



Proceeding Paper Aluminum Beams with Composite Thermal Barriers: Recent Developments in Analysis (Part 2) ⁺

James LaBelle

SB Engineering, LLC, Wauwatosa, WI 53222, USA; jlabelle.sbengr@gmail.com [†] Presented at the 15th International Aluminium Conference, Québec, QC, Canada, 11–13 October 2023.

Abstract: This is Part 2 of an article about analysis developments for aluminum-thermal barrier beams. It includes: (1) *Effective shear modulus*. An FEA study determined the effective shear moduli of the cores of simply supported beams. These beams (tested by others) had glass-fiber-reinforced nylon (polyamide) cores, short overhangs, and two symmetrical concentrated loads. Effective moments of inertia, effective section moduli, and shear flows are presented. (2) *Bending stiffness*. Bending stiffnesses based on a U.S. method are compared to those based on a European method. Two profiles, two shear moduli, and three spans were studied. Results are very similar if face shear deformation is not included.

Keywords: aluminum; thermal barrier; sandwich beam; curtainwall; shear deformation

1. Introduction

As mentioned in Part 1, thermal-barrier beams are used in windows and curtain walls to resist wind loads and reduce energy loss and condensation. These composite beams consist of extruded aluminum sections joined by a structural insulating material. The connection between the thermal barrier (core) and the facing sections resists longitudinal shear, thus enabling the composite behavior of these sandwich beams. Because the core is much less stiff than the aluminum faces, core shear deformation makes these beams more flexible.

Currently, there are two commonly used structural thermal barriers: polyurethane (poured-and-debridged) and glass-fiber-reinforced polyamide (nylon) strips.

Prior research [1,2] resulted in the development of closed-form equations to predict the linear response of this type of composite sandwich beam (simply supported) to four types of symmetrical loading: midspan concentrated, uniform, triangular, and trapezoidal. These four types represent common cases for fenestration products subject to test loading and wind loading. Other more general loading situations, as well as continuous beams and beams with overhangs, can be analyzed with certain types of finite-element software.

The developments presented in Part 2 include an analytical determination of effective core shear moduli for beams (tested by others [3]) with polyamide cores and two symmetric concentrated loads and a comparative study of predicted effective moments of inertia using two methods (U.S. and European).

2. Effective Shear Moduli of Certain Test Beams

A study was made of test results, reported in Huang [3], to analytically determine the effective shear modulus (G_{ce}) for each of the four test geometries (spans). The study focused on the linear portion of the load-deflection plots. These composite aluminum beams had thermal barriers consisting of glass-fiber-reinforced nylon (polyamide).

A discussion of various topics follows below. All dissertation units are SI (metric).



Citation: LaBelle, J. Aluminum Beams with Composite Thermal Barriers: Recent Developments in Analysis (Part 2). *Eng. Proc.* **2023**, *43*, 34. https://doi.org/10.3390/ engproc2023043034

Academic Editor: Mario Fafard

Published: 20 September 2023



Copyright: © 2023 by the author. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/).

2.1. Core Shear Stiffness: Conversion of European Parameter to U.S. Parameter

Using a 100 mm (3.94 in) long specimen, the European method measures the longitudinal deflection of one face relative to the other due to a longitudinal load applied to one face. The parameter *c* equals force (*V*) divided by displacement (δ), divided by sample length (*L*). See Equation (1). Units are N/mm² or lbs/in². Thus, *c* is a measure of the core's longitudinal shear stiffness per unit length. (This meaning of *c* differs from that in AAMA TIR-A8, [2] (pp. 43–61)).

$$c = \frac{V}{\delta L} \tag{1}$$

This can be converted to an effective shear modulus (G_{ce}). Note that shear stress (τ) equals shear modulus (G_c) times shear strain (γ , which is measured in radians). Let D_c = maximum depth of the thermal barrier (see [2]), b = its total average thickness, the longitudinal area = A_L = b L, and $\gamma = \delta/D_c$. Thus:

$$\tau = G_c \gamma = \frac{V}{A_L} = \frac{V}{b L} = \frac{G_c \delta}{D_c}.$$
 (2)

Re-arranging:

$$\frac{V}{\delta L} = \frac{G_c b}{D_c} = c.$$
(3)

Thus, effective $G_c = G_{ce} = c (D_c/b)$.

