engineering proceedings

MDPI

Proceeding Paper

Study of Some Thorium Isotopes near to the Closed Shell (82 and 126) †

Nibras Hayder Hammood Eatiah *, Mohsin Kadhim Muttaleb Al-Jnaby and Ghaidaa A. Hafedh Jaber Hussien

Department of Physics, College of Science, University of Babylon, Babylon 51002, Iraq; sci.mohsin.kadhim@uobabylon.edu.iq (M.K.M.A.-J.); sci.ghaidaa.abdul-hafidh@uobabylon.edu.iq (G.A.H.J.H.)

* Correspondence: nabreshader7080@gmail.com

† Presented at the International Conference on Recent Advances in Science and Engineering, Dubai, United Arab Emirates, 4–5 October 2023.

Abstract: Using the interacting bosons model-one (IBM-1), the nuclear structure of the thorium isotopes ^{224}Th, ^{226}Th, and ^{228}Th were examined in this study, which are near from closed shell 82 and 126. By acquiring this element's energy levels and comparing them to actual values, which provide an indication of these isotopes membership in a specific determination, it is possible to determine that ^{224Th} and ^{226}Th belong to transition region between SU(3) and O(6) but ^{228}Th belong to the SU(3) limit. The ratio of the fourth to the second energy level E_4^+/E_2^+ with other ratios E_6^+/E_2^+ and E_8^+/E_2^+, the order of practical levels, are first exam to determine the limit that belong. Using IBM program to find theoretical energy level and compared with practical one, also the agreement between the theoretical probability of electric transitions B(E2) through IBMT program was investigated. The IBMP program was used to study the surface potential of the nucleus, which provides insight into the deformation that occurs in the nucleus and from the contour lines deviation.

Keywords: interacting boson model-one; electric transitions; potential energy; closed shell

check for updates

Citation: Eatiah, N.H.H.; Al-Jnaby, M.K.M.; Hussien, G.A.H.J. Study of Some Thorium Isotopes near to the Closed Shell (82 and 126). *Eng. Proc.* **2023**, *59*, 229. https://doi.org/10.3390/ engproc2023059229

Academic Editors: Nithesh Naik, Rajiv Selvam, Pavan Hiremath, Suhas Kowshik CS and Ritesh Ramakrishna Bhat

Published: 7 February 2024

Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

The development of the theories of the nucleus, there are two different models. One model is the shell model developed by Mayer and Jensen which deals with single-particle motion of the constituent nucleons in a mean-field potential. The nuclear shell model holds up well near magic nuclei. The model, however, is unable to account for nuclei's rotations and vibrations, which are seen in areas of the nuclear chart that are distant from the magic nuclei. The other model is the working of Bohr and Mottelson together. There are three primary patterns, deformed rotation SU(3), spherical vibration U(5), and γ-soft unstable O(6), are still used as standards to assess the structural integrity of nuclei [1].

The intermediate and heavy atomic nuclei can be well described by the interacting boson model-one (IBM-1). It reproduces the majority of the low-lying states of such nuclei by adjusting a number of parameters [2].

Many nuclear collective phenomena have been successfully addressed by the interacting boson model of Arima and Iachello. The fundamental concept is that neutron and proton bosons with zero spins and 2 spin respectively, can be characterized the low energy collective degrees of freedom in nuclei [3].

2. The Model (IBM)

The shell model and the collective model were the two main pillars of the nuclear theory up to 1970. Feshbach I. and Iachello F. presented the interacting boson model (IBM) in 1973, and in 1974, Arima A. and Iachello F. developed it, a new model based on a three approach, that is group theoretical or algebraic [4].

The IBM is a model that explains how low-lying, positive parity quadrupole-collective states behave when they are present in even-even deformed nuclei for medium and heavy

one. There is no distinction between protons and neutrons in the IBM-1 model. Several extensions of the original IBM-1 framework have been developed, including the IBM-2, which distinguishes between protons and neutrons, and the IBM-3 and IBM-4, which can explain light nuclei [5,6].

