

Article

New Possibilities for Constructing Heuristic Solutions to Problems of Electromagnetic Diffraction

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Abstract: The paper formulates the foundations of a recently developed approach, named the method of fundamental components, intended for constructing heuristic solutions in problems of electromagnetic diffraction, for the first time. The difference between the new method and the known heuristic approaches lies in the application of an adjustment procedure that increases the accuracy. The possibility of the mentioned method for obtaining new results is illustrated with the help of the author's previously published works. The advantages of the new method in constructing high-speed solvers and in the physical interpretation of numerical solutions are shown.

Keywords: electromagnetic diffraction; heuristic approaches; diffraction by polygons and polyhedral; plane angular sector; impedance boundary conditions; physical theory of diffraction



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1. Introduction

The problems of electromagnetic diffraction can be solved in different ways. Mathematically rigorous formulations of the boundary value problems include those for which the existence and uniqueness of the solution have been proven. Unlike mathematically rigorous solutions (analytical or numerical), heuristic approaches are based on physical ideas about the structure and features of the solution. Heuristic solutions play an important role for the construction of high-performance electromagnetic diffraction solvers used in urgent practical problems. At the stage of deriving, all heuristic approaches need verification, i.e., revision using a reliable solution, usually a numerical one. At the stage of application, verification is no longer needed.

1.1. Relevance of Heuristic Solutions

Historically, the theory of diffraction was developed in line with the heuristic [1–3] approaches. In this regard, the works of Huygens, Jung, Fresnel, Helmholtz, Kirchhoff and others can be noted. In the works of Poincaré and Sommerfeld, it was first shown [1] that the solution of problems in the theory of diffraction can be found using the theory of boundary value problems of mathematical physics. This fruitful approach sometimes leads to very difficult solutions. Thus, despite the obvious importance and interest of researchers in a solution to the problem of diffraction by a plane perfectly conducting angular sector, the corresponding solution was obtained more than 100 years later than the solution of the problem of diffraction by a perfectly conducting half-plane [3,4]. Therefore, the development of heuristic methods is still an urgent task.

The need to use heuristic solutions is due to a number of practical problems, such as, for example, diffraction on radar objects, including objects with low radar signature, radio wave propagation in urban environments, etc.

A number of problems can be successfully solved using heuristic approaches, as follows:

- Investigation of relatively large objects that do not have solutions obtained by more rigorous methods. Such objects include, for example, targets with low radar signature,

- non-axisymmetric satellite communication antennas, etc. The propagation of radio waves in urban environments may also be studied using heuristic methodologies.
- Improving performance in calculating urgent problems, including those problems that have solutions obtained using other approaches.
 - Physical interpretation of numerical solutions, which is especially important in cases where they differ from the experimental results.

1.2. Known Heuristic Approaches

The following heuristic approaches are known: the method of geometrical optics (GO) [5], the method of physical optics (PO) [5], the geometrical theory of diffraction (GTD) [6–8] and the method of edge waves (MEW) [9–11]. The term physical theory of diffraction (PTD) can be used for the MEW and, in an extended interpretation, for all heuristic methods.

In heuristic approaches, a balance should be obtained between complexity and accuracy. Rigorous analytical decisions are accurate, but often very complex. The PO approximation can be obtained for any object, but the accuracy may not be enough. However, sometimes, the accuracy of the PO approximation is quite sufficient, as in the case of calculating the amplitude of the main beam and the first side lobe of a reflector antenna.

For relatively large objects, strict calculation is impossible due to limited computer resources. While calculating the scattering of signals on conventional radar targets, it used to be enough to take into account only the contribution of specular points; now, in problems of diffraction on targets with a reduced radar signature, they mainly use the MEW. The same applies to the calculation of propagation in an urban environment. The success of this or that approximation depends on the accuracy required in a particular practical problem.

Heuristic solutions are based on hypotheses, or “postulates” [8]. For example, the GTD postulate says that when scattered by an object of finite dimensions, the field of a finite size edge at the observation point is considered to be the same as if the incident ray field was reflected from an infinite edge. On the infinite edge, you can always find the point of the stationary phase at which the reflection occurs. At the edge of finite dimensions, the stationary phase point does not always exist; therefore, using the GTD, no solution at all can be obtained. This circumstance was the reason for the choice of the MEW (instead of the GTD) for calculating of low radar signature objects made using the “Stealth” technology [10].

The MEW postulate is that for a scatterer of finite dimensions, the field at the finite size edge is considered the same as at the edge of a semi-infinite scatterer with an edge of the same profile. In contrast to the GTD, a solution for the MEW (as well as for the PO method) can always be obtained (although sometimes with an inaccurate amplitude). The amplitude inaccuracies are caused by the fact that the field at the edge of finite dimensions is considered constant and the additional disturbance at the ends of the edge is neglected. In order to take this perturbation into account, it is necessary to investigate the field on a plane angular sector, for example, by numerical methods [12].

1.3. Reference Problems for Constructing Heuristic Solutions

When solving practical problems, both for semi-infinite objects and for objects of finite size, 1D, 2D and 3D reference problems are used.

Types of diffraction reference problems are the following:

- Linear integral (3D): This is a solution in the PO approximation to the diffraction problem on a plane perfectly conducting scatterer. In this case, the far zone condition (FZC) is fulfilled both for the source and for the observation point. The integral over the area of the scatterer is reduced to an integral over its contour [13–15]. If the FZC is not fulfilled, the technique described in [16] can be applied.
- Vertex waves (3D): This is a solution to the diffraction problem for a plane angular sector. The analytical solution is described in [17], the heuristic ones are described in [18–21].

- Solutions to the problem of diffraction by perfectly conducting semi-infinite edges of different profiles (2D) [22–28].
- Reflection and transmission coefficients (1D) R and T for an unbounded plane.
- Solutions to the problems of diffraction by semi-infinite objects with imperfect boundary conditions (2D, 2.5D—oblique incidence) [29–34]: For such structures, heuristic solutions are constructed, despite the presence of more rigorous solutions (both analytical and numerical).

Reference problems can be applied in the form of strict analytical formulas, heuristic formulas, engineering formulas (which may not have physical meaning), within the framework of hybrid approaches (combining elements of numerical and heuristic methods) or in the form of databases (pre-calculated or experimental) [3].

1.4. Motivation to Develop New Heuristic Approaches

The impossibility of using one of the well-known heuristic approaches in solving a practical problem is determined by one of the following factors:

- Insufficient accuracy of known heuristic approaches.
- Insufficient performance of the solver.
- Lack of the required number of rigorous analytical solutions, on the basis of which, a solver can be built for a specific practical problem.
- The need for a physical interpretation of the numerical solution and the identification of physical phenomena that were not taken into account by the old heuristic approaches.

These problems can be overcome by developing a new heuristic approach and constructing refined formulas.

2. New MFC Approach and Its Differences from Traditional Approaches

Suppose there is a specific practical problem for which one needs to construct an efficient (accurate and high-performance) solver. Suppose there are no analytical formulas for the corresponding reference problems, but there are results of numerical calculation of solutions of reference problems or experimental data. This is a common situation. With the help of a recently developed approach—the method of fundamental components (MFC) [3,4]—this situation can be used to construct efficient (compact and accurate) heuristic solutions.

At the first stage in the MFC, primary heuristic formulas are built. They are then subjected to an adjustment procedure. By this term, we mean the correction of primary formulas by functions (which we call fundamental components), based on the comparison of primary formulas with a verification solution.