2.2. Core Shear Stiffness: Discussion

Test results [3] are discussed below.

2.2.1. Strip (Thermal Barrier) Properties

Refer to the dissertation's Appendix in [3] (p. 262) for the width, thickness, and other properties of tested strips in the not-installed condition. The temperature is given as 23 °C (73.4 °F) and the relative humidity as 50%. As tested, the strip cross-section dimensions were: width = 5.18 mm (0.204 in) and thickness = 2.02 mm (0.080 in).

The dissertation [3] (p. 79) indicates that 25% glass fiber-reinforced polyamide (PA66) was used, where PA66 is nylon 6/6. Testing of strips, per an ISO standard, showed E_{cL} = 4477 MPa (649.3 ksi) in the longitudinal direction, which is stated to be the fiber direction. The transverse modulus E_{cT} is reported [3] (p. 80) to be 2681 MPa (388.8 ksi), per the manufacturer. This matches the Appendix value in [3] (p. 262).

2.2.2. Core Shear Stiffness (c Converted to G_{ce}) of Longitudinally Loaded Specimens

See dissertation [3] (pp. 44–45) for plots of test results for core shear stiffness for strips installed in facing sections. The moisture condition of the strips (e.g., dry, equilibrium at 50% relative humidity, or at maximum absorption) was not found in the dissertation. The present focus is on room temperature values because the beam tests were at room temperature. The ten composite specimens were each 100 mm (3.94 in) long. Each strip's thickness (*t*) was 2.02 mm (0.0795 in) at the thin region. Strip depth (D_c) was 24 mm (0.945 in), and the total area of the two strips was 119.7 mm² (0.1855 in²).

Refer to Table 1 for the test range and average of c, as well as the calculated range and average G_{ce} for minimum and average strip widths. The average total core width (b), which is the sum of the strips' average thicknesses, is to be used in the analysis method in [2].

Note that the G_p parameter in [2] (p. 50) has *b* and G_{ce} in the numerator. Refer also to Part 1. Thus, if the product of *b* and G_{ce} is constant, as is true here (within rounding error), then a higher value of G_c paired with a lower (but linearly related) value of *b* would produce the same beam deflection as a lower G_{ce} paired with a higher *b*.

| Test Range c [N/mm ²] (ksi) | Average c [N/mm ²] (ksi) | t [mm] (in) | Computed Range G _{ce} [MPa] (ksi) | Average G _{ce} [MPa] (ksi) | |
|--|---|------------------------------|---|---|--|
| 24 to 46 (3.5 to 6.7) | 33 | 2.02 (0.0795) min. | 140 to 270 (21 to 40) | 200 (28) | |
| | (4.8) | 2.494 (0.0982) average | 120 to 220 (17 to 32) | Average G _{ce} [MPa] (ksi) 200 (28) 160 (23) | |

Table 1. Core Shear Stiffness (*c* and G_{ce}).

For comparison, using the load-displacement plot [3] (p. 45; core shear stiffness), the approximate average *c* equals $(3.5 - 1.5)/\{(1 - 0.5)(100)\} = 0.040 \text{ kN/mm/mm} = 40 \text{ MPa}$ (5.8 ksi). This is equivalent to an approximate average $G_{ce} = 190 \text{ MPa}$ (28 ksi). Both values (*c* and G_{ce}) are less accurate than the averages based on the plot in [3] (p. 44).

2.2.3. Ratio of G_c to E_c

It is interesting to compare the transverse modulus of elasticity (Young's modulus for tension = E_{cT}) of 2681 MPa (389 ksi) for an un-installed strip to the approximate G_{ce} of 120 to 220 MPa (17 to 32 ksi) for a pair of installed strips. The effective shear modulus G_{ce} is only about 4% to 8% of E_{cT} . In contrast, for homogeneous, isotropic solids, $G = E/\{2(1 + \nu)\}$, which theoretically varies from one-half to one-third of *E* for Poisson's ratio (ν) between 0 and 0.5.

