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Quantum-Inspired Photon-Spin Control Framework for Robust Automation of Nonlinear Dynamic Systems Under Uncertain Operating Conditions †

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Abstract

Robust control of nonlinear dynamic systems remains challenging because parametric uncertainty and external disturbances can significantly degrade tracking accuracy and transient performance. This study proposes a Quantum-Inspired Photon–Spin Control Framework (QPSCF) that integrates probabilistic state representation and interference-inspired decision-making directly into the online feedback control process while remaining fully executable on classical computing platforms. Unlike quantum-inspired approaches primarily used for offline controller tuning or heuristic optimization, the proposed framework represents multiple candidate operating states probabilistically and adaptively evaluates competing control actions according to current process conditions. The QPSCF was evaluated on a nonlinear benchmark system under parameter variations of up to ±15% and a 10% external disturbance and compared with conventional PID and Mamdani fuzzy controllers under identical simulation conditions. The proposed controller achieved a settling time of 40.70 s, an overshoot of 0.15%, an RMSE of 4.83, an IAE of 121.54, and an ISE of 1652.41. Compared with PID control, QPSCF reduced RMSE, IAE, and ISE by 19.6%, 29.6%, and 41.9%, respectively. It also reduced the maximum disturbance-induced deviation from 1.80 °C to 0.55 °C and the recovery time from 20.30 s to 5.90 s. These results demonstrate that direct probabilistic decision-making within the feedback loop can improve tracking accuracy and disturbance rejection in nonlinear systems under uncertainty.

Keywords: probabilistic decision-making; interference-inspired inference; uncertainty handling; disturbance rejection; classical computing platform

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1. Introduction

Automation and control technologies constitute the core of modern industrial infrastructure, enabling reliable operation of energy-conversion systems, manufacturing processes, mechatronic platforms, autonomous production facilities, and cyber–physical systems. As industrial processes become increasingly complex, control systems must operate effectively in the presence of nonlinear dynamics, parametric uncertainties, external disturbances, sensor imperfections, and continuously varying operating conditions [1]. These factors complicate the design of high-performance controllers and can degrade closed-loop stability, tracking accuracy, robustness, and energy efficiency. Consequently, developing advanced control methodologies that maintain reliable performance under uncertainty remains an important challenge in modern automation and control engineering [2].

For several decades, proportional–integral–derivative (PID) controllers have remained a dominant solution in industrial control because of their simple structure, ease of implementation, and low computational requirements. Despite their widespread use, conventional PID controllers are based primarily on linear control principles and may exhibit limited performance when applied to highly nonlinear and time-varying processes [3]. In practical applications, parameter variations, unmodeled dynamics, transport delays, and external disturbances may lead to excessive overshoot, prolonged settling times, oscillatory responses, and poor disturbance rejection. To address these limitations, various advanced approaches have been developed, including adaptive, robust, sliding-mode, model predictive, fuzzy-logic, neural-network-based, and hybrid intelligent control methods [4].

Among these methodologies, intelligent control techniques have attracted significant attention because of their ability to approximate nonlinear relationships and operate effectively in situations where accurate mathematical models are unavailable. Fuzzy inference systems provide interpretable rule-based decision-making mechanisms, while artificial neural networks offer powerful nonlinear learning and approximation capabilities [5]. Various hybrid approaches combining fuzzy reasoning, neural computation, evolutionary optimization, and probabilistic decision-making have demonstrated improved performance in many automation applications. Nevertheless, despite these advances, most existing intelligent controllers continue to rely on deterministic state representations and sequential decision-making structures. Under severe uncertainty, multiple feasible control actions may simultaneously exist, yet conventional control algorithms generally evaluate only a single estimated state trajectory, potentially limiting their adaptability and robustness [6].

Recent advances in artificial intelligence have further expanded the capabilities of nonlinear-system modeling, analysis, and intelligent decision-making. Neural-network-based approaches provide flexible mechanisms for approximating complex nonlinear relationships and learning system behavior from data. Beyond conventional data-driven approximation, recent studies have increasingly combined neural networks with symbolic computation and analytical structures. Bilinear neural networks and bilinear residual networks have been developed to construct exact analytical solutions of nonlinear evolution equations [7,8], while fractional sub-equation neural networks integrate analytical fractional structures into neural architectures for solving space-time fractional partial differential equations [9]. More recent neural–symbolic approaches, including neural-network-based symbolic calculation [10], multimodal neurosymbolic reasoning [11], and Artificial Intelligence-Enhanced Mathematical Derivation [12], further demonstrate the potential of combining neural adaptability with symbolic mathematical reasoning for complex nonlinear systems. These developments highlight a broader transition from purely deterministic analytical methods toward data-driven and hybrid intelligent computational paradigms. However, such approaches are primarily directed toward system modeling, analytical solution generation, or data-driven representation rather than probabilistic online feedback decision-making. In contrast, the present QPSCF focuses on embedding probabilistic state representation and interference-inspired decision logic directly into the feedback control process.

Recent advances in quantum-inspired computation have introduced alternative mathematical frameworks for representing uncertainty and complex decision spaces. Unlike physical quantum computing, quantum-inspired methods employ mathematical concepts derived from quantum theory while remaining fully executable on classical computing platforms. Concepts such as superposition, probability amplitudes, interference, and state evolution have shown considerable potential in optimization, machine learning, pattern recognition, and decision-support applications. A key advantage of these approaches is their ability to represent multiple candidate solutions simultaneously within a unified probabilistic framework, enabling a more comprehensive exploration of complex solution spaces under uncertainty [13].

