

Impact of Lewis Number on Natural Convection with Double Diffusion in a Square Cavity Filled with Non-Newtonian Fluid with Viscosity Varying with Temperature [†]

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Abstract

This work focuses on the numerical analysis of double diffusive natural convection inside a square cavity filled with non-Newtonian fluids obeying the power-law model. The viscosity is considered to vary with temperature following an exponential law. The main parameters studied are the Lewis number, thermo-dependent parameter, and behavior index. It was observed that increasing the thermo-dependent parameter intensifies the flow, which leads to a significant improvement in heat and mass transfer within the cavity. When the thermo-dependent parameter is increased from 0 to 3, the average Nusselt number increases by 109% for low Le , while the Sherwood number rises up to 108%, reflecting a significant intensification of heat and mass transfer. This intensification is further enhanced when the behavior index decreases from 1.4 to 0.6, which increases flow intensity due to the decrease in apparent viscosity in high shear areas. Furthermore, a high Lewis number reduces heat transfer but greatly increases mass transfer.

Keywords: Lewis number; thermo-dependent convection; double diffusion; heat and mass transfer; Pearson number; non-Newtonian



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1. Introduction

Natural convection with double diffusion in non-Newtonian heat-dependent fluids is a complex field of study in which temperature and concentration gradients interact with non-linear rheology and temperature-sensitive viscosity. This phenomenon, crucial for applications such as polymer processing, geothermal storage, or biochemical processes, involves coupled mechanisms of heat transfer, mass transfer, and viscous deformation [1–9]. In the study by Onken and Brambilla [10], the phenomenon of salt fingers in the Mediterranean was observed, highlighting an enhancement of double-diffusive transfer under well-defined saline and thermal gradients. In the boundary layers of the atmosphere, double-diffusive convection plays a role in monsoons and cyclones. More and Balasubramanian [11] studied mixing dynamics in stratified fluid layers. They demonstrated that augmenting Rayleigh number leads to an accelerated convective mixing, resulting in enhanced heat transfer and increased sensitivity of mass transport to the initial stratification configuration. Similarly, Javaheri et al. [12] conducted linear stability analysis

of double diffusion, applicable, in particular, to the CO₂ geological storage. This can be applied to other geological formations similar to those in the Alberta basin. When physical parameters remain within similar ranges, modeling CO₂ storage in saline aquifers can reasonably be considered an isothermal process. In this context, thermal effects are negligible; thus, natural convection is mainly controlled by concentration gradients. The study by Saleem et al. [13] investigated thermosolutal convective flow in a solar distiller cooled by external flow. The results showed that simultaneous augmentation in Ra and Le promotes mass and heat transfers, and improves distillation efficiency through the effect of increased evaporation and condensation. Islam et al. [14] considered natural convection in geothermal applications as a means for CO₂ injection into brine-saturated reservoirs with a focus on providing a uniform temperature and CO₂ distribution in confined geometries. Kolsi et al. [15] examined the scenario of power-law fluids within square cavities partially enclosed with porous medium of corrugated interfaces. They showed that an increase in Ra , coupled with a decrease in n , strongly favors mass and heat transfers, with a marked rise of Nu and Sh . Similarly, Nag and Molla [16,17] investigated the effect of thermal dispersion of nanoparticles inside an enclosure containing non-Newtonian nanofluids. They found that an augmentation of Le brings about a decrease in the rate of convective thermal exchange, but also to an augmentation in mass exchange. In a study by Daghab et al. [18], they investigated non-Newtonian heat-dependent fluids inside a partially heated square cavity. They concluded that augmenting Rayleigh number improves heat transfer (Nu), while the pseudoplastic behavior of the fluid ($n < 1$) further accentuates this enhancement. Similarly, Rahmoun et al. [19] analyzed double diffusion inside enclosures filled with thermo-dependent Carreau–Yasuda fluids. They found that an increase in Le significantly enhanced the Sherwood number, and showed that an increase in Pearson number (m) improved mass and heat transfers. Finally, El Hadoui and Kaddiri [20] investigated the aspect ratio role within rectangular cavities and analyzed the impact of aspect ratio on thermal exchange of free convection with temperature-dependent viscosity of nanofluids. Astanina et al. [21] performed numerical studies of free convection in square cavities with fluids whose viscosity is exponentially dependent on temperature. As highlighted by Dumoulin et al. [22], the temperature-dependent effect is significant in non-Newtonian fluids, where temperature-dependent variables complicate the heat and momentum transport. Ramamurthy and Krishnan [23] highlighted the requirement of temperature dependence in non-Newtonian fluids, demonstrating that variation in Rayleigh number significantly affects heat transfer efficiency in typical thermal systems. More studies on the effects of the governing parameters are highlighted in [24–31].

