

Article

Strict Passivity Analysis of UAV Swarm Cooperative Payload Transportation Systems with Time Delays

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Abstract

This paper investigates strict passivity analysis for continuous-time unmanned aerial vehicle (UAV) swarm cooperative payload transportation systems with time delays arising from onboard processing and state acquisition. The UAVs are assumed to have identical dynamic characteristics and to be interconnected through a structurally sparse topology. By exploiting the block-diagonal structure of the subsystem matrices and the sparsity of the structural connectivity matrix (SCM), two necessary and sufficient strict passivity conditions that are mathematically equivalent to the conventional lumped criterion are derived without requiring high-dimensional matrix inversion. Furthermore, two sufficient conditions based on single-UAV parameters are developed to reduce the computational burden for large-scale swarms. Numerical studies based on UAV model parameters and generic subsystem matrices demonstrate the computational advantages of the proposed conditions. For a 14-UAV system, the conventional lumped criterion requires 2069.1 s, whereas the aggregate computation times of the two single-UAV-based conditions are 0.589 s and 0.336 s, respectively. These results demonstrate that the proposed structural decomposition framework can substantially improve the efficiency of strict passivity verification for large-scale UAV swarm systems.

Keywords: cooperative payload transportation systems; strict passivity; sparsity; time delays; unmanned aerial vehicle (UAV) swarm

1. Introduction

With the rapid advancement of swarm robotics, UAV swarms are being deployed in increasingly large and complex cooperative tasks, including agricultural applications using UAV swarms [1,2], UAV swarm cooperative payload transportation [3,4], and UAV swarm formation flight [5,6]. After simplification, these complex systems can be modeled as large-scale networked systems [7]. The UAV swarm cooperative payload transportation system consists of multiple UAVs located at different spatial positions and interconnected through a certain topology [8]. The established interconnections enable the UAVs to exchange information on their positions, velocities and accelerations, leading to mutual interactions among the UAVs. These mutual interactions affect the overall behavior of the UAV swarm cooperative payload transportation system [9]. As the number of UAVs increases, traditional lumped models encounter issues such as substantial computational costs and possible numerical intractability in solving the models [10,11].



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The UAV swarm cooperative payload transportation system is a multi-agent system composed of multiple UAVs coupled with a payload through cables [12,13]. Its dynamic model typically involves state delays, nonlinear couplings and uncertain external disturbances [14]. In such systems, passivity serves as a tool for characterizing the input–output energy properties of a system and provides a systematic theoretical framework for analyzing the stability of large-scale interconnected systems [15–17]. Specifically, passivity constrains the energy gain from the external input to the system output, ensuring that energy cannot be generated spontaneously but can only be dissipated or stored, thereby limiting the amplitude of the system’s response to disturbances [18,19]. For the model studied here, strict passivity is assessed through an input–output energy inequality. Bounds on UAV position errors, payload swings and transient overshoots require additional closed-loop analysis [20,21]. Passivity-based cooperative control provides a framework for studying the input–output properties of interconnected agents [22]. Accordingly, this paper examines the strict passivity of the specified UAV swarm model to characterize its energy dissipation behavior.

Existing studies have applied passivity-based methods to the controller design and stability analysis of UAV swarm cooperative payload transportation systems. Regarding passivity analysis of systems with time delays, reference [23] derived necessary and sufficient conditions for delay-dependent passivity using linear matrix inequalities. For single-UAV modeling and control, reference [24] investigated the mathematical modeling and control of a quadrotor UAV transporting a cable-suspended payload. An interconnection and damping assignment passivity-based control (IDA-PBC) strategy was developed to suppress payload swing during transportation. However, the limited payload capacity of a single UAV restricts its ability to transport heavy loads, making cooperative transportation with multiple UAVs necessary. Reference [25] further investigated the passivity-based formation control problem for multiple UAVs carrying a suspended payload cooperatively and proved the stability of the system equilibrium point when the cables are under tension. Reference [26] designed an observer-based plug-and-play (PnP) control scheme applicable to systems with arbitrary topological interconnection structures. Reference [27] addressed the problem of load swing suppression by employing nonlinear control and input shaping methods to improve trajectory tracking performance. However, the method relies heavily on model parameters and does not sufficiently account for disturbances in complex environments. The aforementioned studies have mainly focused on controller design and system synthesis, demonstrating the fundamental role of passivity-based methods as a tool in cooperative payload transportation systems. However, existing lumped passivity criteria face a substantial computational burden caused by the inversion of high-dimensional matrices as the swarm size increases, which limits their application to large-scale swarm analysis. Reference [28] derived several sufficient conditions for the dissipativity of discrete-time large-scale networked singular systems with time delays by exploiting the structure of the system matrices, demonstrating the potential of structural decomposition for improving computational efficiency. However, the study mainly focuses on the general dissipativity framework and considers discrete-time systems. For UAV swarm systems, specializing the dissipativity framework to strict passivity yields a comparatively simpler linear matrix inequality (LMI) structure. Based on the same structural decomposition approach, necessary and sufficient conditions for the strict passivity of continuous-time UAV swarm cooperative payload transportation systems with time delays are derived. Furthermore, two sufficient verification conditions that rely only on the parameters of a single UAV are established, providing more flexible analysis options than the condition presented in [28]. In contrast to conventional passivity/LMI methods based on the lumped high-dimensional representation, the proposed approach explicitly exploits the block-diagonal subsystem

structure and the sparsity of the SCM. This enables equivalent strict passivity conditions to be derived without high-dimensional matrix inversion. Furthermore, two sufficient conditions depending only on single-UAV parameters are developed to provide a more computationally efficient verification strategy for large-scale UAV swarms. The main contributions of this paper are summarized as follows:

1. By exploiting the block-diagonal structure of the system parameter matrices of the UAV swarm and the sparsity of the SCM, two computationally efficient necessary and sufficient conditions for the strict passivity of the UAV swarm cooperative payload transportation system with time delays are derived. These conditions avoid certain complex matrix operations, particularly the inversion of high-dimensional matrices, thereby significantly improving the computational efficiency of passivity analysis for UAV swarm cooperative payload transportation systems with time delays.
2. Two sufficient conditions that depend only on subsystem parameters are derived, and their computation time depends only on the number of subsystems, thereby improving computational efficiency.

The research framework is outlined as follows. In Section 2, a UAV swarm cooperative payload transportation system composed of N UAVs with time delays is presented. Section 3 presents the analysis results for the strict passivity of the UAV swarm cooperative payload transportation system with state delays. Section 4 validates the effectiveness of the obtained results through two numerical studies. Finally, Section 5 concludes the paper.

2. Problem Formulation and Preliminaries

The mathematical notations adopted in this paper are summarized as follows. Let $\mathbb{R}$ denote the set of real numbers, and $\mathbb{R}^\#$ represent a real vector space with compatible dimensions. The operator $\text{col}\{S_i\}_{i=1}^L$ stacks vectors or matrices S_i ($i = 1, 2, \dots, L$) in a column-wise manner, while $\text{diag}\{S_i\}_{i=1}^L$ constructs a block-diagonal matrix whose i -th diagonal block is S_i . Furthermore, $\{S_{ij}\}_{i=1, j=1}^{i=M, j=N}$ defines an $M \times N$ partitioned matrix, where S_{ij} denotes the block located at the i -th row and j -th column. In this paper, without causing ambiguity between the zero matrix and the zero vector, both are uniformly denoted by 0. I_N denotes the identity matrix associated with N UAVs, and I is employed to represent the identity matrix of appropriate dimensions whenever no ambiguity is introduced. The superscript T stands for matrix or vector transposition. In addition, expressions of the form $(*)^T W X$ and $X W (*)^T$ are interpreted as $X^T W X$ and $X W X^T$, respectively. For symmetric matrices, the symbol “*” is used to denote symmetric counterparts of matrix blocks.

