

## Article

# Hybrid ISMC–FTC Design with PSO Tuning for Finite-Time Stabilization of a 2-DOF Robotic Manipulator

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## Abstract

This paper proposes a hybrid integral sliding mode and finite-time control (ISMC–FTC) strategy for a two-degree-of-freedom (2DOF) robotic manipulator, targeting finite-time stability and high-precision elliptical trajectory tracking. To ensure practical deployment, the controller explicitly enforces actuator saturation limits, joint kinematic bounds, workspace constraints, and prescribed stabilization time requirements. Particle swarm optimization (PSO) is employed to tune the FTC parameters, minimizing convergence time while guaranteeing constraint satisfaction. By integrating ISMC’s inherent robustness against matched disturbances with a PSO-optimized FTC, the scheme eliminates the reaching phase and ensures rapid, finite-time convergence. The simulation results demonstrate that the proposed approach significantly reduces stabilization time for both joints, maintains exceptionally low tracking errors, and enforces all physical constraints under bounded disturbances and model uncertainties through explicit saturation limits and sliding manifold invariance. These results validate the framework’s effectiveness and highlight its potential for safety-critical, high-precision robotic applications requiring guaranteed finite-time performance. Robustness is further validated under simultaneous disturbances and uncertainties, where the controller maintains stable performance and consistently satisfies the required robust stability condition.

**Keywords:** integral sliding mode control (ISMC); finite time control (FTC); particle swarm optimization (PSO); two-degree-of-freedom (2-DOF) robotic arm



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## 1. Introduction

The two-degree-of-freedom (2-DOF) robotic manipulator is a widely used benchmark in nonlinear control research due to its coupled dynamics, compact structure, and practical relevance in industrial automation [1,2]. It effectively captures key challenges present in more complex robotic systems, such as nonlinearity, joint coupling, and sensitivity to external disturbances and model uncertainties [3].

Driven by increasing demands for higher accuracy and faster response, advanced nonlinear control strategies have been developed, among which sliding mode control (SMC) stands out for its strong robustness against uncertainties [4]. However, conventional SMC suffers from high-frequency chattering, which can cause mechanical wear and reduce the lifespan of actuators and joints [5].

To address this issue, integral sliding mode control (ISMC) was introduced, ensuring that system trajectories start directly on the sliding surface—eliminating the reaching phase and enhancing initial robustness. The theoretical foundation of SMC was established by Utkin [4] and successfully applied to robotic systems by Slotine and Li [6]. Sabanovic et al. highlighted the detrimental effects of chattering in precision applications [5]. Plestan et al. proposed enhanced ISMC designs focused on chattering reduction [7]. Uppal et al. and Li et al. demonstrated effective ISMC implementations on 2-DOF systems [1], while De Wit et al. confirmed ISMC's superiority over PID and SMC in terms of robustness and tracking accuracy [3]. More recently, Mobayen [8], Zhang et al. [9] and Nguyen and Nguyen [10] and recent studies on flexible-joint robotic manipulators [11] developed hybrid ISMC-based controllers, although without guaranteeing finite-time convergence.

While ISMC effectively eliminates the reaching phase and provides strong robustness against matched disturbances, it does not inherently guarantee finite-time stability [12]. In fact, when applied to certain robotic systems, ISMC typically transforms the dynamics into a double-integrator form that only ensures asymptotic convergence [13–15]. To address this limitation, a hybrid control strategy is proposed that integrates ISMC with a finite-time controller (FTC). This combination leverages the disturbance rejection capabilities of ISMC while enforcing convergence to the desired trajectory within a prescribed finite time. Notably, prior work has demonstrated that non-smooth control laws, such as those employing signum functions, can enable finite-time convergence within sliding mode frameworks [7].

To further enhance performance, the parameters of the FTC are optimized using the particle swarm optimization (PSO) algorithm. PSO is well suited for this task due to its derivative-free nature, fast convergence, and effectiveness in handling nonlinear, multimodal optimization problems [16,17]. Studies have shown that PSO-tuned controllers achieve significant improvements in key metrics, including reduced settling time and minimized steady-state error. Moreover, PSO has been successfully applied to optimize both sliding mode and finite-time control schemes, demonstrating its versatility in advancing robust, high-performance control designs [18–20].

Recent advances in nonlinear control have integrated sliding mode techniques with adaptive and optimization-based strategies across various electromechanical systems. Hameed, Al-Samarraie, and Humaidi [21] compared flux observers for induction motors, demonstrating that the sliding mode observer achieved the fastest estimation. The same authors [22] designed a one-instant flux observer with barrier function-based ISMC to address latency in flux observation. Ali et al. [23] applied adaptive barrier function-based SMC to a heat exchanger with metal foam, improving settling time and thermal performance. Al-Gadery et al. [24] developed modified sliding mode extremum seeking with barrier ISMC for ABS systems, enhancing robustness and optimization efficiency. S. Al-dahlaky and S. Raafat [25] presented a comparative study of conventional PID, sliding mode control (SMC), and finite-time stability (FTS)-based SMC for a 1-DOF manipulator. Their results demonstrated that integrating FTS with SMC significantly improved convergence speed, disturbance rejection, and tracking performance compared with conventional PID and standard SMC, highlighting the effectiveness of finite-time sliding mode strategies for nonlinear robotic systems. These studies highlight the growing interest in integrating advanced sliding mode techniques with adaptive strategies to address practical constraints [26,27].

The key motivation for the proposed hybrid approach is to bridge the limitations of prior work by integrating robustness against uncertainties (via ISMC) with finite-time convergence (via FTC) [28–30].

The main technical contributions of this work are as follows:

**i. Robust Finite-Time Control Framework:**

A hybrid integral sliding mode control and fault-tolerant control (ISMC-FTC) framework is developed. Unlike conventional ISMC designs that only guarantee asymptotic stability, the proposed framework achieves true finite-time stability while maintaining robustness from the initial time instant.

**ii. Constraint-Aware Optimization:**

Physical constraints—including actuator saturation limits, joint kinematic bounds, and workspace limitations—are explicitly incorporated into the optimization framework. This comprehensive integration of real-world constraints is rarely addressed in the existing finite-time stability (FTS) or ISMC literature [31].

**iii. Progressive PSO Tuning Methodology:**

A practical particle swarm optimization (PSO)-based tuning methodology is introduced, utilizing a progressive objective function. This function dynamically evolves from a time-optimal metric to a comprehensive performance index based on the integral of time-multiplied absolute error (ITAE) alongside strict constraint penalties.

**iv. Comprehensive Experimental/Simulation Validation:**

A detailed, four-case comparative study is presented to demonstrate the practical value and efficacy of the constraint-aware optimization strategy under diverse operating conditions.

**v. Rigorous Theoretical Stability Analysis:**

A complete stability proof is provided, uniquely combining LaSalle’s invariance principle to establish asymptotic behavior and homogeneity theory to verify finite-time convergence.

These contributions collectively bridge the gap between theoretical finite-time guarantees and practical electromechanical feasibility, which has been largely overlooked in the reviewed literature. For instance, studies such as Zhang et al. [32] and Yu and Man [33] focused on finite-time control without addressing sliding mode robustness or actuator constraints. Hong et al. [34] and Levant [35] achieved finite-time convergence through homogeneous and higher-order sliding mode techniques but neglected optimization-based tuning and physical limitations. Çoban [36] and Pan et al. [37] employed integral sliding mode control to eliminate the reaching phase and improve robustness, yet they did not guarantee finite-time convergence. In contrast, the proposed framework uniquely integrates finite-time stability, integral sliding mode robustness, PSO-based optimization, and physical constraint enforcement within unified control architecture.

The remainder of this paper is organized as follows: Section 2 presents the dynamic model of the 2-DOF robotic manipulator and the control problem formulation. Section 3 details the hybrid ISMC–FTC control design and its finite-time stability analysis. Section 4 introduces the PSO-based optimization framework. Section 5 presents simulation results for four cases, while Section 6 provides the conclusions.

## 2. Preliminaries and System Dynamic

The following second-order differential equation can describe the dynamics of a 2-DOF manipulator) [38,39]:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau + D(\theta) \quad (1)$$

where  $\theta \in \mathbb{R}^2$ , vector of joint angles;  $\dot{\theta} \in \mathbb{R}^2$ , vector of joint velocities;  $\ddot{\theta} \in \mathbb{R}^2$ , vector of joint accelerations;  $M(\theta) \in \mathbb{R}^{2 \times 2}$ , inertia matrix, which is symmetric and positive definite;  $C(\theta, \dot{\theta}) \in \mathbb{R}^{2 \times 2}$ , gravitational forces vector;  $G(\theta) \in \mathbb{R}^2$ , matrix of Coriolis and centrifugal forces;  $\tau \in \mathbb{R}^2$ , vector of applied torque/force, representing the control input; and  $D(\theta) \in \mathbb{R}^2$ , vector of disturbances, which may depend on the system state and time. Figure 1 presents a typical schematic for a 2DOF robotic manipulator.