The application of quantum-inspired concepts to automation and control systems has recently emerged as a promising research direction. Existing studies have primarily focused on quantum-inspired optimization for controller tuning, parameter identification, and learning enhancement [14]. Although these approaches have demonstrated promising results, quantum-inspired principles are generally employed as auxiliary optimization tools rather than being directly integrated into the feedback control architecture. Consequently, the potential of quantum-inspired state representation and decision-making for real-time feedback control remains insufficiently explored. In particular, the development of a control framework capable of simultaneously representing multiple possible system behaviors and evaluating competing control actions within a unified feedback structure remains an open research challenge [15].

Furthermore, most existing control methodologies describe system behavior using deterministic state variables that represent a single operating condition at a given instant. Such representations may become inadequate when nonlinear dynamics, parametric uncertainty, and stochastic disturbances interact simultaneously. Under these circumstances, the actual future behavior of the system may evolve along multiple possible trajectories [16]. Therefore, a control framework capable of representing and evaluating alternative system states in parallel could provide substantial improvements in robustness, adaptability, and decision quality [17].

Despite substantial progress in intelligent and quantum-inspired control, an important methodological gap remains. Conventional PID controllers rely on fixed control parameters and may lose performance when nonlinear dynamics, parametric uncertainty, and external disturbances occur simultaneously. Fuzzy and other intelligent controllers can better accommodate nonlinear behavior, but their online decisions are generally generated from deterministic inference structures. Meanwhile, existing quantum-inspired control studies have predominantly employed quantum-inspired concepts for offline optimization, controller tuning, parameter identification, or heuristic search rather than for direct feedback decision-making. Consequently, there remains a need for a control framework that can represent multiple candidate operating states simultaneously, update their relative importance according to current process conditions, and use this probabilistic information directly for online control-action selection. This research gap motivates the development of the proposed QPSCF, in which photon–spin probabilistic state representation and interference-inspired decision-making are incorporated directly into the feedback control loop.

To address these challenges, this study proposes a Quantum-Inspired Photon–Spin Control Framework (QPSCF) for the robust control of nonlinear dynamic systems under uncertain operating conditions. The proposed framework introduces a photon–spin probabilistic state representation in which system states are encoded using photon-inspired energy distributions and spin-inspired probabilistic variables. This representation allows multiple potential system behaviors to coexist within a unified state space, enabling the simultaneous evaluation of alternative control actions [18]. An interference-inspired decision mechanism then evaluates these alternatives according to the current operating conditions and uncertainty level and identifies the most appropriate control action. The selected action is subsequently decoded into a classical control signal and applied to the plant through conventional actuators, forming a hybrid quantum-inspired/classical feedback architecture [19].

Unlike existing quantum-inspired control strategies that primarily employ quantum-inspired algorithms for offline controller tuning or heuristic optimization, the proposed QPSCF integrates photon–spin probabilistic state representation directly into the real-time feedback loop. Rather than relying on a single deterministic representation of the current operating condition, the framework probabilistically encodes multiple candidate state trajectories and evaluates the corresponding control alternatives through an interference-inspired decision mechanism. This formulation extends the role of quantum-inspired computation from auxiliary optimization toward online control-oriented decision-making, while retaining compatibility with conventional industrial automation hardware [20].

The principal contributions of this study are twofold. First, unlike conventional deterministic and existing quantum-inspired control approaches that primarily use quantum-inspired mechanisms for offline optimization, parameter tuning, or auxiliary inference, the proposed QPSCF introduces a photon–spin probabilistic state representation that maintains multiple candidate operating states within the online feedback-control process [21]. Second, an adaptive interference-inspired decision mechanism is formulated to update the relative importance of these candidate states according to the current tracking error and uncertainty and to use the resulting probabilities directly for control-action selection. Thus, the methodological contribution of the proposed framework lies in shifting quantum-inspired computation from predominantly auxiliary optimization toward probability-based online feedback decision-making.

The simulation results indicate that the proposed framework improves robustness, disturbance rejection, and transient performance under the investigated uncertain operating conditions. Compared with the conventional deterministic control strategies considered in this study, the QPSCF provides a probabilistic decision-making mechanism that simultaneously evaluates multiple candidate control actions [22]. These findings support the potential of the proposed framework as an alternative intelligent control strategy for nonlinear dynamic systems subject to parametric uncertainty and external disturbances.

2. Materials and Methods

2.1. Quantum-Inspired Photon-Spin Control Framework

The proposed Quantum-Inspired Photon–Spin Control Framework (QPSCF) was developed to enhance robustness, adaptability, and online decision-making in nonlinear dynamic systems operating under uncertain conditions. The framework combines a quantum-inspired probabilistic state representation with an interference-inspired decision mechanism within a conventional feedback architecture. Unlike quantum-inspired control approaches that primarily use quantum-inspired optimization for offline parameter tuning or heuristic search, the QPSCF incorporates photon–spin probabilistic reasoning directly into the online feedback loop. This formulation enables the controller to continuously evaluate multiple candidate control actions while remaining compatible with conventional industrial automation hardware [23].

The overall control architecture consists of four sequential computational stages. First, process measurements are acquired from the controlled plant using conventional industrial sensors and preprocessed to determine the current operating condition and tracking error. Second, the estimated system state is transformed into a photon–spin probabilistic representation, in which multiple candidate operating states are simultaneously encoded according to their probability amplitudes and relative energy levels. Third, an interference-inspired decision mechanism evaluates the competing candidate states by adaptively assigning probabilistic weights based on the current process dynamics, uncertainty level, and control objective. Finally, the control action with the highest decision probability is decoded into a classical actuator command and applied to the plant, thereby completing the closed-loop feedback cycle [24].