However, the strips are neither homogeneous nor isotropic. The ratio of transverse to longitudinal moduli is 2681/4477, which equals 0.60. To the extent that the fibers are randomly oriented (i.e., some fibers' lengths are oriented between 0° and 90° from the longitudinal direction), the composite material might be approximated for shear distortion as very roughly homogeneous and isotropic on a sufficiently large scale, such as the clear distance between aluminum facing sections. Perhaps in this region of the strips between faces, the shear deformation approaches that of a homogeneous and isotropic material with a minimum E_c of 2681 MPa (389 ksi) and a G_c between 894 and 1340 MPa (130 ksi and 194 ksi)—i.e., 33% and 50% of E_c .

As a further conjecture, there is a significant probability that on a small scale, there is relatively large elastic deformation of the strips at the bearing interface between the aluminum "teeth" (indentations) and the strips due to resisting longitudinal shear force. If sufficiently large, such local deformation could result in an overall G_{ce} of the installed strips that corresponds to approximately 4% to 8% of the tested E_{cT} . In other words, it seems that most of the longitudinal displacement probably occurred at the strips' edges.

The present author is aware of torsion testing, of some un-installed strips, that indicated a G_c at room temperature of about 2275 MPa (330 ksi), for the more flexible of two orientations tested. The value for the other orientation was a few percent higher. However, details of the strips (e.g., E_{cL} , E_{cT} , and moisture absorption) were not given. Clearly, 2275 MPa (330 ksi) greatly exceeds the range of 115 to 221 MPa (17 to 32 ksi) for G_{ce} for installed strips, based on the tests [3] for *c*. Further research is needed to gain a better understanding of this behavioral aspect of installed strips.

2.3. Huang Dissertation [3]: Load-Deflection Plots for Beams (Mullions)

A study by the present author was done to iteratively determine G_{ce} based on several load-deflection plots for room temperature testing. Refer to [3] (p. 69), which has one plot per span (total of four spans). Because there is no table of measured deflection values in the dissertation, approximate values of load at a particular deflection were scaled from the linear portion of the enlarged plots. This procedure provided useful data but reduced accuracy somewhat. Using TB-FEM [4], several cycles of iteration were needed to obtain

approximate values of G_{ce} . Because TB-FEM does not include the shear stiffness of the aluminum facing sections, separate calculations were made to find the approximate portion of the overall deflection that was due only to face shear deformation.

One plot was considered for each of the four tested spans of 2.12 m, 2.62 m, 3.12 m, and 3.64 m (6.96 ft, 8.60 ft, 10.24 ft, and 11.94 ft). These are beams B, C, D, and E, respectively. No nominal thicknesses, nor measurements of actual thicknesses, of the beam's cross-section were given. This also limits the accuracy of the TB-FEM analyses. The test profiles had details (e.g., screw races and gasket-retention slots) that are common for a mullion intended for use in a curtain wall. The dimensions shown for the cross-section (see [3] (p. 36)) appear to be nominal. The beam specimens' overall cross-section dimensions were 179 mm (7.05 in) deep by 90 mm (3.54 in) wide. Span-to-depth ratios ranged from 11.8 to 20.3.

The lower facing-section was approximately a rectangular tube that was about 143 mm deep by 90 mm wide (5.63 in by 3.54 in). The other face was an open profile that was estimated to be 18 mm deep by 50 mm wide (0.71 in by 1.97 in). The polyamide strips were 24 mm (0.9449 in) wide (i.e., core depth = D_c = 24 mm).

Loading consisted of two symmetric concentrated loads (each equal to *P*) on the span, one at the one-third point and the other at the two-thirds point. A couple of TB-FEM runs with 24 elements in the span gave the same deflection as with 48 elements, so 24 elements in the span were used for the rest of the models.

