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Prediction of Random Crack Propagation Life of Thin Porous Plate with Multiple Site Damage †

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Abstract: Based on the crack propagation rate test of LY12CZ aluminum alloy flat plate, the distribution characteristics of Paris formula parameters were characterized by the three-parameter logarithmic normal distribution, and a LY12CZ random crack propagation analysis method was established. By using the Monte-Carlo method to simulate the randomness of crack growth, a prediction model of the random crack propagation life of a thin porous plate with multiple site damage (MSD) was established and programmed. The model can accurately predict the crack growth life and failure probability of thin porous plates with MSD, which provides an effective engineering analysis method for the MSD evaluation of the thin plate structure with multiple holes.

Keywords: thin plate; multiple-site damage; random crack growth life; failure probability

1. Introduction

Multiple site damage (MSD) often occurs in the multi-detail structure of aircrafts, such as fuselage panel structures with a large number of rivet holes and the joint of bolt holes, which will seriously reduce the residual strength of the structure and pose a great threat to the structural safety of the aircraft. In actual use, a large number of aircraft structures have damage defects, and assembly defects, and have experienced multiple incidences of damage caused by various damage sources. Therefore, it is of great engineering significance to carry out multi-crack growth analysis and predict the crack growth life of MSD structure [1–3].

Based on the single crack propagation test of LY12CZ aluminum alloy, the random characteristics of parameters related to crack propagation rate were studied; a probabilistic model for crack propagation life of multi-damage structures with holes was established and programmed; the random life of crack propagation under different stress levels was calculated and analyzed; and the failure probability curve of the structure was established.

2. Crack Growth Rate Test

In order to research the fatigue crack propagation characteristics of LY12-CZ aluminum alloy, a LY12-CZ aluminum alloy thin plate was prepared, a round hole with a diameter of 3 mm was made in the center of the thin plate, and an incision with a length of 3.5 mm was prefabricated on the left and right sides of the round hole. The size of the specimen is shown in Figure 1.

The test specimen was installed on a 1000 kN Instron8801 hydraulic servo fatigue testing machine for the fatigue test. When a crack of 1 mm was generated in the incision, the crack growth rate test was carried out. a_i-N_i test data of the cracks on both sides of the hole were recorded every 5000 cycles until the specimen broke. The test load is as follows: the maximum load P_{max} = 90.7 kN, the stress ratio R = 0.06, the load spectrum is sinusoidal wave of constant amplitude, and the loading frequency f = 10 Hz.

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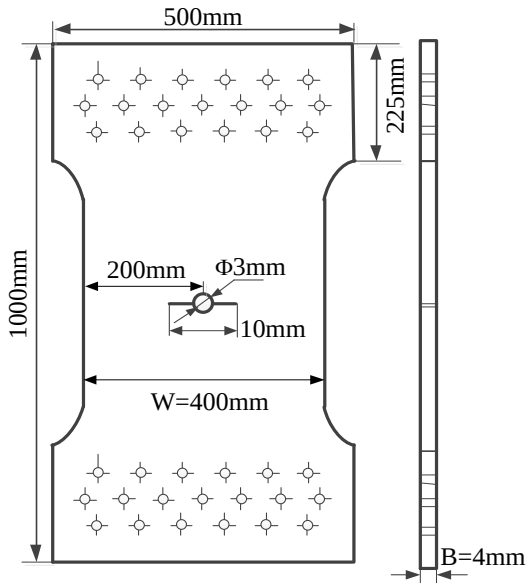


Figure 1. Size of test specimen.

A total of six specimens were tested. a_i-N_i test data of all specimens are shown in Figure 2. It can be seen that the crack growth dispersion of the right and left side of the hole in the same specimen is small, while the crack growth of different specimen has a certain dispersion. The five-point increasing polynomial method was used to process the ai-Ni test data [4], the crack growth rate (da/dN)i was calculated, and the variable range of crack stress intensity factor is calculated as follows:

$$\Delta K = F\frac{\Delta P}{BW}\sqrt{\pi a_i} \quad (1)$$

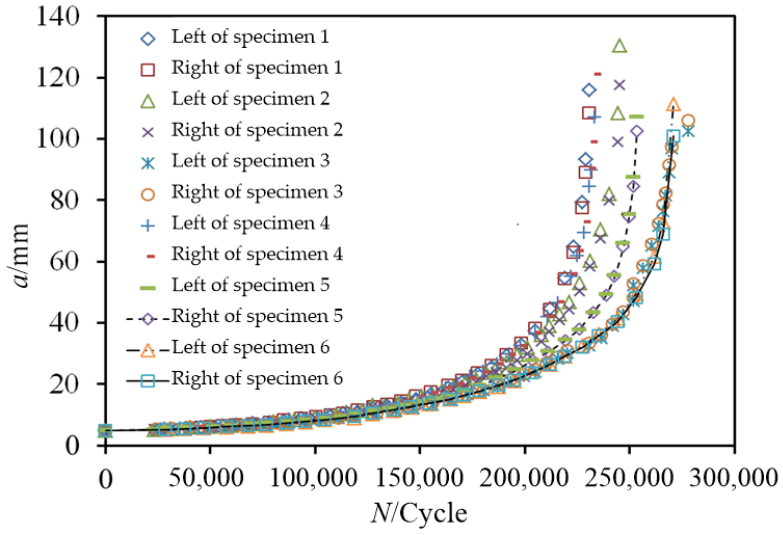


Figure 2. Crack growth test data.

In Formula (1), F is the comprehensive shape factor, and the specific value is obtained by querying the Stress Intensity Factor Manual [5]; $\Delta P = P_{max} - P_{min}$.

3. Prediction Model of Random Crack Propagation Life

3.1. Crack Propagation Parameter Calculation

Paris–Erdogan model is the most widely used fatigue crack growth rate model in engineering.

$$\frac{d_a}{d_N} = C_i(\Delta K)^{n_i} \tag{2}$$

In Equation (2), C_i and n_i are the crack growth rate parameters. The randomness in the crack growth process is simulated by controlling C_i and n_i. Taking logarithms of both sides of Equation (2), there is a linear relationship between the logarithmic crack growth rate lg (da/dN) and the vari-range logarithmic lg (ΔK) of the crack tip stress intensity factor, namely:

$$\lg\left(\frac{d_a}{d_N}\right) = \lg C_i + n_i \lg(\Delta K) \tag{3}$$

After da/dN and ΔK data are obtained, the crack propagation parameters C_i and n_i can be calculated by linear regression analysis. As shown in Figure 3, the analytical results of the crack on the left side of specimen 1.

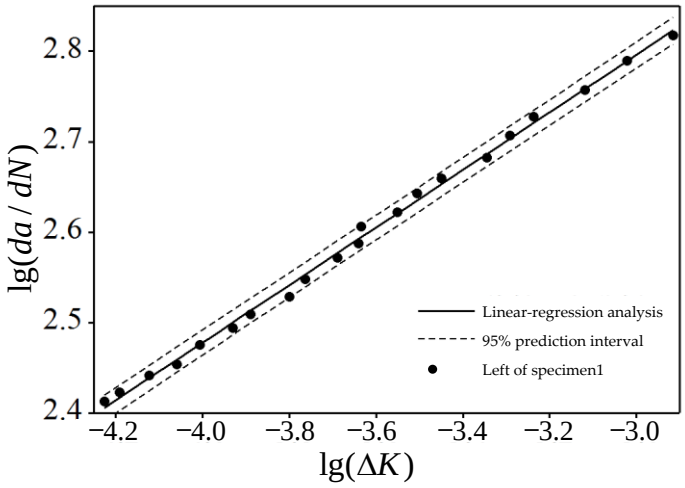


Figure 3. Linear relationship of lg(da/dN)−lg(ΔK) of crack on the left side of specimen 1.

After the above processing and analysis of the test data of the remaining specimens, the fatigue crack growth formula parameters n_i and lgC_i values of each specimen can be obtained. The fitting results of the crack growth rate parameters of all specimens are shown in Table 1.

Table 1. The fitting results of crack growth rate parameters.