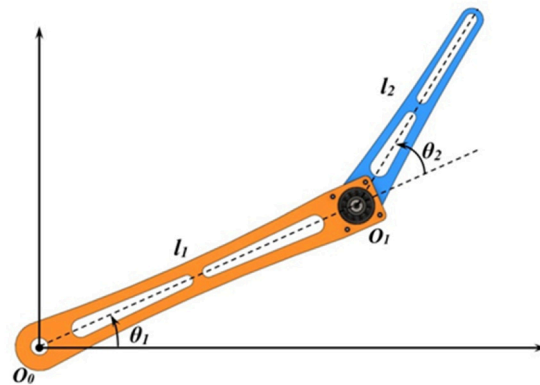


Figure 1. The model of the 2-DOF robotic arm [38].

#### Assumptions on Disturbances and Uncertainties

In this study, the following assumptions are adopted throughout the controller design and robustness analysis [4,38]:

**Matched disturbance** is considered as an additive unknown torque acting directly through the same input channel as the control signal.

**Bounded disturbance** satisfies  $|\Delta_i(t)| \leq \Delta_{\max,i}$ , where  $\Delta_{\max,i}$  denotes the maximum disturbance magnitude.

**Model uncertainty** represents the mismatch between the nominal manipulator model used during controller synthesis and the actual plant parameters. In this study, this uncertainty is represented by variations in the plant dynamic gain and/or the existence of external disturbances [40–42].

The bound  $\Delta_{\max,i}$  is selected based on worst-case analysis of the expected disturbances in the system. In this study, the disturbances are chosen as  $\Delta_1(t) = 3\sin(2\pi t)$  and  $\Delta_2(t) = 2\cos(3\pi t)$ , representing typical bounded disturbances in robotic systems. The bound is set conservatively to  $\Delta_{\max,i} = 4$  to ensure stability under worst-case conditions, as is standard in sliding mode control design [4]. This conservative selection guarantees that the switching gain  $k_i > \Delta_{\max,i}$  satisfies the robustness condition  $k_i > |\hat{\delta}_i|$  for all possible disturbances within the expected range.

To facilitate the control design in the presence of disturbances and parameter mismatches between actual and nominal values, the dynamic equation can be reformulated as follows:

$$\ddot{\theta} = M^{-1}(\theta)\tau + \delta(t, \theta, \dot{\theta}) = M_n^{-1}(\theta)\tau + \hat{\delta}(t, \theta, \dot{\theta}, \tau) \quad (2)$$

where  $M^{-1}(\theta)$  denotes the inverse of the inertia matrix computed using nominal parameters, and  $\hat{\delta}(t, \theta, \dot{\theta})$  represents the total perturbation term, which includes nonlinear effects such as Coriolis forces, centrifugal forces, and gravitational terms, as well as unknown external disturbances.

Assume that [23]

$$M_n^{-1}(\theta)\tau = u \quad (3)$$

where  $u$  is the control input. By this transformation, the system becomes equivalent to a double integrator system [43,44] with an added total disturbance  $\hat{\delta}$ , expressed as follows:



$$\ddot{\theta} = u + \hat{\delta}(t, \theta, \dot{\theta}, \tau) \quad (4)$$

where  $\hat{\delta}(t, \theta, \dot{\theta}, \tau) = D(\theta) + [M_n^{-1}(\theta) - M^{-1}(\theta)]\tau$ , which represents the total lumped disturbance and uncertainty. For a 2DOF manipulator, this transformation can be written separately for each joint:

$$\ddot{\theta}_1 = u_1 + \hat{\delta}_1(t, \theta, \dot{\theta}, \tau) \quad (5)$$

$$\ddot{\theta}_2 = u_2 + \hat{\delta}_2(t, \theta, \dot{\theta}, \tau) \quad (6)$$

This simplification makes the system more tractable for control design as each joint can now be treated as an independent double integrator system subject to disturbances.

### 3. Non-Linear Controller Design

This section presents the complete development of a nonlinear controller designed for a 2-DOF robotic manipulator. It is based on an ISMC strategy, followed by an FTC. Afterward, the section transitions to finite-time control theory, introducing homogeneity concepts and associated lemmas to guarantee convergence.

#### 3.1. Integral Sliding Mode Controller (ISMC) Design

To handle total disturbances by ISMC, its sliding variables are as follows [45–47]:

$$s_i = \dot{\theta}_i + z_i \quad (7)$$

where  $s_i$  are auxiliary variables designed to ensure stability and  $i = 1, 2, 1$  for joint 1 and 2 for joint 2. In this design, we assume that the initial conditions for the sliding variables are  $s_i(0) = 0$ , meaning that the system starts directly on the desired sliding surface.

To ensure  $s_i(0) = 0$ , the auxiliary variable  $z_i$  is initialized as  $z_i(0) = -\dot{\theta}_i(0)$ . This guarantees that the system starts directly on the sliding surface from the initial moment, eliminating the reaching phase. Subsequently,  $z_i$  evolves according to  $\dot{z}_i = -u_{in}$ , which maintains the sliding condition  $s_i(t) = 0$  for all  $t \geq 0$ .

#### 3.2. The Hybrid Controller Design

The control signal  $u$  is divided into two parts, a nominal control part  $u_{in}$  and a discontinuous control part  $u_s$ :

$$u_i = u_{in} + u_{is} \quad (8)$$

The discontinuous control part  $u_{is}$  is designed to ensure reaching the sliding surface and overcome unknown disturbances. It is defined as

$$u_{is} = -k_i \text{sign}(s_i) \quad (9)$$

where  $k_i > |\hat{\delta}_i|$  are design constants that must be greater than the absolute value of the disturbance  $\hat{\delta}_i$  to ensure effective control.

The ISMC switching component is primarily responsible for rejecting matched disturbances and compensating for model uncertainties. Once the sliding manifold is established, disturbance effects are cancelled immediately, resulting in disturbance-decoupled closed-loop dynamics. Subsequently, the FTC component guarantees finite-time convergence of the tracking error toward the desired equilibrium. Therefore, the proposed hybrid ISMC–FTC controller simultaneously achieves robustness against disturbances and uncertainties while preserving finite-time stabilization performance.  $k_i > \Delta_{max}$ , which represents the classical ISMC robustness condition ensuring that the switching authority exceeds the maximum matched disturbance.

Once the ISMC compensates for matched disturbances and reduces the closed-loop dynamics to a simplified structure, each joint can be independently modeled as a double integrator system. Accordingly, the dynamic equation of each joint becomes

$$\ddot{\theta}_i = u_{in}, \forall t \geq t_0 \quad (10)$$

The nominal control part  $u_{in}$  is the effective control signal after disturbance rejection by ISMC. At this stage, the focus shifts to driving  $\theta, \dot{\theta}$  to the origin in finite time.

The finite-time control input  $u_{in}$  is designed using the non-smooth [48,49] homogeneous [50,51] control framework [6], as follows:

**Lemma 1** ([52]). *The control law of finite-time stabilizes for double-integrator system*

$$u_{in} = -l_1 \operatorname{sgn}(x_1)|x_1|^{\alpha_1} - l_2 \operatorname{sgn}(x_2)|x_2|^{\alpha_2}, 0 < \alpha_1 < 1, \alpha_2 = \frac{2\alpha_1}{1+\alpha_1} \quad (11)$$

where  $u_{in}$  the control input  $x_1, x_2$ : first and second states variable of the double integrator.  $l_1, l_2 > 0$  Positive gain that scales the effect of states.  $\alpha_1, \alpha_2$ : A design parameter used to adjust the nonlinear rate of convergence of the states.