Each overhang was 180 mm (7.09 in) long, as tested, and was included in the final computer models. The overhangs did stiffen the span somewhat (the deflection for the shortest span was reduced by 3.2%), relative to a trial run without overhangs. Using a preliminary approximate scaling of the load-deflection plot, the inclusion of the overhangs for the shortest span resulted in $G_{ce} = 110$ MPa (16 ksi) as compared to 141 MPa (20.5 ksi) with no overhangs. Subsequently, for the next phase of the study, an excerpt from an enlarged plot was scaled more accurately. This resulted in a lower G_{ce} for overhangs included (see Table 2).

| Beam | Span [m] (ft) | Measured Deflection * [mm] (in) | Load ** <i>P</i> [kN] (lbs) | δ _{FS} [mm] (in) | δ _{FEM} [mm] (in) | G _{ce} [MPa] (ksi) |
|------|---------------------|--|--------------------------------------|------------------------------|----------------------------------|-----------------------------------|
| В | 2.12 | 10.00 | 8.60 | 0.354 | 9.647 | 75.0 |
| | (6.96) | (0.394) | (1933) | (0.014) | (0.380) | (10.9) |
| С | 2.62 | 10.00 | 5.49 | 0.279 | 9.723 | 223 |
| | (8.60) | (0.394) | (1214) | (0.011) | (0.383) | (32.3) |
| D | 3.12 | 10.00 | 3.49 | 0.212 | 9.787 | 344 |
| | (10.24) | (0.394) | (785) | (0.0083) | (0.385) | (49.9) |
| E | 3.64 | 10.00 | 2.21 | 0.156 | 9.844 | 255 |
| | (11.94) | (0.394) | (497) | (0.0062) | (0.388) | (37.0) |

Table 2. Effective Shear Moduli (*G*_{*ce*}).

* At midspan. See load-deflection plot at [3] (p. 69); ** Loads were read to the nearest 0.01 kN (2.25 lbs) on the plot; δ_{FS} = calculated deflection due to face shear (approximate); δ_{FEM} = [deflection from TB-FEM using G_{ce}] \approx (measured deflection – δ_{FS}).

Most of the cross-section properties of each facing section are given in the dissertation [3] (p. 251). Scaling was used to approximate the dimension from each face's centroid to its core-side extreme fiber. Areas for upper and lower faces, respectively, are 364.11 mm² and 1225.67 mm² (0.5644 in² and 1.8998 in²).

Also listed are the moments of inertia (second area moments) for each face with respect to the neutral axis of the entire composite section. These were converted to I_{o1} and I_{o2} about each face's centroidal axis: 8258 mm⁴ and 3.562×10^6 mm⁴ (0.0198 in⁴ and 8.5574 in⁴). The core area (total of both strips) is 119.7 mm² (0.1855 in²). For a core height (D_c) equal to 24 mm (0.9449 in), this means the average total width is 4.988 mm (0.1964 in). These values were used to find G_{ce} given in Table 2. Two extra analyses (of beams B and E) were run to verify that keeping the product of G_{ce} and total strip width (*b*) constant resulted in the same value of predicted deflection (δ_{FEM}). The input used a higher G_{ce} and a lower *b* (4.04 mm (0.1591 in) = total thickness away from strips' thickened edges). For example, for beam E: (255 N/mm²) (4.988 mm) = 1272 N/mm = (314.8 N/mm²) (4.04 mm). Equivalently: (36.98 ksi) (0.1964 in) = 7.264 kip/in = (45.66 ksi) (0.1591 in).

The required properties were input into TB-FEM [4], using a procedure similar to that in AAMA TIR-A8 [2]. The input value of G_{ce} was iterated until the sum of the face-shear deflection (δ_{FS} ; the large face was assumed to account for all face-shear deflection) and predicted maximum deflection (δ_{FEM}) closely equaled a plot-based deflection (10 mm (0.394 in)). Results are given in Table 2.

The range of G_{ce} values in Table 2 is 75 to 344 MPa (10.9 to 49.9 ksi), and the average is 224 MPa (32.5 ksi). These may be compared with results given previously and based on shear-stiffness tests of short specimens, for which the approximate G_{ce} range was 120 to 220 MPa (17 to 32 ksi) and the average G_{ce} was 160 MPa (23 ksi).