In the process of adjustment, the contributions of known physical factors are consistently taken into account. Based on the difference between a strict and a heuristic solution, one can find the influence of unaccounted phenomena. In this case, we apply the adjustment procedure. One can act in two ways, as follows:

- first, determine the unaccounted phenomenon and find an expression for it, then add it to the primary heuristic formula and carry out verification (as in the MEW);
- or, first, find the difference between the strict solution and the primary heuristic formula, and then find a physical phenomenon that determines this difference (as in the MFC).

This difference significantly increases the number of problems for which heuristic formulas can be constructed. One can obtain a predetermined accuracy, as well as work with formulas for which there are no analytical solutions. So, the MFC has the property of universality.

As the primary heuristic formulas for any practical problem, one can choose the solutions of the simplest diffraction problems, or the heuristic formulas constructed earlier. The fundamental components can be selected for both physical and mathematical reasons.

Earlier, attempts have been also made to refine the primary heuristic formulas. It is possible to apply the multiple re-reflection of the GTD solution at the edges [18,19] or the PTD [10], and also apply the reciprocity principle [31–34]. Sometimes these approaches work well, sometimes they do not. In addition, solutions based on multiple reflections can be cumbersome. It is not always possible to reduce them into compact formulas.

3. Formulas Used in the MFC

3.1. Partitioning the Formula for a 2D Edge

The book [3] contains a general formula for the field $V(\varphi)$ at the observation point (r, φ) , excited by a source located at the point (r_0, φ_0) and scattered by a perfectly conducting wedge with an external aperture angle πn at normal incidence on the TH- or TE-polarized electromagnetic wave:

$$\begin{aligned} V(\varphi) &= v(\varphi - \varphi_0) - v(\varphi + \varphi_0) \text{ (TH)}, \\ V(\varphi) &= v(\varphi - \varphi_0) + v(\varphi + \varphi_0) \text{ (TE)}, \end{aligned} \tag{1}$$

where:

$$v(\psi) \cong \frac{P(w_{sm})}{\sqrt{2\pi ik \frac{rr_0}{r+r_0}}} \frac{\frac{i}{n} \sin \frac{\pi}{n}}{\cos \frac{\pi}{n} - \cos \frac{\psi}{n}} \frac{2i \sqrt{S(w_{sm}) - S(\psi)}}{\exp[iS(w_{sm}) - iS(\psi)]} \int_{\infty \sqrt{S(w_{sm}) - S(\psi)}}^{\sqrt{S(w_{sm}) - S(\psi)}} \exp(iq^2) dq \tag{2}$$

Here, $P(w_{sm})$ is the source field at the saddle point w_{sm} (from the geometric point of view, the saddle point corresponds to the situation when the edge is on the straight line connecting the observation point with the source, with $\varphi \pm \varphi_0 = \pi$), and the eikonal $S(\psi)$ and the eikonal at the saddle point $S(w_{sm})$ are equal, as follows:

$$S(w_{sm}) = k(r + r_0), S(\psi) = k\rho = k\sqrt{(r + r_0)^2 - 2rr_0[1 + \cos \psi]}, \psi = \varphi \mp \varphi_0. \tag{3}$$

The area external to the wedge occupies the space of angles $0 < \varphi < \pi n$. The geometry of the problem is shown in Figure 1.

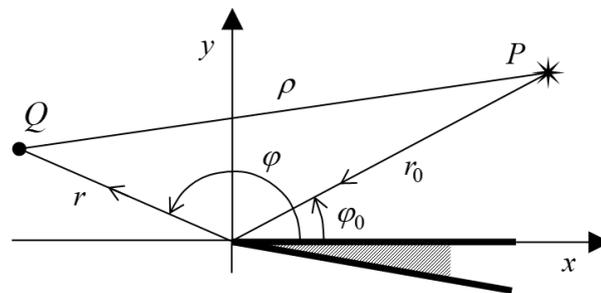


Figure 1. Diagram for the incidence of a point source wave on a wedge. P and Q : source and observation point, φ : scattering angle, r_0 : source distance, φ_0 : angle of incidence, ρ : distance from source to observation point.

In Figure 1, P and Q are the source and observation point, respectively, and other designations are as indicated in Formulas (1)–(3).

Let us consider the integral representation for the scattered field in the case of two saddle points [3] and its particular case for diffraction by a wedge—Expression (2).

Each of the two terms on the right-hand side of Expression (2) consists of four factors. The first factor is as follows:

$$\frac{\sqrt{i}P(w_{sm})}{\sqrt{2\pi k \frac{rr_0}{r+r_0}}} \tag{4}$$

which does not depend on the angular variable and is the product of the field value at the saddle point (i.e., on the shadow boundary) by the factor that determines the dependence of the solution on the distance to the source and the observation point.

The second factor, as follows:

$$\frac{\frac{1}{n} \sin \frac{\pi}{n}}{\cos \frac{\pi}{n} - \cos \frac{\psi}{n}} \tag{5}$$

represents half of the diffraction coefficient. The total diffraction coefficient for a certain type of polarization is obtained by adding or subtracting the values of this factor at the observation point, as follows:

$$\frac{\sin \frac{\pi}{n}}{n} \left(\frac{1}{\cos \frac{\pi}{n} - \cos \frac{\varphi - \varphi_0}{n}} \pm \frac{1}{\cos \frac{\pi}{n} - \cos \frac{\varphi + \varphi_0}{n}} \right). \tag{6}$$

Product of the third and fourth factors, as follows:

$$\frac{2i \sqrt{S(w_{sm}) - S(\psi)}}{\exp[iS(w_{sm}) - iS(\psi)]} \frac{\int_{\infty \sqrt{S(w_{sm}) - S(\psi)}}^{\sqrt{S(w_{sm}) - S(\psi)}} \exp(iq^2) dq}{\infty \sqrt{S(w_{sm}) - S(\psi)}} \tag{7}$$

is the quotient of dividing the Fresnel integral by its asymptotics and characterizes the dependence of the field on the angular distance to the “light-shadow” boundary (or, that which is the same, the “shadow” boundary). Far from the shadow boundary, this product is equal to 1. At the shadow boundary, this factor is zero and compensates for the singularity of half of the diffraction coefficient (5). This compensation results in a field equal to half of the field of geometrical optics at the shadow boundary of a semi-infinite scatterer.

By changing the type of factors (4)–(7), or the parameters included in them, it is possible to construct a variety of heuristic solutions: for two-dimensional or three-dimensional scatterers, finite or infinite size, with or without fulfillment of the far-field condition, for different types of boundary conditions and edge profile.

More information on this topic can be found in [3].

3.2. Linear Integral in the PO Approximation

The linear integral in the PO approximation is used as the basis for the 3D solution of the diffraction problem.

Consider a perfectly conducting plane (with a normal vector \vec{n}) scatterer excited by a plane wave (time dependence $\exp\{ik(\vec{n}', \vec{r})\}$), as follows:

$$\vec{H}^{(i)} = \vec{H}_0 \exp\left\{ik\left(\vec{n}', \vec{R}\right)\right\} = (H_{0x}, H_{0y}, H_{0z}) \exp\left\{ik\left(\vec{n}', \vec{R}\right)\right\}, \tag{8}$$

where $\vec{H}^{(i)}$ and \vec{H}_0 is the magnetic field vector, which depends on the distance and does not depend on the distance, respectively, \vec{n}' is the direction vector of the incident wave, $\vec{R} = \vec{n}'' R$ is the radius vector of the observation point, \vec{n}'' is the direction vector of the observation point, i is the imaginary unit, $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. The spatial position of the vector $\vec{H}^{(i)}$ or \vec{H}_0 in relation to the scatterer depends on the selected type of polarization. The geometry of the problem is shown in Figure 2.