**Remark 1** ([53]). *Clearly, Equation (11) is a homogeneous function of degree  $\alpha_2$  with respect to  $\frac{2\alpha_1}{1+\alpha_1}$ , and it is continuous but not smooth. In the limiting case where  $\alpha_2$  approaches 1, the control becomes the conventional PD law (which is smooth) and thus cannot achieve finite-time convergence.*

To ensure that the angular positions  $\theta_1, \theta_2$  and their velocities converge to the origin in finite time, a finite-time controller (FTC) is implemented. This controller is designed to guarantee rapid convergence and arrival at a prescribed time. In view of the proposed design, the control input takes the form:

$$u_{in} = -l_{1i} \operatorname{sgn}(\theta_i)|\theta_i|^{\alpha_{1i}} - l_{2i} \operatorname{sgn}(\dot{\theta}_i)|\dot{\theta}_i|^{\alpha_{2i}} \quad (12)$$

Equation (12) represents the regulation form of the FTC law, where the desired position is the origin ( $\theta_d = 0$ ). The tracking form, where the desired trajectory is  $\theta_d \neq 0$ , is given in Equation (15) using the tracking error  $e_i = \theta_i - \theta_{d,i}$ , with the same condition for  $\alpha_1$  and  $\alpha_2$  in Equation (11). The double-integrator system can be expressed in the following standard form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ u \end{bmatrix} \quad (13)$$

Here  $x_1$  represents the joint position  $\theta_i$ , and  $x_2$  represents the joint velocity  $\dot{\theta}_i$ . This simplified representation is obtained after the ISMC compensates for disturbances, reducing the original nonlinear dynamics to an equivalent second-order system. Based on this formulation, the finite-time controller is designed to drive both states to the origin within a finite time.

### 3.3. Upper Bound for Finite-Time Convergence

After the ISMC compensates for disturbances, the dynamics for the tracking error of each joint  $i$  simplify to a double-integrator system:

$$\ddot{e}_i = u_{in} \quad (14)$$

where  $e_i = \theta_i - \theta_{d,i}$  is the tracking error for joint  $i$  and  $u_{in}$  is the nominal control input for joint  $i$ .

The FTC law for joint  $i$  is designed as

$$u_{in} = -l_{1i} \cdot \text{sgn}(e_i) \cdot |e_i|^{\alpha_{1i}} - l_{2i} \cdot \text{sgn}(\dot{e}_i) \cdot |\dot{e}_i|^{\alpha_{2i}} \quad (15)$$

with  $0 < \alpha_{1i} < 1$ ,  $\alpha_{2i} = \frac{2\alpha_{1i}}{1+\alpha_{1i}}$ ,  $l_{1i} > 0$ , and  $l_{2i} > 0$ .

A Lyapunov function candidate for joint  $i$  is chosen as [53–56]

$$V_i(e_i, \dot{e}_i) = \frac{l_{1i}}{1+\alpha_{1i}} |e_i|^{1+\alpha_{1i}} + \frac{1}{2} \dot{e}_i^2 \quad (16)$$

This function is positive definite and radially unbounded. Taking the time derivative of  $V_i$  and substituting  $\ddot{e}_i = u_{in}$ ,

$$\dot{V}_i = l_{1i} \cdot \text{sgn}(e_i) \cdot |e_i|^{\alpha_{1i}} \cdot \dot{e}_i + \dot{e}_i \cdot u_{in} \quad (17)$$

Substituting  $u_{in}$  from the FTC law,

$$\dot{V}_i = l_{1i} \cdot \text{sgn}(e_i) \cdot |e_i|^{\alpha_{1i}} \cdot \dot{e}_i + \dot{e}_i [-l_{1i} \cdot \text{sgn}(e_i) \cdot |e_i|^{\alpha_{1i}} - l_{2i} \cdot \text{sgn}(\dot{e}_i) \cdot |\dot{e}_i|^{\alpha_{2i}}]$$

The position-dependent terms cancel exactly because the Lyapunov function and the FTC law are constructed with the same coefficient  $l_{1i}$ :

$$l_{1i} \cdot \text{sgn}(e_i) \cdot |e_i|^{\alpha_{1i}} \cdot \dot{e}_i - l_{1i} \cdot \text{sgn}(e_i) \cdot |e_i|^{\alpha_{1i}} \cdot \dot{e}_i = 0 \quad (18)$$

Therefore,

$$\dot{V}_i = -l_{2i} \cdot \dot{e}_i \cdot \text{sgn}(\dot{e}_i) \cdot |\dot{e}_i|^{\alpha_{2i}} \quad (19)$$

Using the identity  $\dot{e}_i \cdot \text{sgn}(\dot{e}_i) = |\dot{e}_i|$ , we obtain

$$\dot{V}_i = -l_{2i} |\dot{e}_i|^{1+\alpha_{2i}} \leq 0 \quad (20)$$

Thus, we have  $\dot{V}_i \leq 0$ , which proves asymptotic stability by LaSalle's invariance principle [25].

The set  $\Omega$  where  $\dot{V}_i = 0$  is given by

$$\Omega = \left\{ (e_i, \dot{e}_i) \in \mathbb{R}^2 \mid \dot{V}_i = 0 \right\} = \left\{ (e_i, \dot{e}_i) \in \mathbb{R}^2 \mid \dot{e}_i = 0 \right\}. \quad (21)$$

For a trajectory to remain in  $\Omega$  (i.e., for it to be an invariant set), the condition  $\dot{e}_i(t) = 0$  must hold for all times. This implies that its derivative must also be zero:  $\ddot{e}_i(t) = 0$ . Given that  $l_{1,i} > 0$  and  $|e_i|^{\alpha_{1,i}} \geq 0$ , this equation holds if and only if  $|e_i| = 0 \Rightarrow e_i = 0$ .

Therefore, the largest invariant set  $M$  within  $\Omega$  is

$$M = \left\{ (e_i, \dot{e}_i) \in \mathbb{R}^2 \mid e_i = 0, \dot{e}_i = 0 \right\} \quad (22)$$

which is the origin. By LaSalle's principle [57,58], all closed-loop system trajectories converge to this invariant set  $M$ , proving asymptotic convergence to  $(e_i, \dot{e}_i) = (0, 0)$ .

**Lemma 2** ([52]). Suppose that system (1) is homogeneous of degree  $c_i$ . Then the origin of the system is finite-time stable if the origin is asymptotically stable, and  $c_i < 0$ . To prove finite-time convergence for joint  $i$ , it will invoke lemma 1 from [52], which states that if a system is homogeneous of degree  $c_i < 0$  and asymptotically stable, then it is finite-time stable. The closed-loop system (14)–(15) is homogeneous of degree  $c_i = \alpha_{2,i} - 1 < 0$  with respect to the standard dilation  $(\frac{2}{1+\alpha_{1,i}}, 1)$ . Since global asymptotic stability has already been established via LaSalle's principle,

Lemma 1 from [52] directly implies global finite-time stability. Specifically, there exists a positive constant  $c_i > 0$  (dependent on the dynamics of joint  $i$ ) such that

$$|\dot{e}_i|^{1+\alpha_{2,i}} \geq c_i \cdot V_i^{\gamma_i} \quad (23)$$

where  $\gamma_i = \frac{1+\alpha_{2,i}}{2} \in (0, 1)$ . Consequently, the time derivative of the Lyapunov function satisfies

$$\dot{V}_i \leq -l_{2,i}c_i V_i^{\gamma_i} \quad (24)$$

This inequality constitutes the critical condition for finite-time stability of joint  $i$ . From Lyapunov theory for finite-time stability, the  $T_{i,fts}$  from the initial condition  $V_i(0)$  to zero is bounded by

$$T_i \leq \frac{1}{l_{2,i}c_i(1-\gamma_i)} V_i(0)^{1-\gamma_i} \quad (25)$$

This equation provides the theoretical upper bound for the finite-time convergence of joint  $i$ . In accordance with the standard definition of finite-time stability (FTS) [59], this bound depends on the initial energy  $V_i(0)$ , which is a fundamental characteristic of FTS. The time  $T_i$  is directly proportional to the initial energy  $V_i(0)$  of the error and inversely proportional to the strength and efficiency of the controller  $l_{2,i}c_i$ .

#### 4. PSO-Based Parameter Tuning for the Hybrid ISMC-FTC Controller

The PSO will be utilized in this paper to improve the quality of performance of the controlled 2DOF robot by optimizing the effective parameters of the nominal control part. A detailed description of the applied PSO algorithm can be found in [60,61].

