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Monolithic and Decomposition Methods for Optimal Scheduling of Dynamically Adaptive Water Networks †

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Abstract: This paper presents an optimal scheduling problem for coordinating pressure and self-cleaning operations in dynamically adaptive water networks. Our problem imposes a set of time-coupling constraints to manage pressure variations during the transition between operational modes. Solving this time-coupled, nonlinear optimization problem poses challenges for off-the-shelf nonlinear solvers due to its high memory demands. We compare the performance of a decomposition method using the alternating direction method of multipliers (ADMM) with a gradient-based sequential convex programming (SCP) monolithic solver. Solution quality and computational efficiency are evaluated using a model of a large-scale network in the UK.

Keywords: alternating direction method of multipliers; optimal scheduling; sequential convex programming; water distribution networks

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1. Introduction

We recently proposed an optimal scheduling problem for coordinating pressure and water quality operations in dynamically adaptive water networks [1]. The problem integrates pressure management with an operational mode designed to maximize the number of pipes with self-cleaning flow velocities. Such self-cleaning conditions aim to reduce discoloration risk in the network, a key priority for water utilities. To achieve these objectives, we consider the control of two hydraulic actuators: pressure control valves (PCVs) and automatic flushing valves (AFVs). Furthermore, time-coupling constraints manage temporal pressure variations during the transition between control modes.

The resulting time-coupled, nonlinear optimization problem poses challenges for off-the-shelf nonlinear solvers (e.g., IPOPT [2]) due to high memory demands handling the dense Hessian matrices. We address this issue by decomposing the monolithic problem into smaller, independent subproblems and coordinating their solutions using the alternating direction method of multipliers (ADMM) [3]. Although ADMM yielded good-quality feasible solutions, our previous work lacked a performance comparison with other monolithic solution methods. In this paper, we compare ADMM with a gradient-based sequential convex programming (SCP) solver [4] using a large-scale network in the UK.

2. Problem Formulation

We model a water network comprising n_p links, n_n junctions, and n_0 source nodes. Let $\mathcal{T} = \{1, \ldots, n_t\}$ be a set of discrete model time steps. For each time step $t \in \mathcal{T}$, vectors of flow $q_t \in \mathbb{R}^{n_p}$ and head $h_t \in \mathbb{R}^{n_n}$ are governed by steady-state energy and mass conservation equations. The vector of decision variables $u_t \in \mathbb{R}^{n_v+n_f}$ models local losses across n_v PCV links and flushing demands at n_f AFV nodes. Lower and upper bounds define the feasible solution space for continuous variables q_t, h_t, and u_t. These hydraulic

constraints are collected by the set $\mathcal{X}_t$ for all $t \in \mathcal{T}$. Lastly, we impose time-coupling constraints via the set $\overline{\mathcal{X}} = \left\{ h := \left[\{h_t\}_{t\in\mathcal{T}} \right]^T \;\middle|\; \max_{t\in\mathcal{T}}(h_{i,t}) - \min_{t\in\mathcal{T}}(h_{i,t}) \leq \delta,\ \forall i \in \mathcal{N} \right\}$, where $\mathcal{N} = \{1, \ldots, n_n\}$ is the set of model nodes and δ is a user-defined pressure range tolerance.

For each time step $t \in \mathcal{T}$, the objective function either minimizes average zone pressure (AZP) or maximizes self-cleaning capacity (SCC) [1]. We consider AZP as the primary objective due to the continuous nature of background leakage, while SCC serves as the secondary objective, activated within a predefined 1-h window in the daily scheduling horizon (i.e., $\mathcal{T}_{\text{SCC}} \subseteq \mathcal{T}$). Let $x_t := \left[q_t^T,\ h_t^T,\ u_t^T \right]^T$ and $x := \left[\{x_t\}_{t\in\mathcal{T}} \right]^T$. The AZP-SCC scheduling problem is formulated in compact form as

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{t\in\mathcal{T}} f_t(x_t) \\ \text{subject to} \quad & x_t \in \mathcal{X}_t, \quad \forall t \in \mathcal{T} \\ & Hx \in \overline{\mathcal{X}}, \end{aligned} \tag{1}$$

where $f_t := f_{\text{SCC}}$ if $t \in \mathcal{T}_{\text{SCC}}$ and $f_t := f_{\text{AZP}}$ otherwise; and H is a constant matrix defined such that $Hx = h$. The reader is referred to previous work [1] for details on the formulation of f_{SCC} and f_{AZP}. Problem (1) is a nonconvex, nonlinear programming problem.

