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Spatial Scaling Effects in Water Demand †

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Abstract

Residential water demand exhibits stochastic variability across all spatial and temporal scales, making probabilistic approaches essential for realistic modelling. Using scaling laws enables the derivation of statistics on aggregated demand from single-user data while maintaining information about spatial correlation. This study highlights how scaling laws define the dependence of mean and variance on the number of users and also reveals multiscale dynamics. In particular, variance growth at variable exponents and the emergence of multimodal distributions are shown. Furthermore, the Poisson Rectangular Pulse (PRP) model can reproduce these features by introducing frequency-use patterns that reflect the non-homogeneous nature of water demand.

Keywords: water demand; scaling laws; multiscale analysis; PRP models

1. Introduction

PRP stochastic modelling of instantaneous water demand, first proposed by Buchberger et al. [1,2], involves parameter estimation from observed consumption data [3]. A critical challenge arises from spatial scaling effects, which can distort parameter estimates and make them dependent on the number of aggregated users rather than on single-user behaviour. Simulations based on these parameters can adequately reproduce aggregated demand, although the parameters lose physical meaning and cannot be gen-eralised across different scales, as highlighted by Guercio et al. [4].

Empirical analyses of residential consumption data [5,6] also enabled the derivation of scaling laws, which showed that the mean scales linearly with the number of users, whereas second-order statistics can exhibit nonlinear behaviours related to cross-correlations among single-user demands. Moreover, these studies showed that the Pearson cross-correlation coefficient varies with the number of aggregated users in each group, increasing toward a limiting value of 1, which is reached more quickly when the cross-correlation between single-user demands is higher [7].

Scaling laws make it possible to determine water demand statistics at different scales, but an additional hypothesis concerns the probability distribution that best represents demand data. In this regard, the gamma distribution has often been considered in the literature [8,9]. Alternatively, Creaco et al. [10] proposed a beta probability distribution model with tuneable bounds for fitting peak demand at different scales of aggregation, reproducing statistics not only in terms of mean and variance but also in terms of skewness.

The objective of this work is to investigate the multiscale behaviour of instantaneous water demand data and its effects on variance scaling, cross-correlation, and probability

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distributions. Furthermore, the Poisson Rectangular Pulse (PRP) model is employed to provide a numerical explanation of these multiscale properties.

2. Materials and Methods

To investigate the multiscale properties of water demand, we used the dataset of 82 residential households in Latina (Italy), monitored over three years with a temporal resolution of 60 s [3–7]. The sample, consisting of families with similar socio-economic profiles, ensured homogeneity in lifestyle and consumption habits.

Scaling laws were derived to establish the relationships between the mean, variance, and correlation of demand and the number of users at different aggregation levels. The analysis was based on an extensive statistical evaluation of consumption data, distinguishing between weekdays and weekends as well as winter and summer seasons. Special attention was given to the peak hour (here 7:00–8:00), when consumption reaches its maximum and fluctuations can be most pronounced, since this interval typically serves as the reference condition for the design and sizing of water distribution networks.

In parallel, simulations were performed using the PRP model to assess its ability to numerically reproduce the observed scaling effects, specifically to explain the multiscale properties of variance and cross-correlation and probability distributions that emerge un-der specific configurations of the frequency parameter λ of the model.

3. Results and Discussion

3.1. Scaling Laws and Probability Distribution

The analyses confirmed that the scaling law for the mean follows a linear trend with the number of users, as theoretically expected. In contrast, the variance does not grow uniformly but follows variable exponents depending on the minimum and maximum values of the aggregation range. At low aggregation levels, variance increases almost linearly, reflecting weak correlations among users. As the number of users increases, correlations strengthen and the scaling law tends toward a quadratic behaviour (Figure 1).

Number of users		Variance
min	max	exponent
1	10	1.04
1	100	1.22
1	400	1.46
1	700	1.56
1	1358	1.67
100	300	1.68
400	600	1.86
600	1200	1.91
600	1358	1.92
1200	1358	1.93

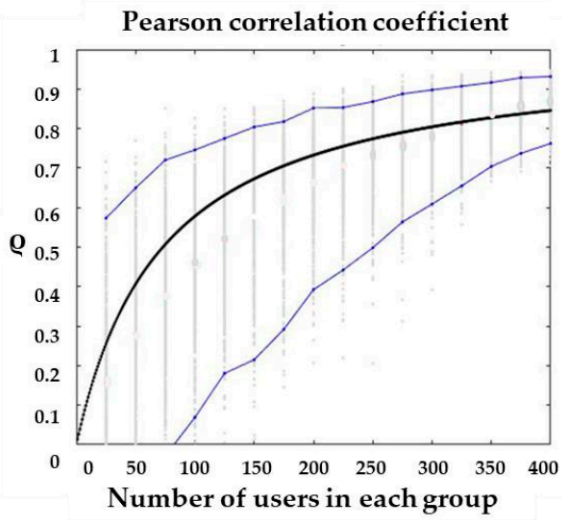

Figure 1. (**Left**): Table reporting the exponent of the variance scaling law as a function of the aggregation range of users. (**Right**): Graph showing the Pearson correlation coefficient across aggregation levels (bold line).

These results demonstrate that variance of water demand is genuinely multiscale: there is no single exponent valid across all aggregation ranges.

