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Using Dichotomous Variables to Model Structural Changes in Time Series: An Application to International Trade †

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Abstract: This research aimed to elucidate the methodology employed in econometric estimations by utilizing dichotomous variables. These variables served a dual purpose: firstly, they denoted an attribute designed to discern structural changes within a linear relationship, and secondly, they quantified the impact and statistical significance of one or more quantitative independent variables on a dependent variable in the presence of such structural changes. The core of this document covers the presentation of the methodology; additionally, an applied analysis of Mexico's foreign trade is included. This analysis delved into estimating the impact and statistical significance of exports on the economic growth across different periods, reflecting significant structural changes in the development of the Mexican economy.

Keywords: dichotomous variables; structural change; stable relationship over time; statistical significance; different periods of time

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1. Introduction

Considering the emergence of structural changes over time, it is crucial to underscore the significance of developing a methodology for quantifying the impact of one or more independent variables on a dependent variable within a linear relationship.

Structural change is a phenomenon that has been studied in various branches due to its intrinsic importance within time series. Some research has focused on problems of this nature in relation to economic cycles, industry, and productivity, among others; such is the case in [1–3]. A study of the state of the art on structural change can be found in [4]; likewise, an application of the methodology presented here can be seen in [5].

The primary objective of this study was to elucidate the formal procedure of econometric estimation. It is noteworthy that estimates and modeling in this vital domain of knowledge should ideally be grounded in economic theory.

However, since this study presents an application focusing on the relationship between trade and economic growth, the analysis of the model was approached in a more generalized manner. The emphasis was predominantly placed on highlighting the causes that trigger structural changes, with a primary focus on formal empirical estimation.

This research, in addition to providing a theoretical explanation of the methodology, presents a simple application focused on the quantification of the impact of the growth rate of Mexican exports on the growth rate of the economy in the face of two important structural changes; namely, the economic crisis of 2009 and the trade friction between China and the United States in 2018.

In this context, the utilization of dichotomous variables to discern attributes within a time series assumes significant importance. By modeling these variables through zeros and ones, different spans or periods separated by a structural change can be distinguished. This analysis extended beyond merely estimating expected values in the presence of an

attribute, such as belonging to a specific period before and after a structural change, akin to a variance analysis model.

This methodology allowed for the estimation of the impact of one or more independent variables on a dependent variable in two linear relationships—prior to and subsequent to the structural change. In the presence of a second structural change, this extends to three linear relationships, and so forth for n variables and m structural changes. This approach provides a comprehensive framework for capturing the intricate dynamics of the relationship under consideration across multiple structural shifts.

Although regression with dichotomous variables has been studied in various texts, it is reviewed here with a focus on structural change [6–9].

2. Structural Change as an Attribute of a Time Series

Dichotomous variables can be used to represent the presence of an attribute of a variable. In this particular context, they can be employed to capture a structural change in a time series by expressing themselves on a nominal scale. These variables are commonly referred to as indicator, binary, categorical, or simply qualitative variables. They adopt values of zeros and ones to denote the absence or presence of a specific attribute. In this instance, the attribute pertains to a change in the time series, which can manifest as a punctual, regime, transitory, or gradual change, as illustrated in [10].

Similarly, numerous techniques can be applied with this type of variable. While maintaining consistency with ANCOVA (analysis of covariance)-type models is crucial, the methodology presented here can also be viewed as the estimation of a regression across specific segments or periods.

Building upon the above, ordinary least-squares regression and its associated assumptions were employed to ascertain the significance and impact of the independent variables. This was achieved through control intercepts and differential intercepts, representing an application of the various procedures outlined in [9].

3. Formalization of the Impact of One or More Variables on Another in the Presence of a Structural Change

This methodology evaluated the impact and statistical significance of a set of independent variables on a dependent variable in different time periods separated by a structural change. In other words, this methodology allowed us to analyze the impact and statistical significance of structural changes over time—for example, a change in the administration of a company or an economy, the evaluation of some policy, or the presence of an economic crisis or health crisis, such as the recent case of the worldwide COVID-19 pandemic.