They pretended that the shell model indicates that the low-lying collective states of such nuclei arise from interacting nucleon pairs coupled with angular momentum L = zero or 2 (s boson and d boson, respectively), with energies(s, d), where s is frequently equal to zero.

The (IBM) is founded on the following presumptions and is based on the spherical shell model and collective model of atomic nucleus [7].

1. Bosons, or pairs of fermions, are used to describe pairs of active nuclear particles or holes that are close to "closed shells".
2. The overall number of bosons affects this model (N).

   Where N = Nπ + Nυ
   Nπ = proton boson number
   Nυ = neutron boson number

   Since they form an anti-symmetric state, identical nucleons have higher angular momentum in even numbers.
3. The variety of shells that are depicted in the shell model are limited to just the straightforward s-shell (L = 0) and d-shell (L = 2) [8,9].

The Hamiltonian of the IBM-1 as a multipole expansion, grouped into different boson-boson interactions as [10,11].

$$\hat{H} = \epsilon\, \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \quad (1)$$

where: $\hat{n}_d = [d^{\dagger}.\tilde{d}]$ the number of d bosons:

$\hat{P} = 1/2\,(\tilde{d}\,.\,\tilde{d}) - 1/2\left(\tilde{s}\,.\,\tilde{s}\right)$ the pairing operator for d and s bosons;

$\hat{L} = \sqrt{10}\,[d^{\dagger} \times \tilde{d}\,]^{(1)}$ the operator of angular momentum;

$\hat{Q} = [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]^{(2)} + \chi[d^{\dagger} \times \tilde{d}]^{(2)}$ the operator of quadrupole;

$\hat{T}_3 = [d^{\dagger} \times \tilde{d}]^{(3)}$ the operator of octupole;

$\hat{T}_4 = [d^{\dagger} \times \tilde{d}]^{(4)}$ the operator of hexapole;

$\epsilon = \epsilon d - \epsilon s$ the boson energy.

The IBM for axially symmetric rotors, spherical and vibrators schematically describes the analytically solvable dynamical symmetries SU(3), U(5), and O(6) [12].

Hamiltonian of SU(3) limit is:

$$\hat{H} = a_1 \hat{L}\hat{L} + a_2 \hat{Q}\hat{Q} \quad (2)$$

and for transition limit $O(6) \rightarrow SU(3)$:

$$\hat{H} = a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} \quad (3)$$

3. Electric Transitions

The absolute electric transition rates serve as a rigorous test for various theories in addition to being a sensitive aspect of nuclear structure. Most of the B(E2) values that are currently known were obtained using coulomb excitation. The following is a possible representation of the electromagnetic transition rates operators' general form [13].

$$\hat{T}^l_m = \alpha_2 \delta_{l_2} [d^{\dagger} \times \tilde{s} + s^{\dagger} \times \tilde{d}]^2_m + \beta_l [d^{\dagger} \times \tilde{d}]^l_m + \gamma_0 \delta_{l0} \delta_{m0} [s^{\dagger} \times \tilde{s}]^0_0 \quad (4)$$

where: l = 0,1,2,3,4,. . ., m = 0,1,2,3,4,. . ., and γ_0, α_2, β_l reflect the particular form of the transition operator and are parameters that specify the different terms in the related operators.

