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Comparison of Inferential Methods for a Novel CMP Model †

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Abstract: In many real-life instances, time series of counts are often exposed to the dispersion phenomenon while at the same time being influenced by some explanatory variables. This paper takes into account these two issues by assuming that the series of counts follow an observation-driven first-order integer-valued moving-average structure (INMA(1)) where the innovation terms are COM-Poisson (CMP) distributed under a link function characterized by time-independent covariates. The second part of the paper constitutes estimating the regression effects, dispersion and the serial parameters using popular estimation methods, mainly the Conditional Least Squares (CLS), Generalized Method of Moments and Generalized Quasi-Likelihood (GQL) approaches. The performance of these estimation methods is compared via simulation experiments under different levels of dispersion. Additionally, the suggested model is used to examine time series accident data from actual incidents in several Mauritius locations.

Keywords: INMA(1); stationary; COM-Poisson; GQL; dispersion

1. Introduction

Undeniably, the COM-Poisson (CMP) model has achieved great popularity in recent years, especially in the modeling of over-dispersed and under-dispersed count data [1–6]. This model has a number of advantages as compared to Weighted or Generalized Poisson distributions [7]. Firstly, this model belongs to the family of exponential densities, and, secondly, it may handle all forms of dispersion. In fact, the structure of the CMP model is made up of a dispersion index, which, under some parameterization, makes the distribution suitable for modeling equi-, over- and under-dispersion. To date, many researchers have referred to this distribution as statistical jargon for modeling many real-life data in sectors such as the linguistic, finance, marketing and road traffic sectors [1–4].

Traditionally, time series of counts have been modeled using two main approaches: the parameter-driven and the observation-driven methods. The parameter-driven approach is concerned with the serial correlations induced by correlated randomly distributed effects, while the observation-driven method focuses on relating the current observation with the previous lagged observation through some binomial thinning functions that build the serial correlation [8]. Over the years, a number of papers have dealt with the suitability of these two approaches, and most of them have focused on the observation-driven method due to its flexibility in specifying likelihood functions for estimating the unknown parameters of a model [9–11]. In the same spirit, Mckenzie et al. [12–14], Al-Osh and Al-Zaid [15] and, lately, Fokianos [16] have designed observation-driven processes to represent time series that may follow the first-order autoregressive (AR(1)), moving average (MA(1)) or ARIMA structures where the innovation terms in these series were assumed to follow the Poisson or the Negative Binomial distribution. However, up to now, there has been no such process developed for the COM-Poisson model. This paper concentrates on developing an

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integer-valued first-order moving-average (INMA(1)) process for the CMP model where the innovation series follow the CMP structure under stationary moments. Note, under this set-up, the stationarity in the process is handled by assuming the mean and variance functions are characterized by some time-constant covariates.

The organization of the paper is as follows: In the next section, the INMA(1) COM-Poisson time series model is introduced along with the derivation of the serial autocorrelation structure. In Section 3, the estimation methodologies are developed, while, in Section 4, a time-independent covariate design is assumed, and a simulation study is conducted in which equi-, over- and under-dispersed count time series are constructed. In Section 5, we analyze a set of real-life data pertaining to accidents occurring between two important regions in Mauritius subject to some time-constant explanatory variables. The conclusion is provided in the last section.

2. A Novel COM-Poisson Model

The COM-Poisson probability function is of the form

$$P(W = w_j) = \frac{(\lambda_j)^{w_j}}{(w_j!)^{\nu}} \frac{1}{Z(\lambda_j, \nu)}, \quad w_j = 0, 1, 2, 3, \ldots, \quad for \quad \lambda_j \geq 0, \nu \geq 0 \tag{1}$$

The weight function $Z(\lambda_j, \nu)$ is given by the formula

$$Z(\lambda_j, \nu) = \sum_{k=0}^{\infty} \frac{(\lambda_j)^k}{(k!)^{\nu}} \tag{2}$$

The mean and the variance of the COM-Poisson model are derived through an iteration process using

$$E(W^{s+1}) = \begin{cases} \lambda_j E(W+1)^{1-\nu} & s = 0, \\ \lambda_j \frac{d}{d\lambda_j} E(W^s) + E(W)E(W^s) & s > 0. \end{cases} \tag{3}$$