The interpretation of the coefficients derived from this methodology was aligned with the conventional interpretation as a rate of change. However, this placed particular emphasis on the division of the time intervals corresponding to the structural change. In this case, dichotomous variables on a nominal scale were utilized, and mutually exclusive values were adopted to signify membership in one time period or another.

With the purpose of obtaining results capable of forming predictions with the best linearly unbiased estimators, the assumptions of a linear regression model were embraced. The model can be expressed as follows:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_t \quad (1)$$

where:

Y: dependent variable;

X_1 and X_2: independent variables;

ε_t: stochastic part of error.

Therefore, the variables X_1 and X_2 have an effect on Y over a period of time.

Assuming a structural change, a dichotomous variable D_1 was then added that accepted values of zero for all the observations before the structural change, and values of one for the opposite case, so that the model can be written in the following form:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \alpha_0 D_1 + \alpha_1 D_1 X_1 + \alpha_2 D_1 X_2 + \varepsilon_t \quad (2)$$

where the parameter α_i is useful for establishing the change that had the impact from one period to another. It was assumed that each structural change will be denoted with a different group of parameters accompanied by a dichotomous variable D_j, where j denotes the number of structural changes.

In this case, the expression that identified the relationship of the independent variables X_1 and X_2 with Y for the period after the structural change is given as follows:

$$Y = (\beta_0 + \alpha_0) + (\beta_1 + \alpha_1)X_1 + (\beta_2 + \alpha_2)X_2 \quad (3)$$

In general, a model for a single study period (without a structural change) can be expressed as follows:

$$Y = \beta_0 + \sum_{i=1}^{n} \beta_i X_i \quad (4)$$

where n is the number of independent variables.

To include a structural change, the following term is added:

$$D_1 [\alpha_0 + \sum_{i=1}^{n} \alpha_i X_i] \quad (5)$$

This can be generalized as follows:

$$D_j [\gamma_0 + n \sum_{i=1}^{n} \gamma_i X_i] \quad (6)$$

Suppose that a relationship of three independent variables with two structural changes is analyzed. Then, the model to estimate this is as follows:

$$Y = \beta_0 + \sum_{i=1}^{n} \beta_i X_i + D_1 [\alpha_0 + \sum_{i=1}^{n} \alpha_i X_i] + D_2 [\gamma_0 + n \sum_{i=1}^{n} \gamma_i X_i] + \varepsilon \quad (7)$$

When finding the value of the parameters using any econometric software, the interpretation is shown below.

The equation for the first period, before any structural changes, is as follows:

$$Y = \beta_0 + \beta_1 X + \beta_2 X_2 + \beta_3 X_3 \quad (8)$$

The equation for the second period, after the first structural change, is as follows:

$$Y = (\beta_0 + \alpha_0) + (\beta_1 + \alpha_1)X_1 + (\beta_2 + \alpha_2)X_2 + (\beta_3 + \alpha_3)X_3 \quad (9)$$

Finally, for the last period, after the second structural change, the equation is given as follows:

$$Y = (\beta_0 + \gamma_0) + (\beta_1 + \gamma_1)X_1 + (\beta_2 + \gamma_2)X_2 + (\beta_3 + \gamma_3)X_3 \quad (10)$$

By employing the principle of ceteris paribus, the impact of each independent variable on the dependent variable was determined for three distinct periods, taking into account the model's assumptions. The subsequent section illustrates a practical example applied to the Mexican economy and its foreign trade, addressing two structural changes.

4. The Impact of Mexican Exports on Economic Growth

Initially, a regression was conducted for the entire study period by examining the relationship between an increase in exports and economic growth using the conventional ordinary least-squares methodology. The resulting estimate is presented in the following equation.

$$DPIB_t = 0.028 + 0.262DX_t + \varepsilon_t \quad (11)$$

Nevertheless, despite the statistical significance of the independent variable, an R^2 of 0.65 was attained, and the model's assumptions were not validated. Specifically, the cumulative sum test and the cumulative sum-of-squares test, or CUSUM and CUSUMQ, respectively, provided evidence of the presence of structural changes.