The electric operators of quadrupole transition expressed as [14]:

$$\hat{T}_m^{(E2)} = \alpha_2[d^\dagger \widetilde{s} + s^\dagger \widetilde{d}]_m^{(2)} + \beta_2[d^\dagger \widetilde{d}]_m^{(2)} \quad (5)$$

where effective charge $e_B = \alpha_2$ and $\beta_2 = \chi\, \alpha_2$

The electric quadrupole reduced transition probabilities B (E2) is defined as:

$$B(E2, l_i \to l_f) = \frac{1}{2l_i + 1} |< l_i \| \hat{T}^L \| l_f >|^2 \quad (6)$$

4. IBM Potential Surface

The potential energy surface (PES) gives the nucleus a final shape and the Hamiltonian function $PE(N, \beta, \gamma)$ is [8,9]:

$$PE(N, \beta, \gamma) = \langle N, \beta, \gamma | H | N, \beta, \gamma \rangle / \langle N, \beta, \gamma \,|\, N, \beta, \gamma \rangle \quad (7)$$

The energy surface has been calculated as a function of boson number (N), deformation (β) and deviation angle (γ):

$$PE(N, \beta, \gamma) = \frac{N\varepsilon_d}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}\left(a_1\beta^4 + a_2\beta^3 \cos 3\gamma + a_3\beta^2 + a_4\right) \quad (8)$$

where a_i are the coefficients, β is a total deformation of the nucleus. The shape is prolate if γ = 0, and if γ = 60, the shape becomes oblate [12]. γ represents the divergence from the focal symmetry and correlates with the nucleus. The equations below provided the potential energy surface for three dynamical symmetries [15,16].

$$PE(N, \beta, \gamma) \propto \begin{cases} U(5): \ \varepsilon_d N \frac{\beta^2}{1+\beta^2} \\ SU(3): \ kN(N-1)\frac{\frac{3}{4}\beta^4 - \sqrt{2}\beta^3 \cos 3\gamma + 1}{(1+\beta^2)^2} \\ O(6): \ k'N(N-1)\left(\frac{1-\beta^2}{1+\beta^2}\right)^2 \end{cases} \quad (9)$$

where $k \propto a_2$ and $\acute{k} \propto a_0$ in Hamiltonian.

5. Results and Discussion

The isotopes of (^{224}Th, ^{226}Th, ^{228}Th) energy levels were extracted using an IBM software program and the Hamiltonian as equation 1, using estimated parameters for transition determination SU(3) and O(6) for first two isotope but pure limit SU(3)for the last one were listed in Table 1 with the parameters in the program in MeV (mega electron volt) unit:

Table 1. The parameters in the program of the Hamiltonian.

The Isotopes	N	*EPS* in MeV	*P.P* in MeV	*L.L* in MeV	*Q.Q* in MeV	$T_3.T_3$ in MeV	$T_4.T_4$ in MeV	Chi Unit Less	SO6 Unit Less
^{224}Th	8	0.0000	0.0013	0.0082	−0.0102	0.0000	0.0000	−0.1700	1
^{226}Th	9	0.0000	0.0013	0.0082	−0.0069	0.0000	0.0000	−0.1700	1
^{228}Th	10	0.0000	0.0000	0.0000	0.1500	−0.5490	0.0008	−0.0242	1

There are decrease in Q.Q value in minus sign with increase of boson number N for first two isotopes, because of increase of the number of neutrons which given unstable motion in nucleus and its properties is between rotation and gamma unstable. The last

one has the properties of rotation motion only and the parameters of the Hamiltonian for SU(3) limit.

The energy levels for ^{224}Th, ^{226}Th, and ^{228}Th that arise from these parameters are illustrated in Figures 1–3 respectively, and they are in good accord with the findings of experiments for ground band. Many energy levels are ambiguous or unknown.

Figure 1 represents the energy levels for ^{224}Th, which have practical energy levels for the ground band only, while through the IBM program, the results for the ground band and the beta band, the ratio of the energy levels are: $E_4^+/E_2^+ = 2.89$, $E_6^+/E_2^+ = 5.8$ and $E_8^+/E_2^+ = 10.7$, so expected limit for this isotope is SU(3). ^{226}Th isotope looks like a ^{224}Th for energy levels as Figure 2 and the ratio of energy levels close to these value too.