From this literature review, it was found that there are no works on the impact of Lewis number on natural double-diffusion convection of non-Newtonian liquids of temperature-dependent viscosity inside square cavities. In order to fill the gap presented in the literature, this study addresses the thermo-dependency of non-Newtonian liquids modeled by the power-law model confined inside square cavities and subjected to uniform concentration and temperature. This work focuses on the Lewis and Pearson numbers impacts on mass and heat transfers.

2. Mathematical Formulation

Figure 1 shows a schematic illustration of the problem in question. It is a square enclosure of size $H' \times H'$ that is filled with non-Newtonian liquids which are governed by the power-law model. The vertical walls are subject to constant concentration and temperature conditions and other walls are impermeable and adiabatic. Assuming a two-dimensional geometry, the problem is treated in the x - y plane, variations along the z axis

being neglected given the invariance of the system in this direction. This assumption simplifies the problem while preserving the essential physical characteristics of the flow.

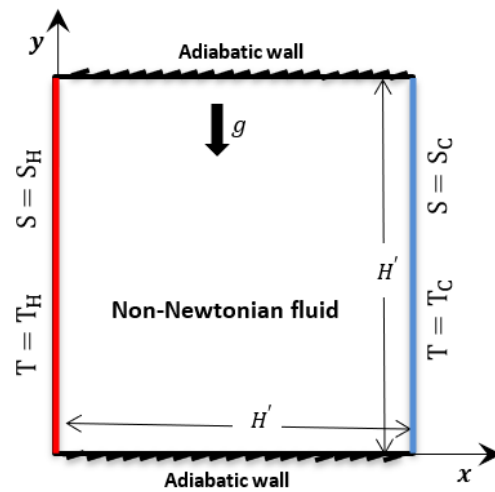


Figure 1. Schematic illustrating the physical problem.

Equations Governing Convection Accompanied by Their Boundary Conditions

The dimensionless equations for natural double diffusion of non-Newtonian liquids with the temperature-dependent viscosity are formulated as follows [32,33]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \text{Pr} \left[\mu_a \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + 2 \frac{\partial \mu_a}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \mu_a}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] - \frac{\partial p}{\partial x}, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \text{Pr} \left[\mu_a \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + 2 \frac{\partial \mu_a}{\partial y} \frac{\partial v}{\partial y} + \frac{\partial \mu_a}{\partial x} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \text{Ra}_T (T + \text{NS}) \right] - \frac{\partial p}{\partial y}, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \nabla^2 T, \quad (4)$$

$$u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} = \frac{1}{\text{Le}} \nabla^2 S, \quad (5)$$

The apparent viscosity is defined as follows [34,35]:

$$\mu_a = e^{-mT} \dot{\gamma}^{n-1}. \quad (6)$$

In Equation (6), $\dot{\gamma}$ denotes the reduced shear rate (dimensionless), while m corresponds to the parameter characterizing the dependence of viscosity on temperature. The shear rate $\dot{\gamma}$ is defined as [36]:

$$\dot{\gamma} = \sqrt{\left(\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + 2 \left(\left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial x} \right)^2 \right) \right)}. \quad (7)$$

