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Modelling the Number of Daily Stock Transactions Using a Novel Time Series Model †

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Abstract: This paper focusses on the impact of the COVID-19 on the Stock Exchange of Mauritius (SEM) by modelling the number of daily stock transactions of two banks. Hence, a non-stationary bivariate integer-valued autoregressive and moving average of order 1 (BINARMA (1,1)) process with COM-Poisson (CMP) innovations (BINARMA (1,1) CMP) is introduced. The conditional maximum likelihood (CML) approach is used to estimate the model parameters. The novel model is applied on the intra-day trading of two banking stocks.

Keywords: non-stationary; autoregressive; moving average; COM-poisson; CML

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1. Introduction

The Stock Exchange of Mauritius Ltd. (SEM, Port-Louis, Mauritius) started trading on 30 March 1989 as a private limited company with the responsibility of promoting a proficient and well-regulated stock market in Mauritius. The SEM changed its status on 6 October 2008 to operate as a public company and has all these years left any stone unturned to be the leading stock exchange market in the African continent. To date, the SEM has positioned itself as an essential capital raising platform for nearly 61 companies operating in the financial, construction, leisure, agricultural and other sectors of the economy. In its internationalization process, the SEM has set-up a multi-currency listing, trading and financial platform and has modernized its listing framework with different multi-asset class financial products. In 2010, the SEM embarked in a new journey by changing its strategic direction and started an internationalization of its operational and regulatory framework. To date, the market capitalization and the annual turnover of the SEM are approximately USD 7.5 billion and USD 302 billion, respectively.

The start of the financial year 2020–2021 was affected by the direct impact of the COVID-19 followed by the first nationwide sanitary confinement from March 2020 to May 2020. With the upliftment of the confinement, the Mauritian economy gradually started its recovery pathway. However, a second nationwide confinement was announced in March 2021 due to the resurgence of the COVID-19 cases with a partial de-confinement plan as from May 2021 up till now. Undeniably, the impact of the COVID-19 was severe such that the SEM has been navigating mostly in the red zone during this pandemic period. In the year 2021, the market started recovering following the announcements of the vaccines against the COVID-19, but the recovery was again affected by the second confinement. Hence, modelling the number of intra-day transactions on the stock market is of upmost importance. Several researchers have also elaborated on the potential covariates that affect these daily stock transactions: the impact of news entering the market, the time of day effect and the day effect [1,2]. Time of the day effect and the impact of news on the market have proved to influence the intensity of daily trading on the stock market [3]. However up till now, there has not been any studies that have incorporated the cross-

correlation between two competing companies affected by the above covariates as well as the COVID-19 news effect.

Quoreshi [4,5] is the first author who tried to model the number of intra-day transactions in the literature by developing a BINMA(p) process and applying the latter on the number of intra-day transactions of AstraZeneca and Ericsson B based on the Generalized Poisson distribution of the innovation series due to the over-dispersed nature of the data. Several authors have recently introduced bivariate processes under the non-stationary negative binomial (NB) and CMP innovations in the literature under either autoregressive or moving average structures (see Jowaheer et al. [6], Mamodekhan et al. [7] and Sunecher et al. [8,9]). As for the estimation of parameters, Quoreshi [4,5] estimated the regression effects using the feasible generalized least squares (FGLS) technique and concluded that the estimates of the model parameters are efficient, but the efficiency of these estimates have been questioned (see Sunecher et al. [8]). Another estimation method which has been frequently used in the literature is the generalized quasi-likelihood (GQL) method which has proved to yield more reliable estimates than FGLS. However, the likelihood-based approach provides the best estimates [10].

Based on the above findings, this paper proposes a novel bivariate integer-valued autoregressive and moving average of order 1 (BINARMA (1,1)) process under non-stationary COM-Poisson (CMP) innovation series where the model parameters are estimated using the conditional maximum likelihood (CML) approach. This novel model is then applied to the intra-day series of two banks listed on the SEM.

Hence, the paper is laid out as follows: In Section 2, the BINARMA (1,1) process with CMP innovations is developed. In Section 3, the CML approach is derived and Section 4 presents the forecasting equations. In Section 5, the BINARMA (1,1) model is applied on the number of intra-day transactions of two banks listed in SEM and is compared with two other competing models. The conclusion is presented in the last section.

2. The Non-Stationary BINARMA (1,1) Process with COM-Poisson Innovations (BINARMA (1,1) CMP)

Consider

$$Y_t^{[1]} = \gamma_{11} * Y_{t-1}^{[1]} + \gamma_{12} * R_{t-1}^{[2]} + R_t^{[1]} \tag{1}$$

$$Y_t^{[2]} = \gamma_{21} * Y_{t-1}^{[2]} + \gamma_{22} * R_{t-1}^{[2]} + R_t^{[2]} \tag{2}$$

where $\gamma_{kj} \in (0,1)$ and $\gamma_{kj}*$ are mutually independent binomial thinning operators such that $\gamma_{kj} * Y_{t-1}^{[k]} = \sum_{i=0}^{Y_{t-1}^{[k]}} Z_i$ where $Z_i \sim Bernoulli(\gamma_{kj})$.