Crack Location	$C/10^{-13}$	lg$C$	n	Correlation Coefficient
Left of specimen 1	2.630	−12.58	3.446	0.985
Right of specimen 1	24.55	−11.61	3.077	0.997
Left of specimen 2	5.248	−12.28	3.320	0.978
Right of specimen 2	1.862	−12.73	3.439	0.984
Left of specimen 3	67.61	−11.17	2.876	0.981
Right of specimen 3	57.54	−11.24	2.909	0.976
Left of specimen 4	19.05	−11.72	3.136	0.954
Right of specimen 4	38.90	−11.41	3.017	0.957
Left of specimen 5	15.14	−11.82	3.155	0.983

Table 1. *Cont.*

Crack Location	$C/10^{-13}$	lg$C$	n	Correlation Coefficient
Right of specimen 5	28.18	−11.55	3.054	0.988
Left of specimen 6	2.344	−12.63	3.504	0.985
Right of specimen 6	22.39	−11.65	3.099	0.980

3.2. Random Crack Growth Model

Linear regression analysis was performed on fatigue crack growth parameters n and lgC in Table 1 (as shown in Figure 4), and it was found that the crack growth characteristics of LY12CZ aluminum alloy were the same as those of other materials [6,7]. Also, there was a strong linear relationship between the two fatigue crack parameters. The linear relationship between the fatigue crack propagation parameters of LY12CZ aluminum alloy was obtained by fitting.

$$\lg C_i = -2.520 n_i - 3.868 \tag{4}$$

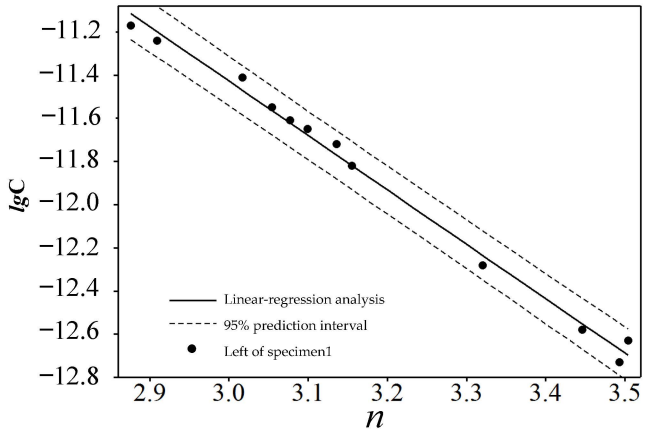


Figure 4. Linear relationship of $n-$lgC.

The three-parameter lognormal distribution can well express the variability of the random parameter n_i, and its probability density function is expressed in the form of the three-parameter lognormal distribution [8].

$$f(n_i) = \frac{0.43429}{(n_i - \tau)\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2} \times [\lg(n_i - \tau) - \mu]^2\right\} \tag{5}$$

In Equation (5), τ is the positional parameter, σ is the shape parameter, and μ is the dispersion parameter. According to the parameter estimation method of three-parameter lognormal distribution proposed in the literature [9] and the test data in Table 1, the parameters are estimated as follows:

$$\tau = 2.374,\ \mu = -0.519,\ \sigma = 0.117 \tag{6}$$

According to the correlation properties of normal distribution, the random value of fatigue crack growth parameter n_i can be obtained by Equation (7).

$$\lg(n_i - \tau) = \mu + \sigma n_{0i} \tag{7}$$

In Equation (7), n_{0i} is the random number generated by the random generator, which follows the standard normal distribution, and the lower corner mark 0 indicates the initial value of n_i. After the random parameter n_i is obtained, the random value of the parameter C_i is obtained by combining with Formula (3).

3.3. Residual Strength Evaluation

The residual strength evaluation of MSD structures is one of the key problems in crack propagation life prediction. The failure of a multi-crack structure should take into account both the instability failure of local cracks and the failure of the whole structure. Therefore, both the net section yield criterion and the critical stress intensity factor criterion should be considered when evaluating the residual strength of an MSD structure.

The true stress of the cracked structure is as follows:

$$\sigma' = \sigma A_g / A_n \tag{8}$$

In Equation (8), A_n and A_g are the net area and gross area of the section where the crack is located, respectively; σ is the distal stress.

When the true stress σ' of the structure is greater than the yield stress of the material, or the K value of the crack a_i is greater than K_{IC}, the number of cycles is the crack growth life of the structure. According to the manual, the fracture toughness of LY12-CZ is generally between 33 and 37 MPa [10]. For the sake of conservatism, the minimum value is 33 MPa.

