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Weakly Dimension-Balanced Hamiltonian Cycle on Three-Dimensional Toroidal Mesh Graph †

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Abstract

The dimension-balanced cycle (DBC) problem is new in graph theory, with applications such as 3D stereogram reconstruction. In a graph whose edges are partitioned into k dimensions, a cycle is dimension-balanced if edge counts across dimensions differ by at most one. When such a cycle is Hamiltonian, it is called a dimension-balanced Hamiltonian cycle (DBH). Since DBHs do not always exist, a relaxed notion—the weakly dimension-balanced Hamiltonian (WDBH) cycle—was considered, allowing a difference of up to three. We prove that WDBH always exists in any 3-dimensional toroidal mesh graph $T_{m,n,r}$ for all positive integers m, n, and r.

Keywords: Hamiltonian; dimension-balanced; 3-dimensional toroidal mesh graph; weakly

1. Introduction

The topology of interconnection networks is modeled using graphs, where vertices represent processors and edges denote the communication links between them. In this study, we follow the standard notation and definitions introduced in Ref. [1].

In a simple graph $G = (V, E)$, a Hamiltonian path is a path that visits each vertex exactly once; a Hamiltonian cycle is a cycle that visits each vertex exactly once [2–7]. Given a graph G whose edge set is partitioned into k dimensions, denoted as $\widetilde{E} = \{E_1(G), E_2(G), \ldots, E_k(G)\}$. A cycle C in G is called a dimension-balanced cycle (DBC for short) with respect to $\widetilde{E}$ if for any pair $i, j \in \{1, 2, \ldots, k\}$, the number of edges in each dimension satisfies $||E_i(C)| - |E_j(C)|| \leq 1$, where $E_i(C) = E(C) \cap E_i(G)$ for $i \in \{1, 2, \ldots, k\}$. If such a cycle is also Hamiltonian, it is referred to as a dimension-balanced Hamiltonian cycle (DBH). If the condition is relaxed into $||E_i(C)| - |E_j(C)|| \leq 3$, then the cycle is called a weakly dimension-balanced (Hamiltonian) cycle (WDBC (WDBH) for short) with respect to $\widetilde{E}$.

The DBH problem has attracted increasing attention in recent years. The DBH problem was developed to address the quality issues in the construction of 3D surface scanning [8–10]. Notable research efforts have examined such cycles in various graph classes, including hypercube and toroidal meshes [11,12]. In 2012, Peng [11] successfully constructed DBHs for two-dimensional toroidal graphs $T_{m,n}$, and in 2018, Wu [13] extended this to the weaker version known as the weak DBH (WDBH).

We examined the class of graphs denoted by $T_{m,n,r}$, known as 3-dimensional toroidal mesh graphs (3D toroidal mesh graphs). The vertex set of such a graph is defined as $V(T_{m,n,r}) = \{(x, y, z) \mid 0 \leq x < m - 1,\ 0 \leq y < n - 1,\ 0 \leq z < r - 1\}$; the edge

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set is given by $E(T_{m,n,r}) = \{(x_1, y_1, z_1)(x_2, y_2, z_2) \mid (x_1 = x_2, y_1 = y_2, |z_1 - z_2| \equiv 1 \pmod r)$ or $(x_1 = x_2, z_1 = z_2, |y_1 - y_2| \equiv 1 \pmod n)$ or $(y_1 = y_2, z_1 = z_2, |x_1 - x_2| \equiv 1 \pmod m)\}$. Let the partition of $E(T_{m,n,r})$ be $\widetilde{E} = \{E_1, E_2, E_3\}$, where $E_1 = \{(x_1, y_1, z_1)(x_2, y_2, z_2) \mid y_1 = y_2, z_1 = z_2, |x_1 - x_2| \equiv 1 \pmod m\}$, $E_2 = \{(x_1, y_1, z_1)(x_2, y_2, z_2) \mid x_1 = x_2, z_1 = z_2, |y_1 - y_2| \equiv 1 \pmod n\}$, and $E_3 = \{(x_1, y_1, z_1)(x_2, y_2, z_2) \mid x_1 = x_2, y_1 = y_2, |z_1 - z_2| \equiv 1 \pmod r\}$. For convenience in further discussions, we define the following three sets of special cross-connections: $\text{cross}^1 = \{(0, j, k), (m-1, j, k) \mid 0 \le j < n, 0 \le k < r\}$, $\text{cross}^2 = \{(i, 0, k), (i, n-1, k) \mid 0 \le i < m, 0 \le k < r\}$, and $\text{cross}^3 = \{(i, j, 0), (i, j, r-1) \mid 0 \le i < m, 0 \le j < n\}$. An illustration of the structure of $T_{5,4,3}$ is shown in Figure 1. Throughout this paper, edges of different dimensions in the figures are represented using three colors: red, blue, and green.