2.1. Mathematical Model for Strict Passivity Analysis of UAV Swarm Systems with Time Delays

Motivated by the UAV dynamics of a cable-suspended payload system in [29], this paper formulates an interconnected UAV state-space model for strict passivity analysis. The mechanical effect of the transported payload on the i -th UAV is represented by the cable-tension force $F_{\text{cable}}^i(t)$, which is included in the generalized external input $\omega^i(t)$ of the UAV dynamics. The generalized external input $\omega^i(t)$ also collects the force and torque components, including the rotor lift, gravity, wind disturbance and control torques, as specified in the force and moment equations below. Thus, the payload–UAV mechanical coupling is represented through the cable-tension contribution within the input. The state vector contains the translational and angular velocities of the UAV, whereas the payload position, velocity and swing angles are not introduced as additional state variables. The subsequent strict passivity analysis is therefore performed for the interconnected UAV swarm under the force and torque inputs containing the cable-tension effect. The UAV swarm under consideration consists of N interconnected UAVs. The i -th UAV is denoted

by Ψ_i , where $i \in \{1, 2, \dots, N\}$. The mathematical model of the i -th UAV Ψ_i is described as follows:

$$\begin{bmatrix} \dot{x}^i(t) \\ \dot{z}^i(t) \\ \dot{y}^i(t) \end{bmatrix} = \begin{bmatrix} A_{TT} & A_{TS} & A_{TP} & A_{TK} \\ A_{ST} & 0 & A_{SP} & A_{SK} \\ A_{ZT} & 0 & A_{ZP} & A_{ZK} \end{bmatrix} \begin{bmatrix} x^i(t) \\ x^i(t-h) \\ v^i(t) \\ \omega^i(t) \end{bmatrix}, \quad (1)$$

where t denotes the continuous-time variable, and i denotes the i -th UAV, with $i = 1, 2, \dots, N$. The linear velocity vector of the UAV in the Earth-fixed coordinate system at time t is given by $\dot{x}_1^i(t) = [\dot{x}_c^i(t) \ \dot{y}_c^i(t) \ \dot{z}_c^i(t)]^T$, while the linear velocity vector in the body-fixed coordinate system is given by $v_B^i(t) = [v_{B,x}^i(t), v_{B,y}^i(t), v_{B,z}^i(t)]^T$. According to three-dimensional rigid-body dynamics, the two coordinate systems associated with $\dot{x}_1^i(t)$ and $v_B^i(t)$ are related by $\dot{x}_1^i(t) = R^i v_B^i(t)$, where the matrix R^i is given by

$$R^i = \begin{bmatrix} c_\theta^i c_\psi^i & s_\theta^i s_\psi^i c_\psi^i - c_\theta^i s_\psi^i & c_\theta^i s_\psi^i c_\psi^i + s_\theta^i s_\psi^i \\ c_\theta^i s_\psi^i & s_\theta^i s_\psi^i s_\psi^i + c_\theta^i c_\psi^i & c_\theta^i s_\psi^i s_\psi^i - s_\theta^i c_\psi^i \\ -s_\theta^i & c_\theta^i s_\psi^i & c_\theta^i c_\psi^i \end{bmatrix}, \quad (2)$$

where $c_\alpha^i = \cos(\alpha^i)$ and $s_\alpha^i = \sin(\alpha^i)$, $\alpha \in \{\varphi, \theta, \psi\}$. The delayed-state term $x^i(t-h)$ in (1) represents an assumed constant effective delay associated with state acquisition and on-board processing, including image acquisition, feature extraction and pose estimation. It does not represent the communication transmission delay between UAVs. In the cooperative transportation scenario, all UAVs adopt the same onboard hardware configuration and state estimation algorithm. Therefore, their time-delay characteristics are consistent in terms of both hardware and processing procedures, making it reasonable to approximate the state delays of all UAVs as a common constant time delay on the time scale considered. Representative onboard perception and state estimation systems have reported end-to-end processing delays on the order of tens of milliseconds, with typical values ranging from approximately 20 to 100 ms depending on the onboard hardware and processing pipeline [30,31]. Therefore, $h \in [20, 100]$ ms can be regarded as a practical reference range for the onboard processing time delay considered in this study. This range provides an illustrative engineering reference for the assumed processing delay. It is not a numerical input to the LMI tests or a restriction required by the theoretical conditions. In this paper, constant time delays are considered for the UAV swarm cooperative payload transportation system to establish a fundamental framework for strict passivity analysis. The extension to time-varying or uncertain time-delay models requires additional assumptions on the delay characteristics and corresponding theoretical derivations. It should be noted that this assumption does not imply that identical hardware necessarily results in exactly the same time delay, but rather represents a reasonable approximation of the time-delay characteristics under the aforementioned operating conditions. For cases involving significant variations in flight states or pronounced differences in time delays among UAVs, time-varying or uncertain time-delay models are required for more accurate characterization. The state vector of the i -th UAV is defined as $x^i(t) = [\dot{x}_c^i(t) \ \dot{y}_c^i(t) \ \dot{z}_c^i(t) \ p^i(t) \ q^i(t) \ r^i(t)]^T$. Here, $\dot{x}_c^i(t)$, $\dot{y}_c^i(t)$ and $\dot{z}_c^i(t)$ denote the linear velocities of the i -th UAV in the inertial coordinate frame along the x , y and z axes, respectively, while $p^i(t)$, $q^i(t)$ and $r^i(t)$ represent the corresponding angular velocities. In the adopted coordinate convention, the resultant force acting on UAV Ψ_i is expressed as $T^i(t) = [-m(R^i)^{-1}G^i(t) + T_{\text{Lift}}^i(t) - F_{\text{cable}}^i(t) + L^i F_{\text{Wind}}^i(t)]$, where $L^i = [k_1^i \sin \beta^i \cos \gamma^i \ k_2^i \sin \beta^i \sin \gamma^i \ k_3^i \cos \beta^i]^T$, $\angle \beta^i$ and $\angle \gamma^i$ denote the attitude angles of the cable of the i -th UAV. The coefficients k_1^i , k_2^i , and k_3^i characterize the influence of wind disturbances on UAV Ψ_i . The control torque inputs τ_φ , τ_θ and τ_ψ act on the UAV

body, and k_{d1} , k_{d2} and k_{d3} denote the aerodynamic rotational damping coefficients along the three body axes. $G^i(t) = \begin{bmatrix} 0 & 0 & g^i(t) \end{bmatrix}^T$, where $g^i(t)$ represents the gravitational acceleration of UAV Ψ_i . The lift force generated by the four rotors of UAV Ψ_i is given by $T_{\text{Lift}}^i(t) = [0, 0, \frac{C_T 4 \rho r^4}{\pi^2} \sum_{j=1}^4 ((\omega_T)_j^i(t))^2]^T$. The generalized external input $\omega^i(t)$ collects the force and torque terms entering the UAV dynamics. These terms include the rotor lift, gravity, cable tension, wind disturbance and control torques, which are grouped according to the input convention used in (1). The external output, internal output and internal input of UAV Ψ_i are denoted by $y^i(t)$, $z^i(t)$ and $v^i(t)$, respectively.

The UAV force description above is motivated by [29]. For the strict passivity analysis, we select the six-dimensional UAV velocity state $x^i(t) = [\dot{x}_c^i(t), \dot{y}_c^i(t), \dot{z}_c^i(t), p^i(t), q^i(t), r^i(t)]^T$ and specify the linear input–state–output subsystem model in (1). Compared with the physical UAV–payload description, payload position, velocity and cable-swing variables are not retained as independent states. The cable-tension force $F_{\text{cable}}^i(t)$ is included as a component of the generalized external input $\omega^i(t)$ in (1) and is not represented by a separate input channel or matrix block. The delayed-state channel $x^i(t-h)$ is introduced to represent the assumed state-acquisition and onboard-processing delay. Hence, (1) is the interconnected UAV model used for strict passivity analysis in this paper, rather than a direct reproduction or an explicitly derived reduction of all the dynamics in [29]. The meanings and numerical choices of the additional delayed-state and generalized-input-to-output coefficients are specified in Section 4.1.

The connectivity relationships between UAVs are given by

$$v(t) = \Phi z(t). \quad (3)$$

This relation describes the modeled interconnection among UAV subsystems and is distinct from the mechanical UAV–payload interaction represented by the cable-tension force. Here, $v^i(t)$ and $z^i(t)$ denote the input vector received from other UAVs and the output vector transmitted to other subsystems, respectively. Define $v(t) = \text{col}\{v^i(t) \mid_{i=1}^N\}$, $z(t) = \text{col}\{z^i(t) \mid_{i=1}^N\}$. The SCM Φ characterizes the interconnection among the UAVs. Following the sparse SCM representation in [32], we assume that each row of Φ contains exactly one nonzero entry equal to 1. For the interconnection class considered in this paper, we further assume that the diagonal blocks of Φ are zero and that no column is identically zero. These assumptions define the topology class used in the main analysis.

