

Multi-Dimensional Taylor Network-Based Predefined-Time Output-Feedback Adaptive Control with Full-State Error Constraints for PMSM Drives in Electric Vehicles [†]

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Abstract

The accelerating adoption of electric vehicles (EVs) has positioned them among the fastest-growing sectors in the electricity market. Since reliability, energy efficiency, and robustness are the fundamental criteria in motor drive selection, the permanent magnet synchronous motor (PMSM) has emerged as a preferred choice for EV applications. Nevertheless, achieving high-performance control of PMSM systems remains challenging due to nonlinear dynamics, parameter uncertainties, and external disturbances. To address these issues, this paper proposes a predefined-time output-feedback tracking control strategy for PMSMs subject to full-state error constraints, unknown nonlinear dynamics, external disturbances, and unmeasured states. Multi-dimensional Taylor Networks (MTNs) are employed to approximate unknown nonlinearities, while MTN-based observers are designed to estimate unmeasured states. The proposed controller integrates predefined-time stability theory, a general potential Lyapunov function, dynamic surface control (DSC), and backstepping to guarantee constraint satisfaction and rapid convergence. A hyperbolic tangent function is incorporated to eliminate singularities, and a predefined-time filter is introduced to mitigate the computational complexity of recursive backstepping. Theoretical analysis based on Lyapunov methods proves that all closed-loop signals remain bounded and that the tracking error converges to zero within a prespecified time. Simulation results confirm the effectiveness, robustness, and practical feasibility of the proposed approach for PMSM-driven EV applications.

Keywords: electric vehicles (EVs); permanent magnet synchronous motor (PMSM); adaptive predefined-time output-feedback controller; Multi-dimensional Taylor Network (MTN); error constraints



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1. Introduction

Permanent magnet synchronous motors (PMSMs) are widely used in electric vehicles, aerospace, and robotics due to their high efficiency, compact structure, and excellent torque characteristics [1]. Despite these advantages, PMSM systems exhibit strong nonlinearity, multivariable coupling, and sensitivity to parameter variations and external disturbances, which complicate accurate control. To improve PMSM performance, numerous control strategies have been developed, including sliding-mode control, disturbance rejection control, adaptive control, and backstepping-based methods [2–5]. Because accurate PMSM

modeling is often difficult in practice, intelligent control approaches such as adaptive fuzzy, adaptive neural-network and Multi-dimensional Taylor Networks (MTNs)-based methods have been extensively investigated [6–8]. Among these control methods, the MTN control method has attracted growing interest, and many interesting results have been reported. Meanwhile, ensuring fast convergence while satisfying state and tracking error constraints has become increasingly important in practical applications. Barrier Lyapunov function-based techniques have been successfully applied to nonlinear systems with full-state constraints, including PMSM systems [9–13]; however, predefined-time stability under such constraints remains insufficiently studied.

Recent advances in finite-time, fixed-time, and predefined-time control provide powerful tools for achieving rapid and predictable convergence [14–16]. Although adaptive finite-time control methods have been proposed for PMSM systems [17], most existing works assume full state availability. Output-feedback and observer-based methods have been explored to address state unmeasurability [18], yet predefined-time Multi-dimensional Taylor Network (MTN) adaptive control for PMSM systems with full-state error constraints and unmeasurable states has not been fully addressed.

Recently, significant progress has been made in predefined-time and fixed-time control strategies for nonlinear systems and PMSM drives. For instance, a deep-neural-network-based output-dependent intermittent control framework was proposed in [19], where deep neural approximators were employed to enhance robustness and learning capability for uncertain nonlinear systems. In addition, an adaptive barrier-function-based predefined-time sliding mode control scheme for PMSM drives was developed in [20], achieving disturbance suppression and predefined-time convergence through barrier Lyapunov techniques. These studies demonstrate the growing interest in combining learning-based approximation and constraint-aware predefined-time control. However, most existing approaches rely on sliding-mode structures or deep neural networks, which may increase implementation complexity or require intensive training procedures. In contrast, the proposed method employs a MTN-based observer together with predefined-time adaptive output-feedback control, providing a computationally efficient alternative while ensuring constraint satisfaction and predictable transient performance.

Motivated by these limitations, this work proposes an output-feedback predefined-time control strategy for PMSM systems with unknown nonlinearities and full-state error constraints. By employing Multi-dimensional Taylor Network approximation and barrier Lyapunov functions, the proposed method guarantees boundedness of all signals and convergence within a user-specified predefined time, as verified through rigorous analysis and simulations. Key contributions are as follows:

An output-feedback predefined-time control strategy is proposed for PMSM systems with unknown nonlinear dynamics and full-state error constraints; by combining a predefined-time dynamic filtering backstepping design with an approximation-based adaptive MTN method, the proposed controller guarantees bounded tracking errors and an explicitly assigned settling time while maintaining a simple structure and significantly reducing computational burden compared with existing methods [21].

2. Problem Formulation and Preliminaries

2.1. Description of the Problem

The mathematical representation of the PMSM system given in [1] can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t) \\ \dot{x}_2 = f_2(x) + x_3 + d_2(t) \\ \dot{x}_3 = f_3(x) + b_4 u_q + d_3(t) \\ \dot{x}_4 = f_4(x) + c_3 u_d + d_4(t) \\ y = x_1 \end{cases} \quad (1)$$

The system states are defined as $x_1 = \theta$, $x_2 = \omega$, $x_3 = i_q$, and $x_4 = i_d$. The associated parameters are $a_1 = \frac{3p\Phi_f}{2}$, $a_2 = \frac{3p(L_d - L_q)}{2}$, $b_1 = -\frac{R_s}{L_q}$, $b_2 = -\frac{pL_d}{L_q}$, $b_3 = -\frac{p\Phi_f}{L_q}$, $b_4 = \frac{1}{L_q}$, $c_1 = -\frac{R_s}{L_d}$, $c_2 = \frac{pL_q}{L_d}$, and $c_3 = \frac{1}{L_d}$. The nonlinear dynamics are described by $f_2(x) = -\frac{B}{J}x_2 + \frac{a_1}{J}x_3 + \left(\frac{a_2}{J} - 1\right)x_3x_4 - \frac{T_L}{J}$, $f_3(x) = b_3x_2 + b_1x_3 + b_2x_2x_4$, and $f_4(x) = c_1x_4 + c_2x_2x_3$. For $i = 1, \dots, 4$, $f_i(x)$ values represent the system nonlinearities, while $d_i(t)$ values denote unknown external disturbances. The control inputs are the d - and q -axis voltages u_d and u_q , respectively, the system output is denoted by y , and the state vector is $x = [x_1, x_2, x_3, x_4]^T$.

Control Objective: For the PMSM system (1), a predetermined-time control method is proposed that handles unknown nonlinear dynamics while enforcing full-state error constraints. This approach ensures the state x reaches a neighborhood of the equilibrium x_e within time T_d , and all tracking errors z_i remain within the bounds set by the user-defined functions $\psi_{x,i}$ and $\psi_{y,i}$.

Assumption 1. Let Π_x^0 be an open set containing the equilibrium states x_e , the initial state x_0 , and the reference trajectory y_d . For system (1), each $f_i(x)$ has derivatives $f_i^p(x)$ that are bounded within the set Π_x for $p = 1, \dots, 4 - i$.