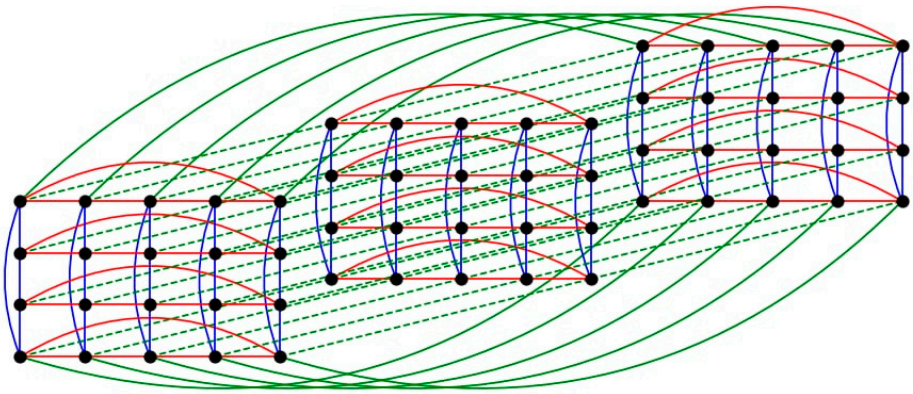


Figure 1. The structure of $T_{5,4,3}$.

Building on the previous foundations, Juan et al. [14] investigated the DBH problem in 3D toroidal mesh graphs with respect to $\widetilde{E}$ in 2019, and the rest of this problem has been solved recently [15]. However, DBHs do not always exist in arbitrary 3D toroidal graphs. Consequently, when a DBH is unattainable, the weaker form, WDBH, becomes a useful generalization. This paper aims to explore the existence conditions of WDBH cycles in 3D toroidal mesh graphs $T_{m,n,r}$ with respect to $\widetilde{E}$, under the constraint that $m, n, r \ge 3$.

2. Background Knowledge

We analyze the properties of the 3D toroidal mesh graph $T_{m,n,r}$, based on the values of m, n, and r mod 6 at first. Then, we present important results.

Let $a = m \bmod 6$, $b = n \bmod 6$, and $c = r \bmod 6$. Considering the graph's inherent symmetry, 56 cases need to be discussed when $m, n, r \ge 3$. In 2015, Peng established an important result regarding d-dimensional toroidal mesh graphs of the form $C_{l1} \times C_{l2} \times \ldots \times C_{ld}$, where each C_{li} is a cycle graph of length l_i, with $1 \le i \le d$. Define the function $\rho(l_1, l_2, \ldots, l_d)$ as the number of odd integers in the set $\{l_1, l_2, \ldots, l_d\}$ [11]. The function $\Psi(l_1, l_2, \ldots, l_d)$ is defined as follows: if $\lfloor (l_1 l_2 \ldots l_d)/d \rfloor$ is even, then $\Psi(l_1, l_2, \ldots, l_d) = (l_1 l_2 \ldots l_d) \bmod d$; else $\Psi(l_1, l_2, \ldots, l_d) = d - ((l_1 l_2 \ldots l_d) \bmod d)$. The results for the DBH problem on the 3D toroidal mesh graph $T_{m,n,r}$ are as follows.

Theorem 1 ([14]). *If any of m, n, or r is divisible by 3, then a DBH exists in $T_{m,n,r}$ with respect to $\widetilde{E}$.*

Theorem 2 ([15]). *If $m, n, r \bmod 3 \neq 0$ and $\Psi(m, n, r) \le \rho(m, n, r)$, then a DBH exists in $T_{m,n,r}$ with respect to $\widetilde{E}$.*

Theorem 3 ([14]). *If* $\Psi(m, n, r) > \rho(m, n, r)$, *there does not exist DBH in* $T_{m,n,r}$ *with respect to* $\widetilde{E}$.

By definition, every DBH is also a WDBH. Table 1 summarizes ten specific cases of toroidal mesh graphs that have been shown not to admit DBHs. Since we are concerned with the cases where $m, n, r \geq 3$, the range of a, b, and c has been restricted to {3, 4, 5, 6, 7, 8}. These cases are of particular interest, as it remains to be determined whether a WDBH exists in each of them. This issue will be examined in detail in the following section.

Table 1. Cases with no DBH in $T_{m,n,r}$ for $m, n, r \geq 3$.

Cases	*a*	*b*	*c*	Cases	*a*	*b*	*c*
1	4	4	4	6	4	8	7
2	4	4	5	7	4	8	8
3	4	4	7	8	8	8	5
4	4	4	8	9	8	8	7
5	4	8	5	10	8	8	8

We present two lemmas that demonstrate how two cycles can be connected in this paper. Figures 2 and 3 illustrate Lemma 1 and Lemma 2.

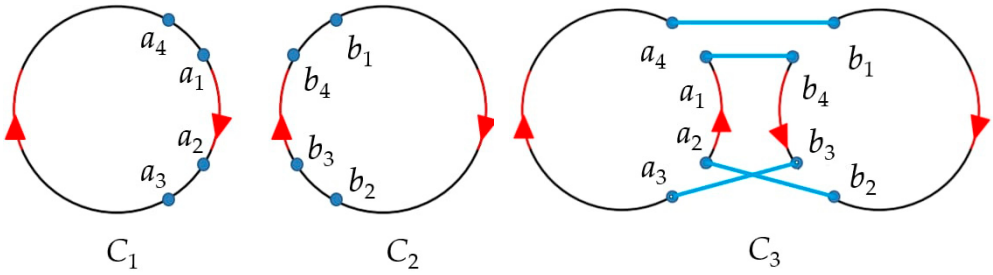


Figure 2. Illustration of Lemma 1.