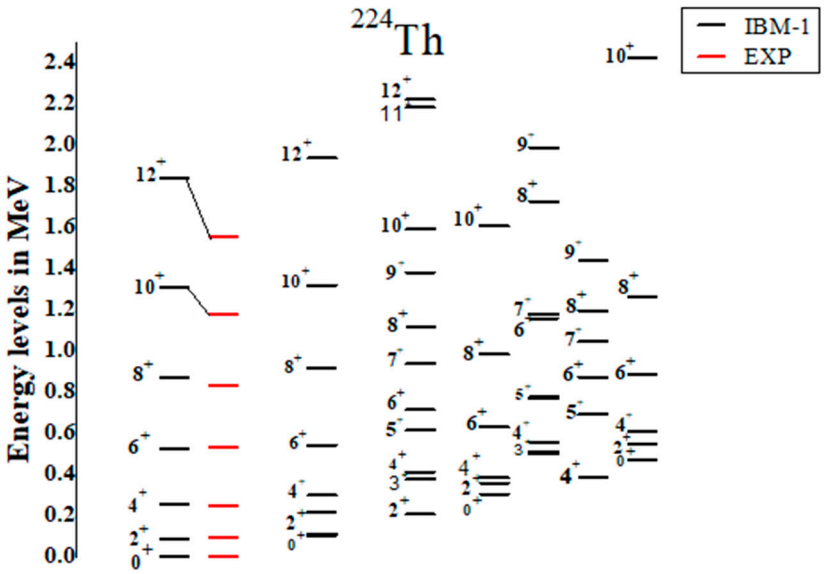


Figure 1. The arrangement of energy levels for ^{224}Th, theoretical and experimental.

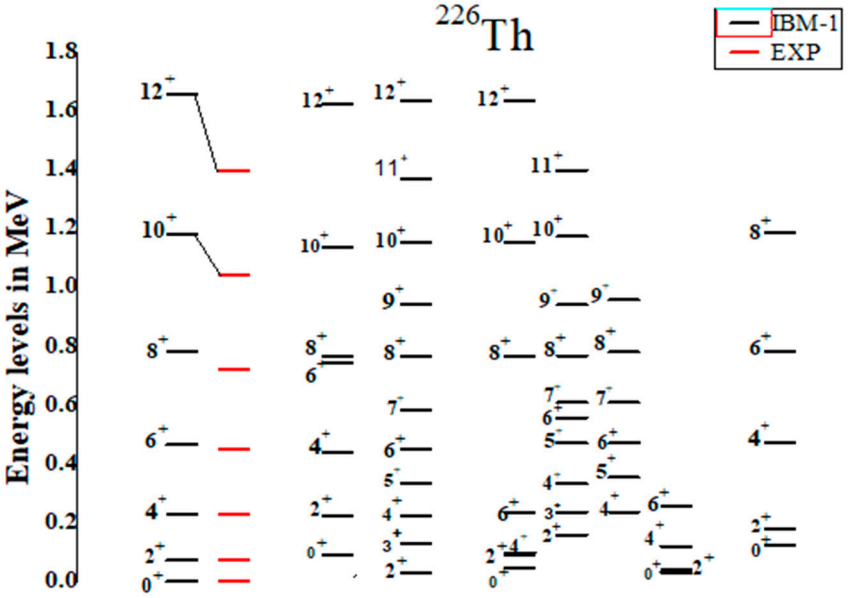


Figure 2. The arrangement of energy levels for ^{226}Th, theoretical and experimental.