For no shear deformation of the core and faces, the upper bound I of the composite section is $6.050 \times 10^6 \text{ mm}^4$ (14.54 in⁴). The calculated values of I_e (for core-shear deformation only; face-shear deflection subtracted from 10 mm) and I'_e (which includes shear deformation of the core and faces) are given in Table 3; see Part 1 at Equations (8) to (10). These are based on the test loads at 10 mm (0.394 in) deflection, including overhangs' effect. Thus, I_e and I'_e would be approximate, but close, for the same beams without overhangs.

| Beam | Span [m] (ft) | I [mm ⁴] (in ⁴) | <i>I</i> e [mm ⁴] (in ⁴) | <i>I'e</i> [mm ⁴] (in ⁴) | I _e /I | I'e/I |
|------|---------------------|---|--|--|-------------------|-------|
| В | 2.12 (6.96) | $6.05 	imes 10^{6} \ (14.54)$ | $4.37 	imes 10^6$ (10.51) | $4.22 	imes 10^{6}$ (10.13) | 0.723 | 0.697 |
| С | 2.62 (8.60) | $6.05 	imes 10^{6} \ (14.54)$ | 5.23×10^{6} (12.56) | $5.08 	imes 10^{6}$ (12.21) | 0.864 | 0.840 |
| D | 3.12 (10.24) | $6.05 	imes 10^{6}$ (14.54) | 5.57×10^{6} (13.39) | $5.46 	imes 10^{6}$ (13.11) | 0.921 | 0.902 |
| Е | 3.64 (11.94) | $6.05 	imes 10^{6}$ (14.54) | 5.57×10^{6} (13.39) | $5.49 	imes 10^{6}$ (13.18) | 0.921 | 0.907 |

Table 3. Moment of Inertia (*I*) and Effective Moments of Inertia (I_e and I'_e).

2.3.1. Reduced Face-Properties

A limited study was made of reduced face properties due to tolerance on aluminum thickness. If the thicknesses were 5% less than nominal, the effects can be approximated by considering 95% of the faces' areas and moments of inertia. For beam B (shortest span = 2.12 m (6.96 ft)), this would result in a G_{ce} of about 107 MPa (15.5 ksi), versus 75.2 (10.9 ksi) for nominal thicknesses. For beam C (2.62 m (8.6 ft) span), reduced face thicknesses would result in a G_{ce} of about 379 MPa (55.0 ksi), versus 223 MPa (32.3 ksi) for nominal thicknesses.

2.3.2. Huang Dissertation [3]: Additional Beam Load-Deflection Plots

Each of the figures in [3] (pp. 159–161) presents three test plots—one for each of the three beam specimens tested at each span length at room temperature. These load-deflection plots, when enlarged, show that there is some variation in stiffness (in the linear region) within each set of three nominally identical specimens.

- For set B, scaling indicates about 4% variation in load (the highest is 4% larger than the smallest) at 10 mm deflection; two of the plots are almost identical at 10 mm;
- At C, variation is about 7%, but the other two plots are almost identical at 10 mm.
- At D, it is about 6%, with the other two plots almost identical at 10 mm;

• At E, it is about 5%, with the other two plots almost identical at 10 mm.

2.4. Stresses Calculated Using TB-FEM Model of Beams

To determine stresses, the effective shear moduli (G_{ce}) in Table 2 were used, which were based on scaled loads from [3] (p. 69). The self-weight of the beam was not included in the loading. For each beam type, Table 4 lists the load (P, at each third point), maximum moment (M), maximum tension stress (f_{TBFEM}), and effective section modulus ($S_{e2} = M/f_{TBFEM}$).

| Beam | Span [m] (ft) | Load P [kN] (lbs) | Moment <i>M</i> [m-kN] (in-kip) | <i>f_{TBFEM}</i> [MPa] (ksi) | S _{e2} [mm ³] (in ³) | S ₂ [mm ³] (in ³) |
|------|---------------------|----------------------------|--|--|---|--|
| В | 2.12 | 8.60 | 6.077 | 114.5 | 53,100 | 63,030 |
| | (6.96) | (1933) | (53.79) | (16.61) | (3.24) | (3.846) |
| С | 2.62 | 5.49 | 4.795 | 83.21 | 57,600 | 63,030 |
| | (8.60) | (1214) | (42.44) | (12.07) | (3.52) | (3.846) |
| D | 3.12 | 3.49 | 3.630 | 61.15 | 59,400 | 63,030 |
| | (10.24) | (785) | (32.12) | (8.869) | (3.62) | (3.846) |
| E | 3.64 | 2.21 | 2.681 | 45.17 | 59,400 | 63,030 |
| | (11.94) | (497) | (23.73) | (6.551) | (3.62) | (3.846) |

Table 4. Moments and Bending Stresses.