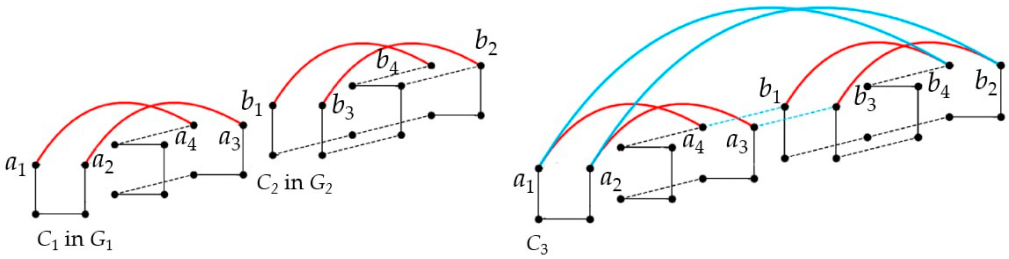


Figure 3. Illustration of Lemma 2. The red edges indicate the deleted edges, while the blue edges represent the newly added edges.

Lemma 1 ([15]). *Let there be two cycles:* $C_1 = \langle a_1, \ldots, a_2, a_3, \ldots, a_4, a_1 \rangle$, $C_2 = \langle b_1, \ldots, b_2, b_3, \ldots, b_4, b_1 \rangle$ *By removing edges* a_4a_1, a_2a_3, b_4b_1, b_2b_3 *and adding new edges a* a_1b_4, b_3a_3, a_4b_1, b_2a_2, *we form a new cycle* C_3 *defined as* $C_3 = \langle a_1, b_4, (b_4, b_3)$*- from* ${C_2}^{-1}$, $b_3, a_3, (a_3, a_4)$*- from* $C_1, a_4, b_1, (b_1, b_2)$*- from* $C_2, b_2, a_2, (a_2, a_1)$*- from* ${C_1}^{-1}, a_1 \rangle$.

Lemma 2 ([15]). *Building on Lemma 1, assume* T_1 *and* T_2 *are Hamiltonian cycles in the toroidal mesh graphs* $G_1 = T_{m,n,r1}$ *and* $G_2 = T_{m,n,r2}$, *respectively. Suppose the edges* a_2a_3, a_4a_1 *lie in the cross*^{3} *direction of* G_1, *and the edges* b_2b_3, b_2b_3 *lie in the cross*^{3} *direction of* G_2. *Then, the added edges* a_1b_4, b_3a_2 *will also lie in the cross*^{3} *direction of the combined graph* $T_{m,n,r1+r2}$.

3. Result and Discussion

Using the modular values of n, m, and r modulo 6, the structure $T_{m,n,r}$ is classified into ten distinct cases, as shown in Table 1. This section constructs the WDBH for each of the cases presented in Table 1, using the model illustrated in Figure 4. At first, we show the main idea of how to construct the WDBH for every case.

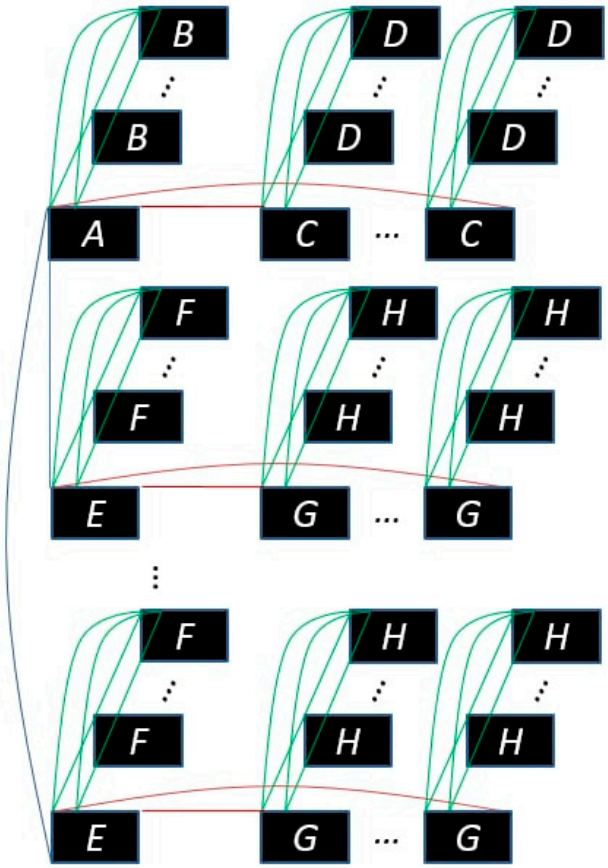


Figure 4. Construction of the WDBH model. Red, blue, and green edges represent edges of different dimensions.