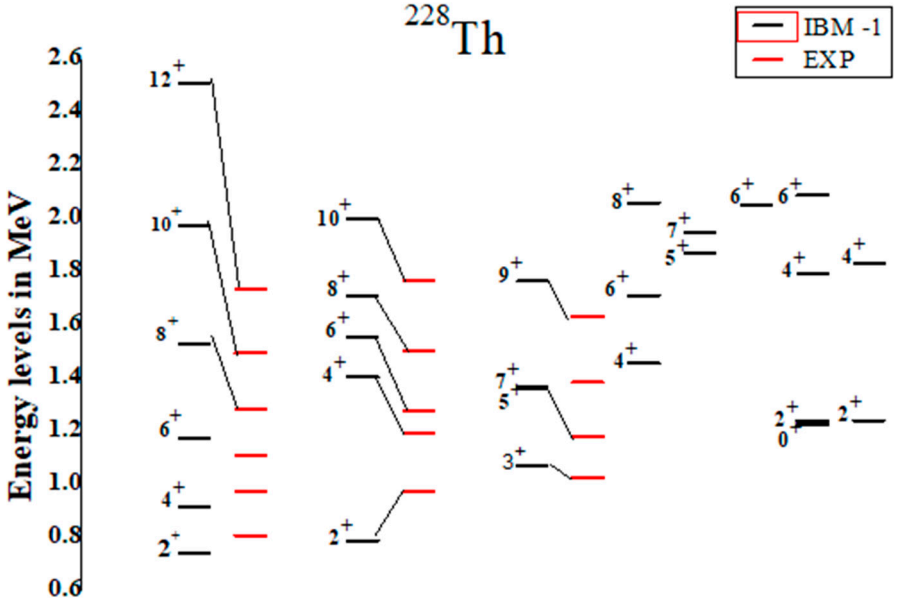


Figure 3. The arrangement of energy levels for ^{228}Th, theoretical and experimental.

The ^{228}Th isotope, it contains practical levels for the ground band, the beta band, and the gamma band as Figure 3. The energy ratios are equal to 3.13, 6.2 and 11.3 respectively and the results from the program were compared with practical value, which denoted to SU(3) limit. The congruence in the practical and theoretical values, this gives the validity of the results, and this congruence was only achieved by adding parameter to the Hamiltonian, which is called symmetry breaking.

Table 2 denoted to the value for exist value for the first electric transition probability from data sheet and their coefficients in IBMT program E2SD, E2DD.

Table 2. Experimental data of B(E2) and its coefficients for isotopes.

The Isotopes	Boson Numbers N	$B(E2;2_1^+\rightarrow 0_1^+)$ e^2b^2	E2SD(eb)	E2DD(eb)
^{224}Th	8	0.77614	0.143292	−0.423866
^{226}Th	9	1.341716	0.188401	−0.557299
^{228}Th	10	0.026489	0.057542	−0.040279

Equations (6) and (7) can be used to determine the electric transition for the isotopes which calculated from IBMT program, and the results are shown in the Tables 3–5 for some selected electric transitions.

Table 6 has the parameters of the coefficients of potential energy surface from the Equation (7), and applying IBMP program. These coefficients such as in the program.

The symmetric shape for two sides: prolate and oblate for ^{224}Th and contour lines for the potential energy denoted in Figure 4. There is asymmetry in both side because the deformation in the distribution of the potential in surface specially in $30 < \gamma < 60$ and the maximum value of potential reach to 1 MeV.

Figure 5 represent to the potential for ^{226}Th, which clear there is asymmetry in both sides and the potential gathered in $\beta < 1$, reach to 800 KeV, but the last Figure 6 for ^{228}Th, refers to asymmetry in both sides and unequal distribution in contour lines with less potential energy than the previous reach to 240 MeV because there is stability in this isotope near the closed shell 126.

Table 3. Some electric transitions B(E2) for ^{224}Th.

$J_i \rightarrow J_f$	B(E2) e^2b^2	
	Exp.	**IBM-1**
$2_1^+ \rightarrow 0_1^+$	0.77614	0.7643948
$2_1^+ \rightarrow 0_2^+$	-	0.0703174
$3_1^+ \rightarrow 2_1^+$	-	0.8552228
$3_1^+ \rightarrow 2_2^+$	-	0.7218213
$3_2^+ \rightarrow 2_1^+$	-	0.0016745
$3_3^+ \rightarrow 2_1^+$	-	0.0008739
$4_1^+ \rightarrow 2_1^+$	-	1.0741140
$4_1^+ \rightarrow 2_2^+$	-	0.0249889
$6_1^+ \rightarrow 4_1^+$	-	1.0869880
$6_1^+ \rightarrow 4_2^+$	-	0.0542134

Table 4. Some electric transitions B(E2) for ^{226}Th.