The proposed framework replaces a single deterministic state representation with a probabilistic description that can retain multiple feasible operating trajectories during control-oriented decision-making. In nonlinear systems subject to parametric uncertainty, external disturbances, and measurement noise, several candidate control actions may be plausible at a given sampling instant. Rather than relying exclusively on a single estimated operating state, the QPSCF maintains a probability-weighted set of candidate trajectories and evaluates their relative relevance before selecting the control action. This representation is intended to improve decision flexibility under uncertain operating conditions [25].

It is important to clarify that the proposed photon-spin representation is a quantum-inspired computational abstraction rather than a model of physical quantum phenomena. Photon-inspired energy encoding and spin-inspired probabilistic variables are used solely as mathematical mechanisms for representing uncertainty and assigning relative importance to competing control alternatives. All computations are performed using conventional numerical algorithms on classical computing platforms; therefore, no physical qubits, quantum measurement, entanglement, or specialized quantum hardware are required. This classical implementation makes the QPSCF conceptually compatible with conventional automation platforms, including programmable logic controllers (PLCs), embedded controllers, industrial computers, supervisory control and data acquisition (SCADA) systems, and digital-twin environments [26].

Compared with conventional deterministic feedback controllers, the proposed framework is designed to provide two main advantages. First, the probabilistic representation enables multiple candidate control actions to be evaluated simultaneously, providing greater flexibility in the presence of parametric uncertainty and external disturbances. Second, the adaptive interference-inspired decision mechanism continuously updates the relative weights of competing candidate states according to the current process information. This mechanism is intended to improve tracking performance, disturbance rejection, and transient response by dynamically prioritizing the most relevant control alternatives. Because the present study is based on numerical simulation, real-time computational feasibility on industrial hardware remains to be experimentally evaluated.

The methodological novelty of the proposed QPSCF does not lie in introducing a new nonlinear plant model or modifying the conventional Mamdani fuzzy controller used for comparison. Instead, it lies in the formulation of the control decision process itself. In contrast to conventional deterministic controllers, which generate a control action from a single estimated operating condition, the QPSCF maintains a normalized probability-weighted ensemble of candidate operating states within each feedback cycle. These candidates are characterized by probability amplitudes, photon-inspired relative weighting, and spin-inspired directional information. Their relative importance is then adaptively updated according to the instantaneous tracking error and uncertainty, after which an interference-inspired evaluation mechanism converts the updated state distribution into control-action probabilities. Thus, the proposed methodological extension is the integration of probabilistic multi-state representation, adaptive state evolution, and probability-based action selection directly within the online feedback loop rather than the use of quantum-inspired computation solely as an offline optimization or parameter-tuning mechanism.

The following subsections present the mathematical formulation of the nonlinear process model, photon-spin probabilistic state encoding, adaptive state-evolution mechanism, and interference-inspired control decision strategy [27].

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2.2. Nonlinear Dynamic System Representation

To evaluate the proposed Quantum-Inspired Photon-Spin Control Framework (QPSCF), a benchmark second-order nonlinear dynamic system subject to parametric uncertainty and external disturbances was considered. Such benchmark systems are commonly used in control studies because they capture key characteristics of industrial processes, including nonlinear dynamics, parameter variations, and disturbance-sensitive behavior, while providing a consistent basis for comparing different control strategies [28].

The controlled process is represented by a nonlinear state-space model applicable to a range of automation-oriented dynamic systems, including thermal processes, chemical reactors, electromechanical systems, and mechatronic plants. The nonlinear system dynamics are expressed as

$$\dot{x}(t) = f(x(t), u(t), d(t), p(t)) \tag{1}$$

where $x(t)$ denotes the system state vector, $u(t)$ is the control input, $p(t)$ represents the uncertain parameter vector, and $d(t)$ denotes the external disturbance acting on the process. The nonlinear function $f(\cdot)$ describes the process dynamics and may include nonlinear interactions among the system states.

The measurable system output is defined as

$$y(t) = h(x(t)) \tag{2}$$

where $h(\cdot)$ denotes the nonlinear output mapping from the system states to the measured process variables.

The control objective is to ensure accurate tracking of a prescribed reference trajectory despite parametric variations and external disturbances. Accordingly, the tracking error is defined as

$$e(t) = r(t) - y(t) \tag{3}$$

where $r(t)$ denotes the reference signal.

To represent realistic industrial operating conditions, the uncertain process parameters are modeled as

$$p(t) = p_0 + \Delta p(t) \tag{4}$$

where p_0 represents bounded parameter variations caused by equipment aging, changes in operating conditions, modeling uncertainties, or environmental effects. In this study, the parameter variations are assumed to remain within $\pm 15\%$ of their nominal values, consistent with the simulation scenarios considered in Section 3.

The proposed control framework is developed under the following assumptions:

- the full system output is measurable through conventional industrial sensors;
- the nonlinear functions $f(\cdot)$ and $h(\cdot)$ are continuous and locally Lipschitz, ensuring existence and uniqueness of the system trajectories;
- external disturbances are bounded and piecewise continuous; and
- parameter variations occur sufficiently slowly relative to the controller sampling interval.

These assumptions provide a consistent basis for the subsequent formulation of the photon-spin probabilistic state encoding and interference-inspired control strategy.