If we find one WDBH (A) and seven Hamiltonian cycles (B through H), then connecting them sequentially according to the construction method shown in Figure 4, we can cover all the various cases in Table 2.

Table 2. Ten cases discussed in this study.

Case	*A* (*a*,*b*,*c*)	*B* (*a*,*b*,6)	*C* (6,*b*,*c*)	*D* (6,*b*,6)	*E* (*a*,6,*c*)	*F* (*a*,6,6)	*G* (6,6,*c*)	*H* (6,6,6)
1	B_1 (**4**,**4**,**4**)	B_2 (**4**,**4**,6)	B_3 (6,**4**,**4**)	B_4 (6,**4**,6)	B_5 (**4**,6,**4**)	B_6 (**4**,6,6)	B_7 (6,6,**4**)	B_8 (6,6,6)
2	B_9 (**4**,**4**,**5**)	B_2	B_{10} (6,**4**,**5**)	B_4	B_{11} (**4**,6,**5**)	B_6	B_{12} (6,6,**5**)	B_8
3	B_{13} (**4**,**4**,**7**)	B_2	B_{14} (6,**4**,**7**)	B_4	B_{15} (**4**,6,**7**)	B_6	B_{16} (6,6,**7**)	B_8
4	B_{17} (**4**,**4**,**8**)	B_2	B_{18} (6,**4**,**8**)	B_4	B_{19} (**4**,6,**8**)	B_6	B_{20} (6,6,**8**)	B_8
5	B_{21} (**4**,**8**,**5**)	B_{22} (**4**,**8**,6)	B_{23} (6,**8**,**5**)	B_{24} (6,**8**,6)	B_{11}	B_6	B_{12}	B_8
6	B_{25} (**4**,**8**,**7**)	B_{22}	B_{26} (6,**8**,**7**)	B_{24}	B_{15}	B_6	B_{16}	B_8
7	B_{27} (**4**,**8**,**8**)	B_{22}	B_{28} (6,**8**,**8**)	B_{24}	B_{19}	B_6	B_{20}	B_8
8	B_{29} (**8**,**8**,**5**)	B_{30} (**8**,**8**,6)	B_{23}	B_{24}	B_{31} (**8**,6,**5**)	B_{32} (**b**,6,6)	B_{12}	B_8
9	B_{33} (**8**,**8**,**7**)	B_{30}	B_{26}	B_{24}	B_{34} (**8**,6,**7**)	B_{32}	B_{16}	B_8
10	B_{35} (**8**,**8**,**8**)	B_{30}	B_{28}	B_{24}	B_{36} (**8**,6,**8**)	B_{32}	B_{20}	B_8

Red bold text denotes values other than 6 and indicates the main discrepancies among the different cases.

During the extension along the first or second dimension, we remove one edge from each of the two Hamiltonian cycles in the same dimension and reconnect them using two new edges, also in the same dimension. As a result, the resulting graph remains a Hamiltonian cycle, and the number of edges in each dimension can simply be summed directly. When extending along the third dimension, a different construction is required. Specifically, we must remove two edges from each of the original Hamiltonian cycles in the third dimension and reconnect them using four edges in the third dimension. To ensure that the resulting structure remains a Hamiltonian cycle, each individual Hamiltonian cycle must satisfy the conditions specified in Lemma 2. According to Lemma 2, this guarantees that the merged graph is indeed Hamiltonian (still one cycle). In terms of edge count, the number of edges in each dimension can simply be summed directly, too.

We present the main theorem of this study as follows.