3. Solution Methods

3.1. Alternating Direction Method of Multipliers (ADMM)

We introduce a global copy of head variables $\overline{h} \in \mathbb{R}^{n_n \times n_t}$ and consensus constraint $Hx - \overline{h} = 0$. Problem (1) is then reformulated as

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{t\in\mathcal{T}} f_t(x_t) \\ \text{subject to} \quad & Hx - \overline{h} = 0, \\ & x_t \in \mathcal{X}_t, \quad \forall t \in \mathcal{T} \\ & \overline{h} \in \overline{\mathcal{X}}. \end{aligned} \tag{2}$$

Let $L\left(x, \overline{h}, y; \rho\right)$ be the augmented Lagrangian of Problem 2, where $y \in \mathbb{R}^{n_n \times n_t}$ is an array of dual variables associated with constraint $Hx - \overline{h} = 0$ and $\rho > 0$ is a fixed penalty parameter. Given a feasible starting point $\left(x^1, \overline{h}^1\right) \in \mathcal{X} \cap \overline{\mathcal{X}}$, ADMM minimizes Problem (2) by performing the following sequence of steps at each iteration k [3]:

$$x_t^{k+1} := \underset{x_t}{\text{argmin}}\ L_t\left(x_t,\ \overline{h}_t^k, y_t^k\right), \quad \forall t \in \mathcal{T} \tag{3}$$

$$\overline{h}^{k+1} := \underset{\overline{h}}{\text{argmin}}\ L\left(x^{k+1}, \overline{h}, y^k\right) \tag{4}$$

$$y^{k+1} := y^k + \rho\left(Hx^{k+1} - \overline{h}^{k+1}\right). \tag{5}$$

A main benefit of ADMM is that Subproblem (3) can be parallelized across time steps $t \in \mathcal{T}$. The algorithm terminates if primal residual $Hx^{k+1} - \overline{h}^{k+1}{}_2 \leq \epsilon$, where $\epsilon > 0$.

3.2. Sequential Convex Programming (SCP)

We implement a tailored SCP solver using gradient-based optimization and a strictly feasible line search strategy. Given a feasible starting point $x^1 \in \mathcal{X} \cap \overline{\mathcal{X}}$, SCP solves a sequence of convex subproblems, formed by linearizing nonlinear terms in the objective function $f(\cdot)$ and hydraulic constraints $\mathcal{X}$ around the current iterate x^k. The subproblem computes valve settings u^{k+1} to form the search direction $du^k = u^{k+1} - u^k$ at iteration k. The following line search is then carried out with initial step size $\alpha = 1$ [4]:

1. Evaluate objective function $f^{k+1} = \sum_{t \in \mathcal{T}} f_t\left(q_t^{k+1}, h_t^{k+1}, u_t^k + \alpha du_t^k\right)$, where q_t^{k+1}, h_t^{k+1} are computed using a hydraulic solver (e.g., EPANET) for all $t \in \mathcal{T}$.
2. If $f^{k+1} < f^k$ and $x^{k+1} \in \mathcal{X} \cap \overline{\mathcal{X}}$, then $u^{k+1} = u^k + \alpha du^k$ and $k \leftarrow k+1$; else set $\alpha = \alpha/2$ and repeat step 1.

This line search strategy guarantees both feasibility and a reduction in objective function. The algorithm terminates when $(f^k - f^{k+1})/f^k \leq \epsilon$, where $\epsilon > 0$.

4. Numerical Experiments and Discussion

We evaluate ADMM and SCP using the Bristol Water Field Lab network (BWFLnet) model [4] (Figure 1). BWFLnet comprises $n_p = 2816$ links, $n_n = 2745$ junction nodes, and $n_0 = 2$ reservoir (inlet) nodes. Control actuators include $n_v = 3$ PCV links and $n_f = 4$ AFV nodes. Here, we optimize their operational settings u_t for each 15-min time step t in the 24-h scheduling horizon $\mathcal{T} = \{1, \ldots, 96\}$. We assume that the SCC mode is activated during the peak demand period, defined by $\mathcal{T}_{\text{SCC}} = \{38, \ldots, 42\} \subseteq \mathcal{T}$. Additionally, we consider tolerances $\delta = \{20,\ 15,\ 10\}$ (in meters) to define time-coupling constraints $\overline{\mathcal{X}}$.

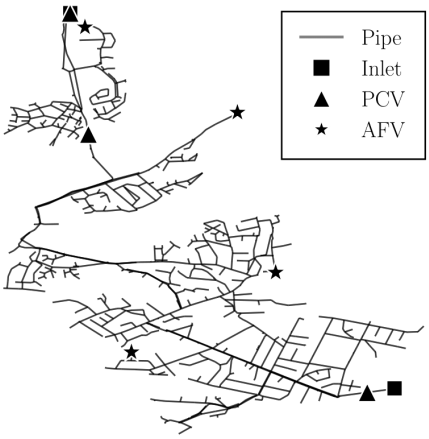


Figure 1. BWFLnet layout and control actuator placement.

Numerical experiments were performed in Julia 1.9.3 with optimization solvers accessed via JuMP 1.12.0 [5]. We solved nonlinear programs using IPOPT [2] and linear programs using Gurobi 10.0.2. Moreover, we set termination tolerances $\epsilon = 10^{-2}$ and $\epsilon = 10^{-3}$ for ADMM and SCP algorithms, respectively, and ADMM penalty parameter $\rho = 10^{-2}$. Table 1 compares objective values and computational (CPU) times using ADMM and SCP to solve Problem (1) with varying δ tolerances.