In the PO approximation, the vector potential is defined as follows [3]:

$$\vec{A}(\vec{R}) = \frac{1}{2\pi} \iint_S \frac{\exp\{ikr\}}{r} \left[\vec{n} \times \vec{H}^{(i)} \right] ds, \tag{9}$$

where $r = \left| \vec{n}'' R - \vec{\rho} \right|$ is the distance between the point on the scatterer and the observation point, $\vec{\rho}$ is the radius vector of the point on the surface of the scatterer, R is the distance from the center of coordinates to the observation point, S is the square of the scatterer. Without loss of generality, we will assume that the plane scatterer is located in the XOY plane. In addition, let the far-field condition $r \cong R - \left(\vec{n}'', \vec{\rho} \right)$ [3,5,21] be satisfied, and then:

$$\vec{A} \cong \frac{1}{2\pi} \frac{\exp\{ikr\}}{R} (-H_{0y}, H_{0x}, 0) I, \quad I = \iint_S \exp\left\{ ik \left(\vec{\Delta}, \vec{\rho} \right) \right\} ds, \quad (10)$$

where $\vec{\Delta} = \left(\vec{n}' - \vec{n}'' \right) - \vec{n} \left[\left(\vec{n}' - \vec{n}'' \right), \vec{n} \right]$ is the projection onto the scatterer surface (in our case, onto the XOY plane) of the difference $\left(\vec{n}' - \vec{n}'' \right)$ between the direction vectors of the incident wave and the observation point.

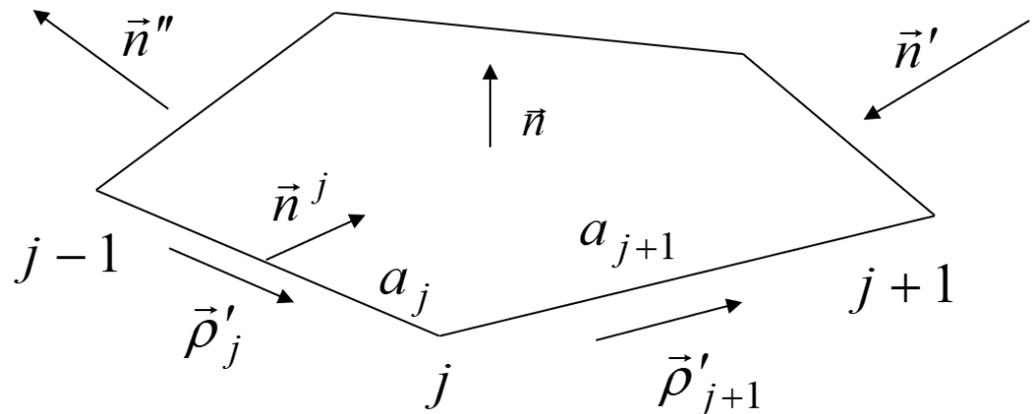


Figure 2. The fall of an electromagnetic wave on a plane polygon.

Applying Stokes' theorem, one can obtain the following [3,14]:

$$I = \iint_S \exp\left\{ ik \left(\vec{\Delta}, \vec{\rho} \right) \right\} ds = \frac{i}{k \left| \vec{\Delta} \right|^2} \oint_C \left(\vec{\Delta}, \vec{n}^i \right) \exp\left\{ ik \left(\vec{\Delta}, \vec{\rho} \right) \right\} dt, \quad (11)$$

where \vec{n}^i is the unit internal normal to the contour C surrounding the scatterer, $\vec{\rho}'$ is the unit vector tangential to the contour and t is the coordinate measured along the contour. Let us denote the phase of the integration point in the exponent, as follows:

$$\Phi = k \left(\vec{\Delta}, \vec{\rho} \right). \quad (12)$$

If the contour is a polygon with N vertices, then:

$$I_j = \frac{i}{k \left| \vec{\Delta} \right|^2} \int_0^{a_j} \left(\vec{\Delta}, \vec{n}^j \right) \exp\left\{ ik \left(\vec{\Delta}, t \vec{\rho}'_j \right) \right\} dt = \frac{ia_j \left(\vec{\Delta}, \vec{n}^j \right) \exp\left\{ ik \left(\vec{\Delta}, \vec{\rho}_j \right) \right\} - \exp\left\{ ik \left(\vec{\Delta}, \vec{\rho}_{j-1} \right) \right\}}{k \left| \vec{\Delta} \right|^2 ik \left(\vec{\Delta}, \vec{\rho}_j - \vec{\rho}_{j-1} \right)},$$

$$\text{or } I = \sum_{j=1}^N I_j, \quad I_j = \frac{ia_j \left(\vec{\Delta}, \vec{n}^j \right) \sin\left[\left(\Phi_j - \Phi_{j-1} \right) / 2 \right]}{k \left| \vec{\Delta} \right|^2 \left(\Phi_j - \Phi_{j-1} \right) / 2} \exp\left\{ i \left(\Phi_j + \Phi_{j-1} \right) / 2 \right\}, \quad (13)$$

where $\Phi_j = k \left(\vec{\Delta}, \vec{\rho}_j \right)$ is the phase of the signal of the j -th vertex with the direction vector, which is the length of the j -th side of the polygon (located between the $j - 1$ th and j -th vertices).

If the observation point is located on the diffraction cone, when $\left(\vec{\Delta}, \vec{\rho}' \right) = 0$, then:

$$\vec{\Delta} \parallel \vec{n}^j, \left| \vec{\Delta} \right| = \left| \left(\vec{\Delta}, \vec{n}^i \right) \right|, \left(\vec{\Delta}, \vec{n}^i \right) = \left(\vec{n}', \vec{n}^i \right) - \left(\vec{n}'', \vec{n}^i \right) = \sin \beta (-\cos \varphi_0 - \cos \varphi), \quad (14)$$

where $(\varphi_0 + \pi)$ and φ are the angles between the projections of the direction vectors \vec{n}' and \vec{n}'' onto the plane perpendicular $\vec{\rho}'$ and the internal normal \vec{n}^j to the contour C , β is the angle between \vec{n}' (or \vec{n}'') and $\vec{\rho}'$ (these vectors make the same angle with the edge, since the diffraction cone does not only \vec{n}' , but also \vec{n}'' directed along the generatrix of the cone, which is specified by the vector \vec{n}'). Finally, we obtain the following:

$$I_j = \frac{ia_j \exp\{i\Phi_j\}}{k \left(\vec{\Delta}, \vec{n}^j \right)} = \frac{-ia_j \exp\{i\Phi_j\}}{k \sin \beta (\cos \varphi_0 + \cos \varphi)}, \quad (15)$$

where $\Phi_{j-1} = \Phi_j$.

Formula (13) are used as a basis for constructing heuristic formulas both for plane three-dimensional objects of finite size and semi-infinite objects.

More information on this topic can be found in [3].