Assumption 2. The functions $\psi_{x,i}$ and $\psi_{y,i}$ are continuous within the compact sets $\Pi_{\psi_{x,i}}$ and $\Pi_{\psi_{y,i}}$ and are at least twice differentiable. These sets are defined as $\Pi_{\psi_{x,i}} = \{\psi_{x,i}^{(2)} : \psi_{x,i}^2 + \dot{\psi}_{x,i}^2 + \ddot{\psi}_{x,i}^2 \leq \Lambda_{\psi_{x,i}}^*\} \subset \mathfrak{R}^3$, $\Pi_{\psi_{y,i}} = \{\psi_{y,i}^{(2)} : \psi_{y,i}^2 + \dot{\psi}_{y,i}^2 + \ddot{\psi}_{y,i}^2 \leq \Lambda_{\psi_{y,i}}^*\} \subset \mathfrak{R}^3$, where $\Lambda_{\psi_{x,i}}^*$ and $\Lambda_{\psi_{y,i}}^*$ are known positive constants, and $\bar{\psi}_{x,i}^{(2)} = [\psi_{x,i} + \dot{\psi}_{x,i} + \ddot{\psi}_{x,i}]^T$, $\bar{\psi}_{y,i}^{(2)} = [\psi_{y,i} + \dot{\psi}_{y,i} + \ddot{\psi}_{y,i}]^T$.

2.2. Multi-Dimensional Taylor Network (MTN)

In this paper, MTNs are used to approximate unknown nonlinear dynamics that occur during the controller design process. The structure of MTN is shown in Figure 1.

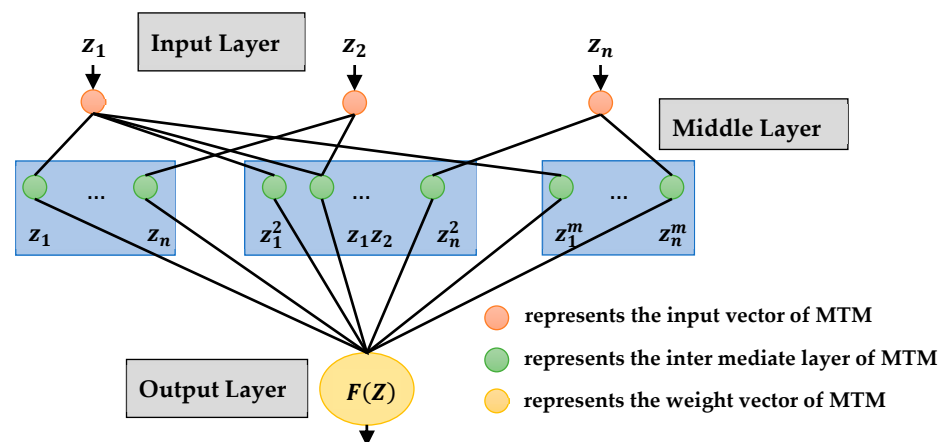


Figure 1. The Structure of MTN.

Lemma 1 ([19]). The continuous function $f(Z) : R^n \rightarrow R$ will be approximated using the specified MTN mentioned below

$$f(Z) = \theta^T \psi(Z) \quad (2)$$

where $Z = [Z_1, Z_2, \dots, Z_l]^T \in \Omega_Z \subset R^q$ denotes the input vector, the input dimension of MTN is denoted by q , the weight vector is denoted by $\theta = [\theta_1, \theta_2, \dots, \theta_l]^T \in R^l$, $l > 1$ is the quantity of nodes within the network, $\psi(Z) = [\psi_1(Z), \psi_2(Z), \dots, \psi_l(Z)]^T$ represents the vector of basic functions, and $\psi_i(Z)$ is chosen to be the frequently utilized Gaussian function. In this case, MTN has the capability to estimate any continuous function $f(Z)$ within the compact set $\Omega_Z \subset R^q$. For any precision $\varepsilon^* > 0$, the following condition is met:

$$f(Z) = \theta^{*T} \psi(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z \subset R^q \quad (3)$$

where the approximation error denoted by $\varepsilon(Z)$ satisfies $|\varepsilon(Z)| \leq \varepsilon^*$. The ideal constant weight vector is denoted by θ^* and is defined as: $\theta^* = \arg \min_{\theta \in R^l} \left\{ \sup_{Z \in \Omega_Z} f(Z) - \theta^T \psi(Z) \right\}$.

Lemma 2 ([1]). For the system $\dot{x}(t) = f(x, t)$, $x(0) = x_0$, a Lyapunov function $V(x)$ satisfies

$$\dot{V} \leq -\frac{\pi}{\lambda T_d} V^{1+\frac{\lambda}{2}} - \frac{A_0 \pi}{\lambda T_d} V^{1-\frac{\lambda}{2}} + \Delta \quad (4)$$

where $0 < \lambda < 1$, $T_d > 0$, $\Delta > 0$; system $\dot{x}(t)$ is a practical predefined-time stable.

2.3. Generalized Potential Lyapunov Function

A generalized potential Lyapunov function, adopted from [1] is employed to address the holomorphic error constraints: $V_{b,i} = \frac{1}{2} \gamma_i^2$, $\gamma_i = \frac{\psi_{x,i} \psi_{y,i} z_i}{(\psi_{x,i} - z_i)(\psi_{y,i} + z_i)}$, where γ_i is the transition error, z_i is the system tracking error, and $\psi_{x,i}, \psi_{y,i} > 0$ satisfy $-\psi_{y,i}(0) < z_i(0) < \psi_{x,i}(0)$. It follows that $\gamma_i = 0$ and $V_{b,i} = 0$ only when $z_i = 0$, while $V_{b,i} > 0$ otherwise. Moreover, as $z_i \rightarrow -\psi_{y,i}$ or $z_i \rightarrow \psi_{x,i}$, both γ_i and $V_{b,i}$ tend to infinity.

3. Main Results

In this section, we first give a MTN state observer to estimate the immeasurable states of the PMSM system (1). Then an adaptive MTN output feedback controller is developed by using the backstepping control design technique and the predefined-time stability theory.

3.1. MTN State Observer Design

Using (5), the approximated PMSM system in (1) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 + d_1(t) \\ \dot{x}_2 = \theta_2^{*T} \psi_2(x) + \varepsilon_2(x) + x_3 + d_2(t) \\ \dot{x}_3 = \theta_3^{*T} \psi_3(x) + \varepsilon_3(x) + b_4 u_q + d_3(t) \\ \dot{x}_4 = \theta_4^{*T} \psi_4(x) + \varepsilon_4(x) + c_3 u_d + d_4(t) \\ y = x_1 \end{cases} \quad (5)$$

where θ_i^* are ideal parameter vectors, $\varepsilon_i(x)$ are approximation errors, and $|\varepsilon_i(x)| \leq \varepsilon_i^*$ are unknown positive constants, with $i = 2, \dots, 4$.

For analytical convenience, system (5) is reformulated as follows:

$$\dot{x} = Ax + Ky + \sum_{i=2}^4 B_i \theta_i^{*T} \psi_i(x) + \varepsilon(x) + Qu + D, \quad i = 2, \dots, 4 \quad (6)$$

where $x = [x_1, x_2, x_3, x_4]^T$, $A = \begin{bmatrix} -l_1 & 1 & 0 & 0 \\ -l_2 & 0 & 1 & 0 \\ -l_3 & 0 & 0 & 0 \\ -l_4 & 0 & 0 & 0 \end{bmatrix} \in R^{n \times n}$, $K = [l_1, l_2, l_3, l_4]^T$, $B_i = [0_{i-1}, 1, 0_{n-i}]^T \in R^n$, $\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4]^T \in R^n$, $Q = [0, 0, b_4, c_3]^T$, $u = [0, 0, u_q, u_d]^T$, $D = [d_1, d_2, d_3, d_4]^T \in R^n$.

Because the system states are not directly measurable, an MTN-based state observer is developed to estimate the unknown states required for control.