Table 1. Results comparison of ADMM and SCP solution methods.

Experiment	Method	Objective Value	CPU Time [s]
$\delta = \infty$ *	ADMM	2859	61.9
	SCP	2869	846
$\delta = 20$ m	ADMM	2875	644
	SCP	2928	901
$\delta = 15$ m	ADMM	3060	487
	SCP	3087	851
$\delta = 10$ m	ADMM	3287	472
	SCP	3279	738

* $\delta = \infty$ corresponds to the unconstrained pressure variation case (i.e., $\overline{\mathcal{X}} = \mathbb{R}$).

The solution qualities achieved by ADMM and SCP are comparable, and both have CPU times suitable for near real-time (i.e., hourly) control applications. However, ADMM

demonstrates a clear computational advantage owing to its use of distributed computing across subproblems, potentially offering better scalability for larger problems.

Figure 2 illustrates the trade-off between pressure range and AZP-SCC objectives across the scheduling horizon. For AZP, this trade-off is evident for pressure range tolerances $\delta \leq 15$ m, which coincides with hydraulic conditions for the no control experiment. On the other hand, SCC performance progressively declines as δ tolerances decrease. Fast solution methods like ADMM and SCP allow network operators to effectively balance this trade-off between AZP-SCC objective and network dynamics in near real-time.

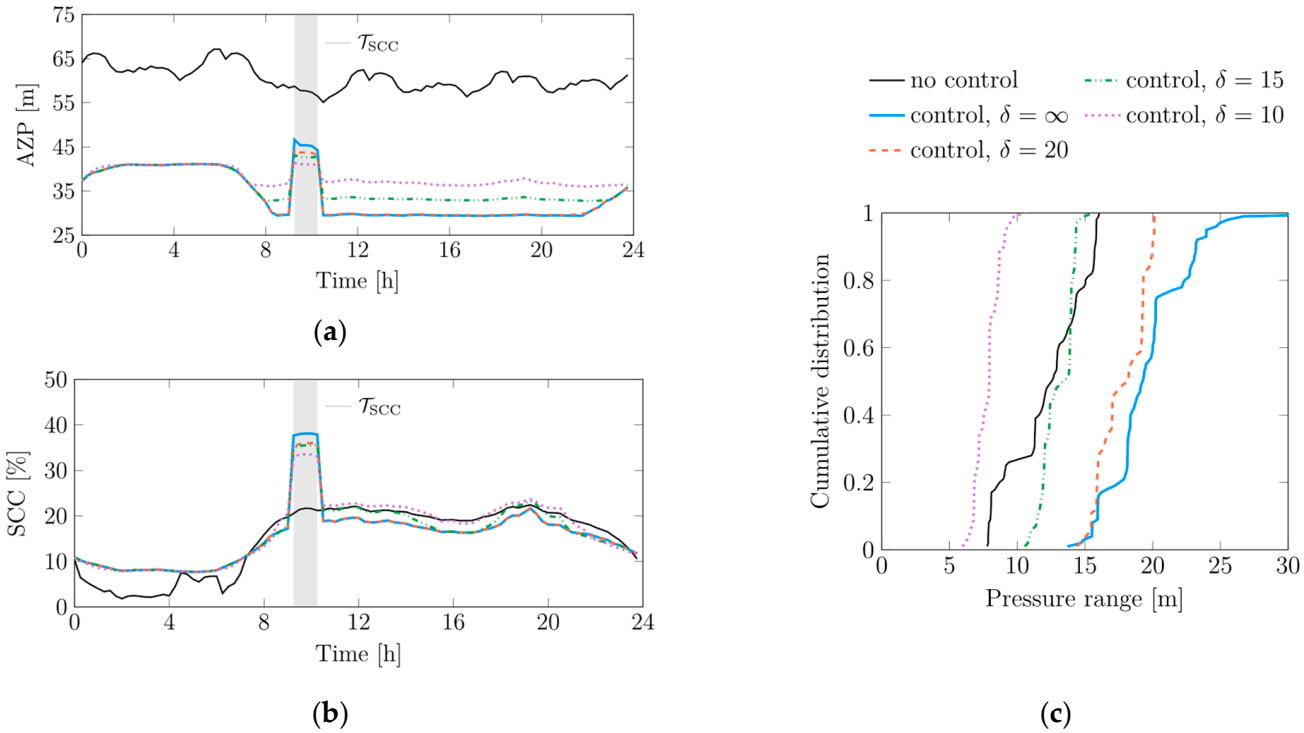


Figure 2. (**a**) AZP and (**b**) SCC objectives values across scheduling horizon $\mathcal{T}$; (**c**) distribution of nodal pressure range for each experiment.

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Conflicts of Interest: The authors declare no conflicts of interest.

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