The larger distance from the composite section's centroid (h_2) is 95.99 mm (3.779 in). The upper bound on the moment of inertia is $I = 6.050 \times 10^6$ mm⁴ (14.535 in⁴), and the associated section modulus $S_2 = I/h_2 = 63,030$ mm³ (3.846 in³). Therefore, for all four of the beams, $84\% \leq S_{c2}/S_2 \leq 94\%$.

For each beam, the stress in Table 4 is the largest absolute value of all stresses calculated for that beam. The table stresses are all tensile. No tensile coupon results were found in [3], but 6063-T6 was used in [3] (p. 35). The minimum yield for this alloy-temper is 172 MPa (25 ksi), so no yielding would be expected. This is consistent with the linear shape of the load-deflection plots below and near the 10 mm (0.394 in) deflection.

Stress Due to Self-Weight

The total cross-section area of the aluminum faces is 1589.8 mm² (2.464 in²), which equates to a uniform load of about 42.8 N/m (2.93 lbs/ft) if strips are included. For a simply supported beam (E), without overhangs, that has a 3.64 m (11.94 ft) span, the bending moment (M_D) is 70.8 kN-mm (627 in-lbs). An upper bound on stress due to self-weight is obtained if the lower face is assumed to carry all the weight: $f_{bD} = 70,800 (74.46)/(3.562 \times 10^6)$, which is 1.48 MPa (0.215 ksi). The stress would be less for shorter spans.

Somewhat more accurately, one could consider G_{ce} to be very large and use the upper bound on section modulus (S_2). Thus:

$$f_{bD} = M_D / S_2 = 70,800 / 63,030 = 1.12 \text{ MPa} (0.163 \text{ ksi})$$
 (4)

Using software (TBSA; see [2]), the maximum stress is 1.16 MPa (0.169 ksi) due to the simply supported beam's self-weight. The associated effective section modulus (S_{e2} , at the bottom of the lower face) is 60,650 mm³ (3.701 in³). TBSA is software that comes with TIR-A8 [2] and implements the closed-form equations for four load cases for simply supported beams without overhangs.

2.5. Shear Flow

The maximum calculated shear flow for each beam type due to load (P = V) is given in Table 5. The effective shear moduli (G_{ce}) in Table 2 were used. The shear flow per TB- FEM [4] is q_{TBFEM} and the upper bound is q, equal to VQ/I. The average longitudinal shear strength ([3] (p. 43)), based on tests of short specimens, was about 43 N/mm (245 lbs/in). The characteristic value, which is a statistical "lower bound" based on a European standard, was approximately 38 N/mm (217 lbs/in). Both the average and characteristic values are substantially greater than the largest value of q_{TBFEM} , equal to 15.5 N/mm (89 lbs/in) in Table 5. The imposed loads (*P*) are associated with a beam deflection of 10 mm (0.394 in), which is well within the linear portion of the load-deflection plots.

| Beam | Span [m] (ft) | Shear V = P [kN] (lbs) | 9твғем [N/mm] (lbs/in) | <i>q</i> [N/mm] (lbs/in) | 9tbfem/q |
|------|---------------------|---------------------------------|------------------------------|--------------------------------|----------|
| В | 2.12 (6.96) | 8.60 (1933) | 13.5 (77.3) | 37.50 (214.1) | 0.361 |
| С | 2.62 (8.60) | 5.49 (1214) | 15.5 (88.8) | 23.94 (136.7) | 0.650 |
| D | 3.12 (10.24) | 3.49 (785) | 11.8 (67.5) | 15.22 (86.9) | 0.777 |
| E | 3.64 (11.94) | 2.21 (497) | 7.59 (43.3) | 9.64 (55.0) | 0.788 |

Table 5. Shear and Shear Flow.