3.4. Multiple Crack Interaction Propagation

The problem of mutual interference in multiple crack propagations is analyzed by cyclic load separation method, and the crack propagation life is predicted by this method. The length of crack growth in the same cycle interval is calculated until the crack grows to the point that the structure does not meet the residual strength, which is the crack growth life. The derivation of the Paris formula used in the calculation of the life prediction is as follows:

$$\frac{da}{dN} = C(\Delta k)^n \tag{9}$$

In Equation (9), the multi-crack stress-intensity factor variation ΔK can be obtained by combining method [11,12], and the calculated expression is as follows:

$$\Delta K = f\Delta\sigma(\pi a) \tag{10}$$

In Equation (10), f is the comprehensive configuration factor of the MSD structure, which is based on the basic principle of the combination of continuous multiplication, as follows:

$$f = f_h f_{oh} f_w f_1 \tag{11}$$

In Formula (11), f_h is the boundary correction factor of the crack initiation to crack; f_{oh} is the boundary correction factor of several holes adjacent to the crack tip to crack; f_w is the boundary correction factor of the finite plate width to crack; f_l is the boundary correction factor of the finite plate length to crack.

When the structure is subjected to uniform tensile stress, the expressions of each boundary correction factor can be obtained from the Stress Intensity Factor Manual and the literature [13,14].

By integrating Equation (9), Equation (12) can be obtained.

$$\Delta a_f = \begin{cases} \left[a_0^{(1-\frac{n}{2})} \frac{\Delta N_f (n-2) C \left(f\Delta\sigma\sqrt{\pi}\right)^n}{2} \right]^{\frac{2}{2-n}}, & n \neq 2 \\ 10^{\Delta N_f C \pi (f\Delta\sigma)^2} a_0, & n = 2 \end{cases} \tag{12}$$

3.5. Prediction of Crack Propagation Life by Monte-Carlo Method

The prediction model of the random crack growth life of MSD structures is established by using the analysis method proposed in Sections 3.1–3.3 and the Monta-Carlo method. The main steps are as follows:

(1) Building the calculation model according to the geometric dimensions.

(2) The stress intensity factor K and its variation were calculated by using the combination method.
(3) A set of random numbers conforming to the standard normal distribution is generated by the random number generator. The random number and a given number of cycles are obtained by Equations (3) and (7). The propagation length of each crack in the cycle is calculated by Equation (12).
(4) According to the crack propagation length, the calculation model is modified, so as to enter the next cycle. The condition of the exit calculation is the residual strength evaluation method described in Section 3.3, and the simulated crack growth life can be obtained.
(5) Repeat the above calculation for L times and obtain (j = 1, 2, . . ., L) life data, and the logarithmic life mean and logarithmic life variance of the structure can be obtained by calculating the data results.

The calculation program is compiled according to the above steps, and the relevant examples are calculated and analyzed.

4. Analysis of Examples

The multi-hole structure has similar details, which is easy to produce MSD, and the crack size is not different. The typical riveted panel structure of the aircraft–finite collinear seven-hole symmetric crack plate structure is used to simulate the calculation, and related tests are carried out. For the studied collinear seven-hole plate (hole serial number from left to right is 1~7; each side of the seven holes has the same length of prefabricated cracks), the geometric size is shown in Figure 5, and the material is LY12CZ. Multiple crack propagation tests were carried out on the same size test pieces, and the *a*-*N* data of each crack was recorded during the test. The stress level and test results are shown in Table 2.

After the completion of the test, 10,000 sample simulation calculations were carried out on the structure using the calculation program established above. The center hole of structural symmetry, namely, hole-4, was studied and analyzed, and the simulation calculation data of the maximum, median, and minimum life in the simulation calculation results were selected for comparison and analysis with the test data. Figure 6 shows the *a*-*N* test data and simulation calculation results of cracks on both sides of hole-4 under three different stress levels. It can be seen from Figure 6 that the random crack growth life prediction model can simulate well the crack growth characteristics and the dispersion under different stress levels.