The assigned boundary conditions in dimensionless formulation are

$$\text{For } y = 0, 1, \forall x \Rightarrow v = u = 0 \text{ and } \frac{\partial T}{\partial x} = 0 \text{ and } \frac{\partial S}{\partial x} = 0, \quad (8a)$$

$$\text{For } x = 0, \forall y \Rightarrow T = 1 \text{ and } S = 1 \text{ and } v = u = 0, \quad (8b)$$

$$\text{For } x = 1, \forall y \Rightarrow T = 0 \text{ and } S = 0 \text{ and } v = u = 0. \quad (8c)$$

The governing Equations (1)–(6) contain several dimensionless parameters essential for system characterization. These parameters include the thermal Rayleigh number, buoyancy ratio, and Lewis, Prandtl, and Pearson numbers, which are, respectively, defined as follows:

$$Ra_T = \frac{g\rho\beta_T\Delta TH^3}{\alpha\mu'_0}; N = \frac{\beta_S\Delta S^*}{\beta_T\Delta T^*}; Pr = \frac{\mu'_0}{\alpha\rho}; Le = \frac{\alpha}{D}; m = -\frac{d\ln\left(\frac{K_T}{K}\right)}{dT}. \quad (9)$$

The current function Ψ is introduced in order to automatically satisfy the incompressibility condition ($\nabla \cdot V = 0$), with the relations [37]:

$$u = \frac{\partial \Psi}{\partial y}; v = -\frac{\partial \Psi}{\partial x}. \quad (10)$$

The maximum value of this function, denoted $|\Psi_{\max}|$, reflects the flow intensity inside the cavity [38].

Mass and heat transfer rates, calculated using the equations below, are expressed by Sherwood and Nusselt numbers [39].

$$Sh = -\int_0^1 (\partial S / \partial x)_{x=0} dy; Nu = -\int_0^1 (\partial T / \partial x)_{x=0} dy. \quad (11)$$

3. Numerical Method

We developed a numerical code on FORTRAN and chose to adopt the finite volume method based on the SIMPLER (Semi-Implicit Method for Pressure-Linked Equations Revised) algorithm, in accordance with the method introduced by Patankar [40], to resolve the system of conservation equations governing the phenomenon under study. This decision stems from several advantages inherent in this approach. Firstly, it offers a high degree of numerical stability, ensuring the reliability of the results obtained, for all combinations of physical parameters considered in the study. In addition, the method demonstrates excellent convergence capability [41].

The size of the mesh grid was determined by the complexity of the solution required. A series of numerical simulations was performed to find the right balance between computational speed and accuracy. As a result of these experiments, a uniform grid of dimensions 100×100 was identified as suitable for accurately modeling the concentration, temperature, and flow fields inside the cavity.

4. Results and Discussion

4.1. Validation of the Numerical Code

In order to guarantee the accuracy and reliability of our calculation code, a rigorous validation was carried out by comparing our numerical results with those reported in the literature. First, for the case of natural convection with a Newtonian fluid, a comparison was made with the works [42,43], whose reference data are shown in Table 1. In addition, double-diffusive convection was studied for specific conditions ($Ra_T = 10^5$, $N = 0.8$, $Le = 1.0$, and $Pr = 6.2$), and the results obtained were compared with those of Nag and Molla [16], as illustrated in Table 2. This correspondence validates the accuracy of the results of the present study and confirms the ability of the numerical model to faithfully reproduce the physical phenomena expected in a natural convection regime dominated by thermal and solutal forces.

Table 1. Nusselt number comparison of the data in [41,42] and the present results for different Rayleigh numbers.