'$\circ$' indicates the binomial thinning operator [11,12] such that $\{\gamma_{kk} \circ Y_{t-1}^{[k]} | Y_{t-1}^{[k]}\} \sim$ Binomial$(Y_{t-1}^{[k]}, \gamma_{kk})$ where:

1. $E(\gamma \circ Y) = \gamma E(Y)$;
2. $\text{Var}(\gamma \circ Y) = \gamma(1-\gamma)E(Y) + \gamma^2 \text{Var}(Y)$;
3. $\text{Cov}(\gamma_1 \circ Y^{[1]}, \gamma_2 \circ Y^{[2]}) = \gamma_1\gamma_2 \text{Cov}(Y^{[1]}, Y^{[2]})$.

As for the innovation terms, $Corr(R_t^{[1]}, R_t^{[2]}) = \alpha_{12}$ where $(R_t^{[1]}, R_t^{[2]})$ follows a bivariate COM-Poisson distribution with $R_t^{[k]} \sim COM - Poisson(\lambda_t^{[k]}, \tilde{\nu}_k)$. Note that $\lambda_t^{[k]} = (\theta_t^{[k]})^{1/\tilde{\nu}_k} - (\frac{\tilde{\nu}_k - 1}{2\tilde{\nu}_k})$, where $\theta_t^{[k]} = \exp(\boldsymbol{x}_t' \boldsymbol{\beta}^{[k]})$ with $\boldsymbol{x}_t = [x_{t1}, x_{t2}, \ldots, x_{tj}, \ldots, x_{tp}]'$ and $\boldsymbol{\beta}^{[k]} = [\beta_1^{[k]}, \beta_2^{[k]}, \ldots, \beta_j^{[k]}, \ldots, \beta_p^{[k]}]'$ for $k \in \{1,2\}$.

Based on the above conditions, the moments are derived as follows:

$$\begin{aligned} E(Y_t^{[1]}) &= E(\gamma_{11} \circ Y_{t-1}^{[1]} + \gamma_{12} \circ R_{t-1}^{[1]} + R_t^{[1]}) \\ &= E(\gamma_{11} \circ Y_{t-1}^{[1]}) + E(\gamma_{12} \circ R_{t-1}^{[1]}) + E(R_t^{[1]}) \\ &= \gamma_{11} E(Y_{t-1}^{[1]}) + \gamma_{12} E(R_{t-1}^{[1]}) + E(R_t^{[1]}) \\ \mu_t^{[1]} = E(Y_t^{[1]}) &= \gamma_{11}\mu_{t-1} + \gamma_{12}\lambda_{t-1}^{[1]} + \lambda_t^{[1]}, \end{aligned} \tag{3}$$

$$\begin{aligned} E(Y_t^{[2]}) &= E(\gamma_{21} \circ Y_{t-1}^{[2]} + \gamma_{22} \circ R_{t-1}^{[2]} + R_t^{[2]}) \\ &= E(\gamma_{21} \circ Y_{t-1}^{[2]}) + E(\gamma_{22} \circ R_{t-1}^{[2]}) + E(R_t^{[2]}) \\ &= \gamma_{21} E(Y_{t-1}^{[2]}) + \gamma_{22} E(R_{t-1}^{[2]}) + E(R_t^{[2]}) \\ \mu_t^{[2]} = E(Y_t^{[2]}) &= \gamma_{21}\mu_{t-1} + \gamma_{22}\lambda_{t-1}^{[1]} + \lambda_t^{[1]}, \end{aligned} \tag{4}$$

$$\begin{aligned} \mathrm{Var}(Y_t^{[1]}) &= Var(\gamma_{11} \circ Y_{t-1}^{[1]} + \gamma_{12} \circ R_{t-1}^{[1]} + R_t^{[1]}) \\ &= Var(\gamma_{11} * Y_{t-1}^{[1]}) + Var(\gamma_{12} * R_{t-1}^{[1]}) + Var(R_t^{[1]}) + 2\mathrm{Cov}(\gamma_{11} * Y_{t-1}^{[1]}, \gamma_{12} * R_{t-1}^{[1]}) \\ &= \gamma_{11}(1-\gamma_{11}) E(Y_{t-1}^{[1]}) + \gamma_{11}^2 \mathrm{Var}(Y_{t-1}^{[1]}) + \gamma_{12}(1-\gamma_{12}) E(R_{t-1}^{[1]}) \\ &+ \gamma_{12}^2 \mathrm{Var}(R_{t-1}^{[1]}) + Var(R_t^{[1]}) + 2Cov(\gamma_{11} * Y_{t-1}^{[1]}, \gamma_{12} * R_{t-1}^{[1]}) \\ &= \gamma_{11}(1-\gamma_{11})\mu_{t-1}^{[1]} + \gamma_{11}^2 \mathrm{Var}(Y_{t-1}^{[1]}) + \gamma_{12}(1-\gamma_{12})\lambda_{t-1}^{[1]} \\ &+ \gamma_{12}^2 \left[\frac{\lambda_{t-1}^{[1]}}{\tilde{v}_k} + \frac{\tilde{v}_k - 1}{2\tilde{v}_k^2} \right] + \left[\frac{\lambda_{t-1}^{[1]}}{\tilde{v}_k} + \frac{\tilde{v}_k - 1}{2\tilde{v}_k^2} \right] + 2\gamma_{11}\gamma_{12} Cov(Y_{t-1}^{[1]}, R_{t-1}^{[1]}) \\ &= \gamma_{11}(1-\gamma_{11})\mu_{t-1}^{[1]} + \gamma_{11}^2 \mathrm{Var}(Y_{t-1}^{[1]}) + \gamma_{12}(1-\gamma_{12})\lambda_{t-1}^{[1]} \\ &+ (1 + \gamma_{12}^2 + 2\gamma_{11}\gamma_{12}) \left[\frac{\lambda_{t-1}^{[1]}}{\tilde{v}_1} + \frac{\tilde{v}_1 - 1}{2\tilde{v}_1^2} \right], \end{aligned} \tag{5}$$