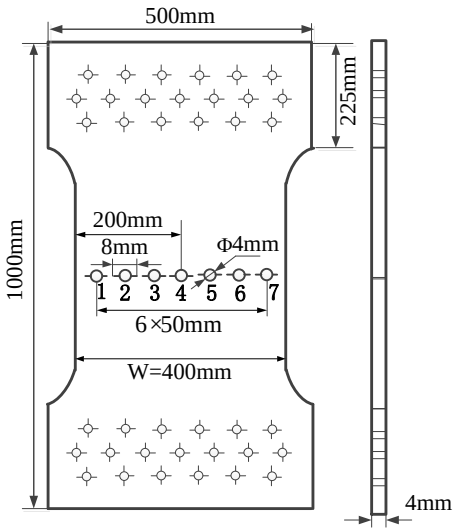
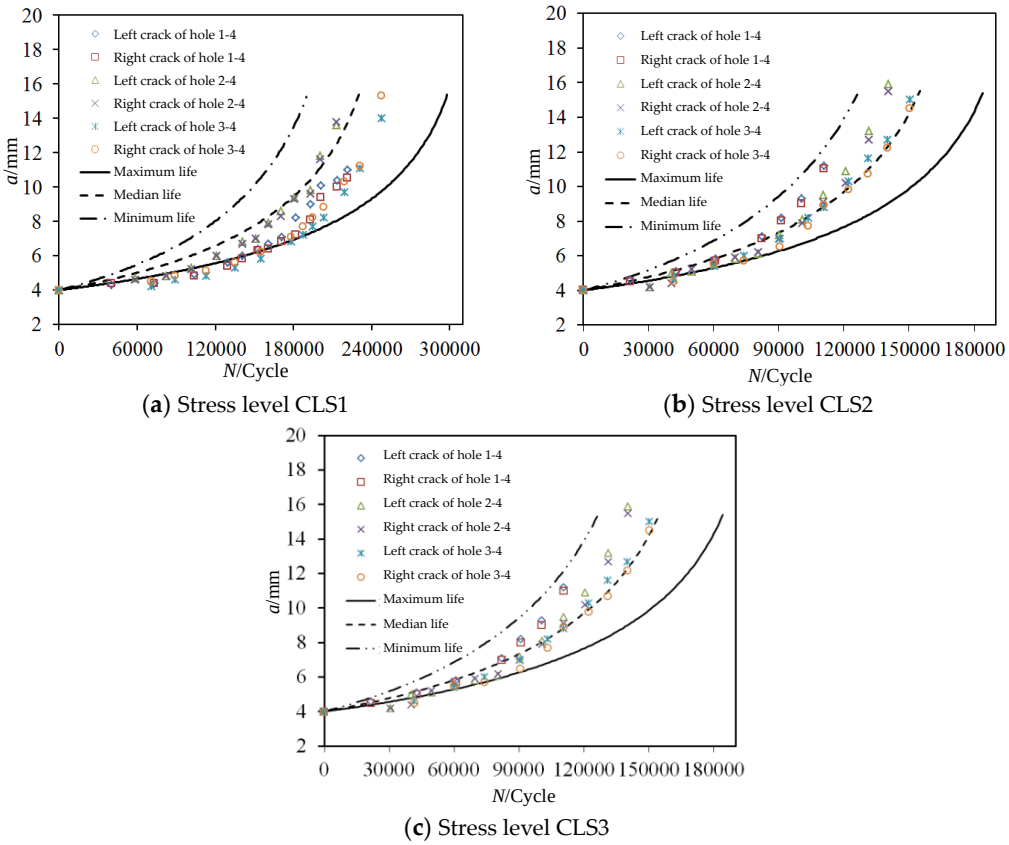


Figure 5. Geometric size of the col linear seven−hole plate structure.

Table 2. Test stress level and results.

Parameter		Different Test Load Spectra		
		CLS1	**CLS2**	**CLS3**
Stress level	F_{max}/kN	82	90	98
	σ_{max}/MPa	51.25	56.25	61.25
	R	0.1	0.1	0.1
	f/Hz	4.5	4.5	4.5
Fatigue life	N/cycles	255,385	170,213	119,416
		236,536	157,688	139,097
		244,704	167,225	128,557

The distribution of the simulated crack growth life N of the seven-hole plate structure under three different stress levels is shown in Figure 7. By comparing the logarithmic life mean and logarithmic life standard deviation with the test values, it is proved that the calculation model is accurate to calculate the crack growth life of the structure. The calculation results are shown in Table 3.



(**a**) Stress level CLS1

(**b**) Stress level CLS2

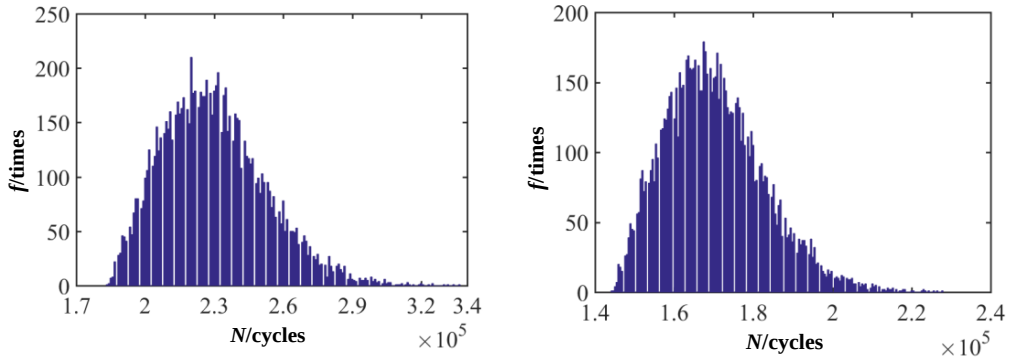
(**c**) Stress level CLS3

Figure 6. *a*-*N* test data and simulation results of cracks on both sides of center hole under different stress levels.

