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Minimization of Water Age in Water Distribution Systems under Uncertain Demand †

Kristina Korder [1], Elad Salomons [2], Avi Ostfeld [2] and Pu Li [1,*]

[1] Process Optimization Group, Technische Universität Ilmenau, 98693 Ilmenau, Germany; kristina.korder@tu-ilmenau.de
[2] Civil and Environmental Engineering, Technion-Israel Institute of Technology, Haifa 32000, Israel; eladsa@cv.technion.ac.il (E.S.); ostfeld@technion.ac.il (A.O.)
* Correspondence: pu.li@tu-ilmenau.de
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Abstract: Most existing approaches to ensuring water quality in water distribution systems (WDSs) are deterministic, i.e., they do not consider uncertainties, although they may have significant impacts on the water quality. It is well recognized that water demand represents a predominant uncertainty in a WDS. In addition, water age is often used as an important parameter to describe the water quality in a WDS and can be influenced by water demand and control elements such as pressure-reducing valves (PRVs). Therefore, the aim of this study is to carry out a probabilistic analysis of the impact of demand uncertainty on the water age in the distribution network. Based on the solution of deterministic optimization to minimize the water age, Monte Carlo simulation will be carried out by sampling the uncertain demand to evaluate the stochastic distribution of water age, as well as other operating variables like pressure and flow. As a result, the probability of violating the constraints of such variables can be determined, with the reliability of the operating strategy (e.g., the settings of the PRVs) given by deterministic optimization provided. In cases of low reliability, it is necessary to modify the operating strategy in order to decrease the probability of constraint violation. For this purpose, a chance-constrained optimization problem is formulated, and its benefits for ensuring the user-defined reliability are studied. A benchmark network is used to verify the proposed approach.

Keywords: optimization; water age; uncertain demand; Monte Carlo simulation; chance constraints

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1. Introduction

To guarantee adequate water quality for consumers, it is essential to incorporate water age into the optimization of water distribution system (WDS) design and operation [1,2], where uncertainties need to be taken into account. Demand is the most uncertain parameter in a WDS [3,4], and therefore it is considered in this study. Robust optimization results in conservative optimization [5], while chance constraint optimization guarantees a certain reliability level of optimization [6,7].

2. Problem Formulation

A WDS consists of n_p pipes that connect n_0 water sources (e.g., reservoirs and tanks) to n_n nodes (e.g., consumer households or industry demands). Our optimization aims to simultaneously minimize the total pressure $p(u,d) \in \mathbb{R}^{n_p}$ and water age $WA(u,d) \in \mathbb{R}^{n_p}$, which depend on the settings of the pressure-reducing valves (PRVs) u and the uncertain demand $d \in \mathbb{R}^{n_n}$, by adjusting the given PRVs [2,7], leading to the following chance-constrained optimization problem:

$$\min_{p,WA} \mathrm{E}\left\{\sum_{i=1}^{n_p} p_i\left(u, d^{(k)}\right) + WA_i\left(u, d^{(k)}\right)\right\} = \min_{p,WA} \frac{1}{N}\sum_{k=1}^{N}\sum_{i=1}^{n_p} p_i\left(u, d^{(k)}\right) + WA_i\left(u, d^{(k)}\right) \quad (1)$$

$$\text{Subject to} \quad \sum_{i=1}^{n_p} q_i\left(u, d^{(k)}\right) = d_i^{(k)} \quad (2)$$

$$\sum_{l=1}^{L} h_{f_l} = 0 \quad (3)$$

$$Pr\left\{0 \le WA\left(u, d^{(k)}\right) \le WA_{max}\right\} \ge \alpha \quad (4)$$

$$Pr\{p_{min} \le p\left(u, d^{(k)}\right) \le p_{max}\} \ge \alpha \quad (5)$$

with sampled demand scenario k, node i and loop l. Equation (2) describes the conservation of flow $q_i \in \mathbb{R}^{n_n}$ to satisfy the demand at each junction (mass balance). The energy balance (3) is expressed as the sum of head losses $h_f \in \mathbb{R}^{n_p}$ around each loop (cf. Bernoulli equation). The chance constraints evaluate the probabilities of satisfying uncertainty-dependent inequality constraints and search for decisions that meet a predefined reliability level [8]. Hence, Equation (4) ensures that the water age should not exceed the maximum allowable value, with a user-defined reliability (confidence) level of α $(0 \le \alpha \le 1)$, to maintain the desired water quality for the consumer. Similarly, Equation (5) provides the pressure chance constraints since the node pressures are also demand-dependent. Demand uncertainties $d^{(k)} \in \Omega$ in WDSs are often modeled using a normal distribution with a known mean value μ and standard deviation σ. The aim of the chance-constrained optimization is to determine an optimal strategy u for WDS operation that minimizes the expected value of the objective function (1).

3. The MCS-Based Probabilistic Analysis Problem

This study proposes using a Monte Carlo simulation (MCS)-based method to evaluate the influences of uncertainties on WDS operations. The principle of MCS in the case of varying demand is shown in Figure 1.