These tests were developed by Page [11] and they analyze the cumulative sum of deviations from a target value. This allows us to determine if the conditional mean of the model is stable when the blue trajectory leaves the red bands, as shown below, with a significance level of 95%.

In Figures 1 and 2 shown below, it should be noted that the cumulative sum of the deviations on the vertical axis is shown with respect to time on the horizontal axis.

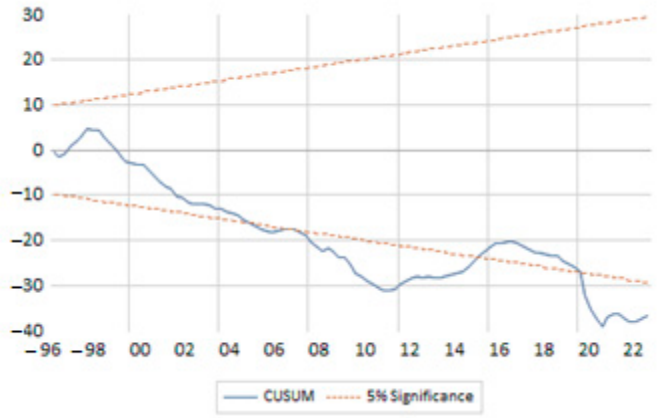


Figure 1. CUSUM test.

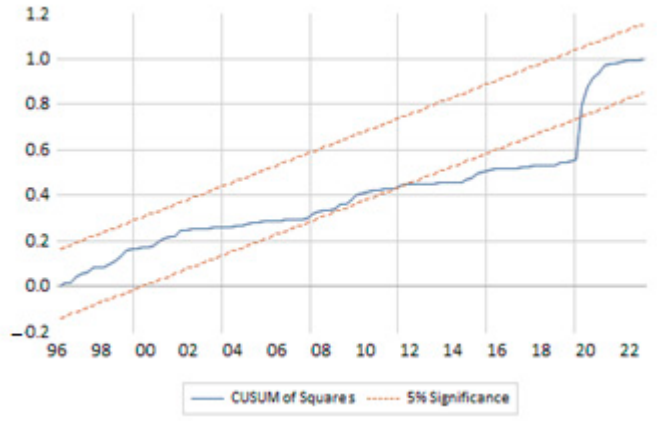


Figure 2. CUSUMQ test.

In this context, and aligned with specific events within Mexico's international trade, it was identified that significant structural changes occurred in the first quarter of 2009 and the second quarter of 2018. These changes can be attributed to the impact of the global financial crisis of 2009 and the commencement of trade friction between the United States and China, as Mexico's principal trading partners.

Given this scenario, the subsequent model was estimated as follows:

$$DGDP_t = \alpha_0 + \alpha_1 DX_t + \beta_0 D_1 + \beta_1 D_1 X_t + \gamma_0 D_2 + \gamma_1 D_2 X_t + \varepsilon_t \quad (12)$$

where:

$DGDP_t$: GDP growth rate of the Mexican economy at time t;

DX_t: growth rate of Mexican exports at time t;

D_1: dichotomous variable that shows the first period before the 2009 crisis (consisting of zeros until the first quarter of 2009 and ones until the second quarter of 2018);

D_2: dichotomous variable that differentiates the last period (composed of zeros until the first quarter of 2018 and ones for the rest of the series);

ε_t: stochastic error term.

The results of the ordinary least-squares regression are shown in Table 1.

Table 1. Model results.

Variable	Parameter	Coefficient	Std. Error	*t*-Statistic	Probability
C	α_0	0.228962	0.274384	0.834459	0.4066
DX	α_1	0.195046	0.020456	9.535084	0.0000
D1	β_0	1.581706	0.378910	4.174352	0.0001
D1 × DX	β_1	−0.09056	0.027414	−3.303424	0.0015
D2	γ_0	−1.91627	0.477476	−4.013346	0.0001
D2 × DX	γ_1	0.158344	0.027334	5.792916	0.0000

In the model, $R^2 = 0.92$ was obtained and it was necessary to include three dummy variables in 2009, 2020, and 2021 to firstly correct for the effects of the global financial crisis, and secondly correct for the effects of the pandemic caused by COVID-19.