Based on work by Sellers et al. [4], the mean and the variance can be estimated by

$$E(W_j) = (\lambda_j)^{1/\nu} - (\frac{\nu - 1}{2\nu}) \quad and \quad V(W_j) = \frac{(\lambda_j)^{1/\nu}}{\nu} \tag{4}$$

where the normalizing constant $Z(\lambda_j, \nu)$ can be computed using

$$Z(\lambda_j, \nu) \simeq \frac{\exp(\nu \lambda_j^{\frac{1}{\nu}})}{\lambda_j^{\frac{\nu-1}{2\nu}} (2\pi)^{\frac{\nu-1}{2}} \sqrt{\nu}} \tag{5}$$

Shmueli et al. [1] demonstrated through numerical tests that these approximations perform satisfactorily for all $\nu > 0$ but particularly well for $\nu < 1$ or $\lambda_j > 10^{\nu}$. With these assumptions and variations, it is simple to confirm that, for $\nu < 1$, there is over-dispersion when $V(W_j) > E(W_j)$, and, for $\nu > 1$, there is under-dispersion when $E(W_j) > V(W_j)$ since $\lambda_j > 0$. Moreover, since $\lambda_j = exp(x_j^T \beta)$ for $x_j = [x_{j1}, x_{j2}, \ldots, x_{jp}]^T$ and $\underset{\sim}{\beta} = [\beta_1, \beta_2, \ldots, \beta_p]^T$, these over- and under-dispersion assumptions are solidly supported. The observation-driven equation for the first-order MA(1) process [14,15] is defined as

$$y_t = \rho * d_{t-1} + d_t \tag{6}$$

where d_t is the innovation term, and $\rho * d_{t-1} d_{t-1} \sim$ Binomial(d_{t-1}, ρ). d_t, y_t and d_{t-1} are independent.

The mean and variance at the t^{th} time point are determined by assuming Y_t is COM-Poisson as follows:

$$E(Y_t) = \theta_t = \lambda_t^{1/\nu} - \frac{\nu - 1}{2\nu} \quad and \quad Var(Y_t) = \sigma_t^2 = \frac{\lambda_t^{1/\nu}}{\nu}, \quad for \quad t = 1, 2, 3, \ldots \tag{7}$$

The mean and variance of the innovation terms d_t are found from Equations (6) and (7) under stationary assumptions, which is due to the presence of time-independent variables: $\lambda_1 = \lambda_2 = \ldots = \lambda_t = \ldots = \lambda_T \Rightarrow \theta_1 = \theta_2 = \ldots = \theta_t = \ldots = \theta_T \Rightarrow \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_t^2 = \ldots = \sigma_T^2$. Then, following the method of Mamode Khan and Jowaheer [17] and assuming d_t is COM-Poisson, the expression for the dispersion index and link function are worked out to obtain the

$$E(d_t) = \frac{\theta_1}{(1+\rho)} \tag{8}$$

and the dispersion

$$\nu^* = \frac{\frac{2\theta_1}{1+\rho} + 1 + \sqrt{(\frac{2\theta_1}{1+\rho} + 1)^2 - 8[\frac{1}{1+\rho^2}[\frac{\theta_1}{\nu} + \frac{\nu-1}{2\nu^2} - \rho(1-\rho)(\frac{\theta_1}{1+\rho})]]}}{4[\frac{1}{1+\rho^2}[\frac{\theta_1}{\nu} + \frac{\nu-1}{2\nu^2} - \rho(1-\rho)(\frac{\theta_1}{1+\rho})]]} > 0 \tag{9}$$

and it is simple to confirm that, under these distributional assumptions of d_t,

$$E(Y_t) = \theta_1 \quad and \quad Var(Y_t) = \frac{\theta_1}{\nu} - \frac{\nu - 1}{2\nu^2} \tag{10}$$

The serial correlation is shown to be

$$Corr(Y_t, Y_{t-k}) = \begin{cases} \frac{\frac{\rho}{1+\rho^2}[\frac{\theta_1}{\nu} + \frac{(\nu-1)}{2\nu^2} - \frac{\rho(1-\rho)\theta_1}{(1+\rho)}]}{\frac{\theta_1}{\nu} + \frac{(\nu-1)}{2\nu^2}} & k = 1, \\ 0 & k \neq 1. \end{cases} \tag{11}$$