where

$$\begin{aligned} \mathrm{Cov}(Y_{t-1}^{[1]}, R_{t-1}^{[1]}) &= \mathrm{Cov}(\gamma_{11} \circ Y_{t-2}^{[1]} + \gamma_{12} \circ R_{t-2}^{[1]} + R_{t-1}^{[1]}, R_{t-1}^{[1]}) \\ &= \mathrm{Cov}(R_{t-1}^{[1]}, R_{t-1}^{[1]}) \\ &= \mathrm{Var}(R_{t-1}^{[1]}) \\ &= \frac{\lambda_{t-1}^{[1]}}{\tilde{v}_1} + \frac{\tilde{v}_1 - 1}{2\tilde{v}_1^2}. \end{aligned} \tag{6}$$

Similarly,

$$\begin{aligned} \mathrm{Var}(Y_t^{[2]}) &= Var(\gamma_{21} * Y_{t-1}^{[2]}) + Var(\gamma_{22} * R_{t-1}^{[2]}) + Var(R_t^{[2]}) + 2\mathrm{Cov}(\gamma_{21} * Y_{t-1}^{[2]}, \gamma_{22} * R_{t-1}^{[2]}) \\ &= \gamma_{21}(1-\gamma_{21})\mu_{t-1}^{[2]} + \gamma_{21}^2 \mathrm{Var}(Y_{t-1}^{[2]}) + \gamma_{22}(1-\gamma_{22})\lambda_{t-1}^{[2]} \\ &+ (1 + \gamma_{22}^2 + 2\gamma_{21}\gamma_{22}) \left[\frac{\lambda_{t-1}^{[2]}}{\tilde{v}_2} + \frac{\tilde{v}_2 - 1}{2\tilde{v}_2^2} \right]. \end{aligned} \tag{7}$$

As for the covariance for the same series,

$$\begin{aligned}
\mathrm{Cov}(Y_t^{[1]}, Y_{t+h}^{[1]}) &= \mathrm{Cov}[Y_t^{[1]}, (\gamma_{11} \circ Y_{t+h-1}^{[1]} + \gamma_{12} \circ R_{t+h-1}^{[1]} + R_{t+h}^{[1]})] \\
&= \gamma_{11}\mathrm{Cov}[Y_t^{[1]}, Y_{t+h-1}^{[1]}] \\
&= \gamma_{11}\mathrm{Cov}[Y_t^{[1]}, (\gamma_{11} \circ Y_{t+h-2}^{[1]} + \gamma_{12} \circ R_{t+h-2}^{[1]} + R_{t+h-1}^{[1]})] \\
&= \gamma_{11}^2\mathrm{Cov}[Y_t^{[1]}, Y_{t+h-2}^{[1]}] \\
&\vdots \\
&= \gamma_{11}^{h-1}\mathrm{Cov}[Y_t^{[1]}, (\gamma_{11} \circ Y_t^{[1]} + \gamma_{12} \circ R_t^{[1]} + R_{t+1}^{[1]})] \\
&= \gamma_{11}^h \mathrm{Var}(Y_t^{[1]}) + \gamma_{11}^{h-1}\gamma_{12}\mathrm{Var}(R_t^{[1]}) \\
&= \gamma_{11}^h \mathrm{Var}(Y_t^{[1]}) + \gamma_{11}^{h-1}\gamma_{12}\left[\frac{\lambda_{t-1}^{[1]}}{\tilde{\nu}_1} + \frac{\tilde{\nu}_1 - 1}{2\tilde{\nu}_1^2}\right]
\end{aligned} \tag{8}$$

and

$$\mathrm{Cov}(Y_t^{[2]}, Y_{t+h}^{[2]}) = \gamma_{21}^h \mathrm{Var}(Y_t^{[2]}) + \gamma_{21}^{h-1}\gamma_{22}\left[\frac{\lambda_{t-1}^{[2]}}{\tilde{\nu}_2} + \frac{\tilde{\nu}_2 - 1}{2\tilde{\nu}_2^2}\right]. \tag{9}$$

