 4, and k = 1/ln(m) = 1/ln(4). $k = 0.72135$, and so calculate the weights $w_j = \frac{(1-e_j)}{\sum_{j=1}^{n}(1-e_j)}$. $w_1 = 0.387$, $w_2 = 0.336, w_3 = 0.247, w_4 = 0.030$. It can be easily seen that $\sum w_j = 1$.

Hence, the weights calculated by the entropy method are $w' = (0.387, 0.336, 0.247, 0.030)$. Proceeding through step 1 to step 4, as in the previous computations with the weights calculated above in the entropy method, the following matrix is obtained:

$$v = \begin{pmatrix} \langle l_{0.967}, (0.436, 0.294)\rangle & \langle l_{0.537}, (0,1)\rangle & \langle l_{0.843}, (0,1)\rangle & \langle l_{0.876}, (0.696, 0.125)\rangle \\ \langle l_{0.893}, (0,1)\rangle & \langle l_{0.901}, (0.880, 0.054)\rangle & \langle l_{0.628}, (0.343, 0.421)\rangle & \langle l_{0.702}, (0,1)\rangle \\ \langle l_{0.744}, (0.515, 0.286)\rangle & \langle l_{0.884}, (0.591, 0.295)\rangle & \langle l_{0.521}, (0,1)\rangle & \langle l_{0.843}, (0.601, 0.358)\rangle \\ \langle l_1, (0.939, 0)\rangle & \langle l_{0.603}, (0.633, 0.279)\rangle & \langle l_{0.612}, (0.674, 0.144)\rangle & \langle l_{0.802}, (0.773, 0.080)\rangle \end{pmatrix}.$$

The positive ideal solution and the negative ideal solution are

$$q^* = (\langle l, (0.939, 0)\rangle, \langle l_{0.901}, (0.880, 0.054)\rangle, \langle l_{0.612}, (0.673, 0.144)\rangle, \langle l_{0.843}, (0.601, 0.358)\rangle).$$

$$q^- = (\langle l_{0.893}, (0,1)\rangle,\ \langle l_{0.537}, (0,1)\rangle,\ \langle l_{0.521}, (0,1)\rangle,\ \langle l_{0.702}, (0,1)\rangle).$$

Calculate the A, B, and C values for all the alternatives in the decision problem, as in computation 1, with entropy weights,

$$A_i = \sum_{i=1}^{n}\left(w_i\left(q_i * - q_{ij}\right)/\left(q_i * - q_i{}^-\right)\right),\ B_i = max\left[w_i\left(q_i^* - q_{ij}\right)/\left(q_i^* - q_i^-\right)\right].$$

$$A_1 = \langle l_{1.092}, (1,0)\rangle, A_2 = \langle l_{0.461}, (1,0)\rangle, A_3 = \langle l_{1.184}, (0.422, 0.511)\rangle, A_4 = \langle l_{0.284}, (1,0)\rangle.$$

$$B_1 = \langle l_{0.336}, (1,0)\rangle, B_2 = \langle l_{0.030}, (1,0)\rangle, B_3 = \langle l_{0.015}, (0.422, 0.511)\rangle, B_4 = \langle l_{0.275}, (0,1)\rangle.$$

And then, $C_j = \nu\left(A_i - A^*\right)/\left(A^- - A^*\right) + (1-\nu)\left(B_j - B^*\right)/\left(B^- - B^*\right)$, where $A^* = \min A_j$; $A^- = \max A_j$; $B^* = \min B_j$; $B^- = \max B_j$.

$$A^* = \langle l_{0.284}, (1,0)\rangle, A^- = \langle l_{1.092}, (1,0)\rangle, B^* = \langle l_{0.275}, (0,1)\rangle,\ B^- = \langle l_{0.336}, (1,0)\rangle.$$

$$C_1 = \langle l_1, (1,0)\rangle, C_2 = \langle l_{2.124}, (1,0)\rangle, C_3 = \langle l_{2.681}, (0.422, 0.511)\rangle, C_4 = \langle l_{0.009}, (0,1)\rangle.$$

C_4 is the lowest value, hence L_4 is the best alternative.

Using the proposed correlation coefficient $K_{LIFS}(C_4, C_1) = 0$, $K_{LIFS}(C_4, C_2) = 0$, and $K_{LIFS}(C_4, C_3) = 0.185$. Hence, the ranking of the alternatives is $L_4 \succ L_3 \succ L_1 = L_2$.