3. Methods of Estimation

In time series settings, the popular methods of estimating the unknown parameters in the INMA(1) model are the Generalized Least Squares (GLS) [18,19] and the Generalized Methods of Moments (GMM) [20,21]. These two techniques are known to provide consistent estimates, with GLS providing more efficient estimates than GMM [18,22,23]. In a recent research paper, Mamode Khan et al. [24] used the Generalized Quasi-Likelihood (GQL) estimation approach [25], which is an extension of the quasi-likelihood method of Wedderburn [26], to estimate the regression effects. This method has been mostly used in longitudinal set-ups, where it was found to yield consistent and efficient parameter estimates as compared to likelihood-based methods. Moreover, Sutradhar et al. [27] showed that this approach yields even asymptotically equally efficient estimates as the maximum likelihood. The GQL approach is flexible and computationally appealing since it requires only the correct specification of the means and variances. This approach is similar to the Generalized Estimating Equation (GEE) introduced by Liang and Zeger [28], which was used to estimate the regression parameters in longitudinal set-ups. However, for the GEE approach, a working covariance structure is assumed. Later, Crowder [29] and Sutradhar and Das [25] illustrated that the GEE approach may not yield such efficient estimates as the independent working structure approach. This finding was contradictory as repeated measures are needed to exhibit significant serial correlation. To overcome this paradox, Sutradhar and Das [25] proposed a robust autocorrelation structure based on the method of moments to estimate the covariance structure and that forms the GQL equation. In this paper, we consider the GQL equation to estimate the regression parameters and the method of moments to calculate the serial correlations. We also compare the performance and efficiency of this approach with CLS and GMM approaches, which has not yet been

achieved in the literature for the INMA(1) CMP time series. As for the CLS approach, the criterion function

$$Q(\psi) = \sum_{t=2}^{T}[y_t - E(y_t d_{t-1})]^2 = \sum_{t=2}^{T}[y_t - \rho d_{t-1} - \frac{(\lambda_1^{1/\nu} - \frac{\nu-1}{2\nu})}{1+\rho}]^2 \tag{12}$$

is minimized.

3.1. Generalized Quasi-Likelihood

The GQL equation is expressed as

$$D^T \Sigma^{-1}(\underset{\sim}{y} - \underset{\sim}{\theta}) = 0 \tag{13}$$

where $\underset{\sim}{y} = [y_1, y_2, \ldots, y_T]$ and $\underset{\sim}{\theta} = [\theta_1, \theta_2, \ldots, \theta_T]$. Assuming a set of p explanatory variables $\underset{\sim}{\beta} = [\beta_1, \beta_2, \ldots, \beta_p]^T$, the derivative matrix is a block diagonal $T \times (p+1)$ matrix and

$$D = \begin{pmatrix} \frac{\partial\theta_1}{\partial\beta_1} & \frac{\partial\theta_2}{\partial\beta_1} & \cdots & \frac{\partial\theta_T}{\partial\beta_1} \\ \frac{\partial\theta_1}{\partial\beta_2} & \frac{\partial\theta_2}{\partial\beta_2} & \cdots & \frac{\partial\theta_T}{\partial\beta_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial\theta_1}{\partial\beta_p} & \frac{\partial\theta_2}{\partial\beta_p} & \vdots & \frac{\partial\theta_T}{\partial\beta_p} \\ \frac{\partial\theta_1}{\partial\nu} & \frac{\partial\theta_2}{\partial\nu} & \cdots & \frac{\partial\theta_T}{\partial\nu} \end{pmatrix}_{(p+1)\times T} \tag{14}$$

where $\frac{\partial\theta_t}{\partial\beta_k} = \frac{(\lambda_t)^{\frac{1}{\nu}} x_j^T}{\nu}$ and $\frac{\partial\theta_t}{\partial\nu} = -\frac{(\lambda_t)^{\frac{1}{\nu}} ln(\lambda_t)}{\nu^2} - \frac{1}{2\nu} + \frac{\nu-1}{2\nu^2}$.