Subsequently, the errors of the model were analyzed and it was determined that the probability value of the Jarque–Bera statistic was 0.46. This implies the normality of the errors at a 95% confidence level; likewise, the probability of the F-statistic in the White test with crossed terms was 0.57, which confirmed the constant variance or homoscedasticity of the errors, and a Durbin Watson statistic of 1.48 with a probability value of 0.08 for the F-statistic were obtained in the Breusch–Godfrey LM test, which indicated the absence of first-order and higher-order autocorrelation.

Interpretation of the Results

For the initial period spanning from the last quarter of 2001 to the first quarter of 2009, in accordance with the demonstrated methodology (during which the first structural change took place), the linear relationship is represented by the following:

$$DGDP_t = \alpha_0 + \alpha_1 DX_t \quad (13)$$

That is to say the following:

$$DGDP_t = 0.22 + 0.19 DX_t \quad (14)$$

For the second period, considering the structural change prompted by the global financial crisis that concluded in the second quarter of 2018, the expression is formulated as follows:

$$DGDP_t = (\alpha_0 + \beta_0) + (\alpha_1 + \beta_1) DX_t \quad (15)$$

Implying the following:

$$DGDP_t = 1.8 + 0.10 DX_t \quad (16)$$

Lastly, for the concluding period delimited by the onset of trade friction between the United States and China, which significantly impacted Mexican trade, the expression is articulated as follows:

$$DGDP_t = (\alpha_0 + \gamma_0) + (\alpha_1 + \gamma_1)DX_t \quad (17)$$

This expression can be reduced as follows:

$$DGDP_t = -1.69 + 0.34DX_t \quad (18)$$

It was evident that the coefficients derived from each of the three periods unveiled the alterations in the linear relationship between economic growth and exports, signaling the presence of two structural changes over time that led to significant shifts.

An approximation of the regression line is given in Figure 3.

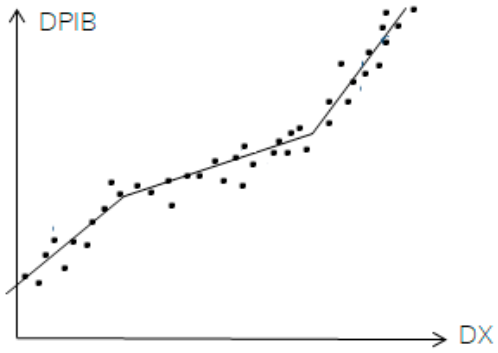


Figure 3. Linear regression in three different periods.

In broad terms, it was evident that the influence of Mexican exports on economic growth decreased by 47% after the first structural change and experienced a substantial increase of 240% after the occurrence of the second structural change.

Some recent methods based on least-squares regression for detecting turning points around structural changes are discussed in [12].

5. Conclusions

The presented methodology holds significant relevance for analyzing the impact of one or more quantitative independent variables on a dependent variable of the same nature amidst crucial structural changes occurring over time. Such was the case for the impact of Mexican foreign trade on the growth rate of its economy in the face of two extremely important structural changes: the global financial crisis of 2009 and the beginning of trade friction between the United States and China, Mexico's main trading partners, with whom it has a strong dependence on the demand and supply sides, respectively, in the links of global value chains.

Within the scope of a quantitative economic analysis, these types of methodologies are often very beneficial for evaluating economic policies, regime changes, or the impacts of phenomena that arise within a theoretical relationship. It is worth noting that one of the limitations that could arise is the evaluation of the model in the face of an inadequate theoretical specification, that is, without theoretical support for the relationship to be studied, in addition to the inconveniences that may arise due to the ordinary least-squares methodology.

In this regard, for the analysis presented here, the variables involved were transformed into growth rates to avoid modeling trends and spuriousness.

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X.L.; writing—original draft preparation, G.C.; writing—review and editing, X.L.; visualization, G.C.; supervision, X.L.; project administration, G.C.; funding acquisition, G.C. and X.L. All authors have read and agreed to the published version of the manuscript.

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