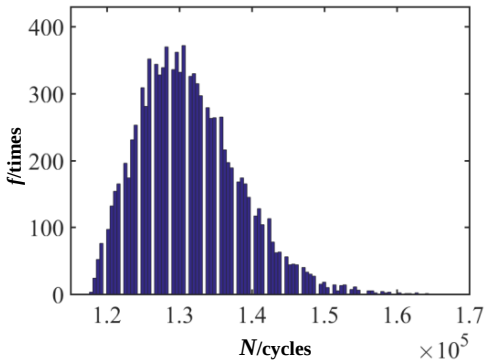
It can be seen from Table 3 that with the increase in stress level, the crack growth life of the structure decreases significantly, which indicates that the stress has a significant effect on the crack growth life of the MSD structure with multiple holes. Figure 8 shows the cumulative probability of structural crack growth life under three stress levels. The cumulative probability curve of the underwater crack growth life of the three stresses, from which the failure probability P and the reliability 1-P of the specific life can be obtained.

Table 3. Comparison of calculation results and test results of seven-hole plate.

σ/MPa	$\overline{\lg N}$	$S_{\lg N}/10^{-3}$	$\overline{lgN'}$	$S_{\lg N'}/10^{-3}$
51.25	5.359	1.70	5.389	0.278
56.25	5.230	1.00	5.217	0.301
61.25	5.118	0.509	5.101	1.40



(**a**) Stress level CLS1 (**b**) Stress level CLS2



(**c**) Stress level CLS3

Figure 7. Simulated calculation of crack propagation life distribution of seven-hole plate under different stress levels.

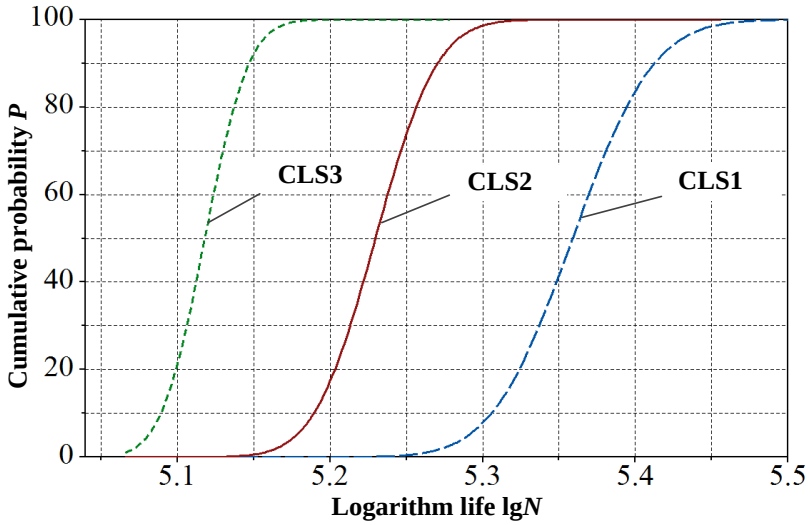


Figure 8. Failure probability of crack propagation of MSD seven-hole plate under different stress levels.

5. Discussion and Conclusions

Through the crack growth rate test of aluminum alloy LY12CZ, parameters *C* and *n* of the Paris formula were calculated. Through research and analysis, it was found that there was a strong linear relationship between parameters *n* and lgC. Based on this, a random crack growth model was established by using the correlation probability analysis method. The randomness of crack growth was simulated by the Monte-Carlo method. The residual strength evaluation method of MSD structures was used to obtain the crack growth life of the structure failure, and the prediction model of the random crack growth life of MSD structures with multiple holes was established. The model can effectively and accurately calculate the crack propagation life of MSD structures with multiple holes and establish the failure probability curve of their lifespans, through which the failure probability and reliability of their specific lifespans can be obtained.

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