The covariance structure $\Sigma = A^{\frac{1}{2}} C(\rho) A^{\frac{1}{2}}$. A is the $T \times T$ variance matrix, and $C(\rho)$ is the general autocorrelation structure represented by

$$C(\rho) = \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{T-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{T-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{T-1} & \rho_{T-2} & \rho_{T-3} & \cdots & 1 \end{bmatrix} \tag{15}$$

where the correlation parameter ρ_l is consistently estimated by a moment-estimating equation of the form

$$\hat{\rho}_l = \frac{\sum_{t=1}^{T-1} \tilde{y}_t y_{(\tilde{t+l})} / (T-1)}{\sum_{t=1}^{T} \tilde{y}_t{}^2 / T} \tag{16}$$

where $\tilde{y}_t = \frac{y_t - \theta_t}{\sqrt{\theta_t}}$. The GQL in Equation (13) is solved using the Newton–Raphson iterative technique, which results in

$$\begin{pmatrix} \hat{\beta}_{r+1} \\ \hat{v}_{r+1} \end{pmatrix} = \begin{pmatrix} \hat{\beta}_r \\ \hat{v}_r \end{pmatrix} + [D^T \Sigma^{-1} D]_r^{-1} [D^T \Sigma^{-1} (\underset{\sim}{y} - \underset{\sim}{\theta})]_r. \tag{17}$$

3.2. Adaptive Generalized Method of Moments (GMM)

The GMM approach developed by Hansen [20] comprises several moment equations that are combined by a quadratic function and thereafter is minimized to estimate the parameters. In this subsection, we adopt the Adaptive GMM procedure of Qu and Lindsay, where only two sets of moments function are considered.

$$g = \begin{bmatrix} D^T(\underset{\sim}{y} - \underset{\sim}{\theta}) \\ \hat{\alpha}^T D^T V(\underset{\sim}{y} - \underset{\sim}{\theta}) \end{bmatrix} \tag{18}$$

and $V = \frac{1}{T}(\underset{\sim}{y} - \underset{\sim}{\theta})(\underset{\sim}{y} - \underset{\sim}{\theta})^T$, and the moment vector (18) is combined to form the quadratic objective function

$$Q(\beta) = g^T C^{-1} g \tag{19}$$

where C is the sample variance of g as follows:

$$\begin{pmatrix} D^T V D & (D^T V^2 D)\hat{\alpha} \\ \hat{\alpha}^T (D^T V^2 D) & \hat{\alpha}^T (D^T V^3 D)\hat{\alpha}. \end{pmatrix} \tag{20}$$

We derive the iterative solution by minimizing function (19) and using the Newton–Raphson technique as follows:

$$\begin{pmatrix} \hat{\beta}_{r+1} \\ \hat{v}_{r+1} \end{pmatrix} = \begin{pmatrix} \hat{\beta}_r \\ \hat{v}_r \end{pmatrix} - \begin{pmatrix} \ddot{Q}(\hat{\beta}_r) \\ \ddot{Q}(\hat{v}_r) \end{pmatrix}^{-1} \begin{pmatrix} \dot{Q}(\hat{\beta}_r) \\ \dot{Q}(\hat{v}_r) \end{pmatrix}. \tag{21}$$

4. Simulations and Results

In this section, the INMA(1) CMP time series is generated for different numbers of time points, and estimation of parameters is carried out using CLS, GMM and GQL methods, as mentioned in the previous section. For the simulation part, data of size $T = 60, 100$ and 500 were generated using Equation (6) for different values of ν. A 2×1 regression vector was considered where, for the first covariate,

$$x_{t1} = \begin{cases} -0.9 & (t = 1, \ldots, T/5), \\ rnorm(1, 0, 1) & (t = (T/5) + 1, \ldots, 3T/5), \\ 0.9 & (T = (3T/5) + 1, \ldots, T), \end{cases}$$

where rnorm generates random numbers from a normal distribution with mean 0 and variance 1 in R, and x_{t2}, the second covariate, follows a standard normal random variable distribution for all $t = 1, 2, \ldots, T$. Note that, for this simulation study, $\beta_1, \beta_2 = 1$ and $\theta_0 = \theta_1$, and 5000 simulations were conducted for each set of ν, ρ and T. The results are shown in Table 1.