The nonlinear state-space representation defined in this subsection provides the mathematical basis for the proposed QPSCF. In the following subsection, this deterministic state description is transformed into a photon–spin probabilistic representation that supports the simultaneous evaluation of multiple candidate operating states under uncertainty [29].

2.3. Photon-Spin Probabilistic State Encoding

A central feature of the proposed Quantum-Inspired Photon–Spin Control Framework is the transformation of the conventional deterministic state description into a probabilistic photon–spin representation capable of retaining multiple candidate operating states simultaneously. Rather than relying on a single estimated trajectory at each sampling instant, the proposed framework maintains a probability-weighted ensemble of candidate trajectories for control-oriented decision-making under nonlinear dynamics, parametric uncertainty, and external disturbances [30].

Within the proposed framework, superposition is used as a computational analogy rather than as a representation of a physical quantum phenomenon. Each candidate operating state is assigned a probability amplitude that characterizes its relative contribution to the probabilistic state representation. Multiple candidate states can therefore be retained simultaneously during the decision-making process until a control action is selected. This formulation provides a structured mechanism for evaluating alternative operating trajectories within a unified probabilistic representation.

The photon-spin probabilistic state vector is defined as

$$\psi(t) = \begin{bmatrix} \psi_1(t) \\ \psi_2(t) \\ \vdots \\ \psi_N(t) \end{bmatrix} \tag{5}$$

where $\psi_\mathrm{i}(t)$ denotes the probability amplitude associated with the i-th candidate operating state and N represents the total number of candidate states considered by the controller.

To preserve probabilistic consistency, the probability amplitudes satisfy the normalization constraint

$$\sum_{i=1}^{N} |\psi_i(t)|^2 = 1 \tag{6}$$

ensuring that the total probability remains conserved throughout the decision-making process.

The probability associated with each candidate operating state is computed according to

$$P_i(t) = |\psi_i(t)|^2 \tag{7}$$

where $P_i(t)$ represents the likelihood that the corresponding operating state provides the most appropriate control decision under the current process conditions.

To assign relative importance to competing candidate states, a photon-inspired energy-encoding mechanism is introduced. The encoded quantity associated with the *i*-th candidate state is expressed as

$$E_i(t) = \gamma|\psi_i(t)|^2 \tag{8}$$

where $\gamma > 0$ is a scaling coefficient that regulates the relative contribution of each candidate state during the subsequent decision-making process.

The encoded quantity does not represent the physical energy of photons; rather, it serves as a computational weighting mechanism for assigning relative importance to competing candidate states according to their probability amplitudes. To further characterize the expected dynamic behavior of each candidate trajectory, every state is additionally assigned a spin variable

$$\sigma_i \in \{-1,+1\} \tag{9}$$

Positive spin-inspired values indicate candidate trajectories evolving toward the desired operating condition, whereas negative values identify trajectories associated with increasing tracking error or unfavorable evolution. The spin-inspired variable therefore provides qualitative directional information that complements the probability-based weighting of the candidate states.

The combination of normalized probability amplitudes, photon-inspired weighting, and spin-inspired directional characterization provides a unified probabilistic representation of multiple candidate operating states. In contrast to a single deterministic state description, this formulation retains information about alternative candidate trajectories until the control decision is generated [31].

The resulting photon–spin probabilistic representation forms the computational basis of the proposed QPSCF. In the following subsection, the encoded candidate states are updated through a quantum-inspired state-evolution process and evaluated using an adaptive interference-inspired decision mechanism to select the control action.

2.4. Quantum-Inspired State Evolution and Decision Mechanism

Following the photon–spin probabilistic state encoding, the candidate states are updated using a quantum-inspired state-evolution mechanism. The purpose of this mechanism is to adapt the probability distribution of the candidate states according to the current tracking error, process uncertainty, and dynamic behavior. The updated states are subsequently evaluated through an interference-inspired decision mechanism to determine the control action.

The state evolution process is represented as

$$\psi(t+1) = U(t)\psi(t) \tag{10}$$

where $U(t)$ denotes the quantum-inspired transition operator.

Unlike physical quantum systems, the transition operator employed in this work is implemented using classical numerical computation. The operator continuously adapts according to tracking error magnitude, uncertainty level, and disturbance intensity.

To maintain probabilistic consistency, the normalization condition

$$\|\psi(t+1)\| = 1 \tag{11}$$

is preserved throughout the control process.

The influence of tracking performance and uncertainty is incorporated through an adaptive weighting function

$$W_i(t) = \exp(-\propto|e(t)| - \beta U_c(t)) \tag{12}$$

where $U_c(t)$ denotes the uncertainty index and α and β are adaptive coefficients.

This weighting mechanism favors states associated with smaller tracking errors and lower uncertainty levels. Consequently, unfavorable trajectories gradually lose influence, whereas promising alternatives become increasingly dominant during state evolution.

The updated probability amplitudes are calculated according to

$$\psi_i^{new}(t) = \frac{W_i(t)\psi_i(t)}{\sqrt{\sum_{k=1}^{N}|W_k(t)\psi_k(t)|^2}} \tag{13}$$

which preserves normalization while emphasizing states with superior control potential.

After state evolution, an interference-inspired inference mechanism is employed to evaluate competing control actions. The interference score associated with the j-th control action is calculated as

$$I_j = \sum_{i=1}^{N} \omega_{ij}\Psi_i \tag{14}$$

where ω_{ij} denotes the interaction coefficient between the i-th state and the j-th control action.

The probability associated with each control action is then determined as

$$D_j = \frac{|I_j|^2}{\sum_{k=1}^{M} |I_k|^2} \tag{15}$$

where M denotes the total number of candidate control actions.