3.3. Diffraction Coefficients for a Perfectly Conducting Half-Plane F and G with a Geometry Pattern

For a perfectly conducting half-plane, the expressions in (6) take the well-known [9] form of singular (at $\varphi \pm \varphi_0 = \pi$) diffraction coefficients, as follows:

$$f(\varphi, \varphi_0) = \frac{1}{2} \left(\frac{1}{-\cos \frac{\varphi - \varphi_0}{2}} - \frac{1}{-\cos \frac{\varphi + \varphi_0}{2}} \right) = \frac{2 \sin \frac{\varphi}{2} \sin \frac{\varphi_0}{2}}{\cos \varphi + \cos \varphi_0} \text{ (TH)}, \quad (16)$$

$$g(\varphi, \varphi_0) = \frac{1}{2} \left(\frac{1}{-\cos \frac{\varphi - \varphi_0}{2}} + \frac{1}{-\cos \frac{\varphi + \varphi_0}{2}} \right) = \frac{-2 \cos \frac{\varphi}{2} \cos \frac{\varphi_0}{2}}{\cos \varphi + \cos \varphi_0} \text{ (TE)}. \quad (17)$$

Taking into account the relation [1], as follows:

$$v^0(r, \psi) = \sin \frac{\psi}{2} v(r, \psi), \text{ where } \psi = \varphi \mp \varphi_0, \quad (18)$$

where $v(\psi)$ is determined by Expression (2), one can construct singular diffraction coefficients for the PO approximation, as follows:

$$f^0(\varphi, \varphi_0) = \frac{1}{2} \left(\frac{\sin \frac{\varphi - \varphi_0}{2}}{-\cos \frac{\varphi - \varphi_0}{2}} - \frac{\sin \frac{\varphi + \varphi_0}{2}}{-\cos \frac{\varphi + \varphi_0}{2}} \right) = \frac{\sin \varphi_0}{\cos \varphi + \cos \varphi_0} \text{ (TH)}, \quad (19)$$

$$g^0(\varphi, \varphi_0) = \frac{1}{2} \left(\frac{\sin \frac{\varphi - \varphi_0}{2}}{-\cos \frac{\varphi - \varphi_0}{2}} + \frac{\sin \frac{\varphi + \varphi_0}{2}}{-\cos \frac{\varphi + \varphi_0}{2}} \right) = \frac{-\sin \varphi}{\cos \varphi + \cos \varphi_0} \text{ (TE)}. \quad (20)$$

3.4. Modifying Functions for a Perfectly Conducting Plane Angular Sector

An analytical solution to the problem of perfectly conducting quarter-plane diffraction appeared only a few years ago [17]. The geometry of the problem is shown in Figure 3.

The polarization transmission coefficients D are determined by the expressions in [17], as follows:

$$\begin{pmatrix} E_{\vartheta}^{\infty} \\ E_{\varphi}^{\infty} \end{pmatrix} = \frac{\exp(ikR)}{kR} \begin{pmatrix} D_{\vartheta\vartheta} & D_{\vartheta\varphi} \\ D_{\varphi\vartheta} & D_{\varphi\varphi} \end{pmatrix} \begin{pmatrix} E_{\vartheta}^{inc} \\ E_{\varphi}^{inc} \end{pmatrix}. \tag{21}$$

In [3,20,21], a heuristic solution to the problem of diffraction by a perfectly conducting plane angular sector is presented. In this case, the principle of “imaginary edge” was used. The imaginary edge is the direction in space in relation to which the directions to the source and to the observation point are disposed on the diffraction cone. A diffraction cone is a set of directions that make up the same angles with the edge [3,9].

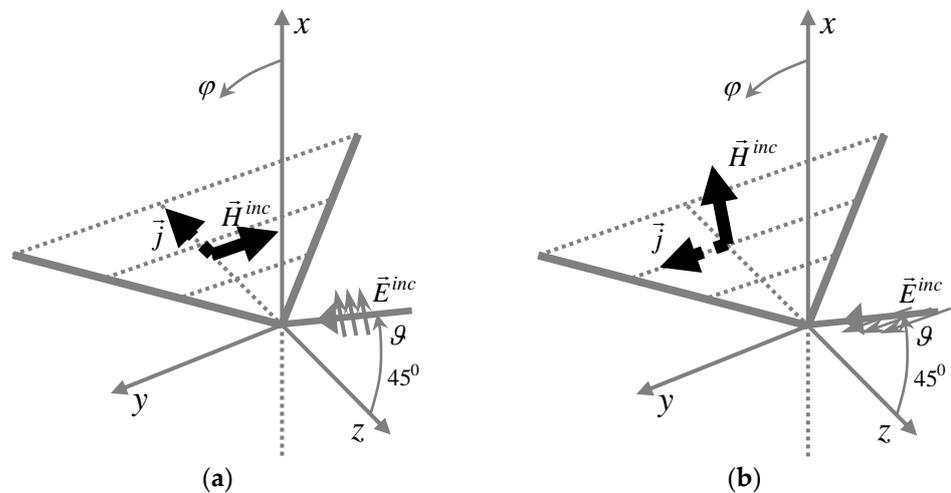


Figure 3. Geometry of the problem of diffraction on a plane angular sector. (a) first type of polarisation, (b) second type of polarisation.

In Figure 4, the dotted line shows an imaginary edge making an angle $\gamma/2$ with the $-z$ direction, while the real edge makes an angle $\beta/2$ with this direction. The currents of physical optics are designated as j_{φ} (a) and j_{ϑ} (b), depending on the types of polarization corresponding to Figure 3a,b. Components of currents directed perpendicularly and parallel to the imaginary edge are designated cr and cp , respectively.

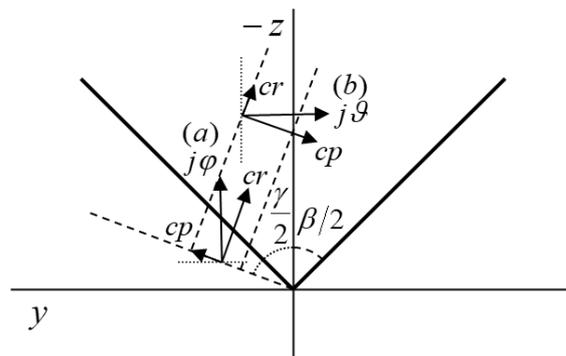


Figure 4. Decomposition of the PO current for the “imaginary” edge. cr : current component perpendicular to the imaginary edge, cp : current component parallel to the imaginary edge.

Figure 5 illustrates the application of an imaginary edge. Figure 5a shows a scheme for solving 2D problem for diffraction of a plane wave at an infinite edge. Waves are scattered only in the directions of the diffraction cones. On the diffraction cones, the GTD and the MEW solutions coincide.

The wave vector of a wave incident on an edge can be divided into two components: one going along the edge, the other is perpendicular to it. This decomposition is shown in Figure 5c.

Figure 5b shows an infinite edge, source position and viewpoint that are outside the diffraction cone, relative to the source. Diffraction cones are also shown, corresponding to situations where the source and observation point are swapped. If the positions of the source and observation points are interchanged, then decomposition becomes different, which leads to a violation of the principle of reciprocity. This violation has to be fought in the classical version of the MEW [9].

In Figure 5d a lilac dotted line shows an imaginary edge. This is the direction on the surface of the scatterer, determined so that the source and receiver are disposed on the diffraction cone in relation to the imaginary edge. Thus, for a solution using an imaginary edge, the reciprocity principle is fulfilled.