Sensitivity Analysis for the VIKOR Method

The process of recalculating outcomes under alternative assumptions to determine the impact of a variable under sensitivity analysis can be useful for various purposes. In this work, the sensitivity analysis is performed on the weight vectors derived from the entropy method. The weights calculated by the entropy method are $w\prime = (0.387, 0.336, 0.247, 0.030)$. Now let us analyze the change in the output when changes are allowed in all the vectors of the weights. Let us allow the small change, $\Delta_1 = 0.052$, for the first vector of the weights.

Following the same computations as are used in sensitivity analysis, using VIKOR, the following decisions (computation 3 to 6) can be obtained, as given in Table 1.

Table 1. Comparison of the proposed methods with different weight vectors.

PROPOSED LIF-VIKOR COMPUTATION METHODS	RANKING OF ALTERNATIVES
Computation 1: Using LIF-VIKOR method and known weights	$L_4 \succ L_3 \succ L_1 = L_2$.
Computation 2: Using LIF-VIKOR method with ENTROPY	$L_4 \succ L_3 \succ L_1 = L_2$.
Computation 3: Using LIF-VIKOR method with SENSITIVITY analysis starting with 1st weight vector	$L_4 \succ L_3 \succ L_1 = L_2$.
Computation 4: Using LIF-VIKOR method with SENSITIVITY analysis starting with 2nd weight vector	$L_4 \succ L_3 \succ L_1 = L_2$.
Computation 5: Using LIF-VIKOR method with SENSITIVITY analysis starting with 3rd weight vector	$L_4 \succ L_3 \succ L_1 = L_2$.
Computation 6: Using LIF-VIKOR method with SENSITIVITY analysis starting with 4th weight vector	$L_4 \succ L_3 \succ L_1 = L_2$.

From the above table, it can be observed that the best alternative is L_4.

Computation 7: Decision-making with Artificial Neural Network

The following ANN procedure will be incorporated in order to use a high-complexity Artificial Neural Network (ANN) to analyze the defuzzyfied decision matrix:

1. Assign input to the matrix. Consider the columns as traits or attributes and the rows as options or alternatives;
2. Normalize the data, ensuring that each attribute contributes evenly by scaling the data;
3. Initiate the ANN—
   - Input layer. The number of attributes or traits is equal to the number of neurons.
   - Hidden layers. More than one layer will be employed for increased complexity.
   - Output layer. Each possibility is scored by a single neuron;
4. Train the ANN. To reduce error, use backpropagation;
5. Assess the outcomes. Choose the option with the greatest output rating.

Pseudo Code for the ANN:

Input: Matrix of alternatives and attributes & Defuzzyfied matrix with the decision data

Step 1: Normalization of the decision matrix

Step 2: Initiate ANN

Input neurons # Number of attributes

hidden neurons = 10 # First hidden layer
hidden neurons = 8 # Second hidden layer
output neurons = 1 # Output layer (single score per row)
Initialize the weighting vector and biases for the ANN
Weights input hidden1 = random initialize (input neurons, hidden neurons layer1)
Weights hidden1 hidden2 = random initialize (hidden neurons layer1, hidden neurons layer2)
Weights hidden2 output = random initialize (hidden neurons layer2, output neurons)
Bias hidden1 = random initialize (hidden neurons layer1)
Bias hidden2 = random initialize (hidden neurons layer2)
Bias output = random initialize (output neurons)
Activation function: define sigmoid(x)=1/(1 + exp(−x))
Step 3: Forward Pass: Define forward pass(row):
Input to hidden1 = sigmoid (dot product (row, weights input hidden1) + bias hidden1)
hidden1 to hidden2 = sigmoid (dot product (input to hidden1, weights hidden1 hidden2) + bias hidden2)
output = sigmoid (dot product (hidden1 to hidden2, weights hidden2 output) + bias output)
return output
# Step 4: Train the ANN (Backpropagation)
for epoch in range:
for row in normalized matrix:
# Calculate forward pass
output = forward pass(row)
# Compute error and backpropagate to update weights
Backpropagate (output, row)
# Step 5: Evaluate and Select Best Alternative
scores = [forward pass(row) for row in normalized matrix]
best alternative = argmax(scores)