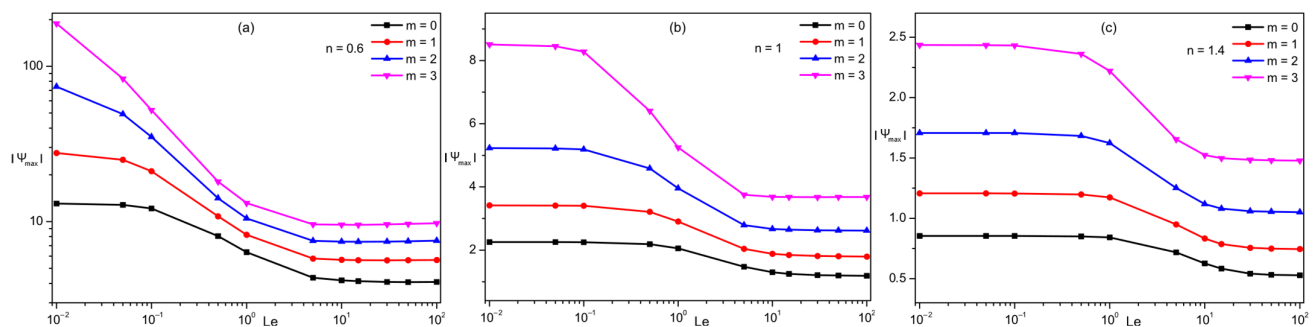
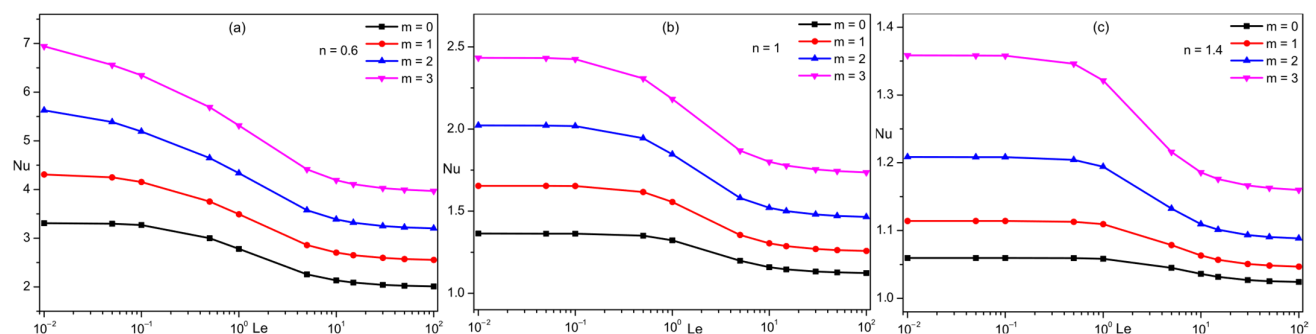
	Wang et al. [42]	Guo et al. [43]	Present Work
Ra		Nu	
10^3	1.117	1.116	1.118
10^4	2.247	2.247	2.247
10^5	4.551	4.534	4.538

Table 2. Nusselt and Sherwood numbers comparison of the data from Nag and Molla [16] and the present results for different n .

	Nag and Molla [16]		Present work	
n	Nu	Sh	Nu	Sh
0.7	11.706	11.706	12.014	12.014
1.0	5.684	5.684	5.688	5.688

4.2. Lewis Number Effect

Figures 2–4 show, in detail, the evolution of the current function ($|\Psi_{max}|$), and the mass transfer rate (Sh) and the heat transfer rate (Nu) as a function of the Lewis number (Le) for selected Pearson numbers (m) and behavior indices (n). These curves highlight the variations in these parameters.

**Figure 2.** Effect of Le on maximum current function ($|\Psi_{max}|$) for selected m and n (a): $n = 0.6$, (b): $n = 1$, and (c): $n = 1.4$, and for $Ra_T = 10^3$, $N = 1$, and $A = 1$.**Figure 3.** Effect of Le on Nu for selected m and n (a): $n = 0.6$, (b): $n = 1$, and (c): $n = 1.4$, and for $Ra_T = 10^3$, $N = 1$, and $A = 1$.