$J_i \rightarrow J_f$	B(E2) e^2b^2	
	Exp.	**IBM-1**
$2_1^+ \rightarrow 0_1^+$	1.341716	1.7095460
$2_1^+ \rightarrow 0_2^+$	-	0.1313972
$3_1^+ \rightarrow 2_1^+$	-	0.0161234
$3_1^+ \rightarrow 2_2^+$	-	0.1522753
$3_2^+ \rightarrow 2_1^+$	-	0.0024018
$4_1^+ \rightarrow 2_1^+$	-	2.4107670
$4_1^+ \rightarrow 2_2^+$	-	0.0330782
$6_1^+ \rightarrow 4_1^+$	-	2.4773540
$6_1^+ \rightarrow 4_2^+$	-	0.0778072

Table 5. Some electric transitions B(E2) for ^{228}Th.

$J_i \rightarrow J_f$	B(E2) e^2b^2	
	Exp.	**IBM-1**
$2_1^+ \rightarrow 0_1^+$	1.382405	1.4301790
$2_1^+ \rightarrow 0_2^+$	-	0.0000000
$3_1^+ \rightarrow 2_1^+$	-	0.0000000
$3_1^+ \rightarrow 2_2^+$	-	0.0116564
$3_2^+ \rightarrow 2_1^+$	-	0.0000002
$4_1^+ \rightarrow 2_1^+$	-	0.0000000
$4_1^+ \rightarrow 2_2^+$	-	0.0002252
$6_1^+ \rightarrow 4_1^+$	-	0.0007039
$6_1^+ \rightarrow 4_2^+$	-	0.0000000

Table 6. The coefficients of potential energy in MeV unit.

The Isotopes	ϵ_s	ϵ_d	$A1$	$A2$	$A3$	$A4$
^{224}Th	−0.0510	0.0390	0.0000	−0.0040	−0.0410	0.0000
^{226}Th	−0.0340	0.0420	0.0000	−0.0030	−0.0280	0.0000
^{228}Th	−0.1210	0.2430	0.0570	−0.0250	−0.0970	0.0000

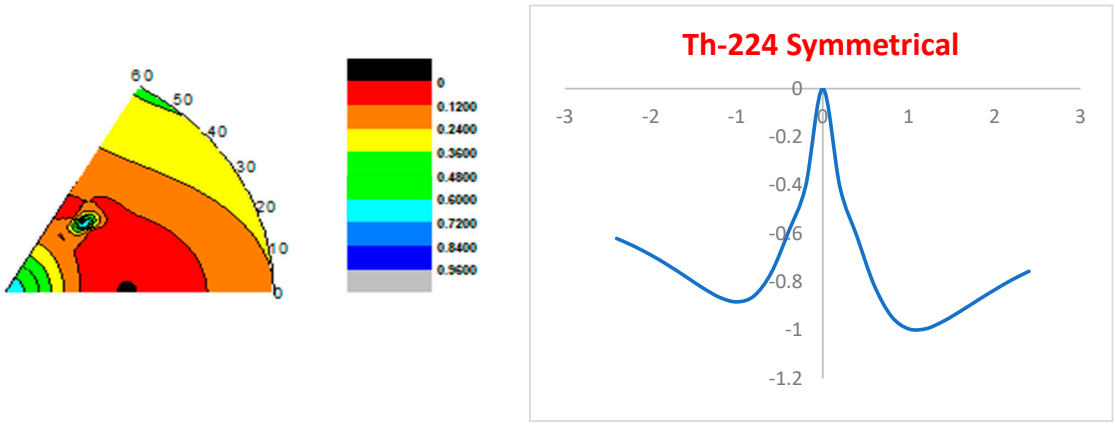


Figure 4. Symmetric shape and potential distribution for ^{224}Th.