Following the procedure, let us start the ANN with the defuzzyfied matrix from computation-1:

$$v = \begin{pmatrix} 1.054 & \underline{0.269} & 0.421 & 1.224 \\ \underline{0.446} & \underline{1.364} & 0.775 & \underline{0.351} \\ 0.986 & 1.090 & \underline{0.260} & 1.043 \\ \underline{1.470} & 0.979 & \underline{1.070} & \underline{1.247} \end{pmatrix}.$$

Step 1. Normalize the Matrix

We will normalize the matrix column-wise so that each column has values scaled between 0 and 1, ensuring that all attributes contribute equally to the analysis. The normalized matrix is given as follows:

$$v = \begin{pmatrix} 0.500 & 0.000 & 0.214 & 0.866 \\ 0.000 & 1.000 & 0.676 & 0.000 \\ 0.450 & 0.777 & 0.000 & 0.706 \\ 1.000 & 0.673 & 1.000 & 1.000 \end{pmatrix}$$

Step 2. Define ANN structure.

- Input layer: Four neurons (one for each attribute).
- Hidden layer 1: Eight neurons.
- Hidden layer 2: Six neurons.
- Output layer: One neuron to compute the score for each row.

Following the above structure, randomly initialize weights and biases:

Weights (Input to Hidden Layer 1): 4×8 matrix.

Weights (Hidden Layer 1 to Hidden Layer 2): 8×6 matrix.

Weights (Hidden Layer 2 to Output): 6×1 matrix.

Biases: One bias for each neuron in the hidden and output layers.

Activation function: Utilize the Sigmoid activation function for the outputs from one layer to the other.

Step 3: Perform forward pass and training

We will implement the forward pass and compute the outputs.

Using these scores, we will identify the best alternative (row with the highest score).

Computations: For each row of the decision matrix, after the hidden layer-1, we get

$$H1 = \begin{pmatrix} 0.811 & 0.888 & 0.832 & 0.800 & 0.713 & 0.707 & 0.809 & 0.788 \\ 0.821 & 0.843 & 0.765 & 0.836 & 0.793 & 0.734 & 0.756 & 0.727 \\ 0.849 & 0.907 & 0.809 & 0.871 & 0.783 & 0.729 & 0.802 & 0.784 \\ 0.905 & 0.966 & 0.906 & 0.925 & 0.883 & 0.767 & 0.863 & 0.889 \end{pmatrix}$$

After the computations of the hidden layer-2, we get

$$H2 = \begin{pmatrix} 0.976 & 0.990 & 0.980 & 0.971 & 0.870 & 0.968 \\ 0.975 & 0.989 & 0.979 & 0.971 & 0.870 & 0.967 \\ 0.978 & 0.991 & 0.982 & 0.974 & 0.878 & 0.971 \\ 0.983 & 0.994 & 0.986 & 0.980 & 0.896 & 0.977 \end{pmatrix}$$

Utilizing the resultant matrix from hidden layer-2, we end up with the final scores of the matrix, as follows:

Row 1, 0.9214; row 2, 0.9213; row 3, 0.9219; row 4, 0.9230.

Hence the best alternative is row 4, with the highest score of 0.9230.

Hence, following the ANN procedure also produces the fourth alternative as the best alternative.

7. Conclusions

In this work, new aggregation operators and correlation measures for Linguistic Intuitionistic Fuzzy Numbers (LIFNs) have been proposed, including a novel correlation coefficient for LIFNs. These are utilized to calculate the closeness coefficients in the decision-making model. Theorems validating the properties of LIFN energies, correlation, and the correlation coefficient have been established. In the LIF-VIKOR method, the proposed correlation coefficient is applied to the final ranking of alternatives. Sensitivity analysis has been conducted for each weight, and LIF-VIKOR has been applied to evaluate its impact. A numerical example is provided, demonstrating the ranking of alternatives through various computations. The entropy method is used to determine weights in the VIKOR method, ensuring consistency in the ranking algorithm. Sensitivity analysis further confirms the robustness of the proposed approach.

Finally, the best alternative in the decision problem is validated using an Artificial Neural Network (ANN) algorithm, and the results are found to be consistent across all proposed methods. This confirms the effectiveness and reliability of the developed approach in decision-making applications.

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