The optimal control decision is selected according to

$$u^* = argmax(D_j) \tag{16}$$

thereby identifying the control action with the highest probability of achieving the desired control objective.

The distinction of this mechanism from conventional fuzzy inference is that simultaneously relevant control alternatives are not combined solely through deterministic membership aggregation and defuzzification. Instead, the QPSCF explicitly maintains a normalized probability distribution over candidate states, updates this distribution according to the instantaneous control condition, and subsequently evaluates the associated control actions through interference-inspired scores. Consequently, the control decision is generated from an adaptively evolving probability distribution rather than from a fixed deterministic rule-aggregation structure. This online probability-update-and-selection cycle constitutes the central methodological element of the proposed framework.

2.5. Feedback Control Strategy and Performance Evaluation

The optimal control action determined by the quantum-inspired interference mechanism is transformed into a conventional actuator command according to

$$u(t) = K_u u^*(t) \tag{17}$$

where $u^*(t)$ denotes the optimal probabilistic control decision and K_u represents the actuator scaling coefficient that adapts the controller output to the physical characteristics of the controlled process. This transformation enables direct integration of the proposed framework with conventional industrial actuators without requiring modification of the existing automation infrastructure.

The generated control signal is applied to the nonlinear process through a standard closed-loop feedback configuration. At each sampling instant, updated process measurements are acquired from the plant, the tracking error is recalculated, the photon-spin probabilistic state representation is reconstructed, and the adaptive decision mechanism generates a new optimal control action. Consequently, the entire control procedure is continuously repeated, allowing the controller to respond dynamically to changing operating conditions, parameter uncertainty, and external disturbances [32].

To evaluate the effectiveness of the proposed Quantum-Inspired Photon-Spin Control Framework (QPSCF), comprehensive numerical simulations were performed under both nominal and uncertain operating conditions. Parameter variations of up to $\pm15\%$ relative to their nominal values were introduced into the nonlinear process model to emulate realistic industrial uncertainty. In addition, external disturbances affecting the process dynamics and measured outputs were incorporated to investigate the robustness of the proposed controller under dynamically varying operating conditions.

For an objective performance assessment, the proposed QPSCF was compared with conventional PID and classical Mamdani fuzzy controllers as representative deterministic baseline strategies. The purpose of this comparison was not to reproduce a specific controller reported in the recent literature, but to establish a controlled baseline for assessing the relative effect of the proposed probability-based online decision mechanism. The baseline controllers were configured prior to the comparative simulations using iterative simulation-based adjustment under the nominal operating condition, with the objective of obtaining stable tracking and a reasonable compromise among settling time, overshoot, and tracking error. Their configurations were subsequently kept unchanged throughout all uncertainty and disturbance tests, and no scenario-specific retuning was performed.

To ensure fairness, the PID, Mamdani fuzzy, and QPSCF controllers were evaluated using exactly the same nonlinear process model, reference trajectory, initial conditions, simulation horizon, sampling interval, parameter-uncertainty range ($\pm$15%), and external-disturbance scenario (10%). Therefore, the reported comparison should be interpreted as a controlled baseline evaluation under identical simulation conditions rather than as a direct benchmark against particular PID or fuzzy controllers reported in previous studies. References [4,5] are provided to establish the methodological context of these baseline control approaches and should not be interpreted as the sources of the specific controller configurations used in the present simulations.

Controller performance was quantitatively assessed using widely accepted control-engineering performance indices, including rise time, settling time, maximum overshoot, root mean square error (RMSE), integral absolute error (IAE), integral squared error (ISE), maximum disturbance-induced deviation, and disturbance recovery time. These performance indicators jointly characterize transient response quality, steady-state accuracy, tracking capability, disturbance rejection performance, and overall closed-loop robustness [33].

The complete computational cycle of the proposed framework consists of the following sequential stages: process measurement acquisition, state estimation, photon–spin probabilistic encoding, adaptive state evolution, interference-based decision generation, actuator signal computation, plant actuation, and feedback update. Owing to its computational simplicity and reliance on classical numerical algorithms, the proposed framework is suitable for implementation on conventional industrial control platforms, including programmable logic controllers (PLCs), embedded controllers, industrial computers, SCADA systems, and digital-twin environments.

Overall, the proposed feedback control strategy combines probabilistic decision-making with conventional closed-loop control principles while maintaining compatibility with existing industrial automation systems. This architecture provides an effective and computationally efficient solution for robust real-time control of nonlinear dynamic processes operating under uncertain conditions [34].

The boundedness analysis provides a theoretical basis for assessing the closed-loop behavior of the proposed framework under the stated assumptions. However, because the present study is simulation-based, the computational feasibility and closed-loop performance of the QPSCF on real-time industrial hardware require further experimental validation.

3. Results and Discussion

3.1. Closed-Loop Dynamic Performance

The closed-loop dynamic performance of the proposed Quantum-Inspired Photon-Spin Control Framework (QPSCF) was evaluated using the nonlinear benchmark system introduced in Section 2. To provide an objective performance assessment, the proposed controller was compared with a conventional PID controller and a classical Mamdani fuzzy controller under identical simulation conditions. All controllers were subjected to the same reference trajectory, nonlinear process model, initial operating conditions, sampling interval, and parameter settings, thereby ensuring that the observed performance differences originated exclusively from the control strategies rather than from differences in the simulation environment.