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + l_1(y - \hat{y}) \\ \dot{\hat{x}}_2 = \hat{\theta}_2^T \psi_2(\hat{x}) + \hat{x}_3 + l_2(y - \hat{y}) \\ \dot{\hat{x}}_3 = \hat{\theta}_3^T \psi_3(\hat{x}) + b_4 u_q + l_3(y - \hat{y}) \\ \dot{\hat{x}}_4 = \hat{\theta}_4^T \psi_4(\hat{x}) + c_3 u_d + l_4(y - \hat{y}) \\ y = x_1 \end{cases} \quad (7)$$

where $\hat{x}_i$ denotes the estimated value of x_i and $\hat{x} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T$. The estimated output signal is denoted by $\hat{y}$, $\hat{\theta}_i$ represents the estimate of the true parameter θ_i^* , and the corresponding estimation error is defined as $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$.

Equation (7) may be rewritten in the following form

$$\dot{\hat{x}} = A\hat{x} + Ky + \sum_{i=2}^4 B_i \tilde{\theta}_i^T \psi_i(\hat{x}) + Qu \quad (8)$$

The error of the state observer is defined as $e = x - \hat{x}$, and the error dynamics associated with the state observer are derived: $\dot{e} = Ae + \sum_{i=2}^4 B_i \tilde{\theta}_i^T \psi_i(\hat{x}) + \varepsilon + D$. The matrix A is Hurwitz and satisfies the Lyapunov equation: $A^T P + PA = -Q$, with Q being any positive definite matrix and P as a positive definite matrix.

3.2. Output-Feedback-Based Predefined-Time Control Design

The error variable is defined as $z_1 = x_1 - y_d$, $z_i = \hat{x}_i - w_{i-1}$, $i = 2, 3$, $z_4 = \hat{x}_4$, $s_i = w_i - \alpha_i$.

In this system, y_d denotes the reference signal tracked by the output y , which serves as a smoothing function. The tracking error is z_1 , while z_i represents the virtual tracking errors. s_i is the filtering error, and w_i and α_i correspond to the filtered output and input signals, respectively.

$$\begin{cases} \dot{\gamma}_1 = \gamma_{1,1} + \gamma_{1,2}(z_2 + s_1 + \alpha_1 + e_2 + d_1 - \dot{y}_d) \\ \dot{\gamma}_2 = \gamma_{2,1} + \gamma_{2,2}(z_3 + s_2 + \alpha_2 + \hat{\theta}_2^T \psi_2(\hat{x}) + l_2 e_1 - \dot{w}_1) \\ \dot{\gamma}_3 = \gamma_{3,1} + \gamma_{3,2}(\hat{\theta}_3^T \psi_3(\hat{x}) + b_4 u_q + l_3 e_1 - \dot{w}_2) \\ \dot{\gamma}_4 = \gamma_{4,1} + \gamma_{4,2}(\hat{\theta}_4^T \psi_4(\hat{x}) + c_3 u_d + l_4 e_1) \end{cases} \quad (9)$$

Let $\gamma_{i,1} = \frac{-\psi_{x,i} \psi_{y,i} z_i^2}{(\psi_{x,i} - z_i)^2 (\psi_{y,i} + z_i)^2} + \frac{\psi_{x,i} \psi_{y,i} z_i^2}{(\psi_{x,i} - z_i) (\psi_{y,i} + z_i)^2}$, $\gamma_{i,2} = \frac{-\psi_{x,i} \psi_{y,i} (\psi_{x,i} \psi_{y,i} + z_i^2)}{(\psi_{x,i} - z_i)^2 (\psi_{y,i} + z_i)^2} > 0$.

To reduce computational complexity, a predefined-time filter can be employed as follows:

$$\dot{w}_i = -\rho s_i^{1+\eta} - \bar{\rho} s_i^{1-\eta} \tanh\left(\frac{\rho s_i^{2-\eta}}{\bar{s}_i}\right) - \omega_i s_i, \quad w_i(0) = \alpha_i(0) \quad (10)$$

where $\omega_i \geq 1$ and $\bar{s}_i > 0$ denote design constants for $i = 1, \dots, 3$. The predefined-time filters are applied at each design step to regulate the virtual control signals.

Step 1: Construct Lyapunov functions using general potential functions.

$$V_1 = e^T P e + V_{b,1} + \frac{1}{2} \tilde{\Theta}^2 \quad (11)$$

where $\tilde{\Theta} = \Theta^* - \hat{\Theta}$ represents the estimation error, with $\hat{\Theta}$ denoting the estimate of Θ^* . Deriving from V_1 , we have

$$\dot{V}_1 = -\lambda_{\min}(Q) \|e\|^2 + 2e^T P \left[\sum_{i=2}^4 B_i \tilde{\theta}_i^T \psi_i(\hat{x}) + \varepsilon + D \right] + \gamma_1 [\gamma_{1,1} + \gamma_{1,2}(z_2 + s_1 + \alpha_1 + e_2 + d_1 - \dot{y}_d)] - \tilde{\Theta} \dot{\hat{\Theta}} \quad (12)$$

Using Young's inequality, we get

$$\begin{aligned} 2e^T P \left[\sum_{i=2}^4 B_i \tilde{\theta}_i^T \psi_i(\hat{x}) + \varepsilon + D \right] &\leq (n+2) \|P\|^2 \|e\|^2 + \sum_{i=2}^4 \tilde{\theta}_i^T \tilde{\theta}_i + \varepsilon^{*2} + D^{*2} \\ \gamma_1 \gamma_{1,2} (z_2 + s_1 + e_2) &\leq \frac{3}{2} \gamma_1^2 \gamma_{1,2}^2 + \frac{1}{2} \|e_2\|^2 + \frac{1}{2} z_2^2 + \frac{1}{2} s_1^2 \\ \gamma_1 \gamma_{1,2} d_1 &\leq \gamma_1^2 \gamma_{1,2}^2 d_1^{*2} + \frac{1}{4} \end{aligned} \quad (13)$$

define $\Theta^* = \max\{\|\theta_i^*\|, \|d_i^*\|\}$, where $i = 1, \dots, 4$. Thus, combining (13) we get

$$\begin{aligned} \dot{V}_1 &\leq -\left(\lambda_{\min}(Q) - (n+2) \|P\|^2 - \frac{1}{2}\right) \|e\|^2 + \sum_{i=2}^4 \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2} z_2^2 + \frac{1}{2} s_1^2 + \varepsilon^{*2} \\ &\quad + \gamma_1 [\gamma_{1,1} + \gamma_{1,2}(\alpha_1 - \dot{y}_d + \frac{5}{2} \gamma_1 \gamma_{1,2} \hat{\Theta})] - \tilde{\Theta} \left(\dot{\hat{\Theta}} - \frac{5}{2} \gamma_1^2 \gamma_{1,2}^2 \right) + D^{*2} + \frac{1}{4} \end{aligned} \quad (14)$$

Designing virtual control signals α_1

$$\alpha_1 = -\frac{\rho}{\gamma_{1,2}} \gamma_1^{1+\eta} - \frac{\bar{\rho}}{\gamma_{1,2}} \gamma_1^{1-\eta} \tanh\left(\frac{\bar{\rho} \gamma_1^{2-\eta}}{\mathfrak{s}_1}\right) - \omega_i s_i - \frac{1}{\gamma_{1,2}} \gamma_{1,1} - \frac{5}{2} \gamma_1 \gamma_{1,2} \hat{\Theta} + \dot{y}_d \quad (15)$$

where $\mathfrak{s}_1 > 0$, $\rho = \frac{\sigma \pi}{\Gamma_\eta T_d}$, $\bar{\rho} = \frac{\bar{\sigma} \pi}{\bar{\Gamma}_\eta T_d}$, $\sigma = (3n+1)^{\frac{\eta}{2}}$, $\bar{\sigma} = \sqrt{2} + 1$, $\Gamma = 2^{1+\frac{\eta}{2}}$, $\bar{\Gamma} = 2^{1-\frac{\eta}{2}}$, $\eta = \eta_1 / \eta_2$, where $\eta_1 > 0$ and $\eta_2 > 0$ being even and odd numbers, fulfillment $\eta_1 < \eta_2$.