The cross-covariance are derived as follows:

$$\begin{aligned}
\mathrm{Cov}(Y_t^{[1]}, Y_t^{[2]}) &= \mathrm{Cov}(\gamma_{11} \circ Y_{t-1}^{[1]} + \gamma_{12} \circ R_{t-1}^{[1]} + R_{t+}^{[1]}, \gamma_{21} \circ Y_{t-1}^{[2]} + \gamma_{22} \circ R_{t-1}^{[2]} + R_t^{[2]}) \\
&= \gamma_{11}\gamma_{21}\mathrm{Cov}(Y_{t-1}^{[1]}, Y_{t-1}^{[2]}) + \gamma_{11}\gamma_{22}\mathrm{Cov}(Y_{t-1}^{[1]}, R_{t-1}^{[2]}) \\
&+ \gamma_{12}\gamma_{21}\mathrm{Cov}(R_{t-1}^{[1]}, Y_{t-1}^{[2]}) + Cov(R_{t-1}^{[1]}, R_{t-1}^{[2]}) + Cov(R_t^{[1]}, R_t^{[2]}) \\
&= \gamma_{11}\gamma_{21}\mathrm{Cov}(Y_{t-1}^{[1]}, Y_{t-1}^{[2]}) + (\gamma_{11}\gamma_{22} + \gamma_{12}\gamma_{21} + \gamma_{12}\gamma_{22})\alpha\sqrt{\lambda_{t-1}^{[1]}}\sqrt{\lambda_{t-1}^{[2]}} \\
&+ \alpha\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}},
\end{aligned} \tag{10}$$

$$\begin{aligned}
\mathrm{Cov}(Y_t^{[1]}, Y_{t+h}^{[2]}) &= \mathrm{Cov}[Y_t^{[1]}, (\gamma_{21} \circ Y_{t+h-1}^{[2]} + \gamma_{22} \circ R_{t+h-1}^{[2]} + R_{t+h}^{[2]})] \\
&= \gamma_{21}\mathrm{Cov}[Y_t^{[1]}, Y_{t+h-1}^{[2]}] \\
&= \gamma_{21}\mathrm{Cov}[Y_t^{[1]}, (\gamma_{21} \circ Y_{t+h-2}^{[2]} + \gamma_{22} \circ R_{t+h-2}^{[2]} + R_{t+h-1}^{[2]})] \\
&= \gamma_{21}^2\mathrm{Cov}[Y_t^{[1]}, Y_{t+h-2}^{[2]}] \\
&\vdots \\
&= \gamma_{21}^{h-1}\mathrm{Cov}[Y_t^{[1]}, (\gamma_{21} \circ Y_t^{[2]} + \gamma_{22} \circ R_t^{[2]} + R_{t+1}^{[2]})] \\
&= \gamma_{21}^h \mathrm{Cov}(Y_t^{[1]}, Y_t^{[2]}) + \gamma_{21}^{h-1}\gamma_{22}\mathrm{Cov}(R_t^{[1]}, R_t^{[2]}) \\
&= \gamma_{21}^h \mathrm{Cov}(Y_t^{[1]}, Y_t^{[2]}) + \gamma_{21}^{h-1}\gamma_{22}\alpha\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}
\end{aligned} \tag{11}$$

and

$$\mathrm{Cov}(Y_t^{[2]}, Y_{t+h}^{[1]}) = \gamma_{11}^h \mathrm{Cov}(Y_t^{[1]}, Y_t^{[2]}) + \gamma_{11}^{h-1}\gamma_{12}\alpha\sqrt{\lambda_t^{[1]}}\sqrt{\lambda_t^{[2]}}. \tag{12}$$

Remark 1. *Under stationary moment conditions, replacing $t = 1$ in Equations (3)–(7) and Equation (10), we have*

$$\mu_1^{[1]} = E(Y_t^{[1]}) = \frac{(\gamma_{12} + 1)\lambda_1^{[1]}}{1 - \gamma_{11}} \tag{13}$$

$$\mu_1^{[2]} = E(Y_t^{[2]}) = \frac{(\gamma_{22}+1)\lambda_1^{[2]}}{1-\gamma_{21}} \tag{14}$$

$$Var(Y_1^{[1]}) = \left(\frac{1}{1-\gamma_{11}^2}\right)\{\gamma_{11}(1-\gamma_{11})\mu_1^{[1]} + \gamma_{12}(1-\gamma_{12})\lambda_1^{[1]} + (1+\gamma_{12}^2+2\gamma_{11}\gamma_{12})\left[\frac{\lambda_1^{[1]}}{\tilde{\nu}_1} + \frac{\tilde{\nu}_1-1}{2\tilde{\nu}_1^2}\right]\}, \tag{15}$$

$$Var(Y_1^{[2]}) = \left(\frac{1}{1-\gamma_{21}^2}\right)\{\gamma_{21}(1-\gamma_{21})\mu_{t-1}^{[2]} + \gamma_{22}(1-\gamma_{22})\lambda_{t-1}^{[2]} + (1+\gamma_{22}^2+2\gamma_{21}\gamma_{22})\left[\frac{\lambda_{t-1}^{[2]}}{\tilde{\nu}_2} + \frac{\tilde{\nu}_2-1}{2\tilde{\nu}_2^2}\right]\}, \tag{16}$$

$$Cov(Y_1^{[1]}, Y_1^{[2]}) = \frac{(\gamma_{11}\gamma_{22}+\gamma_{12}\gamma_{21}+\gamma_{12}\gamma_{22}+1)\alpha\sqrt{\lambda_1^{[1]}}\sqrt{\lambda_1^{[2]}}}{1-\gamma_{11}\gamma_{21}} \tag{17}$$