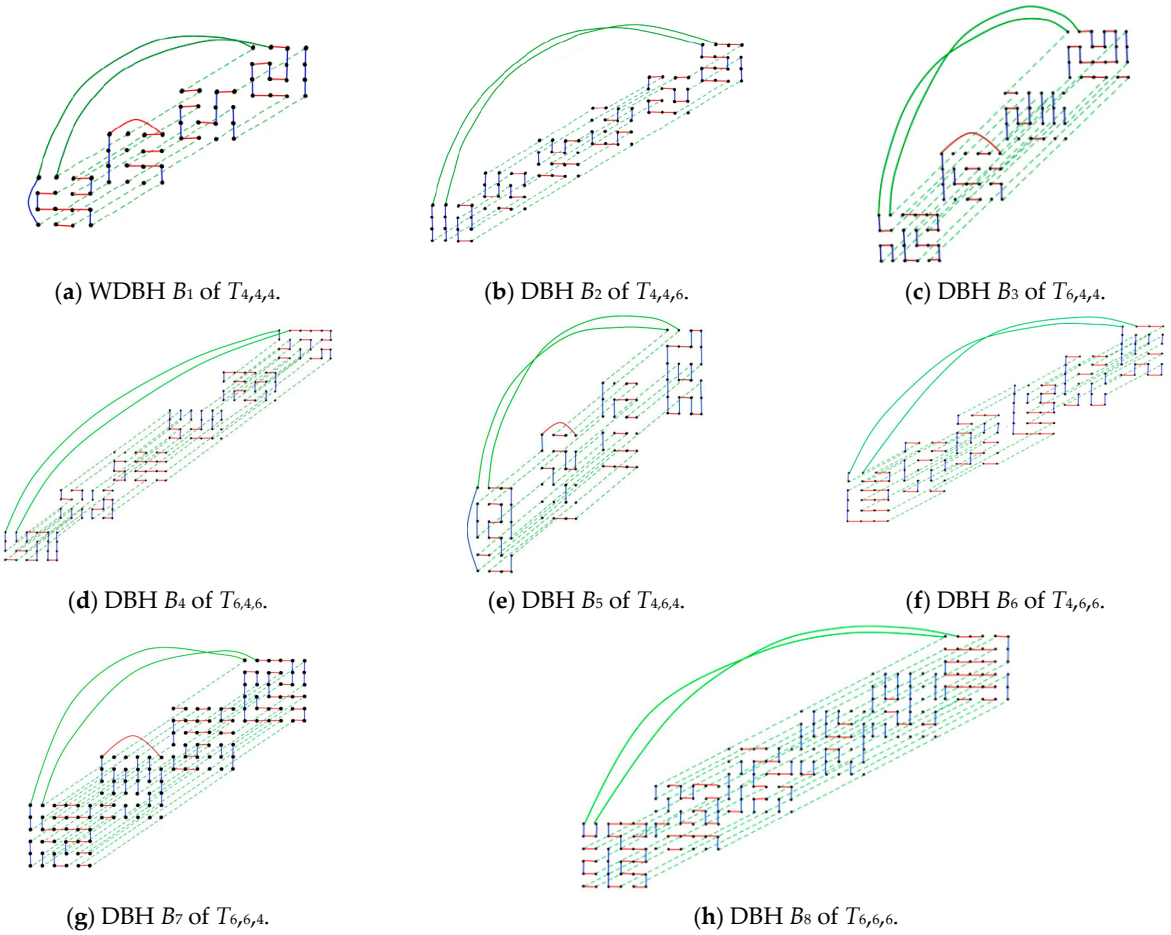
Theorem 4. *For any m, n, or $r \geq 3$, WDBH exists in $T_{m,n,r}$ with respect to $\widetilde{E}$.*

Proof. From the discussion in the previous section, we observe that the validity of this theorem hinges on the ten specific cases listed in Table 1, where no DBH exists. Therefore, to establish this theorem, it is sufficient to prove that each of these ten cases admits a WDBH. Following the construction framework outlined at the beginning of this section, due to space limitations and the similarity in the proof techniques across all cases, we provide a detailed proof for Case 1 only.

Case 1. m mod 6 = 4 and n mod 6 = 4, r mod 6 = 4. Let B_1 (see Figure 5a) be a WDBH of $T_{4,4,4}$. We can see that $|E_1(B_1)| = 20$, $|E_2(B_1)| = 22$, and $|E_3(B_1)| = 22$. Let B_2 (see Figure 5b) be a DBH of $T_{4,4,6}$, where $|E_1(B_2)| = |E_2(B_2)| = |E_3(B_2)| = 32$. After connecting B_2 z times in the third dimension of B_1, it forms a WDBH T_1 in $T_{4,4,(4+6z)}$. Note that $|E_1(T_1)| = 20 + 32z$, $|E_2(T_1)| = 22 + 32z$, and $|E_3(T_1)| = 22 - 2 + 2 + (32 - 2 + 2)z = 22 + 32z$. Let B_3 (see Figure 5c) be a DBH of $T_{6,4,4}$ with $|E_1(B_3)| = |E_2(B_3)| = |E_3(B_3)| = 32$, connecting a DBH B_4 of $T_{6,4,6}$ (with $|E_1(B_4)| = |E_2(B_4)| = |E_3(B_4)| = 48$, see Figure 5d) z times to the third dimension of B_3. It forms a DBH T_2 in $T_{6,4,(4+6z)}$ with $|E_1(T_2)| = |E_2(T_2)| = 32 + 48z$, $|E_3(T_2)| = 32 - 2 + 2 + (48 - 2 + 2)z = 32 + 48z$. Then, connecting T_2 z times to the first dimension of T_1, it forms a WDBH in $T_{(4+6x),4,(4+6z)}$, called T_3. So, $|E_1(T_3)| = 20 + 32z - 1 + 1 + x(32 + 48z - 1 + 1) = 20 + 32z + x(32 + 48z)$, $|E_2(T_3)| = |E_3(T_3)| = 22 + 32z + x(32 + 48z)$.