Substituting (15) into (14), we get

$$\begin{aligned} \dot{V}_1 &\leq -\left(\lambda_{\min}(Q) - (n+2) \|P\|^2 - \frac{1}{2}\right) \|e\|^2 - \rho \gamma_1^{2+\eta} - \bar{\rho} \gamma_1^{2-\eta} + \sum_{i=2}^4 \tilde{\theta}_i^T \tilde{\theta}_i + \frac{1}{2} z_2^2 \\ &\quad + \frac{1}{2} s_1^2 - \tilde{\Theta} \left(\dot{\hat{\Theta}} - \frac{5}{2} \gamma_1^2 \gamma_{1,2}^2 \right) + 0.2785 \mathfrak{s}_1 + \varepsilon^{*2} + D^{*2} + \frac{1}{4} \end{aligned} \quad (16)$$

Step 2: Designing Lyapunov functions

$$V_2 = V_1 + V_{b,2} + \frac{1}{2} \tilde{\theta}_2^T \tilde{\theta}_2 + \frac{1}{2} s_1^2 \quad (17)$$

Utilize the following Lyapunov function

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \gamma_2 \left[\gamma_{2,1} + \gamma_{2,2} \left(z_3 + s_2 + \alpha_2 + \theta_2^{*T} \psi_2(\hat{x}) - \tilde{\theta}_2^T \psi_2(\hat{x}) + \tilde{\theta}_2^T \psi_2(x_2) + \hat{\theta}_2^T \psi_2(x_2) \right. \right. \\ &\quad \left. \left. - \theta_2^{*T} \psi_2(x_2) + l_2 e_1 - \dot{w}_1 \right) \right] - \tilde{\theta}_2^T \dot{\hat{\theta}}_2 - \rho s_1^{2+\eta} - \bar{\rho} s_1^{2-\eta} - \omega_1 s_1^2 - \dot{\alpha}_1 s_1 + 0.2785 \mathfrak{s}_1 \end{aligned} \quad (18)$$

Applying Young's inequality yields

$$\begin{aligned} \gamma_2 \gamma_{2,2} (\theta_2^{*T} \psi_2(\hat{x}) - \theta_2^{*T} \psi_2(x_2)) &\leq 2 \gamma_2^2 \gamma_{2,2}^2 \|\theta_2^*\|^2 + \frac{1}{2} \\ \gamma_2 \gamma_{2,2} \left(z_3 + s_2 - \tilde{\theta}_2^T \psi_2(\hat{x}) + l_2 e_1 \right) &\leq 2 \gamma_2^2 \gamma_{2,2}^2 + \frac{1}{2} z_3^2 + \frac{1}{2} s_2^2 + \frac{1}{2} \tilde{\theta}_2^T \tilde{\theta}_2 + \frac{1}{2} l_2^2 \|e_1\|^2 \\ -\dot{\alpha}_1 s_1 &\leq \frac{1}{2} \dot{\alpha}_1^2 + \frac{1}{2} s_1^2 \end{aligned} \quad (19)$$

where $\bar{\alpha}_1$ is a positive constant, $|\dot{\alpha}_1| \leq \bar{\alpha}_1$, $\dot{\alpha}_1$ is a bounded function.

Accordingly, the virtual control input α_2 and the corresponding parameter adaptation law $\dot{\hat{\theta}}_2$ are constructed.

$$\alpha_2 = -\frac{\rho}{\gamma_{2,2}}\gamma_2^{1+\eta} - \frac{\bar{\rho}}{\gamma_{2,2}}\gamma_2^{1-\eta}\tanh\left(\frac{\bar{\rho}\gamma_2^{2-\eta}}{s_2}\right) - \frac{1}{\gamma_{2,2}}\gamma_{2,1} - \hat{\theta}_2^T\psi_2(x_2) - 4\gamma_2\gamma_{2,2}\hat{\Theta} + \dot{w}_1 - \frac{1}{2\gamma_2\gamma_{2,2}}z_2^2 \quad (20)$$

$$\dot{\hat{\theta}}_2 = -5\hat{\theta}_2 + \gamma_2\gamma_{2,2}\psi_2(x_2) \quad (21)$$

Using Young's inequality, we get $5\tilde{\theta}_2^T\hat{\theta}_2 \leq -\frac{5}{2}\tilde{\theta}_2^T\tilde{\theta}_2 + \frac{5}{2}\theta_2^{*T}\theta_2^*$. We can rewrite $\dot{V}_2$ to get

$$\begin{aligned} \dot{V}_2 \leq & -\left(\lambda_{\min}(Q) - (n+2)\|P\|^2 - \frac{1}{2} - \frac{1}{2}l_2^2\right)\|e\|^2 - \rho\sum_{j=1}^2\gamma_j^{2+\eta} - \bar{\rho}\sum_{j=1}^2\gamma_j^{2-\eta} \\ & + \sum_{j=3}^4\tilde{\theta}_j^T\tilde{\theta}_j - \tilde{\Theta}\left(\dot{\Theta} - \frac{5}{2}\gamma_1^2\gamma_{1,2}^2 + 4\gamma_2^2\gamma_{2,2}^2\right) - \tilde{\theta}_2^T\tilde{\theta}_2 - \rho s_1^{2+\eta} - \bar{\rho}s_1^{2-\eta} \\ & + \frac{1}{2}z_3^2 + \frac{1}{2}s_2^2 + \frac{1}{2}\bar{\alpha}_1^2 + \frac{5}{2}\theta_2^{*T}\theta_2^* + 0.2785\sum_{j=1}^2s_j + \varepsilon^{*2} + D^{*2} + \frac{3}{4} \end{aligned} \quad (22)$$

Step 3: Construct the Lyapunov function as follows:

$$V_3 = V_2 + V_{b,3} + \frac{1}{2}\tilde{\theta}_3^T\tilde{\theta}_3 + \frac{1}{2}s_2^2 \quad (23)$$

Deriving from V_3 , we get

$$\begin{aligned} \dot{V}_3 = \dot{V}_2 + \gamma_3 \left[\gamma_{3,1} + \gamma_{3,2} \left(\theta_3^{*T}\psi_3(\hat{x}) - \tilde{\theta}_3^T\psi_3(\hat{x}) + \tilde{\theta}_3^T\psi_3(x_3) + \theta_3^T\psi_3(x_3) - \theta_3^{*T}\psi_3(x_3) \right. \right. \\ \left. \left. + b_4u_q + l_3e_1 - \dot{w}_2 \right) \right] - \tilde{\theta}_3^T\dot{\hat{\theta}}_3 - \rho s_2^{2+\eta} - \bar{\rho}s_2^{2-\eta} - \omega_2s_2^2 - \dot{\alpha}_2s_2 + 0.2785s_2 \end{aligned} \quad (24)$$

By applying Young's inequality, we obtain

$$\begin{aligned} \gamma_3\gamma_{3,2}(\theta_3^{*T}\psi_3(\hat{x}) - \theta_3^{*T}\psi_3(x_3)) & \leq 2\gamma_3^2\gamma_{3,2}^2\|\theta_3^*\|^2 + \frac{1}{2} \\ \gamma_3\gamma_{3,2}(-\tilde{\theta}_3^T\psi_3(\hat{x}) + l_3e_1) & \leq \gamma_3^2\gamma_{3,2}^2 + \frac{1}{2}\tilde{\theta}_3^T\tilde{\theta}_3 + \frac{1}{2}l_3^2\|e_1\|^2 \\ -\dot{\alpha}_2s_2 & \leq \frac{1}{2}\bar{\alpha}_2^2 + \frac{1}{2}s_2^2 \end{aligned} \quad (25)$$

where $\bar{\alpha}_2$ is a positive constant, $|\dot{\alpha}_2| \leq \bar{\alpha}_2$, $\dot{\alpha}_2$ is a bounded function.