Based on the interconnection structure specified above, we have $\Phi^T \Phi = \Sigma^2$, where $\Sigma^2 = \text{diag}\{\Sigma_j^2 \mid_{j=1}^N\}$, $\Sigma_j^2 = \text{diag}\{r(i) \mid_{i=L_{z,j-1}+1}^{L_{z,j}}\}$, $L_{z,0} = 0$, and $L_{z,j} = \sum_{k=1}^j r_{zk}$. Here, $r(i)$ denotes the number of UAVs directly affected through Φ by the i -th element of the output vector $z(t)$, for $i = 1, 2, \dots, \sum_{k=1}^N r_{zk}$. Under the SCM assumptions adopted in this paper, $r(i) = 1$ for every i , and hence $\Sigma^2 = I$.

For the UAV swarm system considered in this study, the system can be represented by Figure 1. For the convenience of theoretical analysis, each UAV Ψ_i ($i = 1, 2, \dots, N$) is assumed to be a system with the same state delay. In addition, each UAV has its own generalized external input $\omega^i(t)$ and output $y^i(t)$. The SCM Φ characterizes the interconnection relationships among the UAVs through the internal input $v(t)$ and internal output $z(t)$. The generalized external inputs, such as wind disturbances, are included in the UAV swarm model. The strict passivity conditions developed in this paper characterize the input–output energy dissipation property of the interconnected system with time delays. However, evaluating disturbance attenuation performance under specific disturbance characteristics requires additional assumptions and a corresponding robust passivity analysis.

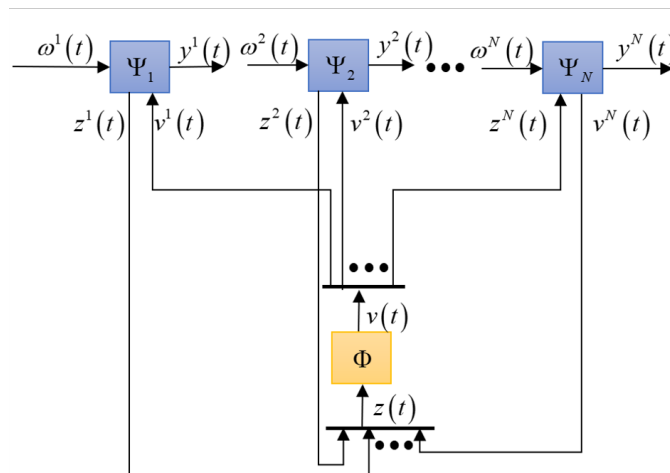


Figure 1. Schematic diagram of the UAV swarm systems with time delays.

For brevity, define the vectors and matrices as follows:

$$A_{*#}^N = \text{diag}\{A_{*#}^i \mid_{i=1}^N\}, \quad (4)$$

where $*, \# = T, S, Z, P, K$. Since $A_{*#}^i$ is identical for each UAV, $A_{*#}^i$ for each UAV is denoted by $A_{*#}$ throughout this paper. By eliminating $v(t)$ and $z(t)$, the dynamic equation of the UAV swarm system Ψ based on the lumped model can be expressed as

$$\begin{bmatrix} \dot{x}(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A_0 & A_1 & B \\ C & 0 & D \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \\ \omega(t) \end{bmatrix}, \quad (5)$$

where

$$\begin{aligned} x(t) &= \text{col}\{x^i(t) \mid_{i=1}^N\}, y(t) = \text{col}\{y^i(t) \mid_{i=1}^N\}, \\ x(t-h) &= \text{col}\{x^i(t-h) \mid_{i=1}^N\}, \omega(t) = \text{col}\{\omega^i(t) \mid_{i=1}^N\}. \end{aligned}$$

The matrix M_N is constructed as follows:

$$M_N = I - (I_N \otimes A_{SP})\Phi, \quad (6)$$

then, the lumped system matrices are given by

$$A_0 = I_N \otimes A_{TT} + (I_N \otimes A_{TP})\Phi M_N^{-1}(I_N \otimes A_{ST}),$$

$$A_1 = I_N \otimes A_{TS},$$

$$B = I_N \otimes A_{TK} + (I_N \otimes A_{TP})\Phi M_N^{-1}(I_N \otimes A_{SK}),$$

$$C = I_N \otimes A_{ZT} + (I_N \otimes A_{ZP})\Phi M_N^{-1}(I_N \otimes A_{ST}),$$

$$D = I_N \otimes A_{ZK} + (I_N \otimes A_{ZP})\Phi M_N^{-1}(I_N \otimes A_{SK}).$$

Remark 1. It should be noted that well-posedness is a fundamental requirement for a system, as an ill-posed system is generally difficult to control or impossible to estimate. Therefore, the system Ψ under consideration is assumed to be well-posed throughout this paper, with M_N^{-1} assumed to exist. The parameter matrices of the lumped analysis method implicitly incorporate the interconnection relationships among the UAVs and contain matrix inversion terms. It should be noted that, in the passivity analysis of the UAV swarm cooperative payload transportation system

with time delays, high-dimensional matrix inversion poses computational difficulties and may even be infeasible [33,34].

2.2. Preliminaries

The following fundamental lemmas are required for the strict passivity analysis of UAV swarm cooperative payload transportation systems with time delays.

Lemma 1 ([35]). For a symmetric matrix $T = \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix}$ of appropriate dimensions, the following conditions are equivalent:

- 1) $T < 0$;
- 2) $T_{11} < 0$, $T_{22} - T_{12}^T T_{11}^{-1} T_{12} < 0$;
- 3) $T_{22} < 0$, $T_{11} - T_{12} T_{22}^{-1} T_{12}^T < 0$.

Lemma 2 ([36]). If the matrices $J_1 \in \mathbb{R}^{n_1 \times n_2}$, $J_2 \in \mathbb{R}^{n_2 \times n_3}$, $J_3 \in \mathbb{R}^{n_2 \times n_1}$ and $Y \in \mathbb{R}^{(n_1+n_3) \times (n_1+n_3)}$ satisfy $\det(I - J_3 J_1) \neq 0$, then, the following inequalities are equivalent:

$$\begin{bmatrix} I \\ J_1(I - J_3 J_1)^{-1} J_2 \end{bmatrix}^T Y \begin{bmatrix} I \\ J_1(I - J_3 J_1)^{-1} J_2 \end{bmatrix} < 0, \quad (7)$$

$\exists \mu \in \mathbb{R}$, such that

$$Y - \mu \begin{bmatrix} J_1 J_2 & J_1 J_3 - I \end{bmatrix}^T \begin{bmatrix} J_1 J_2 & J_1 J_3 - I \end{bmatrix} < 0. \quad (8)$$

Lemma 3 ([37]). Let L and M be matrices with compatible dimensions; there exists a scalar $\gamma > 0$ such that

$$LM + M^T L^T \leq \gamma L L^T + \gamma^{-1} M^T M. \quad (9)$$

Lemma 4 ([38]). For any nonzero vector σ , if $\sigma^T U \sigma = 0$ satisfies $\sigma^T Z \sigma > 0$ for symmetric matrices U and Z of appropriate dimensions, then there must exist a $g \in \mathbb{R}$ such that $Z + gU > 0$, the converse statement also holds.

Lemma 5 ([37]). $G(P_N) < 0$ (P_N is a symmetric matrix to be solved) is an LMI structured as a $Y_1 \times Y_1$ ($Y_1 \geq 1$) block matrix. The coefficient and other constant matrices involved are block diagonal and contain $Y_2 \times Y_2$ ($Y_2 \geq 1$) diagonal blocks. For this LMI, if a fully block feasible solution P exists, then there must exist a feasible solution in block-diagonal form with the appropriate dimensions.

Lemma 6 ([39]). For $C = [c_{ij}] \in W_{m,n}(\mathbb{R})$ and $D = [d_{ij}] \in W_{p,q}(\mathbb{R})$, the Kronecker product (denoted by $C \otimes D$) is defined as the following block matrix:

$$C \otimes D \equiv \begin{bmatrix} c_{11}D & \cdots & c_{1n}D \\ \vdots & \ddots & \vdots \\ c_{m1}D & \cdots & c_{mn}D \end{bmatrix} \in W_{mp,nq}(\mathbb{R}). \quad (10)$$

3. Materials and Methods

3.1. Strict Passivity Analysis of UAV Swarm Systems

Several conditions for the strict passivity of the UAV swarm cooperative payload transportation system with time delays Ψ are derived in this section.