Table 1 shows the simulated mean estimates of the regression, dispersion and correlation parameters under different values of the serial correlation and dispersion parameters. The values in the bracket indicate the standard errors for each regression and dispersion estimate. These converged estimates were obtained by using the iterative processes described in Section 3. With larger sample sizes, the standard errors for each method decline, with the standard errors for the GQL method being lower than those of the GMM and CLS methods. No matter what the correlation parameter, this trend is observed. A few non-convergent simulations are noted, though.

Table 1. Parameter estimates for the COM-Poisson model.

			$\rho = 0.5$				$\rho = 0.9$			
ν	T	**Method**	$\hat{\beta_0}$	$\hat{\beta_1}$	$\hat{v}$	$\hat{\rho}$	$\hat{\beta_0}$	$\hat{\beta_1}$	$\hat{v}$	$\hat{\rho}$
3	60	CLS	0.9827	0.9853	3.1826	0.4865	0.9813	0.9853	3.1404	0.9160
			(0.3024)	(0.3862)	(0.4808)		(0.3168)	(0.3062)	(0.3885)	
		GMM	0.9804	0.9868	3.122	0.4847	0.9815	0.9848	3.112	0.8810
			(0.4191)	(0.4185)	(0.5295)		(0.4503)	(0.4477)	(0.5774)	
		GQL	0.9821	0.9937	3.0401	0.4909	0.9980	0.9956	3.0563	0.8977
			(0.1669)	(0.1020)	(0.1821)		(0.1152)	(0.1180)	(0.1161)	

Table 1. *Cont.*

			$\rho = 0.5$				$\rho = 0.9$			
ν	T	**Method**	$\hat{\beta_0}$	$\hat{\beta_1}$	$\hat{v}$	$\hat{\rho}$	$\hat{\beta_0}$	$\hat{\beta_1}$	$\hat{v}$	$\hat{\rho}$
	100	CLS	0.9843	0.9848	3.1720	0.4886	0.9813	0.9836	3.1605	0.9108
			(0.2034)	(0.2872)	(0.3004)		(0.2550)	(0.2215)	(0.3217)	
		GMM	0.9877	0.9803	3.1099	0.5045	0.9870	0.9839	3.1070	0.8999
			(0.3318)	(0.3302)	(0.4031)		(0.3644)	(0.3812)	(0.4331)	
		GQL	0.9922	0.9941	3.0057	0.5101	1.0554	1.0007	3.0781	0.8927
			(0.1052)	(0.1065)	(0.1067)		(0.0991)	(0.0965)	(0.1837)	
	500	CLS	0.9877	0.9866	3.0463	0.4802	0.9859	0.9815	3.0196	0.9160
			(0.1025)	(0.1918)	(0.2910)		(0.1073)	(0.1076)	(0.2101)	
		GMM	0.9811	0.9814	3.0635	0.5155	0.9880	0.9804	3.0939	0.8853
			(0.2309)	(0.2861)	(0.2158)		(0.2862)	(0.2201)	(0.2194)	
		GQL	1.0024	1.0030	3.0087	0.5058	0.9938	1.0082	3.0044	0.9086
			(0.0642)	(0.0501)	(0.0694)		(0.0481)	(0.0434)	(0.0513)	
2.5	60	CLS	0.980.	0.9890	2.5145	0.4886	0.9891	0.9852	2.5105	0.8974
			(0.4131)	(0.4561)	(0.5908)		(0.4080)	(0.4097)	(0.5223)	
		GMM	0.9845	1.1041	2.4939	0.5104	0.9851	0.9819	2.5089	0.8802
			(0.5411)	(0.5037)	(0.5384)		(0.5249)	(0.5091)	(0.5584)	
		GQL	0.9926	0.9953	2.4915	0.4954	0.9983	0.9972	2.5009	0.8959
			(0.1690)	(0.1245)	(0.1079)		(0.1169)	(0.1159)	(0.1011)	
	100	CLS	0.9807	0.9824	2.5023	0.4876	0.9802	0.9835	2.5030	0.8837
			(0.3814)	(0.3274)	(0.3389)		(0.3948)	(0.3162)	(0.3255)	
		GMM	0.9869	0.9820	2.5058	0.5125	0.9820	0.9881	2.5011	0.8899
			(0.4050)	(0.4303)	(0.4292)		(0.4012)	(0.4075)	(0.4002)	
		GQL	0.9926	0.9960	2.5080	0.4971	0.9978	0.9941	2.5067	0.8943
			(0.0612)	(0.0686)	(0.0655)		(0.0632)	(0.0611)	(0.0680)	
	500	CLS	0.9835	0.9874	2.5061	0.4815	0.9863	0.9804	2.5044	0.8851
			(0.1374)	(0.1880)	(0.1027)		(0.1105)	(0.1553)	(0.1189)	
		GMM	0.9860	0.9858	2.5082	0.5110	0.9854	0.9870	2.5061	0.8880
			(0.2796)	(0.2702)	(0.3321)		(0.2225)	(0.2045)	(0.3241)	
		GQL	0.9937	0.9967	2.5029	0.5066	0.9904	0.9995	2.5022	0.8922
			(0.0193)	(0.0130)	(0.0275)		(0.0134)	(0.0124)	(0.0270)	