The virtual control input u_q and the adaptive update law $\dot{\hat{\theta}}_3$ are designed accordingly.

$$u_q = \frac{1}{b_4} \left[-\frac{\rho}{\gamma_{3,2}}\gamma_3^{1+\eta} - \frac{\bar{\rho}}{\gamma_{3,2}}\gamma_3^{1-\eta}\tanh\left(\frac{\bar{\rho}\gamma_3^{2-\eta}}{s_3}\right) - \frac{1}{\gamma_{3,2}}\gamma_{3,1} - \hat{\theta}_3^T\psi_3(x_3) - 3\gamma_3\gamma_{3,2}\hat{\Theta} + \dot{w}_2 - \frac{1}{2\gamma_3\gamma_{3,2}}z_3^2 \right] \quad (26)$$

$$\dot{\hat{\theta}}_3 = -5\hat{\theta}_3 + \gamma_3\gamma_{3,2}\psi_3(\bar{x}_3) \quad (27)$$

Using Young's inequality, we get $5\tilde{\theta}_3^T\hat{\theta}_3 \leq -\frac{5}{2}\tilde{\theta}_3^T\tilde{\theta}_3 + \frac{5}{2}\theta_3^{*T}\theta_3^*$. $\dot{V}_3$ can be written as

$$\begin{aligned} \dot{V}_3 \leq & -\left(\lambda_{\min}(Q) - (n+2)\|P\|^2 - \frac{1}{2} - \frac{1}{2}l_2^2 - \frac{1}{2}l_3^2\right)\|e\|^2 - \rho\sum_{j=1}^3\gamma_j^{2+\eta} - \bar{\rho}\sum_{j=1}^3\gamma_j^{2-\eta} + \sum_{j=3}^4\tilde{\theta}_j^T\tilde{\theta}_j \\ & - \tilde{\Theta}\left(\dot{\Theta} - \frac{5}{2}\gamma_1^2\gamma_{1,2}^2 + 4\gamma_2^2\gamma_{2,2}^2 + 3\gamma_3^2\gamma_{3,2}^2\right) - \rho\sum_{j=1}^2s_j^{2+\eta} - \bar{\rho}\sum_{j=1}^2s_j^{2-\eta} - \sum_{j=2}^3\tilde{\theta}_j^T\tilde{\theta}_j \\ & + \frac{5}{2}\sum_{j=2}^3\theta_j^{*T}\theta_j^* + \frac{1}{2}\sum_{j=1}^2\bar{\alpha}_j^2 + 0.2785\sum_{j=1}^3s_j + \varepsilon^{*2} + D^{*2} + \frac{5}{4} \end{aligned} \quad (28)$$

Step 4: Constructing Lyapunov functions

$$V_4 = V_3 + V_{b,4} + \frac{1}{2}\tilde{\theta}_4^T\tilde{\theta}_4 \quad (29)$$

The derivative of V_4 can be written as

$$\dot{V}_4 = \dot{V}_3 + \gamma_4 \left[\gamma_{4,1} + \gamma_{4,2} \left(\theta_4^{*T} \psi_4(\hat{x}) - \tilde{\theta}_4^T \psi_4(\hat{x}) + \tilde{\theta}_4^T \psi_4(x_4) + \hat{\theta}_4^T \psi_4(x_4) - \theta_4^{*T} \psi_4(x_4) + c_3 u_d + l_4 e_1 \right) \right] - \tilde{\theta}_4^T \dot{\hat{\theta}}_4 \quad (30)$$

Using Young's inequality

$$\begin{aligned} \gamma_4 \gamma_{4,2} (\theta_4^{*T} \psi_4(\hat{x}) - \theta_4^{*T} \psi_4(x_4)) &\leq 2\gamma_4^2 \gamma_{4,2}^2 \|\theta_4^*\|^2 + \frac{1}{2} \\ \gamma_4 \gamma_{4,2} \left(-\tilde{\theta}_4^T \psi_4(\hat{x}) + l_4 e_1 \right) &\leq \gamma_4^2 \gamma_{4,2}^2 + \frac{1}{2} \tilde{\theta}_4^T \tilde{\theta}_4 + \frac{1}{2} l_4^2 \|e_1\|^2 \end{aligned} \quad (31)$$

The control input is denoted by u_d , while the parameter adaptation laws are given by $\dot{\hat{\theta}}_4$ and $\dot{\hat{\Theta}}$.

$$u_d = \frac{1}{c_3} \left[-\frac{\rho}{\gamma_{4,2}} \gamma_4^{1+\eta} - \frac{\bar{\rho}}{\gamma_{4,2}} \gamma_4^{1-\eta} \tanh\left(\frac{\bar{\rho} \gamma_4^{2-\eta}}{\mathfrak{s}_4}\right) - \frac{1}{\gamma_{4,2}} \gamma_{4,1} - \hat{\theta}_4^T \psi_4(x_4) - 3\gamma_4 \gamma_{4,2} \hat{\Theta} \right] \quad (32)$$

$$\dot{\hat{\theta}}_4 = -5\hat{\theta}_4 + \gamma_4 \gamma_{4,2} \psi_4(x_4) \quad (33)$$

$$\dot{\hat{\Theta}} = -\kappa \rho \hat{\Theta}^{1+\eta} - \bar{\kappa} \bar{\rho} \hat{\Theta}^{1-\eta} + \frac{5}{2} \gamma_1^2 \gamma_{1,2}^2 + 4\gamma_2^2 \gamma_{2,2}^2 + 3\gamma_3^2 \gamma_{3,2}^2 + 3\gamma_4^2 \gamma_{4,2}^2 \quad (34)$$

where $\mathfrak{s}_i > 0, i = 1, \dots, 4, \kappa = 2 + \eta$, and $\bar{\kappa} = 2 - \eta$.

Using Young's inequality: $5\tilde{\theta}_4^T \hat{\theta}_4 \leq -\frac{5}{2} \tilde{\theta}_4^T \tilde{\theta}_4 + \frac{5}{2} \theta_4^{*T} \theta_4^*$. $\dot{V}_4$ can be rewritten as

$$\begin{aligned} \dot{V}_4 \leq & -\tau_4 \|e\|^2 - \rho \sum_{j=1}^4 \gamma_j^{2+\eta} - \bar{\rho} \sum_{j=1}^4 \gamma_j^{2-\eta} + \kappa \rho \hat{\Theta}^{1+\eta} + \bar{\kappa} \bar{\rho} \hat{\Theta}^{1-\eta} - \rho \sum_{j=1}^2 s_j^{2+\eta} - \bar{\rho} \sum_{j=1}^2 s_j^{2-\eta} \\ & - \sum_{j=2}^4 \tilde{\theta}_j^T \tilde{\theta}_j + \frac{1}{2} \sum_{j=1}^2 \bar{\alpha}_j^2 + \frac{5}{2} \sum_{j=2}^4 \theta_j^{*T} \theta_j^* + 0.2785 \sum_{j=1}^4 \mathfrak{s}_j + \varepsilon^{*2} + D^{*2} + \frac{5}{4} \end{aligned} \quad (35)$$

The overall architecture of the proposed predefined-time output-feedback adaptive control scheme is illustrated in Figure 2, which summarizes the interaction between the MTN-based observer, adaptive parameter update laws, constraint mechanism, and controller prior to the detailed mathematical development.