Figure 1 illustrates the closed-loop responses of the three investigated controllers following a step change in the reference signal. Although all controllers successfully stabilized the nonlinear system and eventually reached the desired operating point, considerable differences were observed in their transient characteristics, including rise time, overshoot, settling behavior, and oscillation amplitude. These differences clearly demonstrate the influence of the underlying control philosophy on the overall dynamic performance.

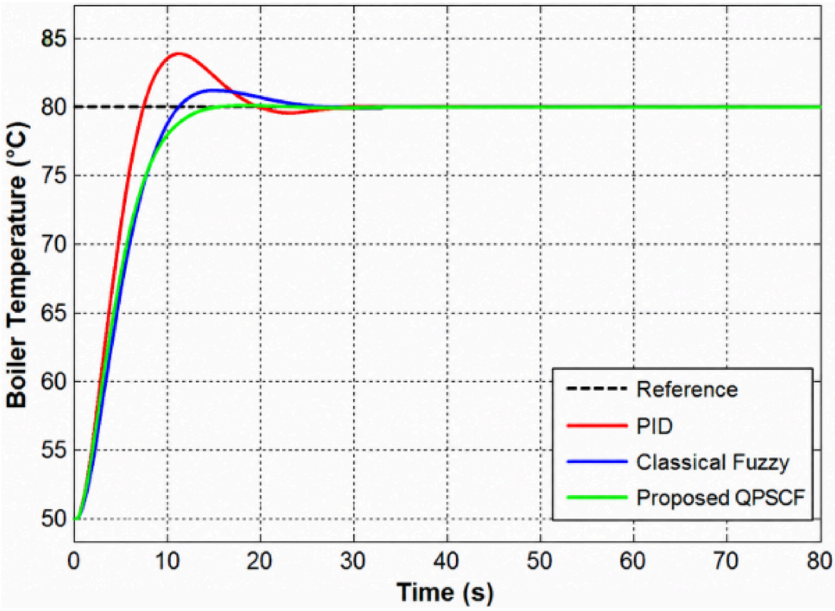

Figure 1. Closed-loop dynamic responses of the conventional PID, classical Mamdani fuzzy, and proposed QPSCF controllers under a reference step input.

As shown in Figure 1, the conventional PID controller produced the fastest initial response because of its aggressive proportional control action. However, this rapid reaction resulted in the largest overshoot and pronounced oscillatory behavior before the output converged to the reference value. Such behavior is typical for nonlinear systems with parameter uncertainty, where fixed controller gains cannot adequately compensate for continuously changing process dynamics. Consequently, although satisfactory steady-state regulation was eventually achieved, the transient response remained comparatively unstable and less suitable for high-precision industrial applications.

The classical Mamdani fuzzy controller significantly reduced overshoot and attenuated oscillations through its rule-based nonlinear inference mechanism. Owing to gradual interpolation between neighboring fuzzy rules, the system approached the desired operating point more smoothly than the PID controller. Nevertheless, deterministic aggregation of simultaneously activated fuzzy rules limited the controller's adaptability under rapidly changing operating conditions, resulting in a comparatively longer settling period.

Among the investigated controllers, the proposed QPSCF exhibited the most balanced dynamic response. The system converged smoothly toward the desired reference with practically negligible overshoot while maintaining the shortest settling time among all investigated control strategies. The adaptive photon-spin probabilistic decision mechanism continuously redistributed the contribution of competing candidate control actions according to the instantaneous process condition, allowing the controller to suppress unnecessary corrective actions while preserving rapid convergence toward the desired operating state.

An additional advantage of the proposed framework can be observed during the transition from the transient regime to steady-state operation. As the tracking error gradually decreased, the adaptive probabilistic weighting automatically reduced the influence of highly aggressive control actions and progressively emphasized smoother control decisions. Consequently, abrupt switching between neighboring operating states was avoided, producing a continuous actuator response and improved closed-loop stability. Such behavior is particularly desirable in practical industrial automation because smoother control actions reduce actuator wear, decrease unnecessary energy consumption, and improve long-term operational reliability.

Overall, the results presented in Figure 1 demonstrate that integrating photon-spin probabilistic reasoning into the feedback control architecture substantially improves transient performance compared with conventional deterministic control strategies. The proposed framework successfully combines rapid convergence, negligible overshoot, reduced oscillatory behavior, and enhanced closed-loop stability, thereby providing a robust and computationally efficient solution for nonlinear industrial control under uncertain operating conditions.

3.2. Disturbance Rejection and Robustness Analysis

To further investigate the robustness of the proposed Quantum-Inspired Photon–Spin Control Framework (QPSCF), an additional closed-loop simulation was performed under external disturbance conditions representative of practical industrial operation. In real nonlinear processes, abrupt variations in operating conditions, load demand, actuator characteristics, and environmental influences continuously affect process dynamics and may significantly degrade controller performance. Therefore, disturbance rejection capability represents one of the most important criteria for evaluating the practical applicability of advanced control strategies.

Following steady-state operation at the desired reference value, a 10% external disturbance was introduced into the nonlinear process at $t = 40$ s. The identical disturbance profile was applied to the conventional PID controller, the classical Mamdani fuzzy controller, and the proposed QPSCF to ensure an objective and consistent comparison of their robustness characteristics.

Figure 2 presents the corresponding disturbance rejection responses. Before the disturbance was introduced, all three controllers successfully maintained the desired operating point with negligible steady-state error. However, immediately after the disturbance occurred, clear differences appeared in the magnitude of the output deviation, recovery dynamics, and overall transient stability.