The variations of $|\Psi_{max}|$ and Nu with the Lewis number in Figures 2 and 3a–c for distinct values of m show a similar pattern, where they decrease with Le and become stable for specific ranges of Le . As can be seen from these figures, for $m = 0$, the low Lewis values remain relatively stable up to ($Le < 10^{-1}$ for $n = 0.6$, $Le < 1$ for $n = 1.0$ and 1.4), and the

increase in m decreases the range of stability due to the thermo-dependent effect, and decreases up to ($Le = 10^{-2}$ for $n = 0.6$, $Le = 10^{-1}$ for $n = 1.0$), and exceeding these values beyond $Le = 10$, these quantities become independent of Le and the double diffusion is dominated by the solutal forces.

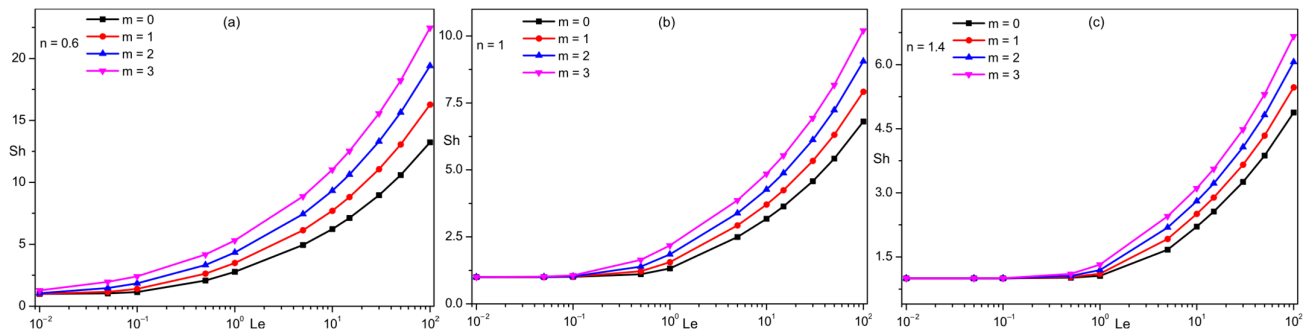


Figure 4. Effect of Le on Sh for selected m and n (a): $n = 0.6$, (b): $n = 1$, and (c): $n = 1.4$, and for $Ra_T = 10^3$, $N = 1$, and $A = 1$.

Concerning the Sherwood number (Figure 4a–c), it is characterized by a considerably different behavior. As expected, for low values of Le , it is characterized by $Sh \approx 1$, indicating that solutal transfer is practically diffusive. When the Lewis number reaches 1, Sh gradually increases asymptotically, synonymous with predominant solutal transfer. The increase in (Sh) with (Le) reflects an improvement in mass transfer in the system, induced by more efficient solutal convection processes.

This behavior can be explained by the fact that at low Le , mass diffusivity is high, which promotes strong coupling between thermal and mass gradients, thereby enhancing convection. However, when Le increases, mass diffusion becomes weak compared to thermal diffusion, which weakens the double-diffusive interaction and limits the intensity of the flow. In addition, the effect of the rheological parameter n is significant: for $n = 0.6$, the pseudoplastic fluid has a lower apparent viscosity in areas of high shear, which amplifies circulation and therefore transfers. The effect of thermo-dependence parameter (m) is then manifested by a reduction in viscosity in hot areas, further promoting flow, resulting in an augmentation up to 109% of Nusselt number, when the thermo-dependence parameter goes from 0 to 3, especially for low Le , but for high Le , it decreases to 97% for shear-thinning fluids, while it results in an augmentation up to 108% of Sherwood number, when the thermo-dependence parameter goes from 0 to 3 for $Le = 0.1$.