The ratio of q_{TBFEM} to q varies from 36% to 79%. The percentage increases as the span increases. This is usual for one value of G_{ce} and it also occurred here, despite the variation in G_{ce} values among spans.

Based on the load-deflection plots ([3] (pp. 159–161)) for beams, longitudinal shear strength (slip load) during testing was reached in the approximate range of 10 kN $\leq P \leq$ 15 kN (2.2 kips $\leq P \leq$ 3.4 kips) for beams B to E. For each beam, the slip load significantly exceeded the corresponding load in Table 5.

3. Comparison of Two Methods for Determining Effective Moment of Inertia

The deflection of a composite aluminum-thermal barrier beam exceeds that based on the moment of inertia (I), as for a similar all-aluminum beam. To account for this, an effective moment of inertia ($I_e < I$) is used. Effective moments of inertia based on TIR-A8 [2] (pp. 43–61) are compared to corresponding values (I_{eE}) based on a European analysis method [5]. Two load types (uniform and triangular) were considered for the U.S. method; the European method is based on one load type (sinus-shaped). For both methods, two profiles (unsymmetric and nominally symmetric, respectively; see Part 1 of this present article in Figure 4), two effective shear modulus values (for urethane cores), and three spans were studied. For the combinations used (and using uniform load for the U.S. method and not including face shear deformation), both methods produced nearly the same values (within 0.4%), with I_e being slightly greater than I_{eE} . Refer to Tables 6 and 7. For symmetric triangular loads, I_e was less than I_{eE} for all combinations by 0.14% or less.

Table 6. Profile K-1 (unsymmetric).

| G _{ce} [MPa] (ksi) | Span [mm] (in) | <i>I</i> [mm ⁴] (in ⁴) | <i>I</i> e [mm ⁴] (in ⁴) | I _{eE} [mm ⁴] (in ⁴) | $[(I_e/I_{eE}) - 1] \ (100\%) \ [\%]$ |
|--------------------------------|----------------------|--|--|---|---------------------------------------|
| 138 | 1219 | 544,500 | 428,220 | 427,550 | 0.16 |
| (20) | (48) | (1.3082) | (1.0288) | (1.0272) | |
| 138 | 1829 | 544,500 | 477,420 | 476.700 | 0.15 |
| (20) | (72) | (1.3082) | (1.1470) | (1.1453) | |

| Table 6. | Cont. | |
|----------|-------|--|
| | | |

| G _{ce} [MPa] (ksi) | Span [mm] (in) | <i>I</i> [mm ⁴] (in ⁴) | <i>I</i> e [mm ⁴] (in ⁴) | I _{eE} [mm ⁴] (in ⁴) | [(I _e /I _{eE}) - 1] (100%) [%] |
|--------------------------------|----------------------|--|--|---|---|
| 138 | 2438 | 544,500 | 502,470 | 501,810 | 0.13 |
| (20) | (96) | (1.3082) | (1.2072) | (1.2056) | |
| 552 | 1219 | 544,500 | 502,470 | 501,810 | 0.13 |
| (80) | (48) | (1.3082) | (1.2072) | (1.2056) | |
| 552 | 1829 | 544,500 | 524,200 | 523,760 | 0.08 |
| (80) | (72) | (1.3082) | (1.2594) | (1.2583) | |
| 552 | 2438 | 544,500 | 532,730 | 532,440 | 0.06 |
| (80) | (96) | (1.3082) | (1.2799) | (1.2792) | |

Table 7. Profile F-1 (symmetric).