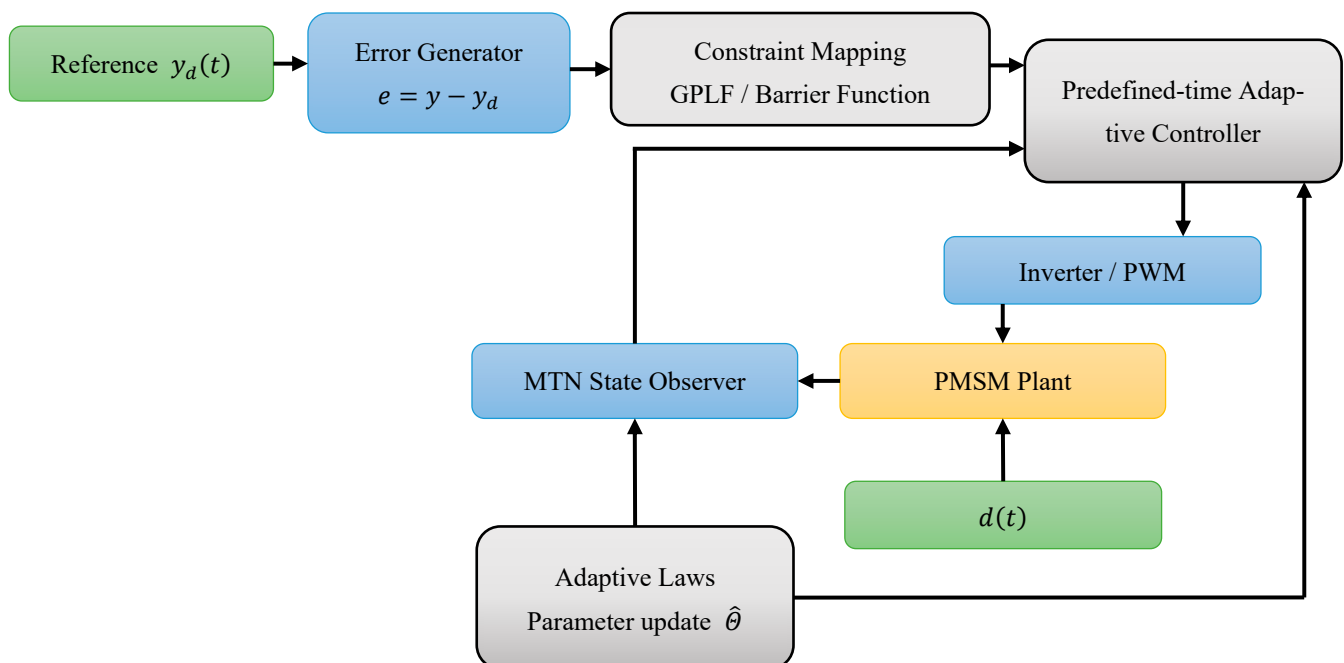


Figure 2. Proposed predefined-time output-feedback adaptive control system block diagram.

3.3. Stability Analysis

Theorem 1. For system (5), under the proposed fuzzy state observer (7), predefined-time filters (10), control inputs (15), (20), (26), (32), and adaptive laws (21), (27), (33), (34), and assuming the constraint functions satisfy Assumptions 1 and 2, the PMSM system achieves predefined-time practical stability. Specifically, the output rapidly tracks the reference signal, all system states and control inputs remain bounded, and the stator currents and angular velocity are strictly confined within the prescribed limits.

Proof. By invoking the inequality presented in [1], we obtain the following result:

$$\begin{aligned}\tilde{\Theta}\hat{\Theta}^{1+\eta} &\leq \frac{1}{\kappa}(\Theta^{*\kappa} - \hat{\Theta}^{\kappa}) \leq \frac{1}{\kappa}\left(2\Theta^{*\kappa} - \tilde{\Theta}^{\kappa}\right) \\ \tilde{\Theta}\hat{\Theta}^{1-\eta} &\leq \frac{1}{\bar{\kappa}}(\Theta^{*\bar{\kappa}} - \hat{\Theta}^{\bar{\kappa}}) \leq \frac{1}{\bar{\kappa}}\left(2\Theta^{*\bar{\kappa}} - \tilde{\Theta}^{\bar{\kappa}}\right)\end{aligned}\quad (36)$$

Then, we apply the following inequality:

$$-\sum_{i=1}^n \frac{\chi_i^2}{Z_i} \leq -\left(\sum_{i=1}^n \frac{\chi_i^2}{2Z_i}\right)^o - \frac{1}{n^{c-1}}\left(\sum_{i=1}^n \frac{\chi_i^2}{2Z_i}\right)^c + \Xi \quad (37)$$

where χ_i is a constant and $|\chi_i| \leq \bar{\chi}_i$, $Z_i > 0$, $0 < o < 1$, $c > 1$, $\Xi = (1-o)o^{o/1-o} + \sum_{i=1}^n \left(\frac{\chi_i^2}{2Z_i}\right)^c$. We can get

$$\begin{aligned}-2e^T Pe &\leq -(e^T Pe)^{1-\frac{\eta}{2}} - (e^T Pe)^{1+\frac{\eta}{2}} + \Xi_e \\ -\tilde{\theta}_j^T \tilde{\theta}_j &\leq -\left(\frac{1}{2}\tilde{\theta}_j^T \tilde{\theta}_j\right)^{1+\frac{\eta}{2}} - \left(\frac{1}{2}\tilde{\theta}_j^T \tilde{\theta}_j\right)^{1-\frac{\eta}{2}} + \Xi_j\end{aligned}\quad (38)$$

where $\Xi_e = \left(\frac{\eta}{2}\right)(1-\frac{\eta}{2})^{2-\eta/\eta} + \bar{e}^{1+\frac{\eta}{2}}$, $\Xi_j = \left(\frac{\eta}{2}\right)(1-\frac{\eta}{2})^{2-\eta/\eta} + (\bar{\theta}_j/2)^{1+\frac{\eta}{2}}$.

Substituting (36) and (38) into (35), and selecting the appropriate design parameters such that $\tau_4 \geq 2\lambda_{\max}(p)$ and $\eta T_d > \max\{\sigma\pi, \bar{\sigma}\pi\}$ holds, we can obtain

$$\begin{aligned}\dot{V}_4 &\leq -\frac{\sigma\pi}{\eta T_d} \left[(e^T Pe)^{1+\frac{\eta}{2}} + \sum_{j=1}^4 \left(\frac{1}{2}\gamma_j^2\right)^{1+\frac{\eta}{2}} + \left(\frac{1}{2}\gamma_j^2\right)^{1-\frac{\eta}{2}} + \sum_{j=1}^2 \left(\frac{1}{2}s_j^2\right)^{1+\frac{\eta}{2}} + \sum_{j=2}^4 \left(\frac{1}{2}\tilde{\theta}_j^T \tilde{\theta}_j\right)^{1+\frac{\eta}{2}} \right] - \frac{\bar{\sigma}\pi}{\eta T_d} \left[\sum_{j=1}^2 \left(\frac{1}{2}s_j^2\right)^{1-\frac{\eta}{2}} \right. \\ &\quad \left. + \sum_{j=2}^4 \left(\frac{1}{2}\tilde{\theta}_j^T \tilde{\theta}_j\right)^{1-\frac{\eta}{2}} + (e^T Pe)^{1-\frac{\eta}{2}} + \sum_{j=1}^4 \left(\frac{1}{2}\gamma_j^2\right)^{1-\frac{\eta}{2}} + \left(\frac{1}{2}\tilde{\Theta}^2\right)^{1-\frac{\eta}{2}} \right] + \Xi_e + \sum_{j=2}^4 \Xi_j + \frac{5}{2} \sum_{j=2}^4 \theta_j^{*T} \theta_j^* \\ &\quad + \frac{1}{2} \sum_{j=1}^2 \bar{\alpha}_j^2 + 0.2785 \sum_{j=1}^4 s_j + 2\rho\Theta^{*2+\eta} + 2\bar{\rho}\Theta^{*2-\eta} + \varepsilon^{*2} + D^{*2} + \frac{5}{4}\end{aligned}\quad (39)$$