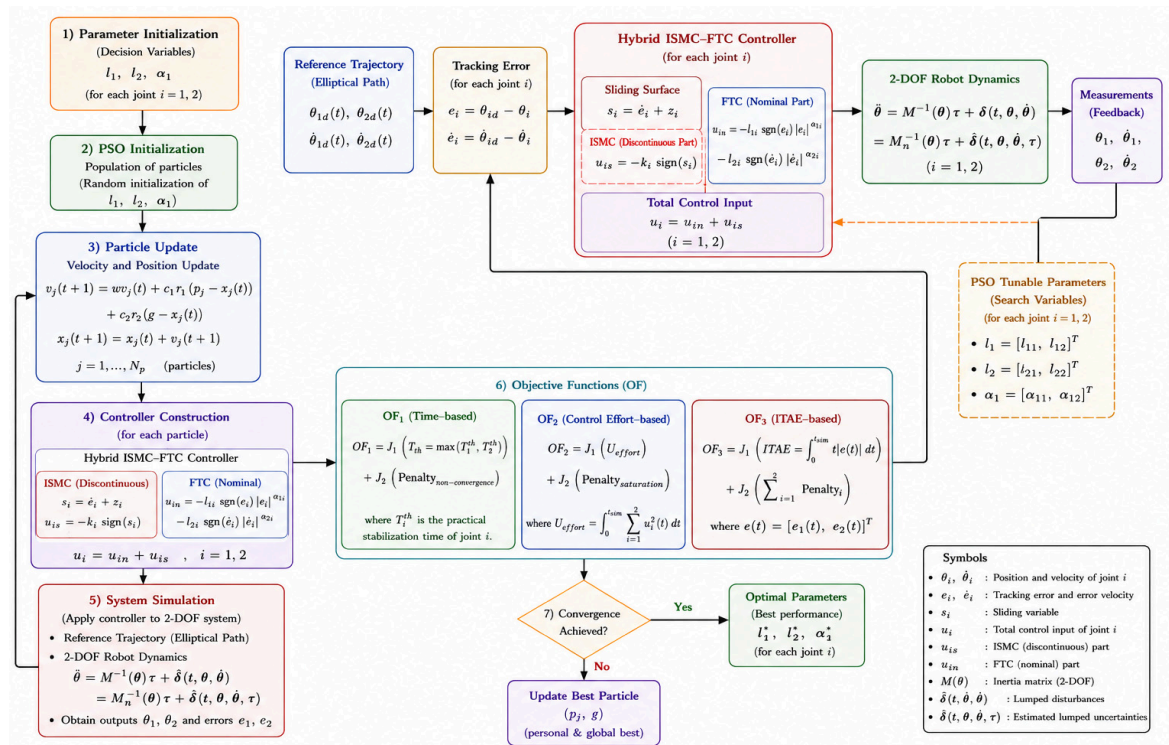
##### 4.1. Implementation of PSO in the Optimization of FTC-ISMIC Controller

In this section, the optimal tuning of the four parameters of the proposed hybrid FTC-ISMIC controller using the PSO algorithm is presented.  $T_i^{\text{prac}}$  denotes the practical stabilization time of joint  $i$ ,  $e(t)$  is the tracking error, and  $J_1$  and  $J_2$  represent the weighting coefficients. In this study, a composite performance index combining the integral of time-weighted absolute error (ITAE) and the practical stabilization time is employed to balance tracking accuracy against convergence speed. The block diagram of the optimization process for the PSO-based FTC-ISMIC controller for the 2-DOF robotic arm is shown in Figure 2.

Table 1 shows the the PSO implementation that follows the standard algorithm introduced by Kennedy and Eberhart [62], with parameter tuning based on Clerc and Kennedy [63].

**Table 1.** Summary table of PSO parameters [62].

| Parameter                    | Value                                   | Description                       |
|------------------------------|-----------------------------------------|-----------------------------------|
| Population size              | 20                                      | Number of particles               |
| Max iterations               | 30                                      | Maximum number of iterations      |
| Inertia weight (w)           | $0.9 \rightarrow 0.95$                  | Damped from 0.9 to $\approx 0.3$  |
| Damping ratio (wdamp)        | 0.95                                    | Inertia weight damping rate       |
| $c_1$ (personal coefficient) | 1.8                                     | Personal acceleration coefficient |
| $c_2$ (social coefficient)   | 1.8                                     | Social acceleration coefficient   |
| Velocity clamping            | Implicit (via bounds)                   | Position bounds enforce limits    |
| Independent runs             | 10                                      | Number of independent runs        |
| Variable bounds              | $\alpha$ : [0.01, 0.99], $l$ : [0.1, 5] | Search space boundaries           |



**Figure 2.** Optimization process of the FTC-ISM-C controller for the 2-DOF robotic arm system.

#### 4.1.1. Time-Based Objective Function

The objective function (OF) is incrementally enhanced across three cases, shifting from a speed-centric formulation to a multi-objective framework that captures comprehensive system performance [64,65].

The initial objective function focused solely on minimizing the theoretical stabilization time:

$$OF_1 = J_1 (T_{th} = \max(T_1^{th}, T_2^{th})) + J_2 (\text{Penalty}_{non-convergence}) \quad (26)$$

where  $J_1 = 0.8$ ,  $J_2 = 0.2$ , and  $\text{Penalty}_{non-convergence} = 10$  if stabilization is not achieved within the simulation time. This form prioritizes fast convergence while penalizing instability.

In this case, the theoretical upper bounds on stabilization time are  $T_1 \leq 5.08$  s and  $T_2 \leq 4.43$  s. Violation of either bound incurs a penalty of +10 in the cost function.

#### 4.1.2. Control-Effort-Aware Objective Function

To account for actuator limitations, a control effort term is incorporated into the objective function. Let  $|u_{in}|$  denote the control input and  $u_{max}$  represent the maximum allowable actuator limit. A penalty function is introduced such that if  $|u| > 0.95 u_{max}$ , an additional penalty term is added to the objective function. This formulation discourages operation near actuator saturation while maintaining fast convergence and system stability.

To enforce actuator saturation limits, the control input is constrained as  $|u_{in}| \leq 6 N.m$ . To discourage operation near this bound, a 10% soft penalty is applied to the objective function  $J$  whenever the control signal exceeds 95% of  $6 N.m$ .

#### 4.1.3. Comprehensive Performance Objective Function

The final and most advanced objective function integrates tracking accuracy, practical stabilization time, and constraint violations:

$$OF_3 = J_1 \left( ITAE = \int_0^{t_{sim}} t|e(t)| dt \right) + J_2 \left( \sum_{i=1}^2 \text{Penalty}_i \right) \quad (27)$$

The objective function combines multiple performance criteria to achieve an appropriate balance between tracking accuracy and constraint satisfaction. Specifically,  $(J_1)$  represents the weighting factor associated with the ITAE performance index, emphasizing tracking accuracy, whereas  $(J_2)$  represents the weighting factor assigned to the constraint violation penalty term. The values  $(J_1 = 0.4)$  and  $(J_2 = 1.0)$  were selected based on preliminary simulation studies and empirical tuning to provide the best trade-off between minimizing the ITAE index and enforcing the constraint penalty term. Here,  $e(t)$  denotes the tracking error, while the penalty function is activated whenever actuator or kinematic constraints are violated.

To provide a formal basis for constraint satisfaction, a constraint-invariance analysis is conducted. The closed-loop system with the proposed hybrid ISMC-FTC controller satisfies the following properties:

$$\mathcal{X} = \left\{ x \in \mathbb{R}^n \mid \left| \theta_i \right| \leq \theta_{\max}, \left| \dot{\theta}_i \right| \leq v_{\max}, \left| u_{\text{in}} \right| \leq u_{\max} \right\} \quad (28)$$

Since the sliding manifold is invariant under the control law (i.e.,  $\dot{s}_i = 0$  when  $s_i = 0$ ), and the control input  $u_{\text{in}}$  is bounded by design ( $|u_{\text{in}}| \leq u_{\max}$ ), the system state  $x(t)$  remains within  $\mathcal{X}$  for all  $t \geq 0$  provided that the initial condition  $x(0) \in \mathcal{X}$ . This invariance property, combined with the explicit saturation limits enforced on the FTC component, ensures that the constraints are strictly satisfied during closed-loop operation.

## 5. Simulation Results

This section presents the simulation results evaluating the performance of the proposed hybrid controller for the 2-DOF robotic manipulator. While the composite ISMC-FTC architecture guarantees closed-loop stability and accurate trajectory tracking, the actual finite-time settling time  $T_{fts}$  is highly sensitive to the selection of key controller gains (e.g.,  $l_{11}$ ,  $l_{21}$ ,  $\alpha_{11}$ ,  $\alpha_{21}$ ). To systematically optimize these parameters and minimize  $T_{fts}$ , PSO is employed as an automated tuning mechanism.