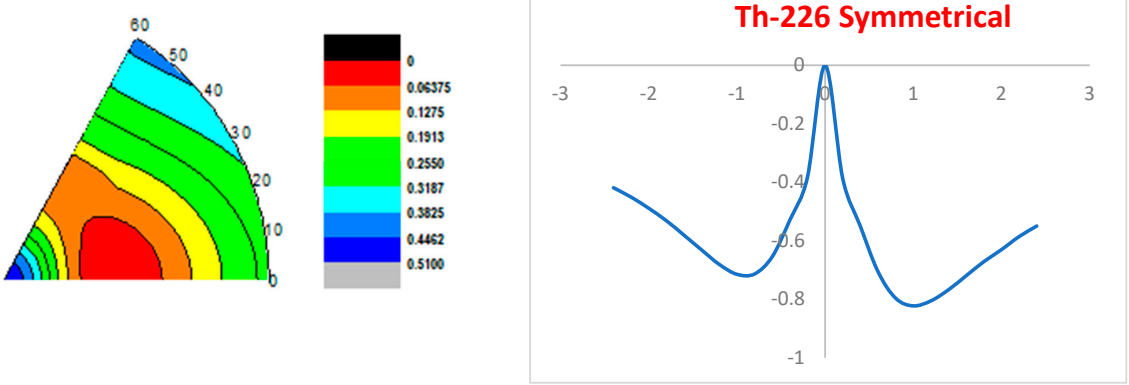


Figure 5. Symmetric shape and potential distribution for ^{226}Th.

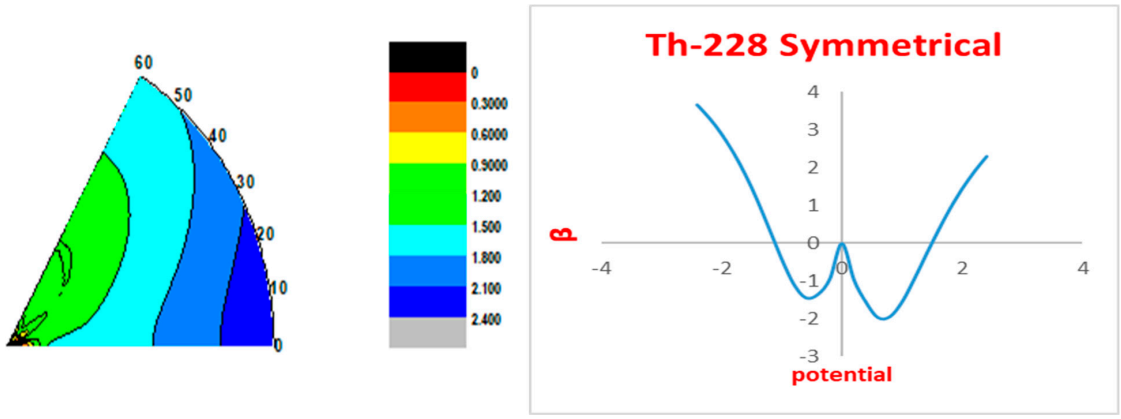


Figure 6. Symmetric shape and potential distribution for ^{228}Th.

6. Conclusions

The estimating numerous energy levels for (^{224}Th, ^{226}Th, and ^{228}Th) is close to the data sheet obtained. Uncertain and unknown levels were estimated from the results using IBM-1 program with a good agreement.

Well calculated B(E2) values and fit with experimental findings data. These electric transitions used to estimate the limit of isotope. However there are differences between them because of how these isotopes' nuclei were deformed.

^{224}Th and ^{226}Th are in SU(3) and O(6) region as expected from the results, and there is deformation in the surface, but SU(3) limit for ^{228}Th.