The observation error e and the parameter estimation error $\tilde{\theta}_j$ are uniformly bounded within a compact set Π . Consequently, there exist unknown positive constants $\bar{e}$ and $\bar{\theta}_j$ such that $\|e^T Pe\| \leq \bar{e}$, $\left\|\tilde{\theta}_j^T \tilde{\theta}_j\right\| \leq \bar{\theta}_j$.

For any $x_i \in R$, there is $0 < p \leq 1$, $1 \leq q \leq \infty$, such that the following inequality holds: $(\sum_{i=1}^n |x_i|)^p \leq \sum_{i=1}^n |x_i|^p$ and $(\sum_{i=1}^n |x_i|)^q \leq n^{q-1} \sum_{i=1}^n |x_i|^q$. Then we can conclude

$$\dot{V}_4 = -\frac{\pi}{\eta T_d} V_4^{1+\frac{\eta}{2}} - \frac{\sigma\pi}{\eta T_d} V_4^{1+\frac{\eta}{2}} + m \quad (40)$$

By invoking the definition of V_4 in (29) together with the stability conditions given in Lemma 2, it follows that the adaptive parameter estimation errors $\|\tilde{\theta}_i\|$ and $\tilde{\Theta}$, the filter error s_i , the transformation error γ_i , and the state estimation error e_i are all uniformly bounded and converge to an arbitrarily adjustable neighborhood of the origin within the predefined time T_d . Consequently, the original system states x_i approach a neighborhood of the equilibrium point in predefined time, since the tracking errors z_i are also driven

into a tunable region around zero within the same T_d , as ensured by the constructed Lyapunov function. $\square$

Remark 1. *Clarification of Predefined-Time Stability.*

To clarify the stability notion adopted in this work, we distinguish among finite-time, fixed-time, and predefined-time stability. A nonlinear system is finite-time stable if the settling time depends on the initial conditions. A system is fixed-time stable if convergence occurs within a uniform upper bound independent of initial states, although this bound is determined by controller parameters and cannot be freely assigned. In contrast, predefined-time stability allows the convergence time to be explicitly specified by a user-defined parameter $T_d > 0$, independent of initial conditions.

In the proposed controller design, the predefined-time parameter T_d is explicitly embedded into the Lyapunov inequality through the predefined-time transformation function. Specifically, the derived Lyapunov function $V(t)$ satisfies an inequality of the form

$$\dot{V}(t) \leq -\frac{c}{T_d} \Phi(V) \quad (41)$$

where $c > 0$ is a design constant and $\Phi(\cdot)$ is a positive definite function ensuring accelerated decay. Integrating this inequality guarantees that the settling time admits an explicit upper bound not exceeding T_d , regardless of the initial condition $V(0)$. Hence, the convergence rate can be prescribed a priori via parameter selection.

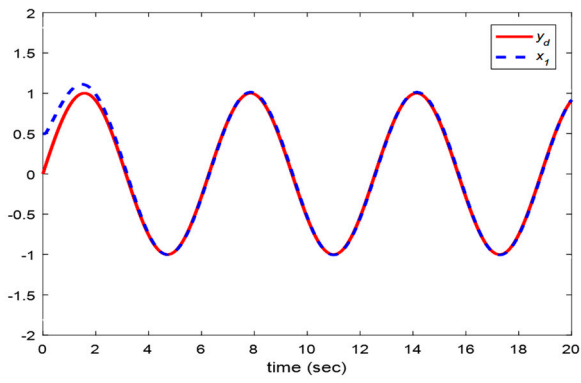
Due to bounded approximation errors introduced by the Multi-dimensional Taylor Network (MTN) and possible implementation uncertainties, an unavoidable residual term ε^* appears in the closed-loop dynamics. Consequently, the tracking error converges to an arbitrarily small compact set rather than exactly zero. Therefore, the proposed scheme guarantees practical predefined-time stability, meaning that all closed-loop signals remain bounded and the tracking error enters a small neighborhood of the origin within the predefined time T_d , provided that the approximation error remains bounded. This interpretation ensures consistency between the theoretical analysis and practical implementation conditions.

4. Simulation Study

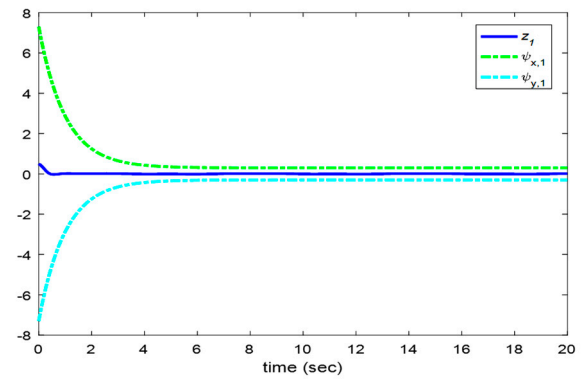
This section presents MATLAB R2016a-based simulation results to demonstrate the effectiveness of the proposed control approach. The PMSM parameters used in the simulations are listed in Table 1.

The control design requires the following parameters: $\eta_1 = 9$, $\eta_2 = 90$, $s_1 = s_2 = 0.3$, $s_3 = s_4 = 0.4$, and $T_d = 10$. The observer gain coefficient l is selected as $l = [l_1, l_2, l_3, l_4]^T = [35, 100, 100, 100]^T$, $\omega_1 = 15$, $\omega_2 = 15$. The initial condition of the observer state is chosen as $\hat{x}_1(0) = 0$, $\hat{x}_2(0) = 0$, $\hat{x}_3(0) = 0$, $\hat{x}_4(0) = 0$. The full-state tracking error is constrained to lie within predefined limits $\psi_{x,1} = 0.3 + 7e^{-t}$, $\psi_{y,1} = 0.3 + 7e^{-t}$, $\psi_{x,2} = 0.5 + 7e^{-t}$, $\psi_{y,2} = 0.5 + 7e^{-t}$, $\psi_{x,3} = 1 + 7e^{-t}$, $\psi_{y,3} = 1 + 7e^{-t}$, $\psi_{x,4} = 0.3 + 7e^{-t}$, $\psi_{y,4} = 0.3 + 7e^{-t}$. The external perturbation is $d_1 = 0.05\sin(0.01t)$, $d_2 = -0.15\sin(0.01t)$, $d_3 = 0.05\sin(0.01t)$, $d_4 = 0.01\sin(0.01t)$. The desired trajectory is given by $y_d = \sin(t)$.