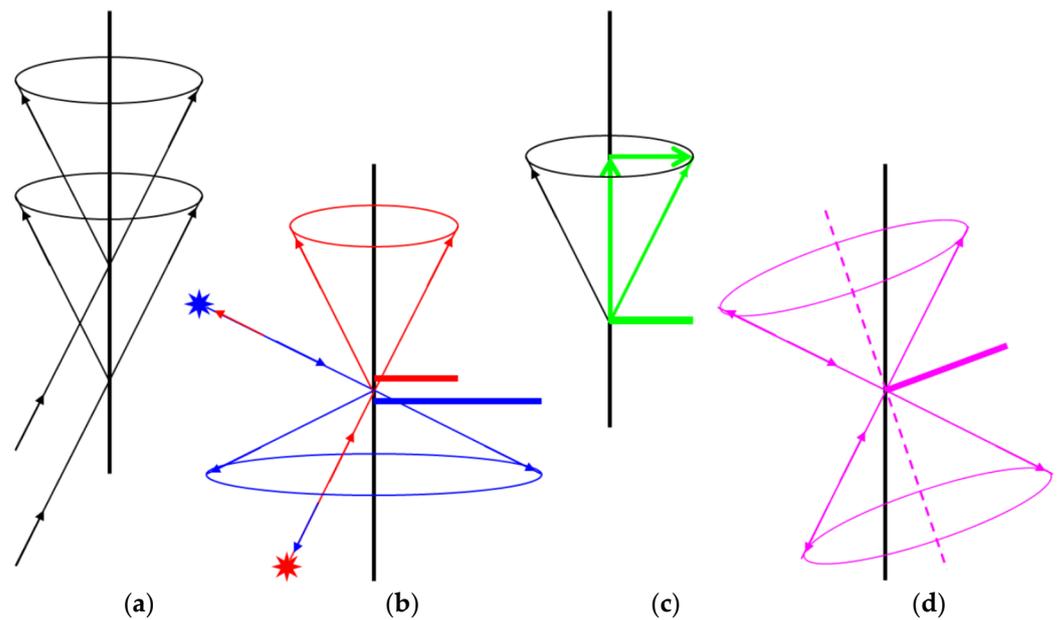


Figure 5. Scheme of using the imaginary edge. (a) a scheme for solving 2D problem for diffraction of a plane wave at an infinite edge, (b) an infinite edge, source position and viewpoint that are outside the diffraction cone, relative to the source, (c) the wave vector of a wave incident on an edge can be divided into two components, (d) a lilac dotted line shows an imaginary edge.

The concept of an imaginary edge unites the approaches of the GTD and the MEW, since the GTD solution on an imaginary edge is used in order to obtain the MEW solution in directions outside the diffraction cones.

A more complete set of formulas describing the heuristic solution of the diffraction problem on a plane angular sector, as well as the calculation results, are given in [3,20,21] and in [35–37]. Calculated data from [17] were used as a verification solution. In this case, we used the linear integral in the PO approximation (13) and the diffraction coefficients (16) and (17), which replaced the diffraction coefficients (19) and (20), contained in the PO expression for the vector potential (10) [3]. Additionally, (in accordance with the principles of the MFC), additional multipliers (modifying functions) obtained as a result of the adjustment were applied, as follows:

$$\begin{aligned}
 D''_{\theta\theta} &= D_{\theta\theta}2cr\gamma^{0.7}, & D''_{\theta\varphi} &= D_{\theta\varphi}0.5cr\gamma^{-0.7}, \\
 D''_{\varphi\theta} &= D_{\varphi\theta}2cr\gamma^{0.7}, & D''_{\varphi\varphi} &= D_{\varphi\varphi}0.5cr\gamma^{-0.7}.
 \end{aligned}
 \tag{22}$$

Formula (22) contain the following functions:

$$cp\gamma = \frac{f(\varphi\gamma, \varphi\gamma_0)}{f^0(\varphi\gamma, \varphi\gamma_0)} = \frac{\sin \frac{\varphi\gamma}{2}}{\cos \frac{\varphi\gamma_0}{2}} = cp\gamma^{-1}, \quad cr\gamma = \frac{g(\varphi\gamma, \varphi\gamma_0)}{g^0(\varphi\gamma, \varphi\gamma_0)} = \frac{\cos \frac{\varphi\gamma_0}{2}}{\sin \frac{\varphi\gamma}{2}}. \quad (23)$$

The index “ γ ” means that the diffraction coefficients were taken on an imaginary edge located at an angle $\gamma/2$ to the coordinate axis (Figure 4).

The use of modifying functions clarifies the heuristic solution of the MFC in comparison with the MEW. The calculation results are shown in Figure 6.

In Figure 6, the worst coincidence with the strict solution [17] (solid gray line) gives the PO approximation, since the D coefficients for the cross polarizations $D_{\varphi\theta}$ and $D_{\theta\varphi}$ vanish. The best match with the strict solution is given by the modified formula in (22). The curves for the D coefficients calculated using the indicated heuristic approaches are shown by red dots and blue dashes next to the strict solution curves.

3.5. Diffraction Coefficients GDC (Generalized Diffraction Coefficients) and PODC (Physical Optics Diffraction Coefficients) for a Semitransparent Half-Plane

The boundary conditions for the impedance half-plane were considered in [38–43]. In papers [4,44], using the MFC, a heuristic solution to the problem of diffraction of an electromagnetic TH—a polarized wave on a half-plane with impedance boundary conditions, such as a thin layer—was obtained. In [45], a heuristic solution of the same problem was obtained for the case of TE polarization.

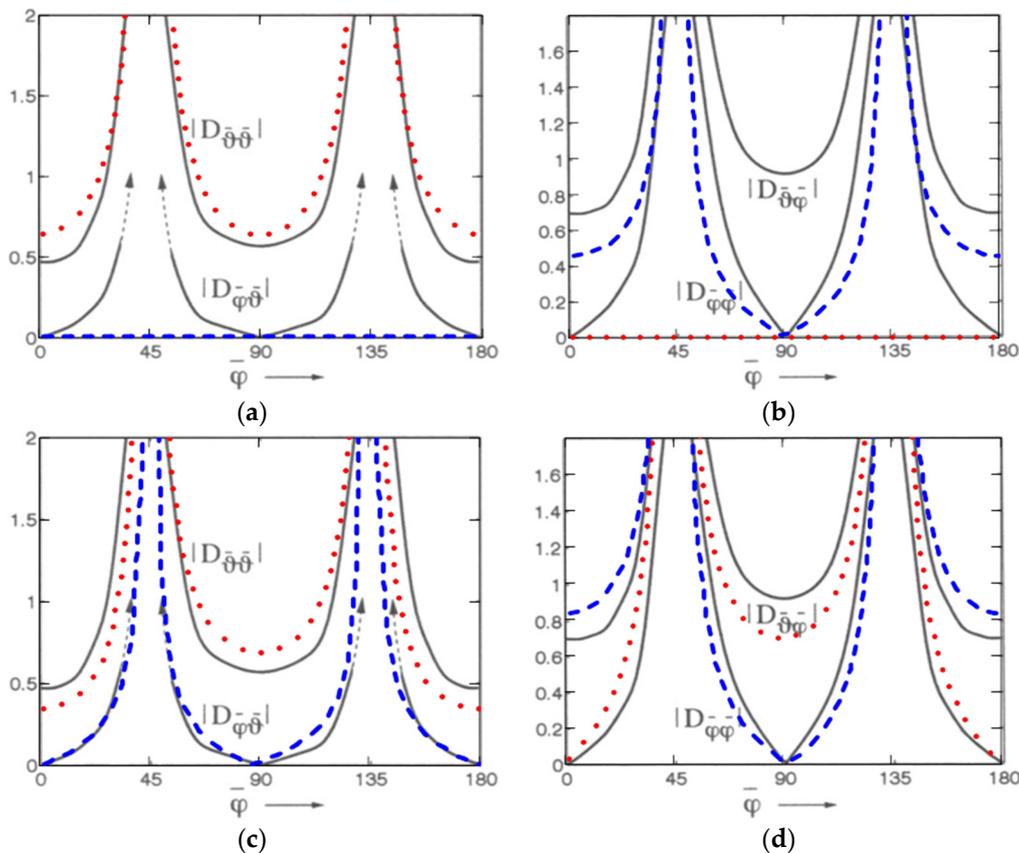


Figure 6. Cont.