The numerical validation is conducted within the MATLAB/Simulink (2020 Ra) environment. The physical parameters of the manipulator, summarized in Table 2, are incorporated into the nonlinear dynamic model to simulate the joint trajectories  $\theta_1$  and  $\theta_2$ . The proposed ISMC-FTC control law is then integrated with the plant dynamics. Finally, a custom MATLAB script implementing the PSO algorithm is interfaced with the Simulink model to iteratively adjust the controller gains, evaluate the composite performance index, and converge to the optimal parameter set.

**Table 2.** Robotic arm system parameters and values [1,2,18].

| Parameter                                   | Value           |
|---------------------------------------------|-----------------|
| $l_1$ (First link length)                   | 1.0 m           |
| $l_2$ (Second link length)                  | 0.8 m           |
| $m_1$ (First link mass)                     | 1.5 kg          |
| $m_2$ (Second link mass)                    | 1.0 kg          |
| $\theta_{\text{limit}}$ (Joint angle limit) | $\pm 1.0$ rad   |
| $v_{\text{limit}}$ (Joint velocity limit)   | $\pm 1.0$ rad/s |
| $r_{\text{max}}$ (Workspace radius)         | 1.8 m           |

In this study, the objective function (OF) is formulated as a composite performance index that combines the integral of time-weighted absolute error (ITAE) and the practical stabilization time. This formulation ensures a balanced trade-off between tracking accuracy and convergence speed.  $J_1$  and  $J_2$  are selected as 0.4 and 0.6, respectively.



The lower and upper bounds for each controller parameters in the minimization of the OF are given in Table 3.

**Table 3.** Upper and lower limits for controller parameters.

| Parameter                  | Bounds    |
|----------------------------|-----------|
| $\alpha_{11}, \alpha_{21}$ | 0.01–0.99 |
| $l_{11}, l_{21}$           | 0.1–80.0  |

To guarantee safe and reliable robot operation, constraints are systematically imposed on different aspects of the system, including the following:

1. **Stabilization Time Constraint:** The closed-loop system must achieve stabilization within predefined time bounds  $T_1 \leq 5.078$  s and  $T_2 \leq 4.43$  s. This constraint guarantees not only closed-loop stability but also a bounded convergence rate, which is essential for real-time robotic applications that demand rapid and predictable responsiveness.
2. **Control Signals:** Limits are applied specifically to the FTC component ( $|u_{in}| \leq 6$  N.m), while preserving the freedom of the ISMC component ( $u_{is}$ ), which is essential for disturbance rejection. This design applies the saturation constraint exclusively to the FTC component, thereby preserving the ISMC's full switching authority and ensuring robust matched-disturbance rejection.
3. **Kinematic Variables and Safe Operating Area:** Constraints are imposed on the kinematic variables to ensure safe operation of the robot. Specifically, the joint angles and velocities are restricted within the limits ( $|\theta_i, \dot{\theta}_i| \leq 1$  rad./second). Furthermore, the overall operating region is characterized by the distance measure  $r = \sqrt{x_{end}^2 + y_{end}^2}$ . This ensures that the robot remains within a bounded safe zone in the angle velocity state space, thereby avoiding violations of physical feasibility and hardware safety.

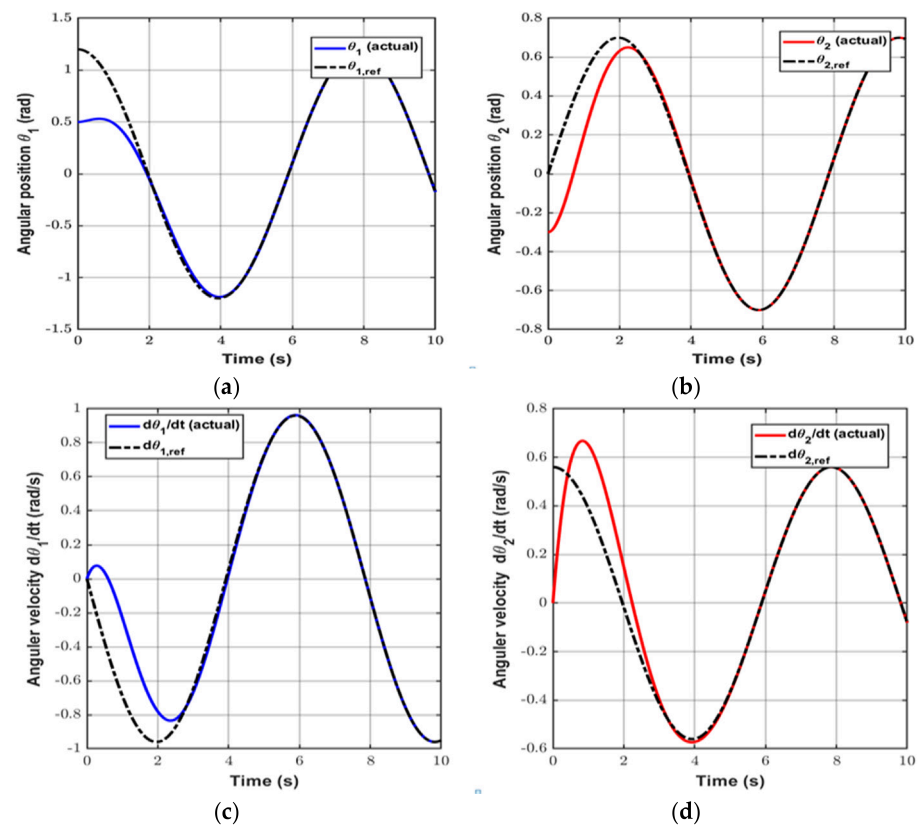
By integrating stabilization time, control effort, and kinematic variables into a unified constraint framework, the proposed system achieves not only robustness and accuracy but also safety, practicality, and guaranteed performance.

#### 5.1. Case I: Without Using PSO Optimization

During the simulation process, the controller parameters were meticulously selected to ensure the desired performance was achieved. From Table 4, the coefficient  $k_i$  within the sliding mode control plays a crucial role in reducing chattering and ensuring that the sliding variables  $s_i$  remain close to zero. The other parameters ( $\alpha_{i1}, l_{ij}$ ) primarily control the predefined  $T_{FTS}$ , according to the relationship specified in Equation (13). Increasing the values of  $\alpha_{i1}$  leads to an increased  $T_{FTS}$ , whereas the values of  $l_{ij}$  influence the system's speed of response. Figure 3a–d and Table 5 demonstrate that the joints  $\theta_i$  and  $\dot{\theta}_i$  accurately track the reference trajectory. Good stability is observed with a strong match to the reference path, and the errors settle into a narrow band around zero over time.

**Table 4.** Desired controller parameters in case I.

| Parameter                    | Value      |
|------------------------------|------------|
| $[k_1, k_2]$                 | [1.5, 1.5] |
| $[\alpha_{11}, \alpha_{21}]$ | [0.9, 0.9] |
| $[l_{11}, l_{12}]$           | [1.9, 1.8] |
| $[l_{21}, l_{22}]$           | [2, 2]     |

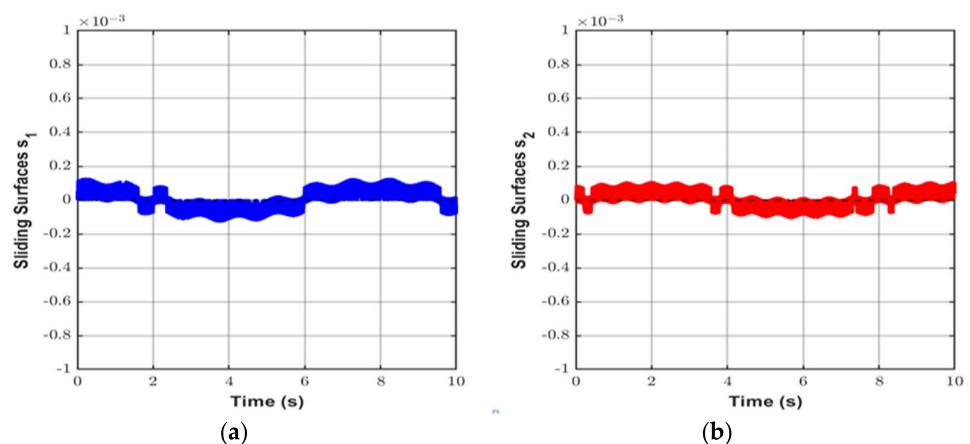


**Figure 3.** Position and velocity tracking performance of the 2-DOF system. (a) Joint 1 angle tracking. (b) Joint 2 angle tracking. (c) Joint 1 velocity. (d) Joint 2 velocity.