In the next stage, we need to add a DBH of $T_{(4+4x),6,(4+6z)}$ y times in the second dimension of T_3. Again, we give a DBH B_5 of $T_{4,6,4}$, as Figure 5e, and connect a DBH B_6 of $T_{4,6,6}$ z times to the third dimension of B_5. It forms a DBH T_4 in $T_{4,6,(4+6z)}$ with $|E_1(T_4)| = |E_2(T_4)| = 32 + 48z$, $|E_3(T_4)| = 32 - 2 + 2 + (48 - 2 + 2)z = 32 + 48z$. Now, let B_7 (see Figure 5g) be a DBH of $T_{6,6,4}$, and B_8 (see Figure 5h) be a DBH of $T_{6,6,6}$. After connecting B_8 z times in the third dimension of B_7, it forms a DBH T_5 in $T_{6,6,(4+6z)}$. Note that $|E_1(T_5)| = |E_2(T_5)| = 48 + 72z$, $|E_3(T_5)| = 48 - 2 + 2 + (72 - 2 + 2)z = 48 + 72z$. After connecting T_5 x times to the first dimension of T_4, it forms a DBH T_6 of $T_{(4+6x),6,(4+6z)}$. Note that $|E_1(T_6)| = 32 + 48z - 1 + 1 + x(48 + 72z - 1 + 1) = 32 + 48z + x(48 + 72z)$, $|E_2(T_6)| = |E_3(T_6)| = 32 + 48z + x(48 + 72z)$. Finally, after connecting T_6 y times to the second dimension of T_3, it forms a WDBH T_7 of $T_{(4+6x),(4+6y),(4+6z)}$. Note that $|E_1(T_7)| = 20 + 32z + x(32 + 48z) + y\,[32 + 48z + x(48 + 72z)]$, $|E_2(T_7)| = 20 + 32z + x(32 + 48z) - 1 + 1 + y\,[32 + 48z + x(48 + 72z) - 1 + 1] = 22 + 32z + x(32 + 48z) + y\,[32 + 48z + x(48 + 72z)] = |E_3(T_7)|$. We have completed the proof of this case. Figure 6 is an illustration for $x = y = z = 1$, that is, $T_{10,10,10}$.

The remaining cases, involving the construction of Hamiltonian cycles on B_9 through B_{36}, is summarized in Figure 7. □



(**a**) WDBH B_1 of $T_{4,4,4}$. (**b**) DBH B_2 of $T_{4,4,6}$. (**c**) DBH B_3 of $T_{6,4,4}$.

(**d**) DBH B_4 of $T_{6,4,6}$. (**e**) DBH B_5 of $T_{4,6,4}$. (**f**) DBH B_6 of $T_{4,6,6}$.

(**g**) DBH B_7 of $T_{6,6,4}$. (**h**) DBH B_8 of $T_{6,6,6}$.

Figure 5. Construction of B_1 to B_8 (**a**–**h**). Red, blue, and green edges represent edges of different dimensions.