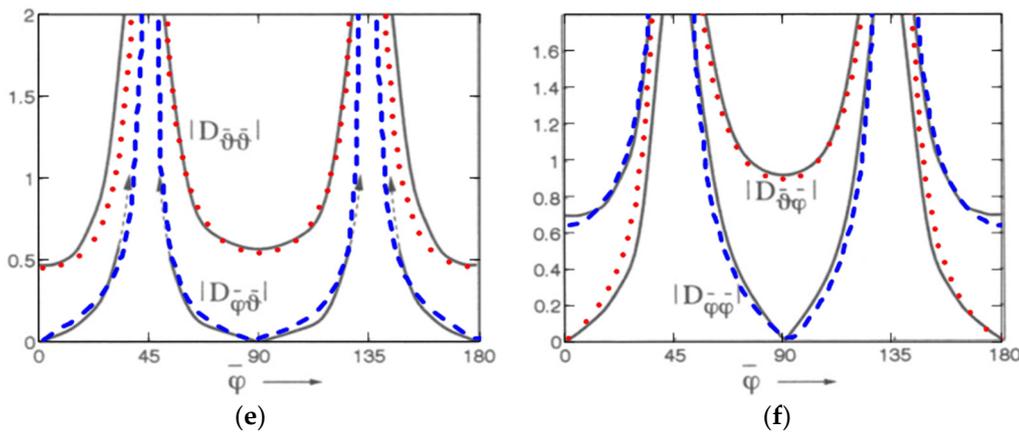


Figure 6. (a,b) Comparison of D coefficients (21) for PO approximation (19), (20); (c,d) D coefficients (21) for method of equivalent edge currents approximation (16), (17); (e,f) D coefficients (22) (red and blue lines) with modifying functions (23) for a plane angular sector. Grey lines correspond to solution [17]. The angle φ is plotted along the horizontal axis (see Figure 1). The vertical axis shows the amplitudes of the D coefficients characterizing the relationship between the polarizations of the incident and scattered fields (21)–(23). $D_{\varphi\theta}$: transmission coefficients D from [17] are determined in (21).. Horizontal lines designate the angle φ as in Figure 3.

Primary heuristic formulas are constructed on the basis of diffraction coefficients (16), (17), (19) and (20), adding to these expressions the reflection, R , and transmission, T , coefficients for an unbounded plane surface. The subscripts “TH” and “TE” indicate the type of polarization when the corresponding vectors are perpendicular to the edge. As the degree of translucency of the half-plane changes from opaque to completely transparent, the moduli of the reflection coefficients R change from 1 to 0.

We will call the resulting diffraction coefficients the GDC, as follow:

$$fg(R_{TH}, T_{TH}, \varphi, \varphi_0) = \frac{1}{2} \left[(1 - T_{TH}) \frac{1}{-\cos \frac{\varphi - \varphi_0}{2}} - R_{TH} \frac{1}{-\cos \frac{\varphi + \varphi_0}{2}} \right], \tag{24}$$

$$gf(R_{TE}, T_{TE}, \varphi, \varphi_0) = \frac{1}{2} \left[(1 - T_{TE}) \frac{1}{-\cos \frac{\varphi - \varphi_0}{2}} + R_{TE} \frac{1}{-\cos \frac{\varphi + \varphi_0}{2}} \right] \tag{25}$$

and the PODC, as follow:

$$fg^0(R_{TH}, T_{TH}, \varphi, \varphi_0) = \frac{1}{2} \left[(1 - T_{TH}) \frac{\sin \frac{\varphi - \varphi_0}{2}}{-\cos \frac{\varphi - \varphi_0}{2}} - R_{TH} \frac{\sin \frac{\varphi + \varphi_0}{2}}{-\cos \frac{\varphi + \varphi_0}{2}} \right], \tag{26}$$

$$gf^0(R_{TE}, T_{TE}, \varphi, \varphi_0) = \frac{1}{2} \left[(1 - T_{TE}) \frac{\sin \frac{\varphi - \varphi_0}{2}}{-\cos \frac{\varphi - \varphi_0}{2}} + R_{TE} \frac{\sin \frac{\varphi + \varphi_0}{2}}{-\cos \frac{\varphi + \varphi_0}{2}} \right]. \tag{27}$$

Verification of these primary formulas showed that when the degree of transparency changes, formulas (24) and (25) transform to formulas (26) and (27) [44,45].

4. Results Obtained Using the MFC

- Diffraction on edges of complex shape [3,4,14,15,24–27].
- Diffraction on a perfectly conducting plane angular sector [3,4,20,21,36,37].
- Diffraction on a half-plane with impedance boundary conditions [3,4,43–45].
- Diffraction of an elastic wave [3,46].
- Electromagnetic wave propagation in urban environment [3,47].
- Diffraction on a perfectly conducting rectangle [3,4,36,37,48]. Adding contribution of a perfectly conducting plane angular sector can improve accuracy.

5. List of Fundamental Components

At the moment, we propose to include the following formulas in the list of fundamental components:

1. Components of the 2D formulas (4)–(7) for the scattered field, $v(\psi)$, (2), (3).
2. Linear integral with emphasis on the contributions of individual edges (13).
3. R and T coefficients for an unbounded flat surface.
4. Diffraction coefficients with R and T GDC (24), (25) and PODC (26), (27). Specific values of R and T are determined by a given type of boundary conditions.
5. The polarization component of the diffraction coefficient [4].
6. The geometric component of the diffraction coefficient [4].
7. Modifying functions $c_{p\gamma}$ and $c_{r\gamma}$ (23) for a plane angular sector.
8. Semitransparency function [4,44,45].

As new tasks are solved, the list of fundamental components will increase. The resulting heuristic formulas can be selected as primary ones for new tasks.

6. Prospects for the Application of New Heuristic Approaches

The difficulties in solving practical problems of diffraction by polygons and polyhedra are as follows:

- Numerical solutions require large computer resources, software and programmers. Modern computers cannot cope with some tasks, so one has to use heuristic approaches.
- Rigorous analytical solutions are lacking for most of the vast variety of edges that do not have analytical solutions.
- The need to improve the performance of solvers for the study of specific types of tasks, such as, for instance, inverse problems.

In connection with the above, the MFC provides new opportunities for solving practical problems. New compact and at the same time exact (i.e., effective) heuristic formulas can be used for physical interpretation of numerical solutions and for constructing high-speed solvers.

7. Discussion

With the help of the MFC, on the basis of numerical solutions, it is possible to create heuristic analytical formulas with different properties. All heuristic formulas are fast. More compact and physically clear formulas can be used for a qualitative description of a numerical solution and its physical interpretation, more complex and accurate formulas are better suited for creating computers designed to solve resource-intensive problems.