Remark 2. *Using the initial values for $t = 1$ in Equations (12)–(16), we compute the values of $\mu_t^{[1]}, \mu_t^{[2]}, Var(Y_t^{[1]}), Var(Y_t^{[2]})$ and $Cov(Y_t^{[1]}, Y_t^{[2]})$ in Equations (3)–(11) iteratively for $t = 2, \ldots, T$.*

3. Conditional Maximum Likelihood Method

In this section, we derive the CML method for estimating the parameters of the BINARMA (1,1) model based on thinning and convolution properties [13]. The conditional density of the proposed BINARMA (1,1) model with COM-Poisson innovations is derived as follows:

$$f_1(k) = \sum_{j_1=0}^{k} \binom{y_{t-1}^{[1]}}{j_1} \binom{r_{t-1}^{[1]} = y_{t-1}^{[1]} - k}{k-j_1} \gamma_{11}^{j_1}(1-\gamma_{11})^{y_{t-1}^{[1]}-j_1}\gamma_{12}^{k-j_1}(1-\gamma_{12})^{y_{t-1}^{[1]}-2k+j_1}, \tag{18}$$

$$f_2(s) = \sum_{j_2=0}^{s} \binom{y_{t-1}^{[2]}}{j_2} \binom{r_{t-1}^{[2]} = y_{t-1}^{[2]} - s}{s-j_2} \gamma_{21}^{j_2}(1-\gamma_{21})^{y_{t-1}^{[2]}-j_2}\gamma_{22}^{s-j_2}(1-\gamma_{22})^{y_{t-1}^{[2]}-2s+j_2}, \tag{19}$$

and a bivariate distribution of the innovation terms $f_3(r_t^{[1]} = y_{t-1}^{[1]} - k, r_t^{[2]} = y_{t-1}^{[2]} - s) = P_{(R_t^{[1]}=r_t^{[1]}, R_t^{[2]} r_t^{[2]})}$, where

$$f_3(r_t^{[1]} = y_{t-1}^{[1]} - k, r_t^{[2]} = y_{t-1}^{[2]} - s) = \left[\frac{(\lambda_t^{[1]})^{y_{t-1}^{[1]}-k}}{((y_{t-1}^{[1]}-k)!)^{\tilde{\nu}_1}}\right]\left[\frac{1}{Z(\lambda_t^{[1]}, \tilde{\nu}_1)}\right]\left[\frac{(\lambda_t^{[2]})^{y_{t-1}^{[2]}-k}}{((y_{t-1}^{[2]}-k)!)^{\tilde{\nu}_2}}\right]\left[\frac{1}{Z(\lambda_t^{[2]}, \tilde{\nu}_2)}\right] \tag{20}$$

where the normalizing constant $Z(\lambda_t^{[k]}, \tilde{\nu}_k) = \sum_{j=0}^{\infty} \frac{\lambda_t^{[k]^j}}{(j!)^{\tilde{\nu}_k}}$.

The conditional density is written as $f((y_t^{[1]}, y_t^{[2]})|(y_{t-1}^{[1]}, y_{t-1}^{[2]}, r_{t-1}^{[1]}, r_{t-1}^{[2]}), \boldsymbol{\theta}) = \sum_{k=0}^{g_1}\sum_{s=0}^{g_2} f_1(k)f_2(s)f_3(r_t^{[1]} = y_{t-1}^{[1]} - k, r_t^{[2]} = y_{t-1}^{[2]} - s)$,

where $\boldsymbol{\theta} = [\gamma_{11}, \gamma_{12}, \gamma_{21}\gamma_{22}, \tilde{\nu}_1, \tilde{\nu}_2, \boldsymbol{\beta}^{[k]}]$ is the vector of unknown parameters, $g_1 = \min(y_t^{[1]}, y_{t-1}^{[1]})$ and $g_2 = \min(y_t^{[2]}, y_{t-1}^{[2]})$.

The conditional likelihood function is given by

$$L(\boldsymbol{\theta}|\boldsymbol{y}) = \prod_{t=1}^{T} f((y_t^{[1]}, y_t^{[2]})|(y_{t-1}^{[1]}, y_{t-1}^{[2]}, r_{t-1}^{[1]}, r_{t-1}^{[2]}), \boldsymbol{\theta}) \tag{21}$$

and after maximizing Equation (22)

$$\log[L(\boldsymbol{\theta}|\boldsymbol{y})] = \log\left[\sum_{t=1}^{T} f((y_t^{[1]}, y_t^{[2]})|(y_{t-1}^{[1]}, y_{t-1}^{[2]}, r_{t-1}^{[1]}, r_{t-1}^{[2]}), \boldsymbol{\theta})\right] \tag{22}$$

we obtain the maximum likelihood estimators of $\boldsymbol{\theta}$ for some starting value of $\boldsymbol{y_0}$.