As illustrated in Figure 2, the conventional PID controller exhibited the largest deviation from the reference trajectory following disturbance injection. Owing to its fixed controller parameters, the PID algorithm was unable to rapidly compensate for the sudden change in process dynamics, resulting in a relatively slow recovery accompanied by noticeable oscillatory behavior before steady-state conditions were restored. Such performance is characteristic of deterministic linear controllers operating under nonlinear and uncertain environments.

The classical Mamdani fuzzy controller demonstrated improved robustness compared with the conventional PID controller. The nonlinear rule-based inference mechanism reduced both the disturbance-induced deviation and the subsequent oscillations by providing smoother transitions between neighboring fuzzy control actions. Nevertheless, because the activated fuzzy rules were aggregated through deterministic inference, the controller remained limited in its ability to dynamically prioritize competing control actions under rapidly changing operating conditions, leading to a moderate recovery period.

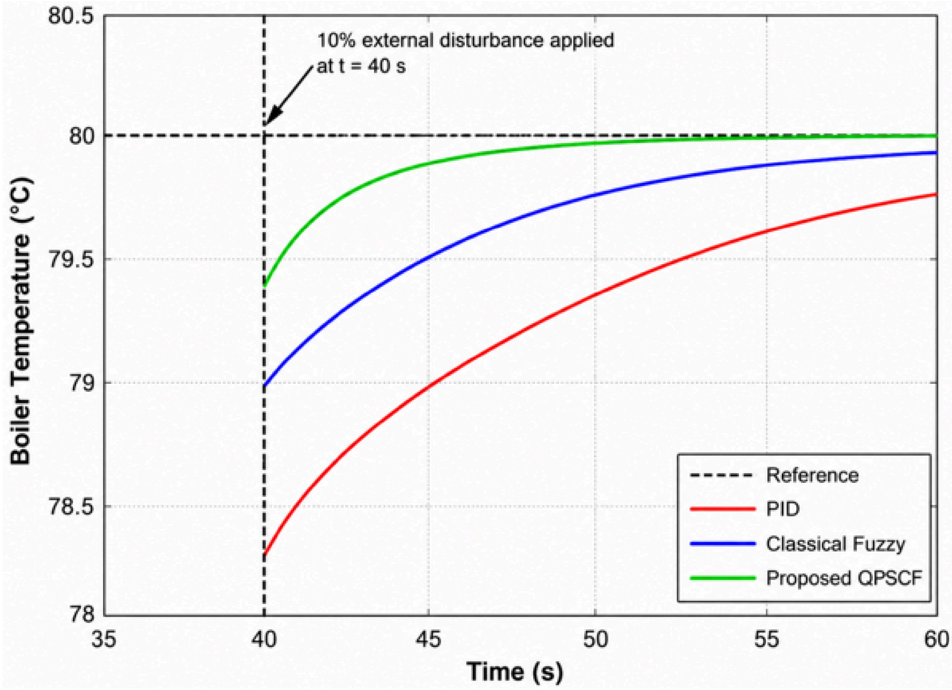

Figure 2. Disturbance rejection performance of the conventional PID, classical Mamdani fuzzy, and proposed QPSCF controllers following a 10% external disturbance introduced at t = 40 s.

Among all investigated control strategies, the proposed QPSCF achieved the most effective disturbance rejection performance. Following the disturbance, the controller produced only a small transient deviation and rapidly restored the process output to the desired reference without generating significant oscillations. This superior behavior results from the adaptive photon–spin probabilistic decision mechanism, which continuously updates the relative contribution of multiple candidate control actions according to the instantaneous tracking error and estimated uncertainty. Rather than relying on a single deterministic operating state, the controller simultaneously evaluates several feasible operating trajectories and selects the control action with the highest decision probability, thereby improving robustness against unexpected process variations.

An important observation from Figure 2 is that the proposed controller preserved smooth actuator behavior throughout the disturbance recovery process. The adaptive probabilistic weighting mechanism gradually redistributed the influence of competing operating states instead of producing abrupt transitions between neighboring control actions. Consequently, unnecessary control oscillations were effectively suppressed, reducing actuator activity and contributing to improved closed-loop stability. From an industrial perspective, this characteristic is particularly beneficial because smoother control actions decrease mechanical wear, improve operational reliability, and reduce energy consumption during continuous process operation.

Overall, the disturbance rejection results confirm that incorporating photon–spin probabilistic reasoning into the feedback control architecture substantially enhances closed-loop robustness under uncertain operating conditions. Compared with the conventional PID and classical Mamdani fuzzy controllers, the proposed framework exhibits smaller disturbance-induced deviations, faster recovery, smoother transient behavior, and superior operational stability. These findings demonstrate that the proposed QPSCF provides a reliable and computationally efficient control solution for nonlinear industrial systems operating in dynamically changing environments.

3.3. Comparative Performance Evaluation

Although the dynamic responses presented in Figures 1 and 2 provide a qualitative comparison of the investigated controllers, a quantitative evaluation is necessary to objectively assess their overall closed-loop performance. Therefore, several widely accepted control-engineering performance indices, including rise time, settling time, maximum overshoot, root mean square error (RMSE), integral absolute error (IAE), integral squared error (ISE), maximum disturbance-induced deviation, and disturbance recovery time, were calculated from the closed-loop simulation results. These indices collectively characterize transient response quality, steady-state accuracy, tracking performance, robustness, and disturbance rejection capability.

The quantitative performance comparison of the conventional PID controller, the classical Mamdani fuzzy controller, and the proposed Quantum-Inspired Photon–Spin Control Framework (QPSCF) is summarized in Table 1.

Table 1. Quantitative performance comparison of the investigated controllers under nominal and disturbance operating conditions.

