Robust Control Design of Multiple Quadrotors Formation [†]

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[†] Presented at the 1st International Conference on Energy, Power and Environment, Gujrat, Pakistan, 11–12 November 2021.

Abstract: In this study, a control strategy is presented for multiple quadrotors, inclusive of sliding mode control and proportional derivative (PD) control, with the goal of providing stability, robustness, reduced disturbance, and formation tracking in uncertain conditions and environments. The presented control technique is based on Newton-Euler equations and satisfying Lyapunov's stability conditions, using sliding mode controller design and PD controller design. The designed control technique was implemented, and the desired results were achieved with minimized position error, orientation error, and distance error, while adhering to Lyapunov's stability conditions.

Keywords: quadrotor; Lyapunov; sliding mode control; formation

Citation: Muzammil, H.; Siddique, N.; Masud, S.A.; Rashad, M.; Raoof, U.; Hussain, I.; Jabbar, W. Robust Control Design of Multiple Quadrotors Formation. *Eng. Proc.* **2021**, *12*, 93. <https://doi.org/10.3390/engproc2021012093>

Academic Editor: Qasim Awais

Published: 27 January 2022

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1. Introduction

With the current use of various quadrotor airplanes, the development of a control strategy for multiple quadrotors has become an interesting issue, drawing increasing consideration. Unlike the fixed-wing airplane, the quadrotor airplane has a wide range of advantages, including basic construction, simple upkeep, and mobility. Because of these favorable circumstances, the quadrotor airplane has been generally used by regular citizens and in the military field. Its uses include airborne photography, geographical overview, calamity alleviation, pipeline investigation, ecological appraisal, etc.

In some unpredictable specific conditions, the use of multiple quadrotor airplanes provides a greater number of options than a solitary airplane can provide. For instance, various airplanes working together can improve the productivity and vigor of the entire control framework. Furthermore, greater airplane coordination may lead to the development of unique hardware that can be applied to more troublesome undertakings.

The development of a control strategy for numerous quadrotor airplanes is very challenging, as it requires additional organized control based on the control strategy for a solitary airplane. In addition, the planning of a control framework and the investigation of the solidness of single quadrotor airplanes are particularly challenging, as the model is based on profoundly nonlinear and solid coupling. Moreover, quadrotors formation is compromised under external disturbances, causing the system to become unstable and the path tracking to be lost.

The objectives of this study include the design of vehicle dynamics, controllers, swarm stability, robustness of formation, formation tracking, and individual interaction. Previous researchers have proposed many techniques for the modelling of multi-quadrotors, including path tracking, formation tracking, attitude stabilization, robust controller design, and sliding mode control. The formation tracking technique was proposed on the basis of sliding mode control [1,2]; a non-linear control design of quadrotors was also proposed [3]; finite-time formation control design has been suggested [4]; control design was proposed to

minimize disturbances [5,6]; robust control design of formation tracking was proposed [7]; and various finite-time control techniques have been proposed [8].

This study proposes a multiple quadrotor formation tracking control. The well-known Newton Euler (N-EF) technique was the proposed target. A controller sliding mode (SMC) control rule was developed to minimize disturbances, unknown parameters, and controller non-linearity. Several parameters were modified in the proposed model to achieve optimal and consistent efficiency. In terms of multi-quadrotor tracking power, the three target parameters were successfully reduced. The robust control setup of several model quadrotors was shown by Newton Euler formalism to control the dynamic quadrotor model.

2. Proposed Methodology

2.1. Dynamic Model of Quadrotor

The mathematical model of the simple quadrotor is provided in Equations (1)–(6). The proposed quadrotor motion can be controlled through the variation of the four rotors by changing the thrust shown in the equations, as well as via the rotors' torque. The proposed study is based on Newton-Euler formalism, which is used for formation tracking of multi-quadrotors. The dynamic model of the proposed Newton-Euler formalism is as follows:

$$\ddot{x} = u_1(\cos\varnothing\sin\theta\cos\varphi - \sin\varnothing\sin\varphi) \quad (1)$$

$$\ddot{y} = u_1(\cos\varnothing\sin\theta\sin\varphi - \sin\varnothing\cos\varphi) \quad (2)$$

$$\ddot{z} = u_1(\cos\varnothing\cos\theta) - g + d(t) \quad (3)$$

$$\ddot{\varnothing} = u_2/l \quad (4)$$

$$\ddot{\theta} = u_3/l \quad (5)$$

$$\ddot{\varphi} = u_4 \quad (6)$$

The state vector for the proposed multi-quadrotor is shown in the following equation:

$$X = [x_{1i} x_{2i} x_{3i} \dots x_{12i}]^T \quad (7)$$

The above equation shows the state space representation of the quadrotor. In addition, the virtual command u_{xi} and u_{yi} , which rotates the thrust vector, is shown by the following equations:

$$u_{xi} = \cos x_{7i} \cos x_{9i} \cos x_{11i} + \sin x_{7i} \sin x_{11i} \quad (8)$$

$$u_{yi} = \cos x_{7i} \sin x_{9i} \sin x_{11i} - \sin x_{7i} \cos x_{11i} \quad (9)$$

2.2. Control Design

The strategy for designing the controller for leader-follower formation, as well as for trajectory tracking, was first to separate the system into two independent subsystems, in the assumption that the closed loop dynamics of rotation are much faster than translational dynamics. Next, three independent controllers were designed, including trajectory tracking and attitude stabilization for the leader as well as for the follower and a formation tracking controller for the follower.

