# Supplement Material

*Table S1: Estimated model parameters for SETAR(k,p,d) model US GDP 1790-2018 using LLR and HP filter for the residuals*

| Gap est. | Model | LR<br>HR | $\hat{\phi}_0$ | $\hat{\phi}_1$ | $\hat{\phi}_2$ | $\hat{\phi}_3$ | $\hat{\sigma}^2$ | AIC |
|---|---|---|---|---|---|---|---|---|
| LLR | SETAR (2,1,0) | 44.05%<br>55.95% | 0.0065<br>-0.0011 | 0.9235***<br>0.8445*** | | | 0.0016 | -1467 |
| | SETAR (2,2,0) | 44.05%<br>55.95% | 0.0094*<br>-0.0029 | 1.2527***<br>1.2254*** | -0.3454***<br>-0.3334*** | | 0.0014 | -1502 |
| | SETAR (2,3,0) | 43.81%<br>56.19% | 0.0093*<br>-0.0017 | 1.2642***<br>1.1214*** | -0.3913***<br>-0.1902 | 0.0397<br>-0.2012** | 0.0013 | -1503 |
| HP | SETAR (2,1,0) | 47.58%<br>52.42% | -0.0011<br>0.0017 | 0.3345***<br>0.3032* | | | 0.0007 | -1662 |
| | SETAR (2,2,0) | 47.58%<br>52.42% | -0.0027<br>0.0008 | 0.3874**<br>0.5088*** | -0.3356***<br>-0.4115*** | | 0.0006 | -1692 |
| | SETAR (2,3,0) | 47.35%<br>52.65% | -0.0030<br>0.0010 | 0.3338**<br>0.4134** | -0.2902**<br>-0.1551 | -0.1069**<br>-0.4764*** | 0.0005 | -1715 |

Notes: P-values, *P(z)<0.1, **P(z)<0.05, ***P(z)<0.01.

Table S2 shows that in the SETAR (2,1,0) model the first coefficient of the HR is significant when using the LLR and insignificant when using the HP filter. This result is robust with respect to other different SETAR specifications. However, this information is only provided using the LLR, while in the HP case the results are inconclusive. In other words, central banks could be misguided by the HP filter and may adjust too quickly to potential "new" economic conditions, which are only implied by the quick adjustment of the HP filter. Those could be interpreted as spurious results (Cogley and Nason, 1995), where the method produces a larger cyclical component than originally present in GDP data. Using a test idea of Fritz et al. (2019a) for trend-stationary vs. difference stationary models, we show that for the GDP data the LLR does not produce spurious results. Similar to the SETAR coefficients displayed in Table S1 for annual data, the semi-SETAR forecast smoothly evolves over time. That is, the output gap is slightly positive in 2017.2 but very close to zero. The forecast points into the direction of a positive gap whose magnitude increases. However, after three quarters comes a turning point and the gap starts to decrease.

*Table S2: Estimated model parameters for SETAR(k,p,d) model US GDP 1790-2018 using LLR and HP filter for the residual growth rates*

| Gap est. | Model | LR<br>HR | $\hat{\phi}_0$ | $\hat{\phi}_1$ | $\hat{\phi}_2$ | $\hat{\phi}_3$ | $\hat{\sigma}^2$ | AIC |
|---|---|---|---|---|---|---|---|---|
| LLR | SETAR<br>(2,1,0) | 49.56%<br>50.44% | -0.0049<br>-0.0041 | 0.1272<br>0.4466*** | | | 0.0016 | -1463 |
| | SETAR<br>(2,2,0) | 49.56%<br>50.44% | -0.0048<br>-0.0043 | 0.1595<br>0.4624*** | -0.1143<br>-0.0430 | | 0.0016 | -1461 |
| | SETAR<br>(2,3,0) | 49.78%<br>50.22% | -0.0043<br>-0.0045 | 0.1614<br>0.4553** | -0.0791<br>-0.0225 | -0.1303<br>-0.0659 | 0.0015 | -1460 |
| HP | SETAR<br>(2,1,0) | 47.79%<br>52.31% | -0.0031<br>0.0023 | -0.1525<br>0.1171 | | | 0.0010 | -1567 |
| | SETAR<br>(2,2,0) | 47.79%<br>52.31% | -0.0032<br>0.0030 | -0.1645<br>0.1586 | -0.2765***<br>-0.2123** | | 0.0009 | -1578 |
| | SETAR<br>(2,3,0) | 47.56%<br>52.44% | -0.0020<br>0.0035 | -0.1896<br>-0.2297* | -0.3147***<br>-0.2233** | -0.2761***<br>-0.1860** | 0.0009 | -1586 |

Notes: P-values, *P(z)<0.1, **P(z)<0.05, ***P(z)<0.01.

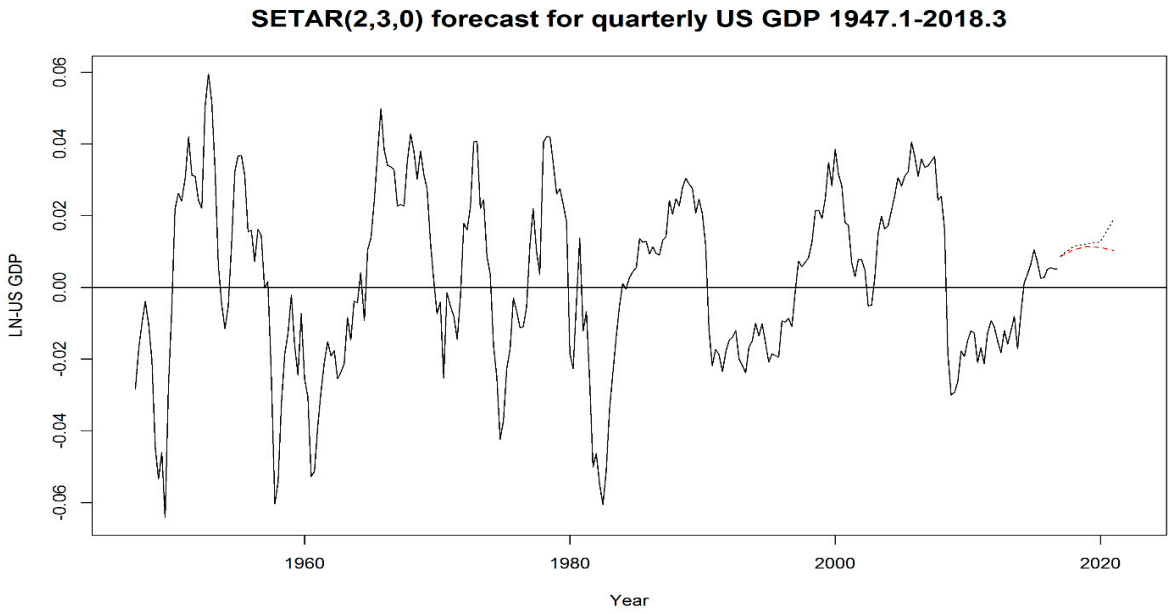


**Figure S1: SETAR(2,3,0) point forecast of the output gap (red dashed) together with the original data (black dotted) for 2017.3-2018.4 using the LLR for quarterly US GDP (black solid) 1947.1-2018.3.**