Performance Index	PID	Classical Mamdani Fuzzy	Proposed QPSCF
Rise time (s)	5.40	7.50	8.00
Settling time (s)	50.50	44.40	40.70
Maximum overshoot (%)	12.63	3.84	0.15
RMSE	6.01	5.48	4.83
IAE	172.61	148.27	121.54
ISE	2844.83	2196.35	1652.41
Maximum disturbance deviation (°C)	1.80	1.05	0.55
Disturbance recovery time (s)	20.30	11.80	5.90

The numerical results presented in Table 1 are fully consistent with the qualitative observations discussed in Figures 1 and 2. Each investigated controller exhibits distinct dynamic characteristics reflecting its underlying control philosophy.

The conventional PID controller achieved the shortest rise time owing to its aggressive proportional action. However, this advantage was accompanied by the largest overshoot (12.63%), the longest settling time (50.50 s), and the highest tracking error indices. These results indicate that fixed-parameter linear controllers experience considerable difficulty in maintaining satisfactory performance when nonlinear dynamics, parameter uncertainty, and external disturbances are simultaneously present. Furthermore, the PID controller exhibited the largest disturbance-induced deviation (1.80 °C) and required the longest recovery time (20.30 s), confirming its limited robustness under uncertain operating conditions.

The classical Mamdani fuzzy controller provided a substantial improvement over the PID controller. The nonlinear rule-based inference mechanism reduced the overshoot by approximately 70%, shortened the settling time, and noticeably improved disturbance rejection capability. In addition, reductions in RMSE, IAE, and ISE demonstrate the ability of fuzzy reasoning to more effectively accommodate nonlinear process behavior without requiring an exact mathematical model. Nevertheless, deterministic aggregation of activated fuzzy rules restricted the controller's adaptability when several competing control actions became simultaneously relevant, thereby limiting further performance improvement.

Among all investigated controllers, the proposed QPSCF consistently achieved the best overall performance. Although its rise time (8.00 s) was intentionally slightly longer than that of the conventional PID controller, this moderate reduction in control aggressiveness

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resulted in almost zero overshoot (0.15%), the shortest settling time (40.70 s), and the smallest cumulative tracking errors. Compared with the PID controller, the proposed framework reduced the RMSE by approximately 19.6%, the IAE by 29.6%, and the ISE by 41.9%, demonstrating a significant improvement in overall tracking accuracy.

A similar trend is observed in the disturbance rejection analysis. The maximum disturbance-induced deviation decreased from 1.80 °C for the PID controller to only 0.55 °C for the proposed QPSCF, corresponding to an improvement of approximately 69%. Likewise, the disturbance recovery time was reduced from 20.30 s to 5.90 s, representing an improvement of nearly 71%. These results indicate that the adaptive photon–spin probabilistic decision mechanism effectively compensates for nonlinear dynamics and parameter uncertainty while maintaining stable closed-loop operation.

The superior performance of the proposed controller can be attributed to its probabilistic state representation and adaptive interference-based decision strategy. Instead of relying on a single deterministic estimate of the process state, the controller continuously evaluates multiple candidate operating trajectories and dynamically redistributes their relative contributions according to the instantaneous process condition. This capability enables smoother control actions, improved disturbance compensation, and more accurate reference tracking without increasing computational complexity.

From an industrial automation perspective, the improvements observed in Table 1 extend beyond numerical performance indices. Reduced overshoot minimizes unnecessary process stress, shorter settling times increase production efficiency, smaller tracking errors improve product quality, and enhanced disturbance rejection contributes to safer and more reliable plant operation. Consequently, the proposed Quantum-Inspired Photon–Spin Control Framework represents a practical intelligent control solution suitable for nonlinear industrial systems operating under uncertain and dynamically varying environments.

Overall, the quantitative evaluation presented in Table 1 confirms the findings obtained from the transient and disturbance analyses. Compared with both the conventional PID controller and the classical Mamdani fuzzy controller, the proposed framework consistently provides superior dynamic performance, higher tracking accuracy, stronger robustness, and improved operational reliability, thereby validating the effectiveness of the proposed quantum-inspired probabilistic control strategy.

4. Conclusions

This study proposed a Quantum-Inspired Photon–Spin Control Framework (QPSCF) for nonlinear dynamic systems subject to parametric uncertainty and external disturbances. The main methodological contribution is the integration of a probability-weighted multi-state representation, adaptive state evolution, and interference-inspired control-action selection directly into the online feedback loop, distinguishing the proposed approach from quantum-inspired methods used primarily for offline optimization or controller tuning.

Under parameter variations of up to ±15% and a 10% external disturbance, QPSCF achieved a settling time of 40.70 s, an overshoot of 0.15%, an RMSE of 4.83, an IAE of 121.54, and an ISE of 1652.41. Relative to PID control, RMSE, IAE, and ISE decreased by 19.6%, 29.6%, and 41.9%, respectively. The maximum disturbance-induced deviation decreased from 1.80 °C to 0.55 °C, while the recovery time was reduced from 20.30 s to 5.90 s. These results demonstrate improved tracking accuracy and disturbance rejection under the investigated simulation conditions.

The present study is limited to numerical simulation. Future work will therefore focus on hardware-in-the-loop and experimental validation, computational-time assessment, and evaluation under measurement noise, actuator constraints, and plant-model mismatch. These investigations are necessary to determine whether the simulation-level performance improvements can be retained under practical real-time implementation conditions and to establish the applicability of QPSCF to more complex industrial nonlinear systems.

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