4. Forecasting Equations

Based on the proposed model, the forecasting equations are derived as follows:

$$\begin{aligned} E(Y_{t+1}^{[1]}|y_t^{[1]}, r_t^{[1]}) &= E(\gamma_{11} \circ Y_t^{[1]}|y_t^{[1]}) + E(\gamma_{12} \circ R_t^{[1]}|r_t^{[1]}) + E(R_{t+1}^{[1]}) \\ &= \gamma_{11} y_t^{[1]} + \gamma_{12} r_t^{[1]} + \lambda_{t+1}^{[1]} \end{aligned} \tag{23}$$

$$\begin{aligned} E(Y_{t+1}^{[2]}|y_t^{[2]}, r_t^{[2]}) &= E(\gamma_{21} \circ Y_t^{[2]}|y_t^{[2]}) + E(\gamma_{22} \circ R_t^{[2]}|r_t^{[2]}) + E(R_{t+1}^{[2]}) \\ &= \gamma_{21} y_t^{[1]} + \gamma_{22} r_t^{[2]} + \lambda_{t+1}^{[2]} \end{aligned} \tag{24}$$

and

$$\begin{aligned} \mathrm{Var}(Y_{t+1}^{[1]}|y_t^{[1]}, r_t^{[1]}) &= \mathrm{Var}(\gamma_{11} \circ y_t^{[1]}|y_t^{[1]}) + Var(\gamma_{12} \circ R_t^{[1]}|r_t^{[1]}) + \mathrm{Var}(R_{t+1}^{[1]}) \\ &= \gamma_{11}(1-\gamma_{11}) y_t^{[1]} + \gamma_{12}(1-\gamma_{12}) r_t^{[1]} + \frac{\lambda_{t+1}^{[1]}}{\tilde{\nu}_1} + \frac{\tilde{\nu}_1 - 1}{2\tilde{\nu}_1^2} \end{aligned} \tag{25}$$

$$\begin{aligned} \mathrm{Var}(Y_{t+1}^{[2]}|y_t^{[2],r_i^{[2]}}) &= \mathrm{Var}(\gamma_{21} \circ y_t^{[2]}|y_t^{[2]}) + Var(\gamma_{22} \circ R_t^{[2]}|r_t^{[2]}) + \mathrm{Var}(R_{t+1}^{[2]}) \\ &= \gamma_{21}(1-\gamma_{21}) y_t^{[2]} + \gamma_{22}(1-\gamma_{22}) r_t^{[2]} + \frac{\lambda_{t+1}^{[2]}}{\tilde{\nu}_2} + \frac{\tilde{\nu}_2 - 1}{2\tilde{\nu}_2^2} \end{aligned} \tag{26}$$

5. Modelling Daily Stock Transactions

This section focusses on the number of daily stock transactions of the two most eminent banking institutions in Mauritius, namely Mauritius Commercial Bank Group Limited (MCB) and State Bank of Mauritius Holdings Ltd. (SBMH), that are listed on SEM. The daily stock transactions refers to the number of times stocks are bought and sold at the prevailing price during the trading session. MCB and SBMH are licensed by the Bank of Mauritius and have the biggest market share in the country. MCB was founded in 1838 and is the oldest and largest banking institution in Mauritius, while SBMH is the second largest commercial bank established in 1973. The total assets of MCB is nearly USD 15.8 billion, with a market capitalization on the SEM of USD 1.5 billion. MCB is owned by almost 22,000 domestic and foreign shareholders, has over 1.1 million individual and institutional clients and employs approximately 3700 staff. On the other hand, the total assets of SBMH is nearly USD 6.6 billion, with a market capitalization on the SEM of USD 253 million. SBMH is owned by almost 18,518 domestic and international shareholders, has over 0.75 million individual and institutional customers and employs approximately 2845 staff. The COVID-19 pandemic since the year 2020 has caused unprecedented disruptions

and created innumerable challenges for both commercial banks. In the wake of this difficult time, many investors of the SEM have been negatively affected and has been navigating in the red zone for quite some time because of the uncertainty prevailing due to the COVID-19 pandemic. MCB and SBMH have not been spared by the pandemic and their performance on the SEM has been affected negatively since the pandemic. Hence, it is of upmost importance to model the number of daily stock transactions of these two banks and provide reliable estimates to the investors so that they can decide whether they need to hold or sell the shares of MCB and SBMH.

The transactions of MCB and SBMH must be inter-related as they operate in the same sector, namely the banking sector and provides the same line of services and financial activities. Thus, we collected data from the several brokers on the number of daily transactions of these two banking institutions from 4 October to 10 December 2021 over 30 min intervals. As far the covariates are concerned, based on previous researchers [1,3–5], the following variables were identified as those influencing the number of daily stock transactions on the SEM: the intervention of any COVID-19 news that affect the financial market, the time of day effect and the day effect.

Hence, in this section, we analyze the intra-day stock transactions of MCB and SBMH using a novel time series model, namely the BINARMA (1,1) model with CMP innovations. The Stock market data for the number of daily transactions were collected from the SEM for MCB and SBMH within 30 min interval from 4 October to 10 December 2021, amounting to 450 paired observations. In the same line, the covariates that influence these daily stock transactions were recorded as follows: information on COVID-19 news (x_{t1}) where 1 refers for any new COVID-19 information which influence the stock trading of SBMH and MCB and 0 for no COVID-19 information, Friday effect (x_{t2}) where 1 refers to trading conducted on Fridays and 0 for trading conducted on Mondays, Tuesdays, Wednesdays and Thursdays and time of the day effect (x_{t3}) where 1 refers to the trading effected during the time period 12:00–13:30 and 0 for the trading between 09:00 and 12:00. Normally, the SEM operates from Monday to Friday only between 09:00 to 13:30 and is closed on public holidays. Based on 450 paired observations, the time series plots and the descriptive statistics are shown in Figures 1–4 and Table 1.