To design the trajectory tracking control, the position error was considered, which is defined as $\tilde{\xi} = \xi - \xi_d$; using a switching function and stability conditions, the equivalent control becomes

$$u_{eq} = [(TRe_3)_d]_{eq} = m \left(g e_3 + \ddot{\tilde{\xi}}_d - k_1 \dot{\tilde{\xi}} - k_2 \tilde{\xi} \right) \quad (10)$$

To add uncertainty and disturbance in the system, a discontinuity was added to keep the system dynamics to the surface i.e., $\sigma_1 = 0$

$$\dot{\sigma}_1 = k_1 \dot{\xi} + k_2 \bar{\xi} + \frac{1}{m(TRe_3)_d} - ge_3 - \ddot{\xi}_d = -L_{\xi} Sgn(\sigma_1) \quad (11)$$

where L_{ξ} is greater than zero. Next, the discontinuity control equation becomes

$$(TRe_3)_d = u_{eq} - mL_{\xi} Sgn(\sigma_1) \quad (12)$$

Then, the robustness of the controller could be analyzed by considering a model with bounded uncertainties, as shown by the following:

$$m\ddot{\xi} = (TRe_3)_d - mge_3 - m\ddot{\xi}_d + \Delta f(\xi) \quad (13)$$

where $||\Delta f(\xi)|| < 1$.

Using the Lyapunov function and applying the control law, the Euler angles can be shown as

$$\Phi_d = \arcsin\left(-\frac{Rd_y - Rd_x \tan(\psi_d)}{\sin(\psi_d) \tan(\psi_d) + \cos(\psi_d)}\right) \quad (14)$$

$$\Phi_d = \arcsin\left(\frac{Rd_x - \sin(\Phi_d) \sin(\psi_d)}{\cos(\Phi_d) \cos(\psi_d)}\right) \quad (15)$$

Attitude stabilization control consists only of a proportional derivative (PD), i.e., a PD controller that acts as the error, defined as

$$= k_{d0} \dot{\bar{\phi}} - k_{p0} \bar{\phi} \quad (16)$$

where k_{d0} and $k_{p0} \in \mathbb{R}^+$.

To formulate the formation tracking control, define the orientation error $e_{\psi} = \psi_F - \psi_L$, and achieve formation errors close to zero, a control law was designed, in which follower velocities are considered as control inputs. These formation error dynamics can be shown as

$$\dot{\chi} = F(\chi) + G(\chi)v \quad (17)$$

where

$$F(\chi) = \begin{bmatrix} e_y \omega_L + \gamma_1 \\ e_x \omega_L + \gamma_2 \\ e_{\psi} \end{bmatrix} \quad (18)$$

$$G(\chi) = \begin{bmatrix} -ce_{\psi} & se_{\psi} & 0 \\ -se_{\psi} & -ce_{\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

Implementing stability conditions and control law, the equivalent controller would become

$$\dot{\sigma}_2 = \dot{\chi} + k_f \chi = F(\chi) + G(\chi)v_{eq} + k_f \chi = 0 \quad (20)$$

Ideally, in the absence of uncertainty and disturbance, the equivalent control would become,

$$v_{eq} = G^{-1}(\chi) \left(-F(\chi) - k_f \chi \right) \quad (21)$$

3. Simulation Results and Discussion

The proposed formation tracking is divided into two scenarios. The first scenario is a fixed-time trajectory; the second scenario is a varying-time trajectory. The formation tracking control of a multi-quadrotor is presented in Figure 1a, tracking the trajectory of a multi-quadrotor. The overall performance of the proposed control design was effective.

Moreover, the contribution of the multi-quadrotors robust control was also added in the proposed system. Stabilization of formation tracking control is very important to provide long-term control for multi-quadrotors. Therefore, the proposed model focused on the stabilization factor to improve the validity of formation tracking control for multi-quadrotors. Stabilization of formation tracking control for multi-quadrotors is presented in Figure 1b. The two cases considered in this study are discussed below.

Figure 1. (a) tracking with constant reference; (b) stabilization of formation tracking.

In Figure 2a, case 1, the system is simulated without the involvement of a sliding mode controller. The response of the system is a bit irregular and unstable because of uncertainty and disturbances that occur outside, which affect the response of the system and provide overshoot in the curves. In Figure 2b, case 2, we introduce a sliding mode controller. The role of the sliding mode controller was to reduce the uncertainty and disturbances that occur in the background and to provide stable output. The data acquisition implementation, output correction, and computation of the control law for this design were implemented. To improve quadrotors' performance, the law of control can be applied. Therefore, the engagement of control law is very important for the control design.

Figure 2. (a) case 1; (b) case 2.

4. Conclusions

This research proposed a control strategy for the monitoring of multi-quadrotors. The proposed goal was achieved by the well-known Newton-Euler method. A control rule based on the sliding mode controller (SMC) was presented to reduce the disturbances, unknown parameters, and non-linearity of the controller system. The proposed model amends several parameters, so that optimum and consistent performance may be achieved. The results of the simulation show that the proposed model exceeds the current model in real world scenarios. The proposed model was also validated with MATLAB/SIMULINK. The three target parameters were successfully minimized in terms of multi-quadrotor tracking power.

Author Contributions: Conceptualization, H.M. and N.S.; methodology, H.M.; software, H.M.; validation, H.M., S.A.M. and N.S.; formal analysis, N.S.; investigation, I.H.; data curation, N.S. and M.R.; writing-original draft preparation, H.M.; writing-review and editing, H.M., W.J. and U.R.; supervision, N.S. All authors have read and agreed to the published version of manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data can be acquired by requesting the correspondence author.

Conflicts of Interest: The authors declare no conflict of interest.

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