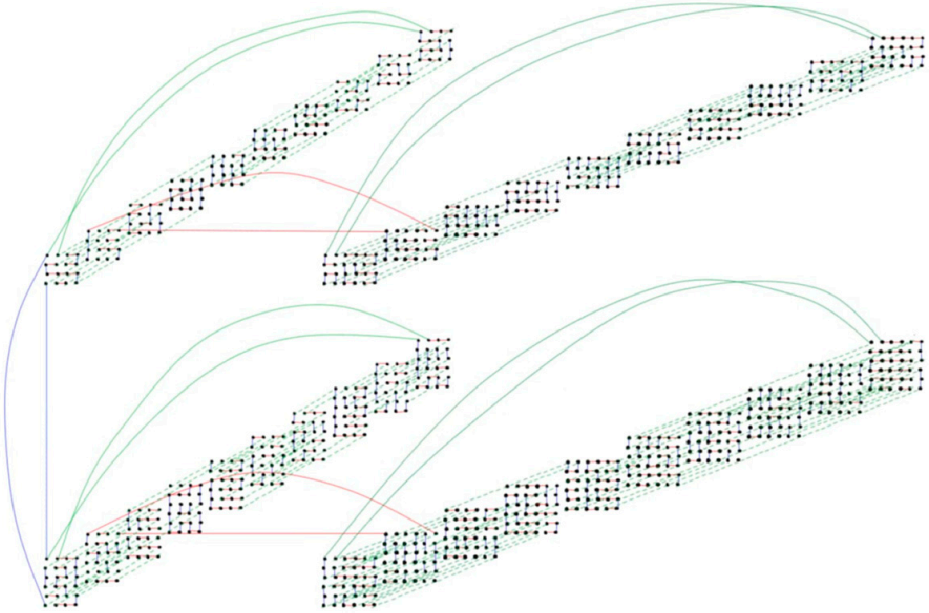
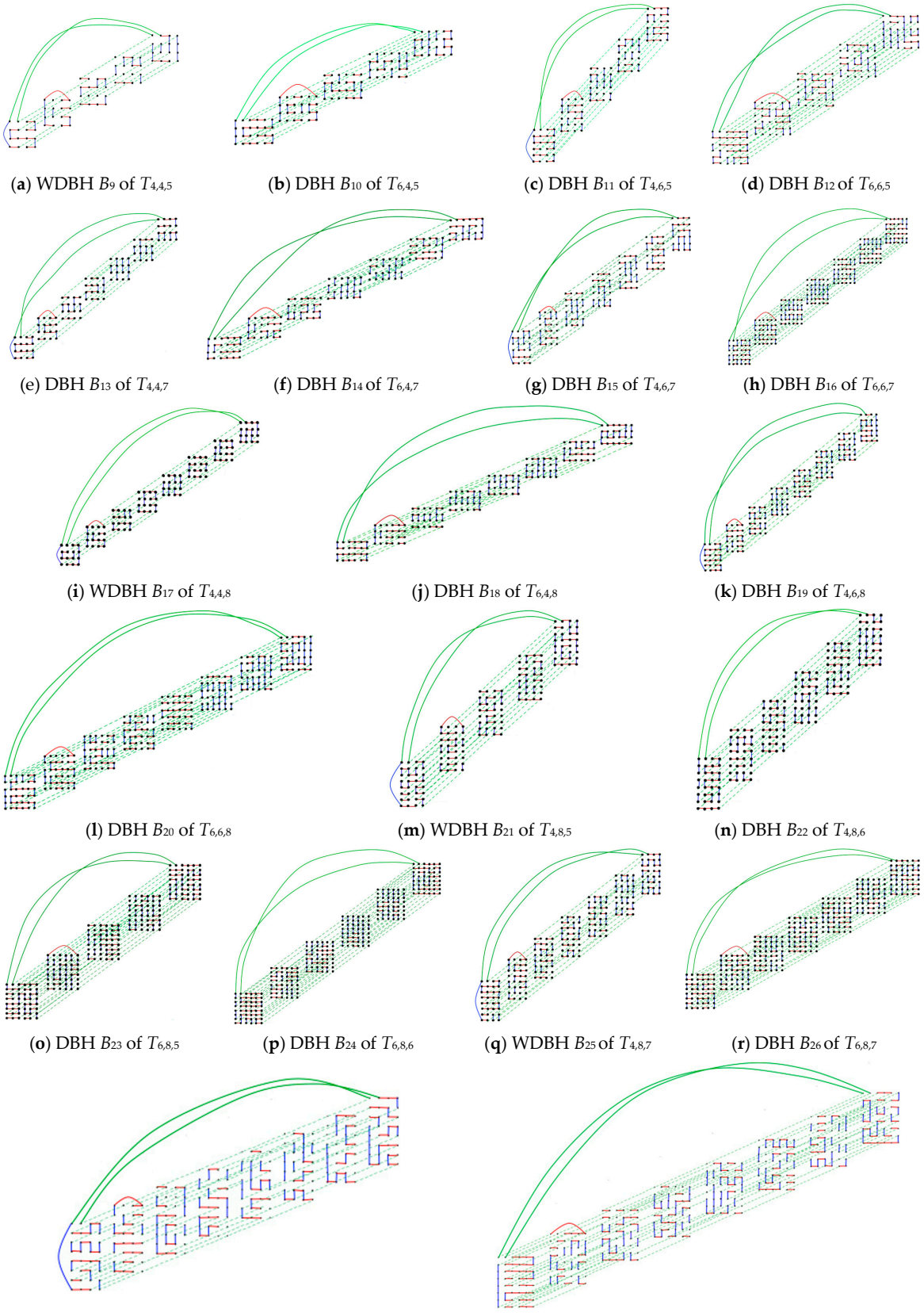


Figure 6. Constructed WDBH for $T_{10,10,10}$. Red, blue, and green edges represent edges of different dimensions.



(**a**) WDBH B_9 of $T_{4,4,5}$ (**b**) DBH B_{10} of $T_{6,4,5}$ (**c**) DBH B_{11} of $T_{4,6,5}$ (**d**) DBH B_{12} of $T_{6,6,5}$

(e) DBH B_{13} of $T_{4,4,7}$ (**f**) DBH B_{14} of $T_{6,4,7}$ (**g**) DBH B_{15} of $T_{4,6,7}$ (**h**) DBH B_{16} of $T_{6,6,7}$