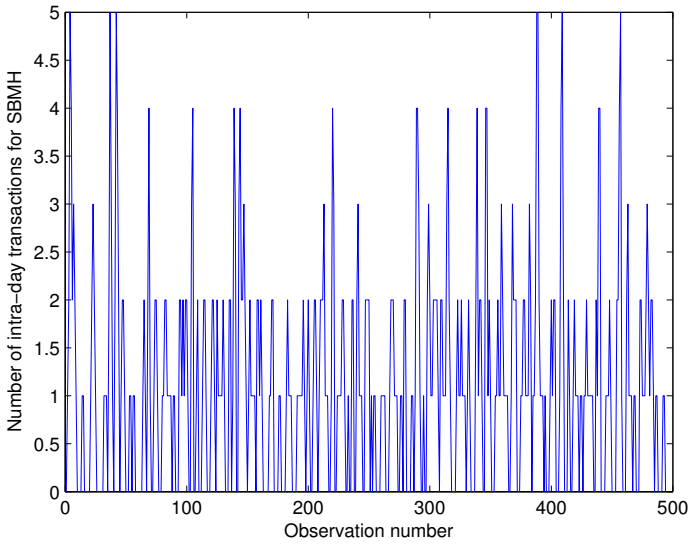


Figure 1. Time series plot for SBMH.

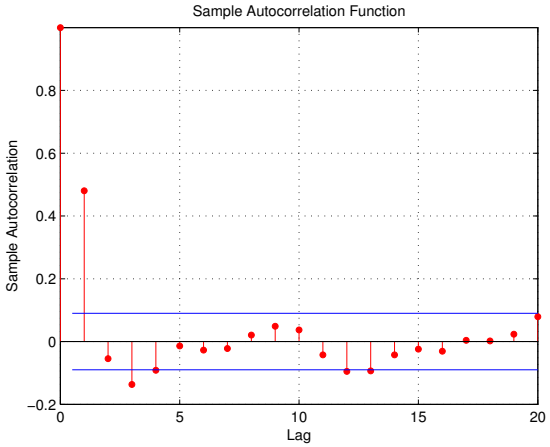


Figure 2. ACF plot for SBMH.

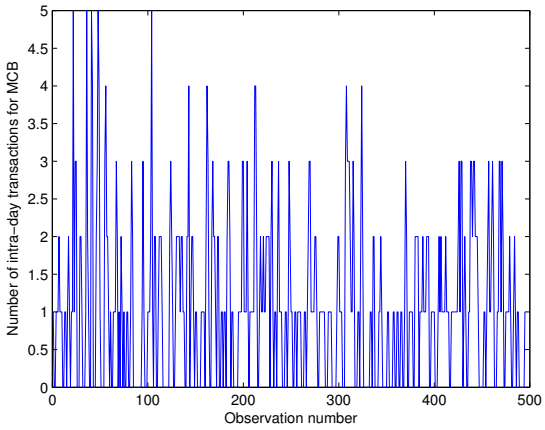


Figure 3. Time series plot for MCB.

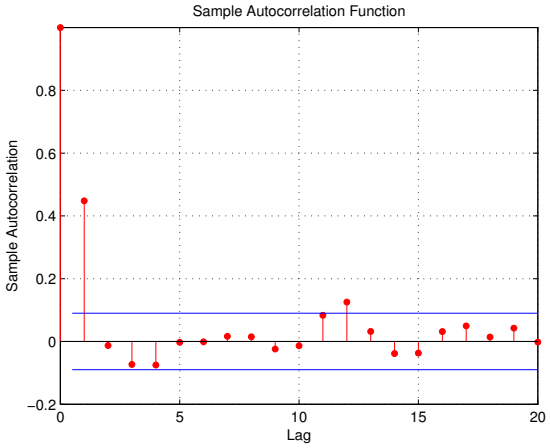


Figure 4. ACF plot for MCB.

Table 1. Summary statistics for the intra-day transactions of SBMH and MCB.

	Sample Mean	Sample Variance	Sample Lag-1	Sample Cross-Correlation
SBMH	1.0765	1.4191	0.5867	0.1617
MCB	1.0393	1.0154	0.5116	

From Table 1, we can conclude that the SBMH data series is slightly over-dispersed while the MCB data series is slightly under-dispersed. Both series have an average sample lag-1 correlation, with a sample cross-correlation of 0.1617, which confirms both the existence of relationship between as well as within the two series. As for the ACF plots, we observe that for both series that lag-1 has the highest peak. Thus, the proposed BINARMA (1,1) model with CMP innovations is used to model the in-sample daily trading of SBMH ($Y_t^{[1]}$) and MCB ($Y_t^{[2]}$) between 4 October and 3 December 2021, totalling 405 paired observations, while the out-sample from 5 December to 10 December 2021 is used to validate the model. We also apply the bivariate integer-valued autoregressive model of order 1 with CMP innovations (BINAR(1)CMP) developed by Jowaheer et al. [6] and the bivariate integer-valued moving average model of order 1 with CMP innovations (BINMA(1)) developed by Mamodekhan et al. [7] on the intra-day series. Under $\lambda_t^{[k]} = \exp(\hat{\beta}_0^{[k]} + \hat{\beta}_1^{[k]} x_{t1} + \hat{\beta}_2^{[k]} x_{t2} + \hat{\beta}_3^{[k]} x_{t3})$, the estimates of the covariate effects under the application of the three models are shown in Table 2.