Lemma 7 ([40]). *The UAV swarm cooperative payload transportation system with time delays Ψ is admissible and strictly passive if and only if there exist symmetric positive-definite matrices $P_N = P_N^T > 0$ and $Q_N = Q_N^T > 0$ such that the following conditions hold:*

$$\begin{bmatrix} P_N A_0 + A_0^T P_N + Q_N & P_N A_1 & P_N B - C^T \\ A_1^T P_N & -Q_N & 0 \\ B^T P_N - C & 0 & -D - D^T \end{bmatrix} < 0. \quad (11)$$

For system Ψ , the interconnection relationships among UAVs are implicitly embedded in the system parameter matrices. According to the Kronecker product-based decomposition, the matrices A_0 , A_1 , B , C and D are typically sparse. Consequently, the strict passivity analysis based on lumped system models leads to high-dimensional matrix inversion and numerical stability issues. To address these challenges, a computationally efficient set of conditions equivalent to Lemma 7 is further derived as follows:

Theorem 1. *The UAV swarm cooperative payload transportation system with time delays Ψ is admissible and strictly passive if and only if there exist symmetric positive-definite matrices $P_N = P_N^T > 0$, $Q_N = Q_N^T > 0$ and a scalar $\delta > 0$ such that*

$$\begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} - \delta \begin{bmatrix} D_2^T D_1^T D_1 D_2 & D_2^T D_1^T (D_1 D_3 - I) \\ * & (D_3^T D_1^T - I)(D_1 D_3 - I) \end{bmatrix} < 0. \quad (12)$$

When the UAV parameter matrices are identical, Lemma 6 yields that

$$Y_{11} = \begin{bmatrix} P_N(I_N \otimes A_{TT}) + I_N \otimes A_{TT}^T P_N + Q_N & P_N(I_N \otimes A_{TS}) & -I_N \otimes A_{ZT}^T + P_N(I_N \otimes A_{TK}) \\ * & -Q_N & 0 \\ * & * & -I_N \otimes (A_{ZK}^T + A_{ZK}) \end{bmatrix}$$

$$, Y_{12} = \begin{bmatrix} P_N(I_N \otimes A_{TP}) & 0 & P_N(I_N \otimes A_{TP}) \\ 0 & 0 & 0 \\ -I_N \otimes A_{ZP} & 0 & -I_N \otimes A_{ZP} \end{bmatrix}, \quad Y_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$D_1 = I_3 \otimes \Phi,$$

$$D_2 = (I_N \otimes A_{ST}) \oplus (I_N \otimes 0_{6 \times 6}) \oplus (I_N \otimes A_{SK}),$$

$$D_3 = I_{3N} \otimes A_{SP}.$$

Proof. Inequality (12) can be combined into

$$\begin{bmatrix} Y_{11} - \delta D_2^T D_1^T D_1 D_2 & Y_{12} - \delta D_2^T D_1^T (D_1 D_3 - I) \\ * & Y_{22} - \delta (D_3^T D_1^T - I)(D_1 D_3 - I) \end{bmatrix} < 0. \quad (13)$$

Furthermore, inequality (13) can be rewritten as

$$\begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} - \delta (*)^T \begin{bmatrix} D_1 D_2 & D_1 D_3 - I \end{bmatrix} < 0. \quad (14)$$

According to the definitions of D_1 , D_3 and M_N , one has

$$I - D_3 D_1 = I - [I_3 \otimes (I_N \otimes A_{SP})](I_3 \otimes \Phi) = I_3 \otimes M_N. \quad (15)$$

Since the system is assumed to be well-posed, M_N is nonsingular. Therefore, $\det(I - D_3 D_1) = \det(I_3 \otimes M_N) = \det(M_N)^3 \neq 0$. Hence, the nonsingularity condition required in Lemma 2 is satisfied. By Lemma 2, inequality (14) can be equivalently transformed into

$$(*)^T \begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} \begin{bmatrix} I \\ D_1(I - D_3 D_1)^{-1} D_2 \end{bmatrix} < 0. \quad (16)$$

Inequality (16) can be expanded as follows:

$$\begin{aligned} & Y_{11} + Y_{12} D_1 (I - D_3 D_1)^{-1} D_2 + \left(Y_{12} D_1 (I - D_3 D_1)^{-1} D_2 \right)^T \\ & + \left(D_1 (I - D_3 D_1)^{-1} D_2 \right)^T Y_{22} \left(D_1 (I - D_3 D_1)^{-1} D_2 \right) < 0. \end{aligned} \quad (17)$$

Substituting $D_1, D_2, D_3, Y_{11}, Y_{12}, Y_{22}$ into inequality (17) yields

$$\begin{bmatrix} P_N A_0 + A_0^T P_N + Q_N & P_N A_1 & P_N B - C^T \\ A_1^T P_N & -Q_N & 0 \\ B^T P_N - C & 0 & -D - D^T \end{bmatrix} < 0. \quad (18)$$

The sufficiency proof is thus completed. By substituting the parameter matrices of the UAV swarm into inequality (12), inequality (18) can be readily obtained through straightforward matrix manipulations. Furthermore, inequality (18) can be equivalently rewritten in the form of inequality (17). In view of Lemma 2, the necessity part is also established. Therefore, the proof of Theorem 1 is completed. $\square$

Remark 2. Theorem 1 is equivalent to Lemma 7. In contrast, the strict passivity conditions derived in Theorem 1 fully exploit the parameter structure of the UAV swarm cooperative payload transportation system with time delays, where the matrix $I_N \otimes A_{**}(*, \# = T, S, Z, P, K)$ is block-diagonal and the UAV SCM Φ is sparse. This approach avoids the inversion of high-dimensional matrices, thereby significantly improving computational efficiency. Furthermore, the resulting conditions may provide a basis for subsequent passive-controller synthesis.

Theorem 2. The UAV swarm cooperative payload transportation system with time delays Ψ is admissible and strictly passive if and only if there exist symmetric positive-definite matrices $P_N = P_N^T > 0$ and $Q_N = Q_N^T > 0$ and a scalar $k > 0$ such that

$$\begin{aligned} & (*)^T \begin{bmatrix} Q_N & 0 & 0 & P_N & 0 \\ 0 & -Q_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I_N \otimes I_6 \\ P_N & 0 & 0 & 0 & 0 \\ 0 & 0 & -I_N \otimes I_6 & 0 & 0 \end{bmatrix} \times \\ & \begin{bmatrix} 0 & I_N \otimes I_6 & 0 & 0 \\ 0 & 0 & I_N \otimes I_6 & 0 \\ 0 & 0 & 0 & I_N \otimes I_6 \\ I_N \otimes A_{TP} & I_N \otimes A_{TT} & I_N \otimes A_{TS} & I_N \otimes A_{TK} \\ I_N \otimes A_{ZP} & I_N \otimes A_{ZT} & 0 & I_N \otimes A_{ZK} \end{bmatrix} - k \times \\ & (*)^T \begin{bmatrix} I & -\Phi \\ -\Phi^T & \Sigma^2 \end{bmatrix} \begin{bmatrix} I_N \otimes I_6 & 0 & 0 & 0 \\ I_N \otimes A_{SP} & I_N \otimes A_{ST} & 0 & I_N \otimes A_{SK} \end{bmatrix} < 0. \end{aligned} \quad (19)$$

Proof. Lemma 1 can be reformulated as follows:

$$(*)^T \begin{bmatrix} Q_N & 0 & 0 & P_N & 0 \\ 0 & -Q_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I \\ P_N & 0 & 0 & 0 & 0 \\ 0 & 0 & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ A_0 & A_1 & B \\ C & 0 & D \end{bmatrix} < 0, \quad (20)$$

due to

$$\begin{bmatrix} A_0 & A_1 & B \\ C & 0 & D \end{bmatrix} = \begin{bmatrix} I_N \otimes A_{TT} & I_N \otimes A_{TS} & I_N \otimes A_{TK} \\ I_N \otimes A_{ZT} & 0 & I_N \otimes A_{ZK} \end{bmatrix} + \begin{bmatrix} I_N \otimes A_{TP} \\ I_N \otimes A_{ZP} \end{bmatrix} \Phi M_N^{-1} \begin{bmatrix} I_N \otimes A_{ST} & 0 & I_N \otimes A_{SK} \end{bmatrix}, \quad (21)$$

the matrices Λ_1 , N_1 and L_1 are defined as follows:

$$\Lambda_1 = (*)^T \begin{bmatrix} -Q_N & 0 & 0 & -P_N & 0 \\ 0 & Q_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_N \otimes I_6 \\ -P_N & 0 & 0 & 0 & 0 \\ 0 & 0 & I_N \otimes I_6 & 0 & 0 \end{bmatrix} \times \quad (22)$$

$$\begin{bmatrix} 0 & I_N \otimes I_6 & 0 & 0 \\ 0 & 0 & I_N \otimes I_6 & 0 \\ 0 & 0 & 0 & I_N \otimes I_6 \\ I_N \otimes A_{TP} & I_N \otimes A_{TT} & I_N \otimes A_{TS} & I_N \otimes A_{TK} \\ I_N \otimes A_{ZP} & I_N \otimes A_{ZT} & 0 & I_N \otimes A_{ZK} \end{bmatrix},$$

$$N_1 = \begin{bmatrix} \Phi M_N^{-1}(I_N \otimes A_{ST}) & 0 & \Phi M_N^{-1}(I_N \otimes A_{SK}) \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad (23)$$

$$L_1 = \begin{bmatrix} I & -\Phi \end{bmatrix} \begin{bmatrix} I_N \otimes I_6 & 0 & 0 & 0 \\ I_N \otimes A_{SP} & I_N \otimes A_{ST} & 0 & I_N \otimes A_{SK} \end{bmatrix}. \quad (24)$$