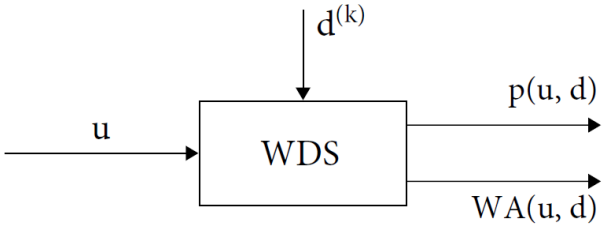


Figure 1. Input/output diagram using MCS.

The PRV setting u and the sampled demand values $d^{(k)}$ in each scenario k influence the hydraulics of the WDS, including pressure p and water age WA. The standard normally distributed random variable $d_s \sim N(0,1)$ can be transformed into the uncertain demand distribution $d \sim N(\mu, \sigma^2)$ by

$$d = \sigma d_s + \mu \quad (6)$$

based on samples which are generated by a random number generator in MATLAB [8].

4. Case Study

As a case study, we examine a simple gravity-based water distribution network consisting of one reservoir, five nodes, six pipes and a PRV located between nodes 5 and 3, as shown in Figure 2.

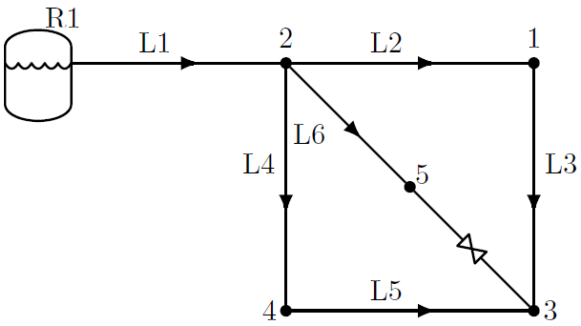


Figure 2. A simple network with one reservoir, five nodes, six pipes and a PRV on pipe L6.

The pipes in the network have a diameter of $D = 0.2$ m, a length of $l = 1000$ m and a roughness of $C = 100$. The reservoir is located at an elevation of $e_R = 120$ m, while all the nodes in the network are at an elevation of $e = 0$ m. Nodes 2 and 5 have zero demand, while nodes 1 and 4 have a demand of $d_{N1,\,N4} = 25$ LPS. The demand of node 3 is assumed to be uncertain and follows a Gaussian distribution with an expectation of $\mu = 50$ LPS and a standard deviation of $\sigma = 5$ LPS (i.e.10%). The EPANET-MATLAB Toolkit [9] is used to run the MCS for 96 h with 5000 samples for the demand values at node 3 while changing the PRV setting from 0 m to 100 m in 10 m increments.

The simulation results demonstrate that higher water consumption has a positive effect on water age, as expected. However, to optimize the WDS operations, additional measures should be taken despite the high consumption. It is observed that closing the PRV on pipe L6 results in a lower pressure and water age at node 3 (see Figure 3) compared to the case when the PRV is completely open (i.e., PRV setting = 100 m) due to the significantly higher flow in pipes L3 and L5.

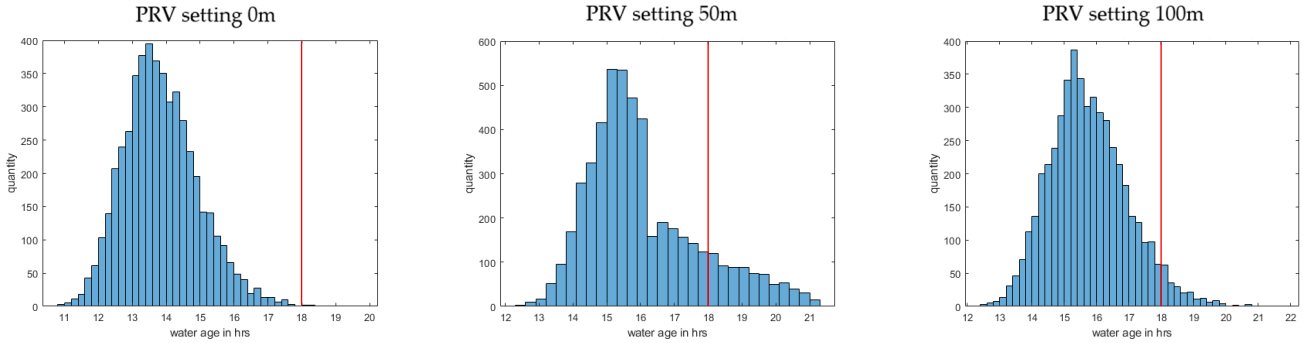


Figure 3. Water age distribution of node 3 with PRV settings of 0 m, 50 m and 100 m.

Let us assume the water age constraint in this example is $WA_{max} = 18$ h (see the red line in Figure 3). We can evaluate the probability of violating constraint (4). The highest violation (14.38%) is achieved with a PRV setting of 50 m (Figure 3 middle). The water age constraint is violated by 4.38% with an open PRV (Figure 3 right), while the constraint is fulfilled to 99.86% with a closed PRV (Figure 3 left).