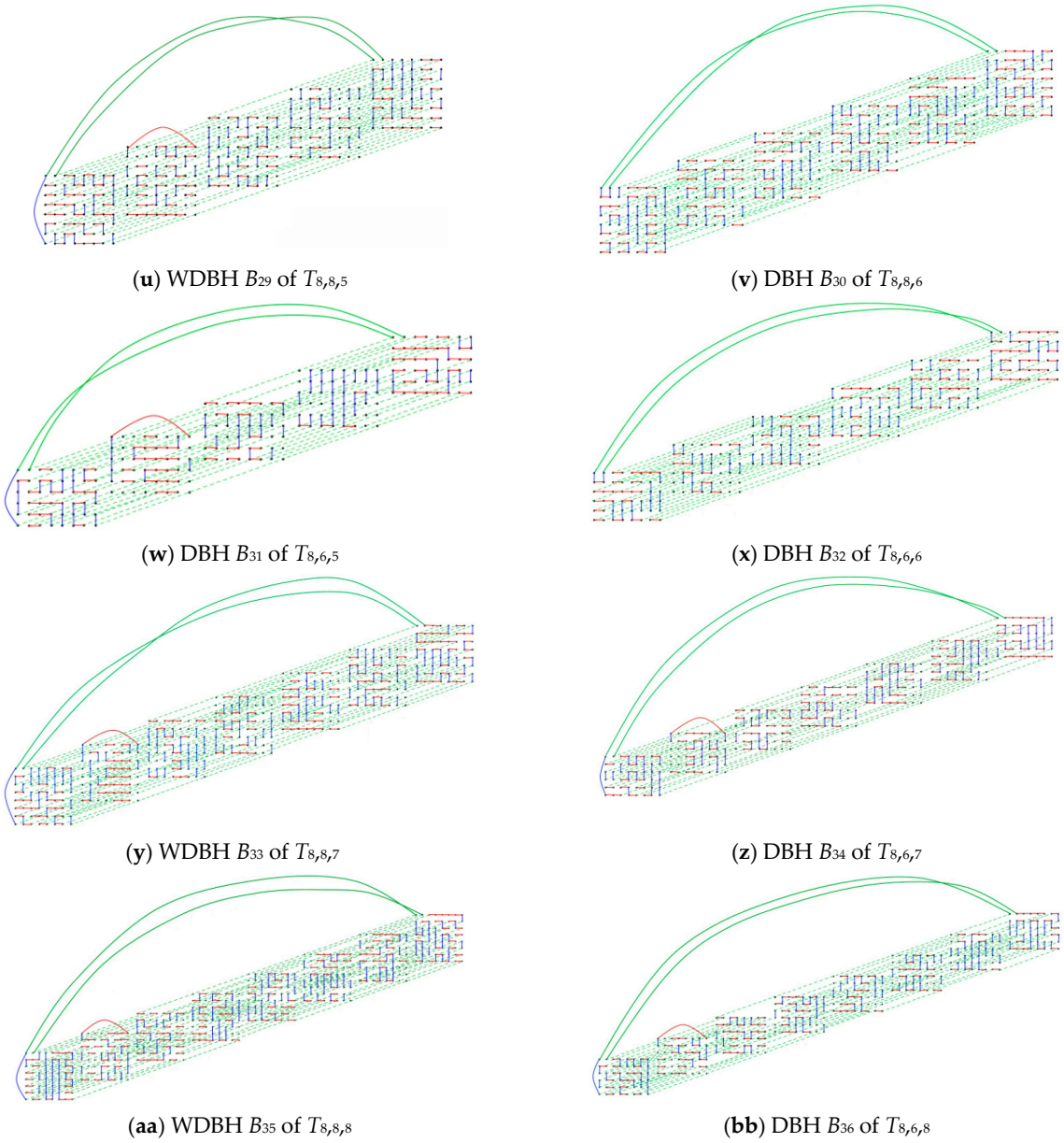
(**i**) WDBH B_{17} of $T_{4,4,8}$ (**j**) DBH B_{18} of $T_{6,4,8}$ (**k**) DBH B_{19} of $T_{4,6,8}$

(**l**) DBH B_{20} of $T_{6,6,8}$ (**m**) WDBH B_{21} of $T_{4,8,5}$ (**n**) DBH B_{22} of $T_{4,8,6}$

(**o**) DBH B_{23} of $T_{6,8,5}$ (**p**) DBH B_{24} of $T_{6,8,6}$ (**q**) WDBH B_{25} of $T_{4,8,7}$ (**r**) DBH B_{26} of $T_{6,8,7}$

(**s**) WDBH B_{27} of $T_{4,8,8}$ (**t**) DBH B_{28} of $T_{6,8,8}$

Figure 7. *Cont.*



(**u**) WDBH B_{29} of $T_{8,8,5}$ (**v**) DBH B_{30} of $T_{8,8,6}$

(**w**) DBH B_{31} of $T_{8,6,5}$ (**x**) DBH B_{32} of $T_{8,6,6}$

(**y**) WDBH B_{33} of $T_{8,8,7}$ (**z**) DBH B_{34} of $T_{8,6,7}$

(**aa**) WDBH B_{35} of $T_{8,8,8}$ (**bb**) DBH B_{36} of $T_{8,6,8}$

Figure 7. Construction of B_9 to B_{36}. Red, blue, and green edges represent edges of different dimensions.

4. Conclusions

We investigate the existence of WDBHs in three-dimensional toroidal mesh graphs $T_{m,n,r}$ for all m, n, $r \geq 3$. We merge two Hamiltonian cycles by removing one or two edges from each and reconnecting them. When removing one edge per cycle, the new Hamiltonian cycle is straightforward to construct; when removing two edges, careful connection is essential to avoid disconnected cycles (which need to satisfy Lemma 2). We process the ten cases where DBHs are known not to exist. By applying the construction method in Figure 4 and using the B_1 to B_{36} cycles shown in Figures 5 and 7, along with Table 2, we successfully built a WDBH for each case. This confirms that a WDBH exists in every three-dimensional toroidal mesh graph. This is an important result, especially compared with the two-dimensional case, where WDBHs are known to be absent in several configurations. It raises a compelling question for future research on a higher-dimensional toroidal mesh graph with WDBH. This direction offers a promising avenue for further theoretical exploration.

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