Substituting (21) into inequality (20) yields

$$N_1^T \Lambda_1 N_1 > 0. \quad (25)$$

When $m = N_1 \zeta$ (where ζ is a vector of appropriate dimension), for any $m \neq 0$, there exists $L_1 m = 0$ such that $m^T \Lambda_1 m > 0$.

According to Lemma 4, there exists a scalar k such that

$$\Lambda_1 + k L_1^T L_1 > 0. \quad (26)$$

Multiplying both sides of inequality (26) by -1 yields inequality (19), which completes the necessity proof.

Multiplying the left and right-hand sides of inequality (19) by m^T and m , respectively, yields $m^T(-\Lambda_1)m - km^T L_1^T L_1 m < 0$. Since $L_1 N_1 = 0$ and $\zeta \neq 0$, it follows that $m^T(-\Lambda_1)m < 0$. This corresponds to inequality (20) and completes the sufficiency proof through straightforward matrix manipulation. $\square$

Remark 3. Theorems 1 and 2 are equivalent and introduce no additional conservatism. In contrast to the existing inequality (11), inequalities (12) and (19) depend linearly on the scalar parameters

δ or k on the left-hand side, while explicitly revealing the extended structure of the system Ψ . In addition, both Theorems 1 and 2 fully exploit the inherent structural properties of the UAV swarm systems. When combined with the DSDP5 algorithm, strict passivity of UAV swarm systems with time delays can be efficiently checked via these two results. Moreover, based on Theorems 1 and 2, computationally efficient conditions that depend on individual UAV parameters can also be further established. When the subsystem parameters are different, $I_N \otimes A_{*#}$ in Theorems 1 and 2 can be written in the form of the block-diagonal matrix in (4). According to [28], Theorems 1 and 2 can also be applied to strict passivity analysis when the subsystem parameters are different.

Remark 4. When the matrix Σ^2 in inequality (19) is replaced by $\Phi^T \Phi$, Theorem 2 remains valid for UAV interconnection matrices Φ of arbitrary structural form. Under the sparse SCM structure considered in this paper, $\Phi^T \Phi = \Sigma^2$ has a structured diagonal form, which is favorable for exploiting the sparsity of the interconnection structure. For a general interconnection matrix Φ , $\Phi^T \Phi$ may contain additional nonzero off-diagonal elements, resulting in less sparse matrix inequalities and an increased computational burden. Nevertheless, the proposed formulation still avoids the direct inversion of the high-dimensional matrix required by the conventional lumped criterion.

3.2. Passivity Analysis of Parameters for a Single Unmanned Aerial Vehicle

As the required payload capacity increases, a larger UAV swarm may be required, causing the dimensions of the system parameter matrices to grow linearly with the number of UAVs. However, due to the limited memory capacity of computational devices, the high-dimensional matrix operations involved in Theorems 1 and 2 may become infeasible to implement. Therefore, under memory-constrained conditions, avoiding the computational bottleneck induced by the increasing dimensionality of the system parameter matrices becomes an urgent issue. To address this problem, strict passivity conditions depending solely on the parameters of a single UAV are derived from Theorems 1 and 2. In the local inequalities below, the subsystem matrices and the decision matrices P and Q refer to the UAV being tested and may differ among UAVs. The scalar coupling parameters k_1 and k_2 are selected consistently across the swarm. When the UAV parameters are identical, as in the UAV numerical example, one local LMI can be solved and its result reused for all UAVs.

Theorem 3. The UAV swarm cooperative payload transportation system with time delays is strictly passive if, for each UAV, there exist positive-definite matrices P and Q of appropriate dimensions and constants k_1 and k_2 satisfying either $k_1 \geq k_2 \geq 0$ (or $k_2 \leq k_1 \leq 0$), such that the following linear matrix inequalities hold:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} - \begin{bmatrix} k_1 D_2^T D_2 & k_1 D_2^T D_3 \\ * & k_1 D_3^T D_3 - k_2 I_6 \end{bmatrix} < 0, \quad (27)$$

where

$$Y_{11} = \begin{bmatrix} PA_{TT} + A_{TT}^T P + Q & PA_{TS} & -A_{ZT}^T + PA_{TK} \\ * & -Q & 0 \\ * & * & -A_{ZK}^T - A_{ZK} \end{bmatrix},$$

$$Y_{12} = \begin{bmatrix} PA_{TP} & 0 & PA_{TP} \\ 0 & 0 & 0 \\ -A_{ZP} & 0 & -A_{ZP} \end{bmatrix}, \quad Y_{22} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_2 = A_{ST} \oplus 0_6 \oplus A_{SK}, \quad D_3 = I_3 \otimes A_{SP}.$$

Proof. From Theorem 1, it follows that

$$\begin{bmatrix} D_2^T D_1^T D_1 D_2 & D_2^T D_1^T D_1 D_3 \\ * & D_3^T D_1^T D_1 D_3 + I \end{bmatrix} = (*)^T \begin{bmatrix} D_1 D_2 & D_1 D_3 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} (*)^T, \quad (28)$$

let $X = \begin{bmatrix} 0 & I \end{bmatrix}^T$ and $Y = \begin{bmatrix} D_1 D_2 & D_1 D_3 \end{bmatrix}$. Then, for any $\alpha > 0$, it follows from Lemma 3 that

$$\begin{bmatrix} D_2^T D_1^T D_1 D_2 & D_2^T D_1^T (D_1 D_3 - I) \\ * & (D_3^T D_1^T - I)(D_1 D_3 - I) \end{bmatrix} \geq (1 - \alpha) \begin{bmatrix} 0 \\ I \end{bmatrix} (*)^T + (1 - \alpha^{-1}) (*)^T \begin{bmatrix} D_1 D_2 & D_1 D_3 \end{bmatrix}. \quad (29)$$

From inequality (29), it follows that

$$\begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} - \delta \left((1 - \alpha) \begin{bmatrix} 0 \\ I \end{bmatrix} (*)^T + (1 - \alpha^{-1}) (*)^T \begin{bmatrix} D_1 D_2 & D_1 D_3 \end{bmatrix} \right) < 0. \quad (30)$$

Let $k_1 = -(1 - \alpha^{-1})\delta$ and $k_2 = (1 - \alpha)\delta$. Then, it is evident that $k_1 = \alpha^{-1}k_2$. Specifically, when $\alpha \leq 1$, one has $k_1 \geq k_2 \geq 0$, whereas for $\alpha \geq 1$, it holds that $k_2 \leq k_1 \leq 0$. Consequently, invoking Lemma 5 completes the proof of Theorem 3. $\square$

Remark 5. Theorem 3 is derived from Theorem 1 and enables strict passivity verification using only the parameters of a single UAV. However, the dimension of the resulting parameter matrix is excessively large. Therefore, Theorem 4 is derived by further reducing the dimension of the LMI.