When comparing two scenarios with low consumption ($d_{S1} = 47.86$ LPS) and high consumption ($d_{S2} = 54.63$ LPS) at node 3, a peak in the water age curve (see Figure 4) is observed with the medium PRV settings (30–60 m). This can be explained by the fact that at a PRV setting = 30 m, the valve is virtually closed, causing the water age at node 5 to increase. By increasing the PRV setting to 50 m, the water age at node 5 is reduced, as water can now flow through the PRV at almost all demand values at node 3. However, the water age at node 3 increases with a higher PRV setting.

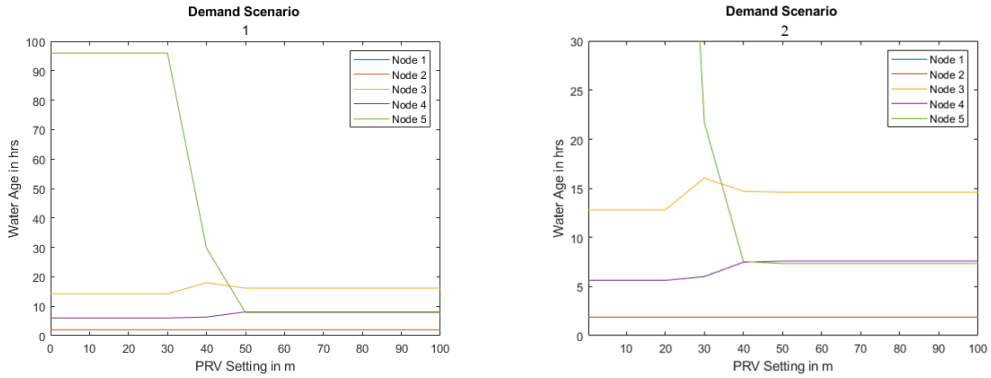


Figure 4. Water age values with changing PRV settings (0–100 m) in scenarios 1 and 2.

These results demonstrate that random demand values have considerable impacts on the pressure and water age in a network, necessitating appropriate pressure and water age management under uncertainty. For the simple WDS (Figure 2), it is recommended to keep the PRV closed to stay within the water age and pressure limits with high reliability.

5. Conclusions

This study models the problem of WDS operation under uncertainty with chance constraints. To highlight the stochastic features of WDS operation under uncertainty, we present a probabilistic analysis using MCS. Our analysis focuses on examining and interpreting water age constraints with probability or reliability. Numerical solution of the chance-constrained optimization problem will be our next step to establish a relationship between optimality and reliability with the optimal PRV settings.

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References

1. Office of Water (4606). Distribution System Water Quality—Maintaining a Disinfectant Residual. In *Technical Report EPA/815/F-21/003*; U.S. Environmental Protection Agency: Washington, DC, USA, 2021.
2. Korder, K.; Cao, H.; Salomons, E.; Ostfeld, A.; Li, P. Simultaneous Minimization of Water Age and Pressure in WDS by PRVs. *Water Resour. Manag.* **2024**, *38*, 3561–3579. [CrossRef]
3. Dandy, G.; Wu, W.; Simpson, A.; Leonard, M.A. Review of Sources of Uncertainty in Optimization Objectives of Water Distribution Systems. *Water* **2023**, *15*, 136. [CrossRef]
4. Lansey, K.E.; Mays, L.W.; Tung, Y.-K. Water Distribution System Design Under Uncertainties. *J. Water Resour. Plan. Manag.* **1989**, *115*, 630–645. [CrossRef]
5. Perelman, G.; Ostfeld, A. Adjustable Robust Optimization for Water Distribution System Operation Under Uncertainty. *Water Resour. Res.* **2023**, *59*, e2023WR035508. [CrossRef]
6. Stuhlmacher, A.; Mathieu, J.L. Chance-Constrained Water Pumping to Manage Water and Power Demand Uncertainty in Distribution Networks. *Proc. IEEE* **2020**, *108*, 1640–1655. [CrossRef]
7. Raza, N.; Moazeni, F. Chance-constrained vulnerability assessment of smart water distribution systems against stealthy false data injection attacks. *Int. J. Crit. Infrastruct. Prot.* **2024**, *24*, 100645. [CrossRef]

8. Li, P. *Prozessoptimierung unter Unsicherheiten*, 1st ed.; Oldenbourg Wissenschaftsverlag: München, Germany, 2007. [CrossRef]
9. Eliades, D.G.; Kyriakou, M.; Vrachimis, S.; Polycarpou, M.M. EPANET-MATLAB Toolkit: An Open-Source Software for Interfacing EPANET with MATLAB. In Proceedings of the 14th International Conference on Computing and Control for the Water Industry (CCWI), Amsterdam, The Netherlands, 7–9 November 2016. [CrossRef]

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