| G _{ce} [MPa] (ksi) | Span [mm] (in) | <i>I</i> [mm ⁴] (in ⁴) | <i>I</i> e [mm ⁴] (in ⁴) | <i>I_{eE}</i> [mm ⁴] (in ⁴) | $[(I_e/I_{eE})-1] \ (100\%) \ [\%]$ |
|--------------------------------|----------------------|--|--|---|-------------------------------------|
| 138 | 1219 | 524,060 | 323,200 | 322,080 | 0.35 |
| (20) | (48) | (1.2590) | (0.7765) | (0.7738) | |
| 138 | 1829 | 524,060 | 400,080 | 398,680 | 0.35 |
| (20) | (72) | (1.2590) | (0.9612) | (0.9578) | |
| 138 | 2438 | 524,060 | 443,450 | 442,160 | 0.29 |
| (20) | (96) | (1.2590) | (1.0654) | (1.0623) | |
| 552 | 1219 | 524,060 | 443,450 | 442,160 | 0.29 |
| (80) | (48) | (1.2590) | (1.0654) | (1.0623) | |
| 552 | 1829 | 524,060 | 483,790 | 482,920 | 0.18 |
| (80) | (72) | (1.2590) | (1.1623) | (1.1602) | |
| 552 | 2438 | 524,060 | 500,390 | 499,810 | 0.12 |
| (80) | (96) | (1.2590) | (1.2022) | (1.2008) | |

However, the TIR-A8 [2] method also provides equations for the effect of shear deformation of the facing sections for several symmetric load cases (uniform, triangular, trapezoidal, and midspan concentrated). The adjusted effective moment of inertia is denoted as I'_e , which is less than the corresponding I_e . For variables other than span held constant, the ratio I'_e / I_e decreases as the span decreases. The largest effect occurs for profile K-1 (uniformly loaded), for which I'_e is about 4.0% less than I_e for a span of 1219 mm (48 in) and $G_{ce} = 552$ MPa (80 ksi). For the same conditions for the symmetric profile F-1, I'_e is about 3.9% less than I_e .

4. Conclusions

Part 2 briefly introduced the topic of aluminum beams with composite thermal barriers. For the linear portion of the load-deflection behavior, Section 2 has:

- Converted the European core-stiffness parameter to the equivalent U.S. parameter;
- Discussed uninstalled-strip material properties versus installed (effective) shear modulus (*G*_{ce}) of strips; *G*_{ce} values are substantially smaller than uninstalled (*G*_c) values;
- Determined (using FEA, for four tested beam spans) effective moments of inertia, as well as discussed flexural stiffness variations due to face dimensional tolerances and among nominally identical test specimens. G_{ce} values vary significantly among spans; test values of short-specimen shear stiffness also vary significantly;
- Determined, with FEA, the stresses and effective section moduli for the four test spans. Calculated stresses are well below the minimum yield for the alloy-temper used.

• Calculated the shear flows for the test spans; all are significantly less than the upper bound (q = VQ/I) and well below average and characteristic test strengths.

In Section 3, as expected, the results for I_e for uniform and triangular loads in the U.S. method and I_{eE} for a sinus-shaped load agree with the European standard's statement (Note 2 in [5]) that the method is highly accurate for constant and triangular loads, even though it is based on a sinus-shaped load. However, if face shear is included, as much as 4% added deflection occurs for a uniform load.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No physical testing was conducted and hence no new data are available.

Conflicts of Interest: The author declares no conflict of interest.

References

- LaBelle, J. Structural Behavior of Aluminum/Elastomer Sandwich Beams. Doctoral Dissertation, University of Wisconsin-Milwaukee, Milwaukee, WI, USA, 1990.
- LaBelle, J.; Davies, M. Structural Design Method for Composite Aluminum/Elastomer Beams. In *Structural Performance of Composite Thermal Barrier Framing Systems*; Section 7.5; AAMA TIR-A8-16; American Architectural Manufacturers Association (AAMA; now FGIA): Schaumburg, IL, USA, 2016.
- Huang, S. Investigation of Composite Façade Mullions. Doctoral Dissertation, University of Technology Sydney, Sydney, Australia, 2014. Available online: https://opus.lib.uts.edu.au/handle/10453/29258 (accessed on 21 April 2023).
- 4. Wolf, K. *TB-FEM*. Engineering Software Utilizing a Special-Purpose Finite Element for Sandwich Beams (with User's Guide). Unpublished work, 1996.
- prEN 14024:2021; Metal Profiles with Thermal Barrier—Mechanical Performance—Requirements, Proof and Tests for Assessment (Draft). CEN-CENELEC Management Centre: Brussels, Belgium, 2021.

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.