**Table 5.** Performance metrics in case I.

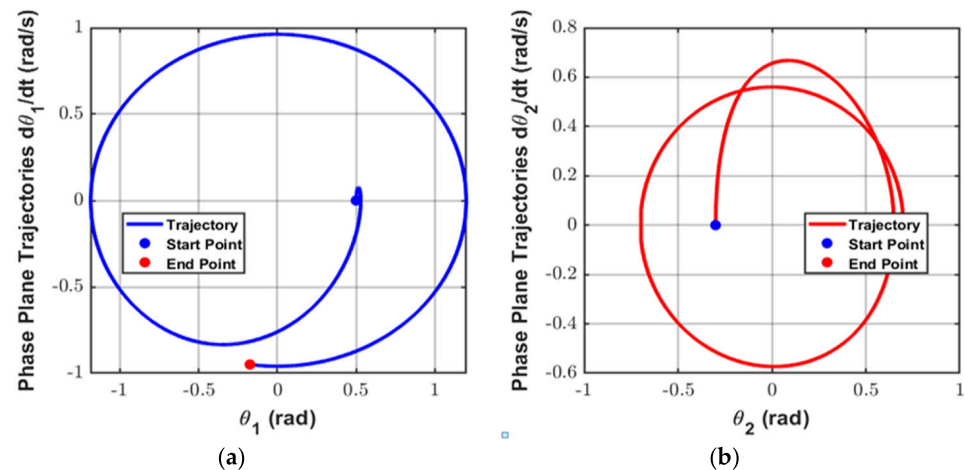
| Performance Metrics | $\theta_1$ | $\theta_2$ |
|---------------------|------------|------------|
| $T_{i\_FTS}$        | 5.078 s    | 4.43 s     |
| ISE                 | 0.3617     | 0.1742     |
| IAE                 | 0.7835     | 0.5858     |
| ITAE                | 0.6683     | 0.5473     |
| RMSE                | 0.9021     | 0.1320     |

Figure 4a,b show the sliding variables remaining very close to zero throughout the operation. This confirms the ISMC strategy's effectiveness in achieving the sliding state almost instantly, which eliminates the reaching phase and provides robust disturbance rejection.



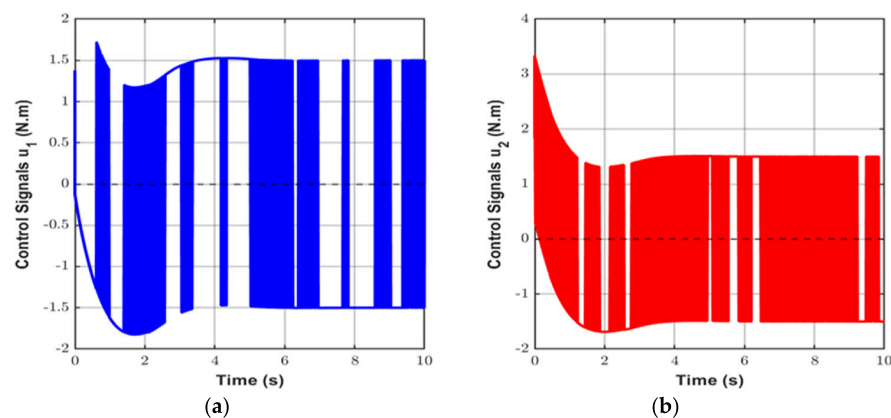
**Figure 4.** Evolution of sliding variables. (a) Sliding variable  $s_1$ . (b) Sliding variable  $s_2$ .

The phase plane trajectories for both joints are presented in Figure 5, showing the trajectories of (a) and (b) exhibit smooth spiral patterns converging toward the origin, denoting the intended equilibrium point. The ongoing convergence of the trajectories signifies stable system dynamics and efficient energy dissipation over time. Notwithstanding the system's nonlinear dynamics, both joints demonstrate consistent convergence without divergence or instability, hence affirming the robustness of the suggested control technique.



**Figure 5.** Phase plane trajectories of the 2-DOF manipulator joints. (a) Joint 1 phase portrait. (b) Joint 2 phase portrait.

In addition, Figure 6a,b depict the control signals containing high-frequency components (chattering) resulting from the “sign (s)” function in the ISMC part. This is an expected and acceptable trade-off given the significant benefits in terms of robustness and disturbance cancellation.



**Figure 6.** Control input signals showing chattering behavior. (a) Total control input  $u_1$ . (b) Total control input  $u_2$ .

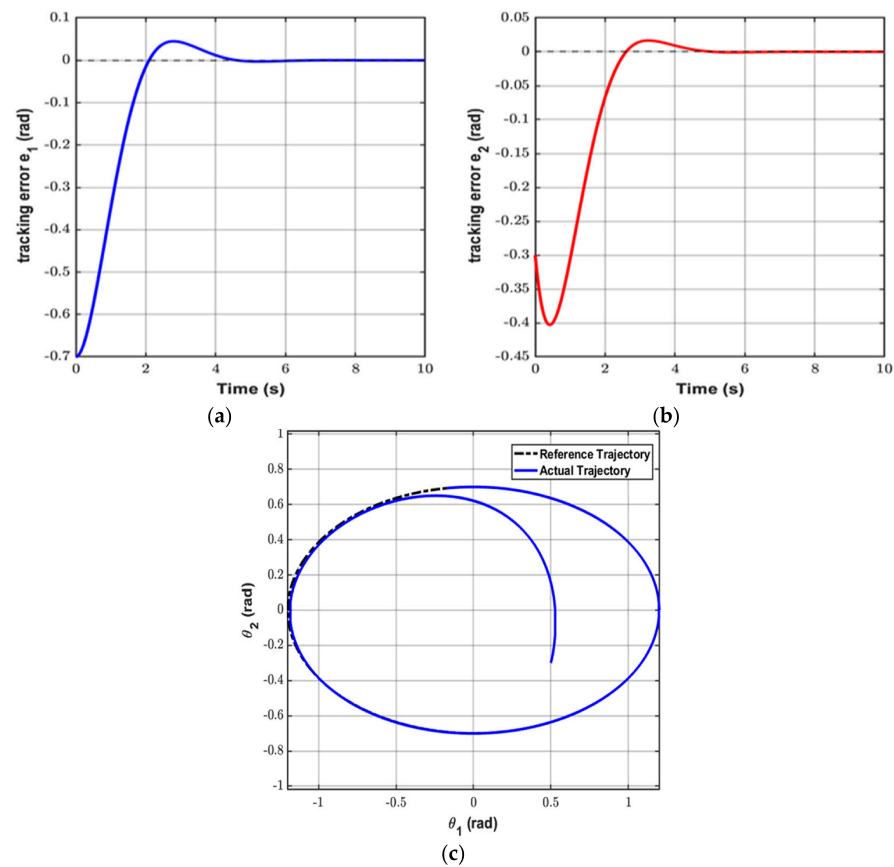
Figure 7a–c illustrate the gradual decrease of tracking errors towards zero, indicating continuous improvement in tracking performance over time.

## 5.2. Case II: PSO-Based Control Design for Specified-Time Stabilization of Robotic Manipulators

In this case, PSO algorithm was employed to automatically tune the parameters of the FTC controller from Equation (12), with the objective of achieving faster system stabilization. The controller parameters  $\alpha_{11}, \alpha_{21}, l_{11}, l_{21}$  are optimized as they directly govern the system's convergence rate and transient dynamic response. Specifically, the following are noted:

- The parameters  $\alpha_{i1}$  govern the nonlinear convergence characteristics of the finite-time controller.

- The parameters  $l_{ij}$  determine the aggressiveness of the control action and, consequently, the speed of response after disturbance rejection.



**Figure 7.** Tracking error convergence for both joints. (a) Joint 1 tracking error. (b) Joint 2 tracking error. (c) Elliptical trajectory tracking.

Manual tuning of these parameters is challenging due to their nonlinear and coupled influence on system performance. Excessively small values result in sluggish response, while excessively large values may induce chattering or destabilize the system. Therefore, PSO was utilized as an intelligent optimization technique to search the parameter space efficiently and identify the optimal set that minimizes the practical stabilization time while satisfying performance and stability requirements. The simulation results obtained using the PSO algorithm are presented in Table 6.

**Table 6.** Resulted parameters from PSO in case II.

| Parameter                    | Value              |
|------------------------------|--------------------|
| $[k_1, k_2]$ fixed           | $[1.5, 1.5]$ fixed |
| $[\alpha_{11}, \alpha_{21}]$ | $[0.9, 0.9]$       |
| $[l_{11}, l_{12}]$           | $[8.89, 2]$        |
| $[l_{21}, l_{22}]$           | $[9.08, 1.8]$      |

The optimization objective was to achieve stabilization times strictly less than those of case 1:

- $T_1 < 5.08$  s for joint 1;
- $T_2 < 4.43$  s for joint 2.