There is decrease in the deformation of the nuclei within increase in boson number and near the closed shell.

Author Contributions: Methodology and writing—original draft preparation, N.H.H.E.; validation, M.K.M.A.-J.; formal analysis, G.A.H.J.H.; investigation, M.K.M.A.-J.; resources, G.A.H.J.H.; data curation, M.K.M.A.-J.; writing—review and editing, G.A.H.J.H.; visualization, M.K.M.A.-J.; supervision, G.A.H.J.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All the data used in the experiment have been made available in the present article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Arima, A.; Iachello, F. *The Interacting Boson Model*; Cambridge University Press: Cambridge, UK, 1987.
2. Arima, A.; Iachello, F. The Interacting Boson Model. *Annu. Rev. Nucl. Part. Sci.* **1981**, *31*, 75–105. [CrossRef]
3. Al-Khudair, F.H.; Subber, A.R.; Jaafer, A.F. Shape transition and triaxial interaction effect in the structure of $^{152–166}$Dy isotopes. *Commun. Theor. Phys.* **2014**, *62*, 847–858. [CrossRef]
4. Jaber, G.A.H.; Muttaleb, M.K. Studying the Isotopes nearby Closed Shell of Kr, Xe and Hg Using Interacting Boson Models. *J. Univ. Babylon Pure Appl. Sci.* **2020**, *28*, 173–183.
5. Hady, H.N. Transitions symmetries shape in $^{176–196}$ Pt isotopes with interacting boson mode. *J. Univ. Babylon* **2013**, *21*, 2162–2171.
6. Yazar, H.R.; Uluer, I. Negative parity states and some electromagnetic transition properties of even-odd erbium isotopes. *Phys. Rev. C* **2007**, *75*, 034309. [CrossRef]
7. Hossain, I.M.; Sharrad, A.F.; Abdullah, H.Y.; Salman, A.D.; Al-Dahan, N. Yrast states and B(E2) values of even $^{100–102}$Ru isotopes using interacting boson model (IBM-1). *Chiang Mai J. Sci.* **2015**, *42*, 996–1004.
8. Frankfurt, L.; Strikman, M. Hard nuclear processes and microscopic nuclear structure. *Phys. Rep.* **1988**, *160*, 235–427. [CrossRef]
9. Motleb, M.C. A Study of Nuclear Structure of $^{98–108}$Ru Even-Even Isotops by the Interacting Boson Model-1. Master's Thesis, University of Babylon, Hilla, Iraq, 2005.
10. Hammad, M.M.; Fawaz, S.M.; El-Hammamy, M.N.; Motaweh, H.A.; Doma, S.B. q-deformed vibrational limit of interacting boson model. *J. Phys. Commun.* **2019**, *3*, 085019. [CrossRef]
11. Sharrad, F.I. Determination of The $^{108–112}$Pd Isotopes Identity Using Interacting Boson Model. *Nucl. Phys. At. Energy* **2017**, *18*, 313–318.
12. Proskurins, J. *The Study of Nuclear Shape Phase Transitions and Quantum Chaos in The Frameworks of Geometrical and Algebraic Models of Even-Even Nuclei*; University of Latvia: Riga, Latvia, 2010.
13. Casten, R.F.; Warner, D.D. The Interacting Boson Approximation. *Rev. Mod. Phys.* **1988**, *60*, 389–406. [CrossRef]
14. Casten, R.; Casten, R.F. *Nuclear Structure from a Simple Perspective*; Oxford University Press on Demand: Oxford, UK, 2000; Volume 23.
15. Subber, A.; Hamilton, W.D.; Park, P.; Kumar, K. An Application of The Dynamic Deformation Model to The Tellurium Isotopes. *J. Phys. G* **1987**, *13*, 161–175. [CrossRef]
16. Pfeifer, W. An introduction to the interacting boson model of the atomic nucleus. *arXiv* **1998**, arXiv:nucl-th/0209039.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.