The increase in the thermo-dependence parameter (m) amplifies this effect, which could be attributed to increased interaction between thermal and mass gradients, while the decrease in the behavior index results in a decrease in mass and heat exchange rates and flow intensity.

Physically, this can be explained by a reduction in mass diffusivity at high Le , which accentuates concentration gradients, reinforcing convective mass transport. The increase in m amplifies this effect by decreasing viscosity in hot zones, which promotes fluid movement and intensifies transfers. Overall, these results highlight that high Le values, combined with strong thermo-dependence (high m) and pseudoplastic behavior (low n), accentuate the imbalance between heat and mass transfer, resulting in mainly solute convection.

4.3. Analysis of Flow Fields, Isotherms, and Isoconcentrations

Figure 5 shows the profiles of streamlines (ψ , left), isotherms (T , center), and isoconcentrations (S , right) for two values of Lewis number $Le = 5$ and $Le = 10^2$, as a function of distinct thermo-dependence parameters $m = 0$ (red lines) and $m = 3$ (black lines), and for

different behavior indices of the non-Newtonian fluid $n = 0.6, 1.0$ and 1.4 , corresponding, respectively, to the top, middle, and bottom rows of the figure. Simulations are carried out under the following conditions: $A = 1$, $Ra_T = 10^3$, and $N = 1$.