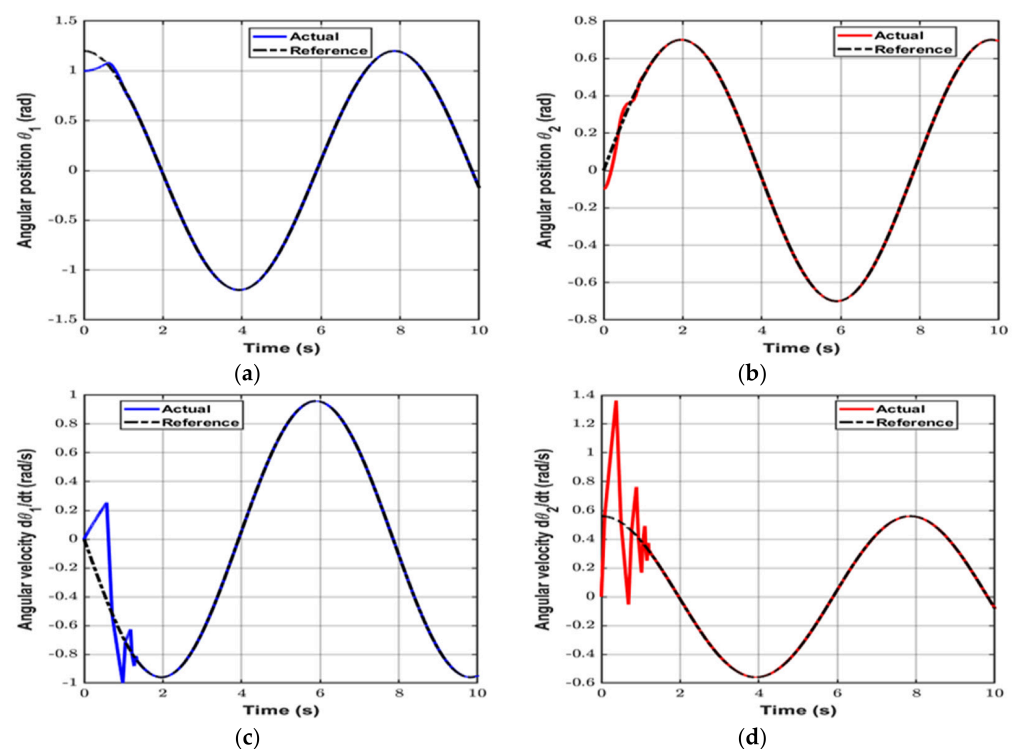
Simulation results after PSO-based tuning demonstrated a remarkable improvement in dynamic performance. The practical stabilization times were reduced to

- $T_1 = 1.05$  s (an 83.3% reduction);
- $T_2 = 0.95$  s (a 78.6% reduction).

This five-fold enhancement in convergence speed underscores the effectiveness of PSO in automating the controller design process and achieving ultra-fast finite-time stability. In addition, a significant improvement in tracking accuracy was achieved. Table 7 shows that the performance indices such as ISE, IAE, and ITAE measure the cumulative error between the actual and reference trajectories. A reduction of more than 80–95% in these indices indicates that the robot tracks the elliptical trajectory with very high precision, and the instantaneous tracking errors are very small. These results confirm a substantial enhancement in tracking accuracy and a significant reduction in cumulative error, indicating that the system not only responds faster but also follows the reference trajectory with higher fidelity. Figure 8 shows that the joint angles and velocities in Figure 8a–d accurately track the elliptical reference trajectory. Although transient overshoots are observed in the initial velocity responses, particularly in Figure 8d, they are mainly caused by the aggressive finite-time nominal control action during the initial transient. These overshoots decay rapidly without sustained oscillations, while the ISMC component maintains robustness and accurate steady-state tracking.

**Table 7.** Performance metrics for case II compared with case I.

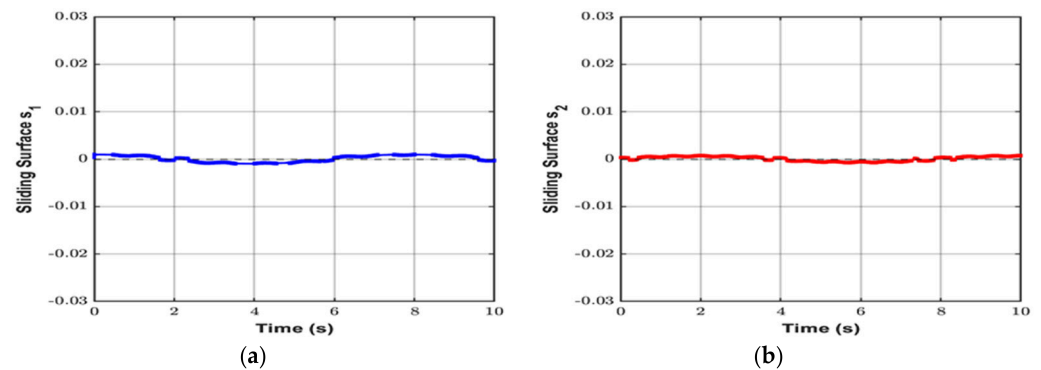
| Performance Metrics | Case2: $\theta_1$ with PSO | Case2: $\theta_2$ with PSO | Improved $\theta_1$ from Case 1 | Improved $\theta_2$ from Case 1 |
|---------------------|----------------------------|----------------------------|---------------------------------|---------------------------------|
| $T_{i\_FTS}$        | 1.05 s                     | 0.95 s                     | <b>83.3%</b>                    | 78.6%                           |
| ISE                 | 0.0126                     | 0.0927                     | 96.5%                           | <b>46.8%</b>                    |
| IAE                 | 0.0429                     | 0.0787                     | 94.5%                           | 86.6%                           |
| ITAE                | 0.1530                     | 0.0355                     | 77.2%                           | 93.5%                           |
| RMSE                | 0.8410                     | 0.0199                     | 6.0%                            | 84.8%                           |



**Figure 8.** Position and velocity tracking of the elliptical trajectory. (a) Joint 1 angle tracking. (b) Joint 2 angle tracking. (c) Joint 1 velocity. (d) Joint 2 velocity.

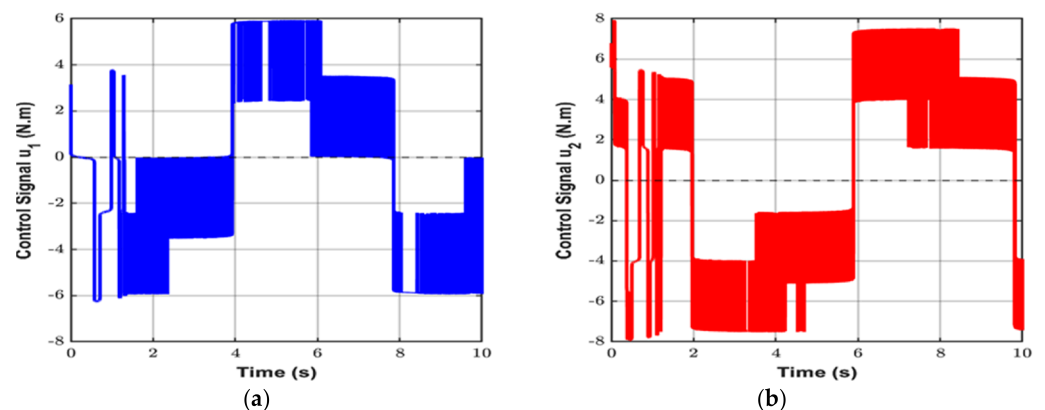
Figure 9a,b illustrate the sliding variables ( $S_1$ ) and ( $S_2$ ), which remain very close to zero throughout the operation. This confirms that the system operates on the sliding surface from the initial time, effectively eliminating the reaching phase.

Figure 10a,b show the control signals, which contain high-frequency oscillations due to the discontinuous sign(s) function. Although this introduces chattering, it is an acceptable trade-off given the improved robustness and disturbance rejection capability.