Theorem 4. the UAV swarm cooperative payload transportation system with time delays are strictly passive if, for each UAV, there exist positive-definite matrices P and Q of appropriate dimensions and constants k_1 and k_2 satisfying either $k_1 \leq k_2 \leq 0$ (or $0 \leq k_1 \leq k_2$), such that the following linear matrix inequalities hold:

$$\begin{aligned} (*)^T \begin{bmatrix} Q & 0 & 0 & P & 0 \\ 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I \\ P & 0 & 0 & 0 & 0 \\ 0 & 0 & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & I_6 & 0 & 0 \\ 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & I_6 \\ A_{TP} & A_{TT} & A_{TS} & A_{TK} \\ A_{ZP} & A_{ZT} & 0 & A_{ZK} \end{bmatrix} - \\ (*)^T \begin{bmatrix} k_1 I_6 & 0 \\ 0 & -k_2 I_6 \end{bmatrix} \begin{bmatrix} I_6 & 0 & 0 & 0 \\ A_{SP} & A_{ST} & 0 & A_{SK} \end{bmatrix} < 0. \end{aligned} \quad (31)$$

Proof. Clearly, the following relation holds:

$$\begin{bmatrix} I & 0 \\ 0 & \Phi^T \Phi \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \Phi^T \end{bmatrix} \begin{bmatrix} 0 & \Phi \end{bmatrix}. \quad (32)$$

Select the matrices $M = \begin{bmatrix} I \\ 0 \end{bmatrix}$, $U = \begin{bmatrix} 0 & \Phi \end{bmatrix}$. It follows from Lemma 3 that there exists an $\eta > 0$ such that

$$\begin{bmatrix} 0 & -\Phi \\ -\Phi^T & 0 \end{bmatrix} \geq -\eta \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} - \eta^{-1} \begin{bmatrix} 0 \\ \Phi^T \end{bmatrix} \begin{bmatrix} 0 & \Phi \end{bmatrix}. \quad (33)$$

Adding (32) and (33) yields

$$\begin{bmatrix} I & -\Phi \\ -\Phi^T & \Phi^T \Phi \end{bmatrix} \geq (1-\eta) \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} + \left(1 - \frac{1}{\eta}\right) \begin{bmatrix} 0 \\ \Phi^T \end{bmatrix} \begin{bmatrix} 0 & \Phi \end{bmatrix}. \quad (34)$$

According to the conditions in Theorem 2 and (34), it follows that

$$\begin{aligned} (*)^T \begin{bmatrix} Q & 0 & 0 & P & 0 \\ 0 & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I_6 \\ P & 0 & 0 & 0 & 0 \\ 0 & 0 & -I_6 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & I_6 & 0 & 0 \\ 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & I_6 \\ A_{TP} & A_{TT} & A_{TS} & A_{TK} \\ A_{ZP} & A_{ZT} & 0 & A_{ZK} \end{bmatrix} - k(*)^T \times \\ \left((1-\eta) \begin{bmatrix} I \\ 0 \end{bmatrix} (*)^T + (1-\eta^{-1}) \begin{bmatrix} 0 \\ \Phi^T \end{bmatrix} (*)^T \right) \begin{bmatrix} I_N \otimes I_6 & 0 & 0 & 0 \\ I_N \otimes A_{SP} & I_N \otimes A_{ST} & 0 & I_N \otimes A_{SK} \end{bmatrix} < 0. \end{aligned} \quad (35)$$

Let $k_1 = (1-\eta)k$ and $k_2 = -(1-\eta^{-1})k$. Then, it is clear that $k_2 = \eta^{-1}k_1$. By Lemma 5, Theorem 4 is thus proved. $\square$

Remark 6. Compared with Theorem 3, Theorem 4 reduces the matrix dimension and improves computational performance. In the strict passivity analysis of the UAV swarm cooperative payload transportation system with time delays, Theorems 3 and 4 have significant advantages over Theorems 1 and 2 in terms of computational efficiency. This is because the computation time of Theorems 3 and 4 depends only on the number of subsystems, while the dimensions of the subsystems remain unchanged. For UAVs with known but different parameters, the global conditions are evaluated using the corresponding block-diagonal subsystem matrices. The local conditions in Theorems 3 and 4 are checked for each UAV using its own matrices and local decision variables, with the scalar coupling parameters selected consistently across the swarm. Identical subsystem matrices, as used in the UAV numerical example, allow the same local LMI to be solved only once. These checks establish strict passivity for a specified parameter realization. Feasibility for one realization does not guarantee feasibility for every possible parameter variation. Because the inequalities are strict, a feasible certificate is preserved under sufficiently small perturbations of the model matrices, provided that the interconnection remains well-posed. No quantitative tolerance for uncertainty in the physical UAV parameters is claimed, as no parameter uncertainty bounds are specified here. Theorems 3 and 4 may provide a useful basis for future distributed or PnP controller synthesis.

Remark 7. It should be noted that the conditions in Theorems 1 and 2 are obtained through equivalent transformations of the conventional lumped strict passivity criterion. Therefore, they preserve the original feasibility property and do not introduce additional conservatism. For large-scale UAV swarm systems, subsystem-level conditions are further derived from Theorems 1 and 2 by utilizing the structural characteristics of the system. Although these conditions may introduce certain conservatism due to decomposition, they significantly reduce the computational burden and provide a practical balance between conservatism and scalability.

4. Numerical Studies and Results

The numerical studies in this section consider simulations with two different subsystem parameter settings. Section 4.1 investigates whether the proposed method improves the computation time of strict passivity analysis for UAV swarm cooperative payload transportation systems with time delays under identical UAV dynamic parameters. Section 4.2 extends the study based on the simulation method in [28], and verifies whether the obtained results can improve the computation time of strict passivity analysis for general large-scale networked time-delay systems with different subsystem parameters.

4.1. Numerical Simulations Based on UAV Parameters

The numerical simulations were conducted on a personal computer equipped with an AMD Ryzen 7 7800X3D 8-Core Processor (4.20 GHz) and 32.0 GB of RAM. DSDP5 was used as the LMI solver. In this numerical study, all UAVs are assigned the same dynamic parameters listed in Table 1. Figure 2 shows the simplified model of a single UAV based on the model in [29]. Since the UAVs have identical subsystem matrices in this example, the local LMI for each of Theorems 3 and 4 needs to be solved only once, and the resulting certificate can be reused for all UAVs. Let t_{local} be the time for one local solve. For comparison with the global formulations, the reported serial aggregate time is estimated as $T_{\text{agg}}(N) = Nt_{\text{local}}$.

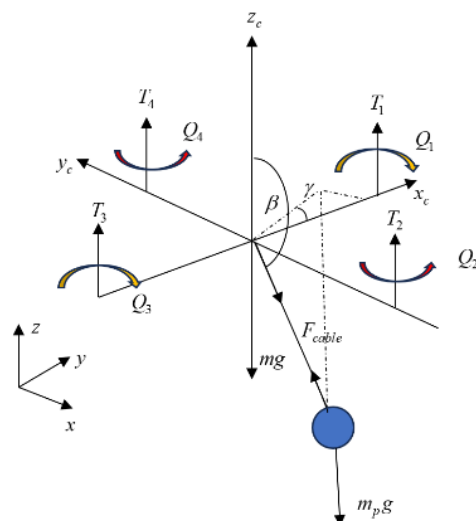


Figure 2. Schematic of a single UAV carrying a cable-suspended payload.

The payload is shown to illustrate the physical transportation scenario. Its dynamics are not included as independent states in the strict passivity analysis model. UAV parameters are listed in Table 1.

Table 1. UAV parameters.

Symbol	Description	Numerical Value
K_t	Coefficient of air friction	10×10^{-15}
C_T	Thrust factor	0.07428
r	Propeller radius	0.5 m
g	Gravitational acceleration	9.81 m/s^2
m_a	Mass of a propeller	0.65 kg
m_b	Mass of four propellers	2.6 kg
m	Total mass of unmanned aerial vehicle	3 kg
I_{xx}	Moment of inertia around x axis	$0.03 \text{ kg} \times \text{m}^2$
I_{yy}	Moment of inertia around y axis	$0.05 \text{ kg} \times \text{m}^2$
I_{zz}	Moment of inertia around z axis	$0.10 \text{ kg} \times \text{m}^2$
k_{d1}	Aerodynamic damping coefficient	0.05
k_{d2}	Aerodynamic damping coefficient	0.05
k_{d3}	Aerodynamic damping coefficient	0.03
φ	Roll angle	30°
θ	Pitch angle	20°
ψ	Yaw angle	45°

Following [29], the translational and rotational dynamics of a UAV carrying a suspended payload are used as the physical basis of the numerical model. The generalized

external input $\omega^i(t)$ contains the force and torque components entering the UAV dynamics, including the rotor lift, gravity, cable tension, wind disturbance and control torques. The effect of the transported payload is specifically represented by the cable-tension contribution $F_{\text{cable}}^i(t)$ within this generalized external input. The state variables used in the passivity analysis are the translational and angular velocities of each UAV. The payload position, velocity and swing angles are not introduced as additional state variables. In this model, the mechanical effect of the payload on each UAV is represented by the cable-tension force.