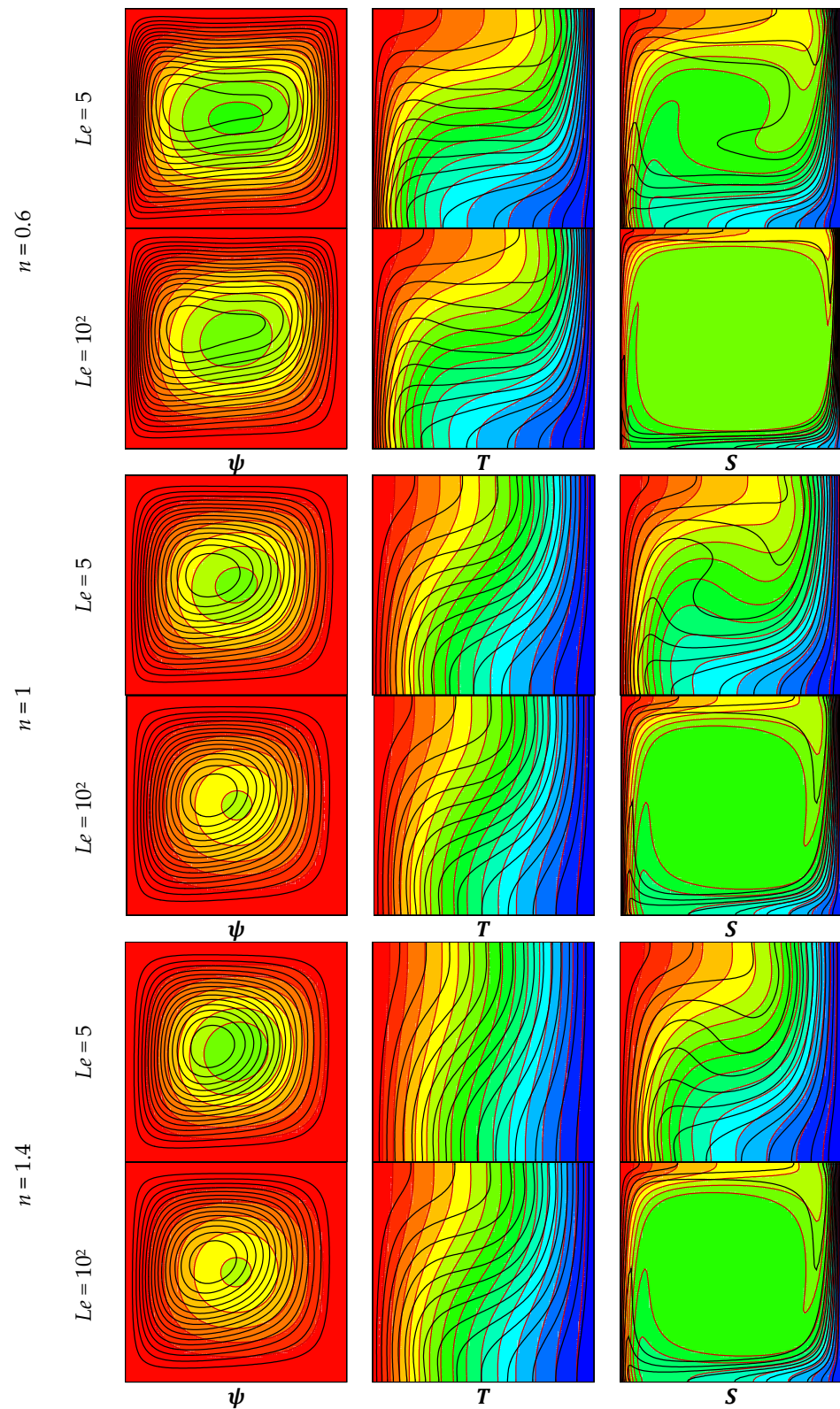


Figure 5. Profiles of streamlines (ψ) on the left, isotherms (T) in the center, and isoconcentrations (S) on the right, for cases $m = 0$ (red lines) and $m = 3$ (black lines), and for $n = 0.6$ (top row), 1 (middle row), and 1.4 (bottom row).

Streamline analysis reveals the presence of a main convection cell of generally circular shape, both in the presence and absence of viscosity thermo-dependence. However, in the absence of viscosity thermal dependence ($m = 0$), this cell remains symmetrical with respect to the center of the cavity, which is typical of classical natural flow. When viscosity depends on temperature ($m = 3$), this symmetry is broken, and the convection cell tends to shift towards the hot wall (left), as a direct consequence of the local decrease in viscosity and hence flow resistance in high-temperature regions.

The effect of thermo-dependence becomes more pronounced as the parameter m increases and the index n decreases. Indeed, a lower value of n (pseudoplastic fluid) implies a lower apparent viscosity in high shear zones, facilitating flow. This behavior translates into increased deformation of the flow lines, concentrating fluid movement next to the left sidewall, where temperature is highest. This intensification of convection is thus the result of a synergy between the rheological effect (via n) and thermo-dependence (via m).

The isotherms show an overall stratified configuration, indicating a predominantly conductive heat transfer mode. The behavior of isoconcentrations is similar, but is more influenced by the value of the Lewis number Le . At $Le = 5$, isoconcentrations are relatively diffuse, reflecting active mass transfer. When $Le = 100$, isoconcentrations become tighter and show more pronounced deformation in the zones close to the hot wall, particularly for $n = 0.6$ and $m = 3$. This increased deformation results from the combination of amplified convective transport and slower diffusion (due to the larger Le).

5. Conclusions

This work numerically explores the effect of Lewis number on double-diffusive natural convection with viscosity varied as a function of temperature in the two-dimensional square enclosure filled with non-Newtonian power-law fluids, heated and salted horizontally using uniform temperatures and concentrations. Flow, mass, and heat transfer are analyzed according to the values of the governing parameters, namely Pearson number, (m), behavior index (n), and Lewis number, (Le). The key results are summarized as follows:

- The high Lewis number enhances mass transfer and reduces heat transfer and flow intensity, and this effect is amplified by increased thermo-dependence (high values of (m)).
- Variations according to (n) show that the flow intensity and heat and mass transfer increase with decreasing n .
- The increase in the thermo-dependence parameter (m) improves the flow intensity and heat and mass transfer due to the decrease in the apparent viscosity.

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