5. Application

In the last five years, the number of road traffic accidents in Mauritius has decreased due to strict legal measures against careless drivers and installation of speed cameras and police patrols on the motorways. In Mauritius, one of the routes which is the most used by the working population is the route connecting the capital city with the Ebene Cybercity since the Ebene Cybercity is one of the most industrialized regions of the island. Additionally, this area is also situated near the University of Mauritius and links Mauritius to several other routes going to the south or the international airport. Connected to this, we collected data from the central statistical office from January 2016 to February 2020 and recorded the fortnightly number of accidents occurring between Port-Louis and Ebene Cybercity, subject to time-independent covariates such as the number of traffic lights (TL), number of roundabouts (RA) and number of speed cameras (NSC). Here, the univariate time series is composed of the fortnightly number of accidents between January 2016 and February 2020, which make up a total of 108 observations, and all these are subject to the covariates mentioned above. Note that, in our definition of accidents, we consider any minor or serious accident that may consist of a collision of cars, buses or motorcycles. Table 2 provides the summary statistics of the number of accidents.

Table 2. Summary statistics for the number of fortnightly accidents.

	Mean	Variance	$\hat{\rho_1}$	$\hat{\rho_2}$	$\hat{\rho_3}$	$\hat{\rho_4}$
Fortnightly Accident	5.578	10.623	0.411	0.1471	0.0055	0.0003

From Table 2, we can observe that the data are over-dispersed as the variance exceeds the mean, and there is a lag 1 correlation of 0.411. Thus, to analyze these data, we may apply the proposed INMA(1) model.

As shown in Table 3, the speed cameras and number of traffic light coefficients are negative, which implies that, as more cameras and traffic lights are placed over this area, fewer accidents are expected. In fact, from the estimates obtained, the number of accidents is reduced by 0.00145 percent. On the other hand, the reduction in the number of accidents by the installation of cameras is huge, with a reduction rate of 0.685 percent. As for roundabouts, this model displays a positive sign, which means that the more roundabouts there are on the motorway, the more accidents are prone to take place. In fact, from the records, it is revealed that, at the Reduit roundabout, around 1 to 3 accidents occur each week that include the collision of cars. As for the over-dispersion parameter, the estimates are significant and exhibit quite huge dispersion, which matches the summary statistics. The $lag-1$ autocorrelation coefficient is also reliable.

Table 3. Fortnightly number of accidents and estimates of the regression/serial correlation and dispersion parameters.

Method	Intercept	TL	NSC	RA	Dispersion	$\hat{\rho}$
GQL_{CMP}	1.879 (0.1238)	−0.00145 (0.1328)	−1.155 (0.1411)	0.524 (0.1234)	0.7913 (0.0312)	0.389

6. Conclusions

This paper treats the modeling of the discrete time series process using an INMA(1) structure where the innovation terms follow the COM-Poisson model under a stationary distributional assumption. In such set-ups, the parameters of interest include the regression coefficients, the serial correlation parameters and the dispersion index. The estimation of these effects is carried out using the CLS, GMM and a GQL approaches. The CLS and GMM estimating equations are conventional, while the GQL specifically finds the estimates of the regression parameters and the dispersion index, and the serial parameters are obtained through a robust method of moments. From the simulation studies, it can be concluded that GQL estimated these parameters with lower standard errors than the other two techniques. The model was also applied to real-life accident data with reliable estimates.

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