Table 2. Intra-day transactions for MCB and SBMH: Estimates of the regression parameters.

Models	Series	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
BINARMA (1,1) CMP	$Y_t^{[1]}$	0.2268	0.2633	0.1482	0.1399
	s.e	(0.1975)	(0.0581)	(0.0288)	(0.0295)
	$Y_t^{[2]}$	0.1543	0.2456	0.1297	0.1193
	s.e	(0.2259)	(0.0447)	(0.0350)	(0.0587)
BINMA(1)CMP	$Y_t^{[1]}$	0.2122	0.2530	0.1533	0.1264
	s.e	(0.2089)	(0.0615)	(0.0314)	(0.0322)
	$Y_t^{[2]}$	0.1650	0.2627	0.1074	0.1317
	s.e	(0.2451)	(0.0566)	(0.0416)	(0.0661)
BINAR(1)CMP	$Y_t^{[1]}$	0.2412	0.2490	0.1231	0.1211
	s.e	(0.2159)	(0.0691)	(0.0375)	(0.0388)
	$Y_t^{[2]}$	0.1856	0.2659	0.0926	0.1262
	s.e	(0.2517)	(0.0612)	(0.0487)	(0.0701)

From the regression coefficients Table 2, we observe that the estimates of the covariates obtained using the BINARMA (1,1) CMP have lower standard errors compared to those obtained using BINAR(1)CMP and BINMA(1)CMP and hence, we interpret only the estimates of the BINARMA (1,1) CMP model. We can also notice that all the explanatory variables are significant, thus confirming their influence on the number of intra-day transactions of MCB and SBMH. Since the pandemic of COVID-19 started in the year 2020, any news entering the domestic market pertaining to confinement, number of COVID-19 cases in Mauritius, potential vaccines against COVID-19, the gradual recovery of the economy and the lifting of the confinement have caused an increase in the number of inra-day stock transactions of both MCB and SBMH. For SBMH, as news filter in the market, we expect an increase in the stock transactions of 30.1 percent and 27.8 percent for MCB. From these figures, we can conclude that there are more trading for SBMH stocks than MCB stocks as MCB is a more robust bank than SBMH. The other two explanatory variables, namely the Friday and time effects, have also affected the number of intra-day transactions of MCB and SBMH, but their influence were lower than the COVID-19 news effect. From the correlation Table 3, we can confirm that there exists a relationship within and between the number of daily trading of MCB and SBMH.

Table 3. Intra-day transactions for MCB and SBMH: Estimates of the dependence parameters.

Models	Parameters	$\hat{\gamma}_{11}$	$\hat{\gamma}_{22}$	$\hat{\gamma}_{12}$	$\hat{\gamma}_{21}$
BINARMA (1,1) CMP	Correlation	0.3117	0.1096	0.0825	0.3988
	s.e	(0.0440)	(0.0322)	(0.0533)	(0.0564)
BINMA(1)CMP	Correlation	0.3182	0.1112	0.0875	0.3893
	s.e	(0.0488)	(0.0351)	(0.0561)	(0.0590)
BINAR(1)CMP	Correlation	0.3021	0.1178	0.0802	0.4049
	s.e	(0.0498)	(0.0377)	(0.0581)	(0.0595)

Using the forecasting Equations (23) and (24), we compute the one-step ahead forecast of the number of stock trading of SBMH and MCB based on the out-sample trading data between 5 December to 10 December 2021, totalling 45 paired observations and the corresponding root mean square errors (RMSEs) for three models with CMP innovations are shown in Table 4:

Table 4. RMSEs for the out-sample number of intra-day transactions of MCB and SBMH.

Models	RMSE $Y_t^{[1]}$	RMSE $Y_t^{[2]}$
BINARMA (1,1) CMP	0.1511	0.1285
BINMA(1)CMP	0.1598	0.1357
BINAR(1)CMP	0.1655	0.1469

From Table 4, we observe that the BINARMA (1,1) CMP provide better RMSEs than BINAR(1)CMP and BINMA(1)CMP.

6. Conclusions

This paper considers the modeling of the intra-day transactions of two most prestigious banking companies: MCB and SBMH in Mauritius and how the COVID-19 pandemic has affected the stock transactions of these two commercial banks. Since the time series data of one bank is over-dispersed and the other one is under-dispersed, we develop a BINARMA (1,1) process with CMP innovation terms to model these data. In this paper, a CML approach is used to estimate the regression and correlation parameters for the two series. This novel BINARMA (1,1) CMP model together with the CML were then applied to estimate the regression and correlation effects of the intra-day transactions where it was found to yield significant estimates for the time of trade, Friday and COVID-19 news effect. The forecasting equations were also developed and they yield reliable estimates for the volume of transaction for the two series based on the real figures.

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