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Moreover, probability density functions of water demand during the peak hour showed a transition from unimodal shapes, typical of low aggregation, to multimodal distributions as the number of aggregated users increased (Figure 2). This multimodality reflects the existence of user subgroups exhibiting non-synchronous yet partially correlated behaviours.

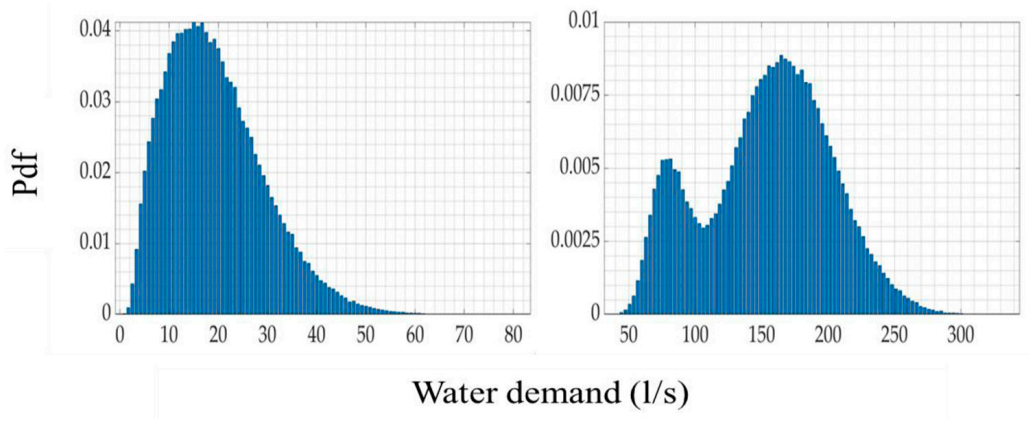

Figure 2. Probability density function of water demand for increasing levels of aggregation: 100, 800 users (from left to right).

3.2. PRP Model

PRP simulations further clarified these results. Both single-frequency scenarios and simulations with different frequencies assigned to separate user groups—but not overlapping in time—produced regular unimodal distributions. In contrast, when frequencies were grouped into distinct subintervals within the same time window, the simulations reproduced the emergence of multimodal distributions. The exponent of the variance scaling law reflects this behaviour, being close to 1 in case (a) and about 1.91 in case (b), which denotes strong correlation.

This highlights the importance of considering non-homogeneous temporal structures in probabilistic water demand modelling and indicates that, during peak periods, analysis at the hourly scale should be replaced by a finer temporal resolution to capture the underlying dynamics (Figure 3).

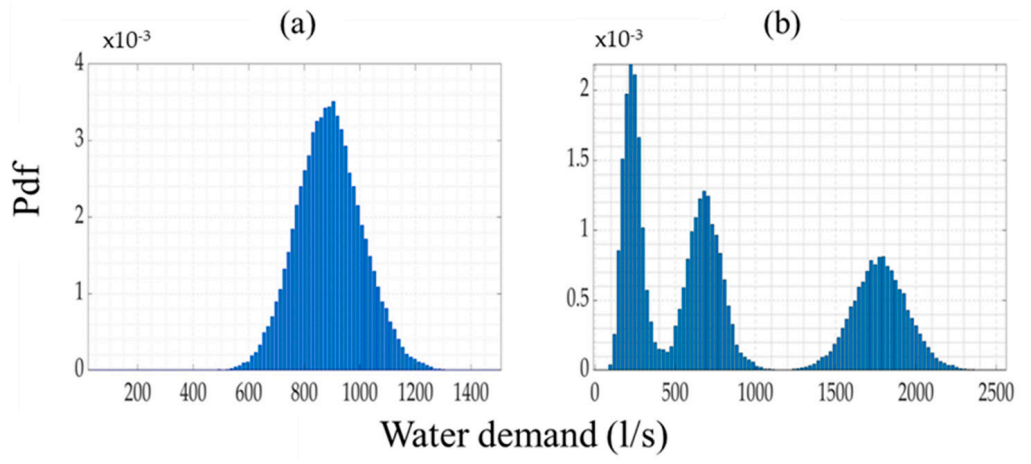

Figure 3. (**a**) PRP simulation without subintervals and with three randomly distributed frequency parameters λ; (**b**) PRP simulation within a time window divided into three subintervals, each associated with a distinct λ parameter.

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4. Conclusions

This study has shown that residential water demand at peak hour exhibits genuine multiscale behaviour, with statistical properties that cannot be captured by a single scaling exponent across all aggregation levels. While the mean grows linearly with the number of users, the variance follows exponents that change with aggregation, reflecting the scale-dependent nature of cross-correlations between groups with different numbers of users. Probability distributions further highlight this multiscale character, evolving from unimodal to multimodal forms as aggregation increases, thus revealing the presence of subgroups of users with non-homogeneous consumption characteristics.

The PRP model successfully reproduced these features when multiple frequency parameters were introduced, confirming its ability to capture both the differentiated variance growth and the emergence of multimodality. This result emphasises the importance of incorporating non-homogeneous temporal structures into stochastic demand model-ling, especially during peak hours when high variability and partial synchrony among users play a critical role.

Overall, the findings contribute to a better understanding of the spatial scaling effects in residential water demand and highlight the need for probabilistic models that explicitly account for multiscale variability. In practical terms, scaling laws and the PRP model can support utilities in deriving aggregate demand scenarios from individual user data, improving demand forecasts across various spatial scales. This enables better peak demand estimation and supports more reliable water network designs and real-time management strategies under variable consumption patterns.

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