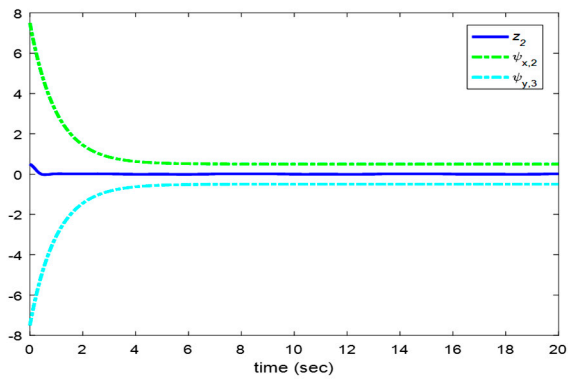
The simulation results presented in Figure 3a–g verify the effectiveness of the proposed control scheme. The rotor angle accurately tracks the reference signal within the predefined time, while all state errors and control inputs remain bounded. In particular, the rotor angle, angular velocity, and stator current errors are maintained within prescribed limits, and the control voltages exhibit stable behavior, demonstrating the stability, effectiveness, and practical feasibility of the proposed predefined-time output-feedback controller for PMSM systems.



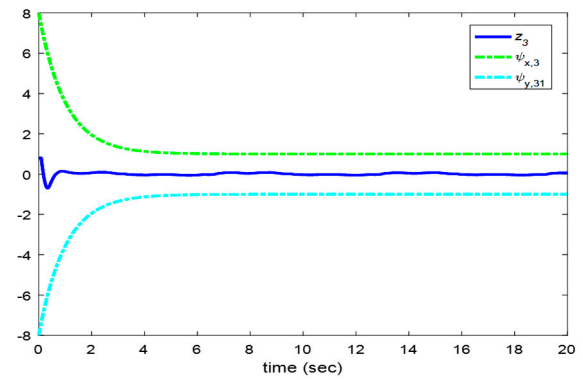
(a)



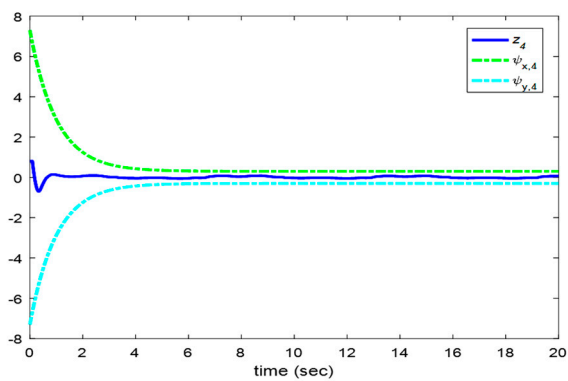
(b)



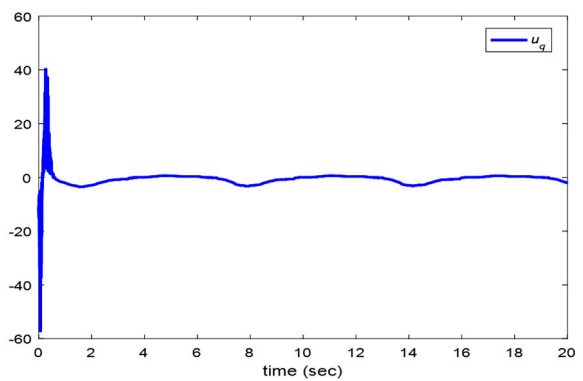
(c)



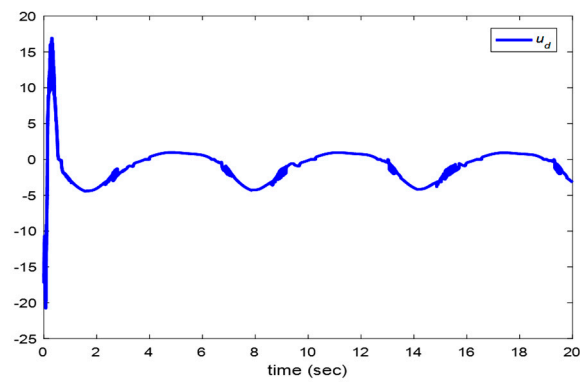
(d)



(e)



(f)



(g)

Figure 3. (a) System output x_1 and y_d ; (b) Error trajectory z_1 curve; (c) Error trajectory z_2 curve; (d) Error trajectory z_3 curve; (e) Error trajectory z_4 curve; (f) Control input u_q ; (g) Control input u_d .

Table 1. The values of the PMSM system parameters.

Performance	Description	Values
J	Rotor moment of inertia	0.86 kg·m ²
B	Friction coefficient	0.0217 N·M/(rad/s)
Φ	Magnet flux linkage of inertia	0.1245 Wb
L_q	q-axis stator inductor	0.68 H
L_d	d-axis stator inductor	0.58 H
n_p	Number of pole pairs	4
R_s	Armature resistance	0.0003 Ω

It can be observed that during the initial interval ($0 - 2$ s), the system output x_1 does not perfectly match the reference signal y_d . This behavior corresponds to the transient adaptation phase, where the MTN-based observer and adaptive parameters are adjusting to estimate the unknown nonlinear dynamics from non-equilibrium initial conditions. During this period, the controller simultaneously performs state estimation, parameter adaptation, and constraint enforcement, which results in a temporary tracking deviation. As the adaptation process progresses, the tracking error decreases rapidly and the system output converges toward the reference trajectory within the predefined time bound T_d , consistent with the theoretical stability analysis.

4.1. Verification of Predefined-Time Convergence

According to the theoretical analysis presented in the stability section, the proposed control scheme guarantees predefined-time convergence characterized by the design parameter T_d , which provides an upper bound on the settling time independent of initial conditions. The tracking-error responses shown in Figure 3b–e demonstrate that all error trajectories converge before the prescribed time T_d . This behavior is consistent across the considered operating conditions and confirms the predefined-time convergence property predicted by the Lyapunov-based analysis. Therefore, the simulation results provide numerical verification of the theoretical predefined-time stability guarantee.

4.2. Constraint Satisfaction Under Disturbances

The proposed controller incorporates a generalized potential Lyapunov function to enforce full-state error constraints throughout system operation. From a theoretical perspective, this formulation guarantees bounded error evolution in the presence of bounded disturbances. As observed in Figure 3b–e, all state-error trajectories remain strictly within the prescribed constraint boundaries during both transient and steady-state phases, even when disturbances are applied. These results confirm that the constraint-handling mechanism remains effective under non-ideal operating conditions, in agreement with the theoretical analysis.

4.3. Discussion of Practical Advantages

A key advantage of the proposed predefined-time control strategy is the ability to explicitly prescribe the convergence time through the parameter T_d , which differs from conventional asymptotic or fixed-time controllers where convergence speed cannot be directly assigned. The simulation results illustrate rapid transient response, bounded control inputs, and strict satisfaction of state constraints, highlighting the practical effectiveness of the proposed method. These observations demonstrate the benefits of predefined-time control in achieving predictable performance while maintaining robustness and constraint compliance.

5. Conclusions

This paper investigated a predefined-time control strategy for PMSM systems subject to unknown nonlinear dynamics and full-state error constraints, with particular attention to the challenges posed by unmeasurable states. To address state unavailability, a MTN-based state observer was constructed, while a MTN was employed to approximate unknown nonlinearities. By integrating generalized potential Lyapunov functions with dynamic surface control and nonlinear filtering techniques within an adaptive backstepping framework, an output-feedback control scheme with predefined-time stability and full-state error constraints was developed. Rigorous stability analysis demonstrated that the proposed method guarantees boundedness of all closed-loop signals, predefined-time convergence, and strict satisfaction of state error constraints. Simulation results further confirmed the effectiveness and superiority of the proposed control strategy.

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