For the UAV swarm considered in this paper, the interaction among individual UAVs is further described by introducing the internal input $v^i(t)$ and internal output $z^i(t)$, which satisfy the interconnection relation $v(t) = \Phi z(t)$. The generalized external input and output of each UAV are denoted by $\omega^i(t)$ and $y^i(t)$, respectively. Accordingly, the numerical UAV subsystem is represented through the current-state, delayed-state, internal-interconnection and generalized-external-input channels in (1), providing the model used for the subsequent strict passivity analysis.

In Equation (1), the matrix parameters associated with each UAV are defined as follows:

$$\begin{aligned} A_{TT} &= \begin{bmatrix} A_{TT}^{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & A_{TT}^{22} \end{bmatrix}, \quad A_{TT}^{11} = \frac{-K_t R R^T}{m} \approx 0_{3 \times 3}, \quad A_{TK} = \begin{bmatrix} A_{TK}^{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & A_{TK}^{22} \end{bmatrix}, \\ A_{TK}^{11} &= \frac{R}{m}, \quad A_{ST} = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \quad A_{TP} = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}, \\ A_{TT}^{22} &= -\begin{bmatrix} k_{d1} & 0 & 0 \\ 0 & k_{d2} & 0 \\ 0 & 0 & k_{d3} \end{bmatrix} A_{TK}^{22}, \quad A_{SP} = \begin{bmatrix} A_{SP}^{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}, \quad A_{ZP} = \begin{bmatrix} A_{ZP}^{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & -I_{3 \times 3} \end{bmatrix}, \\ A_{ZK} &= \begin{bmatrix} A_{ZK}^{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & A_{ZK}^{22} \end{bmatrix}, \quad A_{SP}^{11} = -A_{ZP}^{11} = R^{-1}, \quad A_{TS}^{11} = \begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{bmatrix}, \\ A_{TS}^{22} &= \begin{bmatrix} \alpha_{pp} & \alpha_{pq} & \alpha_{pr} \\ \alpha_{qp} & \alpha_{qq} & \alpha_{qr} \\ \alpha_{rp} & \alpha_{rq} & \alpha_{rr} \end{bmatrix}, \quad A_{ZK}^{11} = \begin{bmatrix} \beta_{xx} & 0 & 0 \\ 0 & \beta_{yy} & 0 \\ 0 & 0 & \beta_{zz} \end{bmatrix}, \quad A_{ZT} = \begin{bmatrix} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \\ A_{TS} &= \begin{bmatrix} A_{TS}^{11} & 0_{3 \times 3} \\ 0_{3 \times 3} & A_{TS}^{22} \end{bmatrix}, \quad A_{SK} = \begin{bmatrix} 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & A_{SK}^{22} \end{bmatrix}, \quad A_{ZK}^{22} = A_{SK}^{22} = A_{TK}^{22} = \begin{bmatrix} I_{xx}^{-1} & 0 & 0 \\ 0 & I_{yy}^{-1} & 0 \\ 0 & 0 & I_{zz}^{-1} \end{bmatrix}. \end{aligned}$$

Here, A_{TS} is the coefficient matrix of the delayed state $x^i(t-h)$. In A_{TS}^{11} , the diagonal coefficients α_{xx} , α_{yy} and α_{zz} describe delayed-state self-coupling among the translational velocity components, while the off-diagonal entries describe cross-coupling among these components. Likewise, the diagonal coefficients α_{pp} , α_{qq} and α_{rr} in A_{TS}^{22} describe delayed-state self-coupling among the angular velocity components, and its off-diagonal entries describe the corresponding cross-coupling. The coefficients β_{xx} , β_{yy} and β_{zz} are entries of A_{ZK}^{11} and specify the translational generalized-input-to-output gains. These α - and β -coefficients are coefficients of the specified linear subsystem model and are not additional physical parameters taken directly from [29]. The mass, moments of inertia and aerodynamic damping coefficients used in the subsystem matrix blocks are listed in Table 1. Fixed values of the α - and β -coefficients are used for the numerical example. The strict passivity conditions are formulated for general subsystem matrices and do not depend on these particular values.

The matrix parameters used in the numerical study are specified below. The matrix blocks determined by the physical UAV parameters are calculated using the values in

Table 1. In particular, the rotation matrix R is evaluated from the listed roll, pitch and yaw angles and held constant when constructing the subsystem matrices. The α - and β -coefficients are fixed model coefficients specifying the delayed-state and generalized-input-to-output channels, respectively. Accordingly, the reported LMI feasibility results apply to this fixed-attitude parameter realization.

$$R = \begin{bmatrix} 0.6645 & -0.4915 & 0.5630 \\ 0.6645 & 0.7333 & -0.1441 \\ -0.3420 & 0.4698 & 0.8138 \end{bmatrix}, \quad A_{TT}^{22} = \begin{bmatrix} -1.6667 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -0.3 \end{bmatrix},$$

$$A_{TK}^{11} = \begin{bmatrix} 0.2215 & -0.1638 & 0.1877 \\ 0.2215 & 0.2444 & -0.0480 \\ -0.1140 & 0.1566 & 0.2713 \end{bmatrix}, \quad A_{ZK}^{22} = A_{SK}^{22} = A_{TK}^{22} = \begin{bmatrix} 33.3333 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix},$$

$$A_{TS}^{11} = \begin{bmatrix} -0.1 & 0.02 & 0 \\ 0.02 & -0.1 & 0 \\ 0 & 0 & -0.2 \end{bmatrix}, \quad A_{TS}^{22} = \begin{bmatrix} -0.05 & 0.01 & 0 \\ 0.01 & -0.05 & 0 \\ 0 & 0 & -0.03 \end{bmatrix},$$

$$A_{ZK}^{11} = \begin{bmatrix} 0.21 & 0 & 0 \\ 0 & 0.13 & 0 \\ 0 & 0 & 0.22 \end{bmatrix}, \quad A_{SP}^{11} = -A_{ZP}^{11} = \begin{bmatrix} 0.6645 & 0.6645 & -0.3420 \\ -0.4915 & 0.7333 & 0.4698 \\ 0.5630 & -0.1441 & 0.8138 \end{bmatrix}.$$

For Theorems 3 and 4, identical UAV matrices allow a local LMI solution to be reused across the swarm. Table 2 lists the computational time of the five analysis methods for different numbers of UAVs. Based on the data in Table 2, Figure 3 plots the computational time of each method as a function of the number of UAVs.

Table 2. Aggregate computation time for strict passivity verification of UAV swarm cooperative payload transportation systems with time delays.

Number of UAVs	Computation Time (s)					
	N	Lemma 7	Theorem 1	Theorem 2	Theorem 3	Theorem 4
2	2	0.072	0.211	0.069	0.076	0.047
4	4	9.955	0.509	0.348	0.167	0.099
6	6	18.668	1.374	1.048	0.254	0.148
8	8	67.472	2.813	2.055	0.339	0.198
10	10	249.504	6.367	3.471	0.432	0.241
12	12	873.648	14.069	6.836	0.505	0.298
14	14	2069.1	24.08	12.58	0.589	0.336

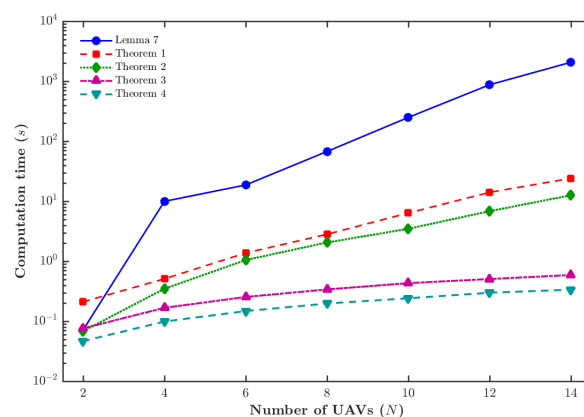


Figure 3. Computation time for different numbers of UAVs.

In addition to the computation times reported in Table 2, the feasibility of the corresponding LMIs was also checked for each tested swarm size. Feasible LMI solutions were obtained for Lemma 7 and Theorems 1–4 for all tested swarm sizes. Theorems 1 and 2 have the same feasibility status as Lemma 7 for the same parameter setting and swarm size. The feasibility of Theorems 3 and 4 was checked separately.