One can set the accuracy depending on the purposes of the calculation. In the case of a study of propagation in urban environment, a less accurate approximation can be more appropriate. On the contrary, in the case of radar problems, it is better that the approximate (heuristic) solution is as accurate as possible, and this can be achieved using the adjustment procedure.

The novelty of the MFC in comparison with the PO, GTD and MEW lies in the way in which heuristic formulas are constructed. In the MFC, on the basis of the primary heuristic formulas GDC and PODC, it is possible to isolate and exclude from further consideration the determining factors, such as the influence of singularities at the “light-shadow” boundaries and the coefficient of reflection from an unbounded surface. Then, they focus on the adjustment procedure. As a result, within a specific formulation of the problem, it is possible to identify and describe the subtle physical features of the diffraction process, which have not yet been included in the initial heuristic formulas. For example, using the MFC during the tuning procedure, it was possible to detect and describe a characteristic of the diffraction solution, such as the semitransparency function.

The formulas for singular diffraction coefficients (16), (17), (19), (20), (24)–(27) are compact, physically clear and accurate.

The physical clarity of the formulas constructed using the MFC, taking into account their speed and accuracy, gives the method an advantage over alternative approaches: both over engineering formulas and over hybrid numerical methods, as well as over algorithms based on working with databases obtained for pre-calculated reference tasks.

Heuristic formulas are obtained on the basis of a numerical solution of the diffraction problem in a specific formulation, but the same technique can be extended to other problems. In addition, if the numerical results differ from the experiment due to the inaccurate formulation of the problem, then using the developed technique, appropriate corrections can be made so that the final heuristic formulas better correspond to the experiment. In this case, it is possible to identify physical characteristics that are absent in the numerical solution.

The unification of the MFC approach also lies in the fact that for different formulations of problems, for all types of scatterers and all types of geometry, we use the same technique for constructing heuristic solutions by adjusting the primary heuristic formulas. In this work, we construct a heuristic solution for a scatterer with boundary conditions of a specific type (a half-plane with impedance boundary conditions, such as a thin layer), but this technique for studying the diffraction process on two-dimensional structures based on a numerical solution can be extended to edges with a different profile and other types of boundary conditions.

Unlike known heuristic approaches, the MFC formulas can have a predetermined accuracy. Unlike rigorous analytical solutions, the MFC formulas have a uniform structure, can be guaranteed to be created in a predetermined time frame and are well subject to algorithmization. These qualities help improve the efficiency of application packages.

The sequence of steps in determining the scattered field is as follows:

1. conducting of ray tracing;
2. recognition and classification of the type of obstacle in the path of the beam;
3. account according to the selected formula [47].

For the development of this approach, numerical solutions of two-dimensional and three-dimensional problems of diffraction on standard semi-infinite scatterers are required. Unfortunately, such solutions are not yet available in most well-known commercial packages.

The MFC approach can be used to calculate diffraction for large 3D objects, for which it is impossible to obtain a numerical solution. The procedure for adjusting the heuristic solution is carried out on a smaller scatterer. Having achieved the required accuracy, one can further apply the heuristic formula for a larger scatterer, which can no longer be calculated by rigorous numerical methods. In this case, the performance of heuristic formulas will not change, and the accuracy will even increase compared with the smaller size scatterer.

8. Conclusions

Heuristic formulas, constructed on the basis of the newly developed MFC, provide new opportunities for solving practical problems.

On the one hand, new opportunities allow an increase in the performance of solvers due to the accuracy and compactness of heuristic formulas constructed using the MFC.

On the other hand, new opportunities allow one to carry out a physical interpretation of numerical solutions with the aim of a deeper study of the physical features of the diffraction process. This becomes possible due to the decomposition of heuristic formulas into physically clear basic components.