Figure 3 compares the computation times of the different methods for different numbers of UAVs. Since Lemma 7 involves matrix blocks of lower dimensions, the computation times of the different methods are relatively close when the number of UAVs is small. As the number of UAVs increases, the computational advantages of Theorems 1–4 over the conventional lumped criterion become increasingly pronounced, mainly because high-dimensional matrix inversion is avoided. It can also be observed from Figure 3 that Theorem 2 is slightly more computationally efficient than Theorem 1 because it involves matrix blocks of lower dimensions. As the number of UAVs further increases, the computational efficiency advantages of Theorems 3 and 4 become more pronounced. Since the UAVs have identical subsystem matrices in this example, the local LMI associated with each of Theorems 3 and 4 needs to be solved only once, and the resulting certificate can be reused for all UAVs. The computation times reported for Theorems 3 and 4 are estimated serial aggregate times calculated as $T_{\text{agg}}(N) = Nt_{\text{local}}$, where t_{local} denotes the computation time for one local LMI solve. The dimension of each local LMI remains unchanged as the number of UAVs increases. This explains why Theorems 3 and 4 retain a clear computational advantage as the number of UAVs increases.

The proposed framework is developed for a general UAV swarm system consisting of N interconnected subsystems, rather than a specific swarm size. Therefore, the obtained conditions remain applicable to UAV swarm systems with different numbers of UAVs. The computational scalability with respect to the number of UAVs is further discussed. For Theorems 1 and 2, the obtained conditions are established based on the overall system representation. Therefore, the dimensions of the involved matrices increase with the swarm size, leading to increased computational complexity. However, by exploiting the block-diagonal structure of subsystem matrices and the sparse interconnection topology, the computational burden is significantly reduced compared with the conventional lumped criterion. For Theorems 3 and 4, the verification only involves individual UAV subsystem parameters. Hence, the computation for each subsystem remains almost unchanged, and the total computational cost mainly depends on the number of subsystems, showing an approximately linear growth with the swarm size. Therefore, the proposed subsystem-level conditions provide a scalable verification strategy for large-scale UAV swarm systems with different sizes.

The UAV example in this section uses identical physical parameters. For specified UAVs with known but different subsystem matrices, the conditions can be checked using their respective matrices, subject to the interconnection assumptions and the requirements of the corresponding theorems.

4.2. Extended Simulations for Large-Scale Systems

Based on a general large-scale networked interconnected system, this study conducts a strict passivity analysis of a UAV swarm cooperative payload transportation time-delay system with identical UAV dynamic parameters and derives the corresponding mathematical conditions. The simulation results in Section 4.1 show that the four derived methods can effectively improve the computational efficiency of strict passivity analysis for UAV swarm cooperative payload transportation systems with time delays. Furthermore, to further investigate the applicability of the proposed conditions to heterogeneous systems, this section considers general large-scale interconnected systems with a larger number

of subsystems and different subsystem parameters. The performance of the proposed methods in strict passivity analysis is evaluated through numerical simulations. The numerical simulations in this section are conducted based on the research method for general large-scale networked interconnected systems presented in [28].

To further evaluate the computational efficiency and scalability of the four methods as the number of subsystems increases, this subsection considers generic subsystem parameter matrices of dimension 2×2 . These matrices are artificially constructed for computational-performance evaluation only and are not intended to represent the physical dynamics of an actual UAV. In contrast to the UAV-specific subsystem matrices considered in Section 4.1, the 2×2 matrices used here do not retain the physical state variables, dynamic couplings, or parameter meanings of the UAV model. Instead, they are used to construct general large-scale interconnected systems with heterogeneous subsystem parameters. Accordingly, to match the dimension of the generic subsystem matrices, the identity matrix I_6 in Theorems 1–4 is replaced by I_2 . The parameters of each subsystem are independently and randomly generated from a continuous uniform distribution over the interval $[-0.9, 0.9]$. These generic heterogeneous tests assess computational behavior. They do not establish a robustness margin for uncertainties in physical UAV parameters. For the numerical study, the SCM Φ was generated as a representative sparse topology, with one nonzero entry in each row and each column. The tested subsystem sizes are $N \in \{2, 5, 10, 20, 30, 40, 50\}$. For each value of N , ten independent random trials are performed to obtain the average computation time and standard deviation. The corresponding data are presented in Tables 3 and 4, respectively, while the experimental results are shown in Figure 4a,b.

Table 3. Average value of computation time.

Number of Subsystems N	Average Value of Computation Time (s)				
	Lemma 7	Theorem 1	Theorem 2	Theorem 3	Theorem 4
2	0.0042	0.034	0.0287	0.0039	0.0036
5	0.0195	0.1001	0.0797	0.0094	0.0091
10	0.2419	0.6882	0.4274	0.0173	0.0169
20	10.6387	5.2491	3.6669	0.0358	0.0334
30	166.3854	31.4654	18.5534	0.0509	0.0496
40	714.9535	123.4929	64.4766	0.0671	0.0673
50	2206.3721	366.4253	163.2647	0.0862	0.0845

Table 4. Standard deviation of computation time.

Number of Subsystems N	Standard Deviation of Computation Time (s)				
	Lemma 7	Theorem 1	Theorem 2	Theorem 3	Theorem 4
2	0.0015	0.0012	0.0018	0.00047	0.00055
5	0.0084	0.0096	0.0096	0.00075	0.00071
10	0.0527	0.017	0.0101	0.00077	0.00093
20	1.7789	0.0411	0.0275	0.0011	0.0015
30	65.6408	0.5863	0.1981	0.0014	0.0012
40	205.9481	0.8898	0.3502	0.0025	0.0014
50	330.8274	1.3542	0.6694	0.0036	0.0022

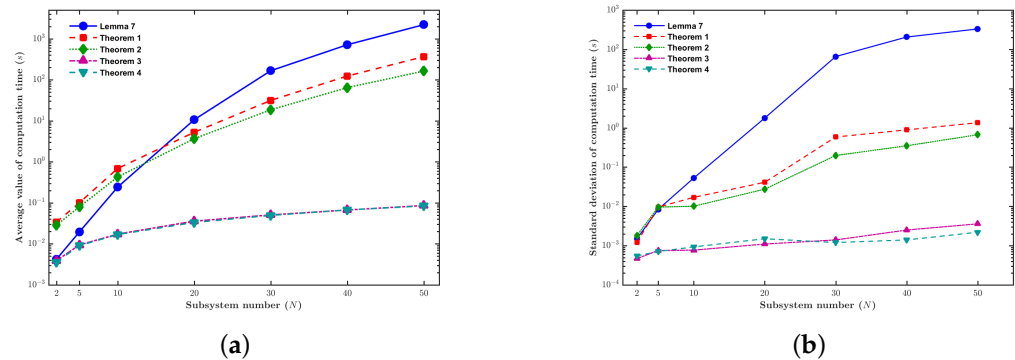


Figure 4. (a) Average value of computation time. (b) Standard deviation of computation time.

Figure 4a,b compare the mean computation times and standard deviations of the five formulations for the sampled generic interconnected systems. At $N = 50$, the computation times reported for Theorems 3 and 4 are lower than those for Theorems 1 and 2 in these tests. These timing results do not establish strict passivity feasibility for every sampled system or a quantified robustness margin for uncertainty in physical UAV parameters.

5. Conclusions

This paper analyzed the strict passivity of continuous-time UAV swarm cooperative payload transportation systems with time delays. In the specified interconnected linear model, the mechanical effect of the payload enters each UAV through the cable-tension component of the generalized external input. By exploiting the block-diagonal subsystem matrices and the sparse SCM, Theorems 1 and 2 provide necessary and sufficient conditions equivalent to the lumped criterion without high-dimensional matrix inversion. Theorems 3 and 4 give sufficient conditions based on individual UAV subsystem matrices. For the tested systems, the proposed conditions reduce computation time at larger swarm sizes.

The model assumes a square SCM with one nonzero entry equal to 1 in each row and column and zero diagonal blocks. A directed ring with one-to-one internal input and output connections satisfies this assumption, whereas a star that broadcasts the same output component to multiple UAVs or a weighted combination of several outputs into one input component does not. The state delays are approximated by a common constant. The UAV numerical example reports LMI feasibility and computation time, while the generic heterogeneous matrix study assesses computation time. These studies do not quantify robustness to unknown physical parameter errors or establish nonlinear, closed-loop payload-transport performance.

Future work will extend the analysis to denser and weighted topologies, time-varying or uncertain delays, and physically parameterized swarms with known heterogeneous UAV dynamics. It will also investigate distributed passive-controller synthesis. Nonlinear simulations and experiments will require explicit payload dynamics, characterization of onboard delay variations, and identification of individual UAV masses and moments of inertia to evaluate payload tracking, cable tension and swing behavior in closed loop.

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