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References

1. Hönl, H.; Maue, A.W.; Westpfahl, K. *Handbuch der Physik*; Flügge, S., Ed.; Springer: Berlin/Heidelberg, Germany, 1961; Mir: Moscow, Russia, 1964; Volume 25/1, p. 218.
2. Kravtsov, Y.A.; Zhu, N.Y. *Theory of Diffraction: Heuristic Approaches*; Alpha Science Int.: Oxford, UK, 2010.
3. Vesnik, M.V. *The Method of the Generalized Eikonal. New Approaches in the Diffraction Theory*; Walter de Gruyter GmbH: Berlin, Germany; Boston, MA, USA, 2015; ISBN 978-3-11-031112-9.
4. Vesnik, M.V. Physical Interpretation of Strict Solutions of Diffraction Problems by Heuristic Relations. *J. Math. Sci.* **2019**, *239*, 751–770. [[CrossRef](#)]
5. Born, M.; Wolf, E. *Principles of Optics*; Pergamon Press: Oxford/London, UK, 1968.
6. Keller, J.B. Geometrical Theory of Diffraction. *J. Opt. Soc. Am.* **1962**, *52*, 116–130. [[CrossRef](#)]
7. James, G.L. *Geometrical Theory of Diffraction for Electromagnetic Waves*; Peter Peregrinus Ltd.: London, UK, 1976.
8. Borovikov, V.A.; Kinber, B.E. *Geometrical Theory of Diffraction*; IEEE: London, UK, 1994.
9. Ufimtsev, P.Y. *Method of Edge Waves in the Physical Theory of Diffraction*; Sovetskoe Radio, US Air Force Foreign Technology Division: Moscow, Russia, 1962; pp. 1–1154.
10. Ufimtsev, P.Y. *Theory of Edge Diffraction in Electromagnetics*; Tech Science Press: Encino, CA, USA, 2003; ISBN 0-9657001-7-8.
11. Ufimtsev, P.Y. *Fundamentals of the Physical Theory of Diffraction*; John Wiley & Sons, Inc.: Hoboken, NJ, USA, 2007.
12. Ozturk, A.K.; Paknys, R.; Trueman, C.W. Vertex Diffracted Edge Waves on a Perfectly Conducting Plane Angular Sector. *IEEE Trans. Antennas Propag.* **2011**, *59*, 888–897. [[CrossRef](#)]
13. Gordon, W.B. Far field approximations to the Kirchhoff-Helmholtz representations of scattered fields. *IEEE Trans. Antennas Propag.* **1975**, *23*, 590–592. [[CrossRef](#)]
14. Vesnik, M.V.; Ufimtsev, P.Y. An Asymptotic Feature of Corner Waves Scattered by Polygonal Plates. *Electromagnetics* **1992**, *12*, 265–272. [[CrossRef](#)]
15. Vesnik, M.V. Elimination of Infinities in Diffraction Coefficients of Physical Optics Current's Components for a Shadow Contour of a Scatterer. In Proceedings of the 1995 International Symposium on Electromagnetic Theory, St. Petersburg, Russia, 23–26 May 1995; pp. 407–409.
16. Johansen, P.M.; Breinbjerg, O. An Exact Line Representation of the Physical Optics Scattered Field: The Case of a Perfectly Conducting Polyhedral Structure Illuminated by Electric Hertzian Dipoles. *IEEE Trans. Antennas Propag.* **1995**, *43*, 689–696. [[CrossRef](#)]
17. Klinkenbusch, L. Electromagnetic Scattering by a Quarter Plane. In Proceedings of the 2005 IEEE Antennas and Propagation Society International Symposium, Washington, DC, USA, 3–8 July 2005; Volume 3B, pp. 163–166.
18. Albani, M.; Capolino, F.; Maci, S. Vertex diffraction coefficient for a quarter plane. *URSI Int. Symp. EM Theory Pisa Italy* **2004**, 1146–1148.
19. Albani, M.; Capolino, F.; Maci, S. Diffraction at the vertex of a quarter plane. *Ant. Prop. Soc. Int. Symp. IEEE* **2004**, 1991–1994.
20. Vesnik, M.V. On a Possibility of Constructing a Refined Heuristic Solution for the Problem of Diffraction by a Plane Angular Sector. *J. Commun. Technol. Electron.* **2011**, *56*, 531–543. [[CrossRef](#)]
21. Vesnik, M.V. Efficiency of Different Heuristic Approaches to Calculation of Electromagnetic Diffraction by Polyhedrons and other Scatterers. *Radio Sci.* **2014**, *49*, 945–953. [[CrossRef](#)]
22. Capolino, F.; Albani, M.; Maci, S.; Tiberio, R. Double Diffraction at a Pair of Coplanar Skew Edges. *IEEE Trans. Antennas Propag.* **1997**, *45*, 1219–1226. [[CrossRef](#)]
23. Albani, M.; Capolino, F.; Maci, S.; Tiberio, R. Diffraction at a Thick Screen Including Corrugations on the Top Face. *IEEE Trans. Antennas Propag.* **1997**, *45*, 277–283. [[CrossRef](#)]
24. Vesnik, M.V.; Kravtsov, Y.A. Subsection 5.1.7 Diffraction by Bodies with Wedges: Method of Generalised Eikonal (MGE). In *Theory of Diffraction: Heuristic Approaches*; Kravtsov, Y.A., Zhu, N.Y., Eds.; Alpha Science International Ltd.: Oxford, UK, 2010.
25. Vesnik, M.V. The Analytical Solution for the Electromagnetic Diffraction on 2-D Perfectly Conducting Scatterers of Arbitrary Shape. *IEEE Trans. Antennas Propag.* **2001**, *49*, 1638–1644. [[CrossRef](#)]
26. Vesnik, M.V. Analytical solution for the 2D diffraction of electromagnetic wave by a truncated wedge. *J. Commun. Technol. Electron.* **2012**, *57*, 1061–1072. [[CrossRef](#)]
27. Vesnik, M.V. Use of the generalized eikonal in the analytic solution of the problem of electromagnetic-wave diffraction by a 2D perfectly conducting half-plate. *J. Commun. Technol. Electron.* **2008**, *53*, 131–142. [[CrossRef](#)]
28. Vesnik, M.V. Using of 2-Dimensional Solution in 3-Dimensional Problems. *Radiotekhnika Elektron.* **1993**, *38*, 1416–1423.
29. Raman, C.V.; Krishnan, K.S. The Diffraction of Light by Metallic Screens. *Proc. R. Soc. Lond. A* **1927**, *116*, 254–267. [[CrossRef](#)]
30. Shmoys, J. Diffraction by a Half-Plane with a Special Impedance Variation. *IRE Trans. Antennas Propag.* **1959**, *7*, 88–90. [[CrossRef](#)]
31. Luebbers, R.J. Finite conductivity uniform GTD versus knife edge diffraction in prediction of propagation path loss. *IEEE Trans. Antennas Propag.* **1984**, *32*, 70–76. [[CrossRef](#)]
32. Holm, P.D. A New Heuristic UTD Diffraction Coefficient for Nonperfectly Conducting Wedges. *IEEE Trans. Antennas Propag.* **2000**, *48*, 1211–1219. [[CrossRef](#)]
33. El-Sallabi, H.M.; Ioannis, T.; Rekanos, I.T.; Vainikainen, P. A New Heuristic Diffraction Coefficient for Lossy Dielectric Wedges at Normal Incidence. *IEEE Antennas Wirel. Propag. Lett.* **2002**, *1*, 165–168. [[CrossRef](#)]

34. Soni, S.; Bhattacharya, A. New heuristic diffraction coefficient for modeling of wireless channel. *Prog. Electromagn. Res. C* **2010**, *12*, 125–137. [[CrossRef](#)]
35. Vesnik, M.V. Physical interpretation of the numerical solution for the diffraction problem of an electromagnetic wave on a plane perfectly conducting scatterer. *J. Radioelectron.* 2017. (In Russian). Available online: <http://jre.cplire.ru/jre/apr17/7/text.pdf> (accessed on 8 November 2021).
36. Vesnik, M.V. Properties of Diffraction Coefficients in the Problem of Diffraction by 3D Plane Polygon. I. Principles of the Method of Fundamental Components. *J. Commun. Technol. Electron.* **2020**, *65*, 1236. [[CrossRef](#)]
37. Vesnik, M.V. Properties of Diffraction Coefficients in the Problem of Diffraction by 3D Plane Polygon. II. Application of the Method of Fundamental Components in Practical Problems. *J. Commun. Technol. Electron.* **2020**, *65*, 1353. [[CrossRef](#)]
38. Senior, T.B.A. Half plane edge diffraction. *Radio Sci.* **1975**, *10*, 645. [[CrossRef](#)]
39. Senior, T.B.A. Diffraction tensors for imperfectly conducting edges. *Radio Sci.* **1975**, *10*, 911. [[CrossRef](#)]
40. Voitovich, N.N.; Katsenelenbaum, B.Z.; Korshunova, E.N.; Pangonis, L.I.; Pereyaslavets, M.L.; Sivov, A.N.; Shatrov, A.D. *Electromagnetics of Antennas with Semitransparent Surfaces. A Method of Constructive Synthesis*; Nauka: Moscow, Russia, 1989.
41. Bankov, S.E. *Integrated Microwave Optics*; Fizmatlit: Moscow, Russia, 2018.
42. Klionovski, K.; Bankov, S.E. Electromagnetic wave scattering by a half-plane with generalized semitransparent boundary conditions. *Waves Random Complex Media* **2020**, 1–12. [[CrossRef](#)]
43. Bankov, S.E.; Vesnik, M.V.; Kravchenko, V.F. Heuristic Solution to the Diffraction Problem on a Superconducting Half-Plane. *J. Commun. Technol. Electron.* **2020**, *65*, 398–405. [[CrossRef](#)]
44. Vesnik, M.V. Using the Method of Base Components for a Heuristic Solution to the Diffraction Problem on a Half-Plane with Nonideal Boundary Conditions. *J. Commun. Technol. Electron.* **2019**, *64*, 1211–1217. [[CrossRef](#)]
45. Vesnik, M.V.; Bankov, S.E. Heuristic solution to the problem of diffraction of a TE-polarized electromagnetic wave on a semitransparent half-plane. *Waves Random Complex Media* **2021**, 1–12. [[CrossRef](#)]
46. Vesnik, M.V. Analytical heuristic solution for the problem of elastic wave diffraction by a polygonal flat 3D scatterer. In Proceedings of the International Conference Days on Diffraction, St. Petersburg, Russia, 27–31 May 2013; p. 27.
47. Vesnik, M.V. Calculation of Diffraction Attenuation on Radio Waves Propagation Paths in Urban Environment. *Phys. Bases Instrum.* **2018**, *7*, 73–79. (In Russian) [[CrossRef](#)]
48. Vesnik, M.V.; Kalinichev, V.I. Features of electromagnetic diffraction on a perfectly conducting flat rectangle. In Proceedings of the Vth All-Russian Microwave Conference, Moscow, Russia, 28 November 2017; pp. 114–118. (In Russian)