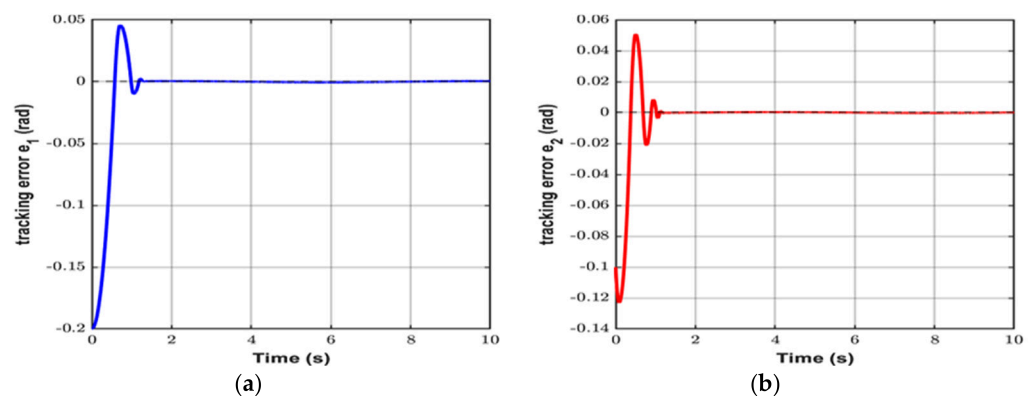


**Figure 9.** Sliding surface behavior for both joints. (a) Sliding variable  $S_1$ . (b) Sliding variable  $S_2$ .

Figure 11a shows the tracking error rapidly decreasing and remaining near zero, demonstrating excellent steady-state accuracy. In addition, Figure 12b illustrates the convergence of the PSO fitness function, which decreases monotonically, confirming stable and effective optimization.

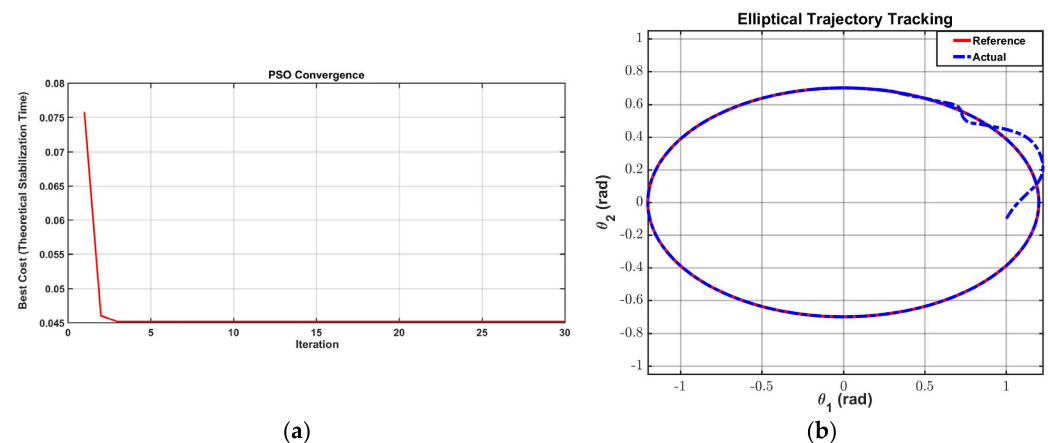


**Figure 10.** Control input signals showing chattering characteristics. (a) Total control input  $u_1$ . (b) Total control input  $u_2$ .



**Figure 11.** Optimization convergence and tracking error. (a) Tracking error  $e_1$ . (b) Tracking error  $e_2$ .





**Figure 12.** PSO convergence behavior and elliptical trajectory tracking in the  $(\theta_1, \theta_2)$  plane. (a) PSO fitness function convergence. (b) Elliptical trajectory tracking with r circle.

### 5.3. Case III: Design of a Hybrid ISMC-FTC Controller with Control Signal Constraints Using PSO

In this study, the constraints imposed on the control signals were successfully achieved using the optimization strategy. Specifically, the bounds of the finite time control  $u_{in}$  were maintained within the specified limit  $|u_{in}| \leq 6 \text{ Nm}$  reflecting the precise implementation of the constraints. Furthermore, the value of the integral sliding mode control  $u_{is}$  remained fixed at  $|u_{is}| \leq 4 \text{ Nm}$  as required, confirming the stability of this part of the system.

These values originate from the total control effort, which is constrained  $|u_{total}| \leq 10 \text{ Nm}$  and through the disturbances imposed during the simulation as  $\Delta_1(t) = 3 \sin(2\pi t)$  and  $\Delta_2(t) = 2 \cos(3\pi t)$ . So, let  $|k_i| = 4 = |u_{is}|$ . Consequently, the remaining control effort allocated to the finite time control component is given by  $|u_{in}| = 10 - |u_{is}|$ , resulting in  $|u_{in}| \leq 6 \text{ Nm}$ . This ensures a proper distribution of control authority while respecting actuator limitations and maintaining system stability.

During the optimization process, only  $(\alpha_{11}, \alpha_{21}, l_{11}, l_{12}, l_{21}, l_{22})$  parameters were adjusted as they are the ones that directly influence the value of the FTC control  $u_{in}$ . In the PSO algorithm, the parameters were optimized to achieve faster stabilization times while adhering to the predefined constraints on the control signals.

The simulation results obtained using the algorithm are presented in Table 8. Table 8 shows that although applying these constraints led to a slight increase in the practical stabilization time  $T_{fts}$  compared with the unconstrained case, the overall performance remained acceptable and good. This can be observed from the various performance metrics listed in Table 9, where all indicators showed noticeable improvement or maintained good stability under the applied constraints.

**Table 8.** Resulted parameters from case III.

| Parameter                    | Value              |
|------------------------------|--------------------|
| $[k_1, k_2]$ fixed           | $[1.5, 1.5]$ fixed |
| $[\alpha_{11}, \alpha_{21}]$ | $[0.7, 0.6]$       |
| $[l_{11}, l_{12}]$           | $[1.1274, 1.51]$   |
| $[l_{21}, l_{22}]$           | $[0.6014, 1.51]$   |

During the optimization process, tracking accuracy improved significantly, especially for joint 2, with ISE decreasing by 94.6%. However, for joint 1, the IAE increased substantially, indicating a longer error convergence period. On the other hand, the ITAE results show that in case III, the system does not oscillate for a prolonged time but instead settles smoothly, with improvement rates ranging from 16% to 50%. Regarding RMSE, a significant

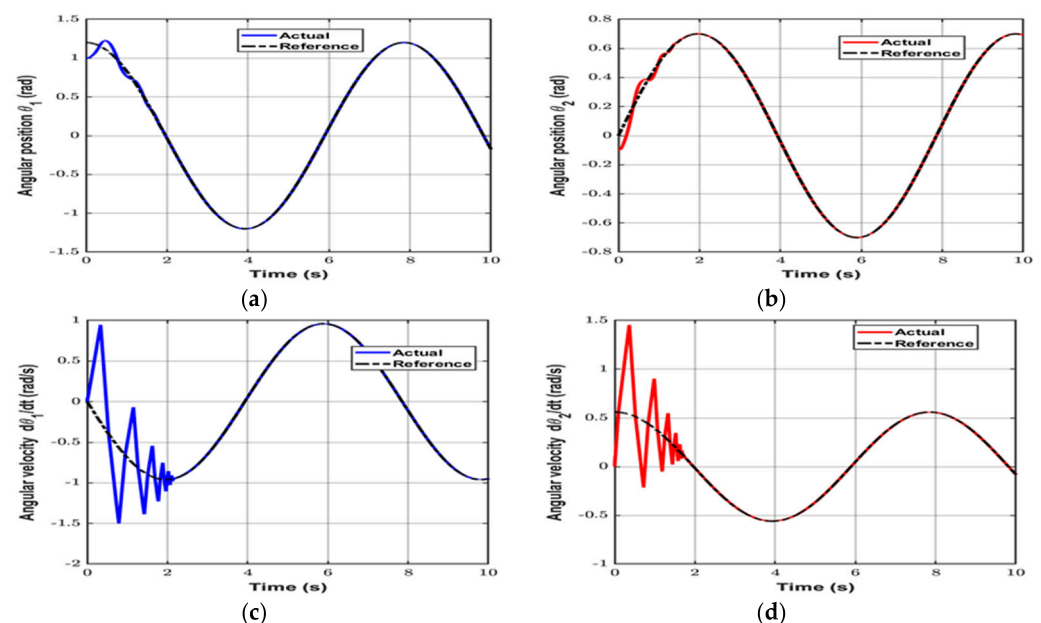
improvement was observed for joint 1, while for joint 2, a slight increase of 12% occurred, which remains within acceptable limits.

**Table 9.** Performance metrics for case III and comparison with case II.

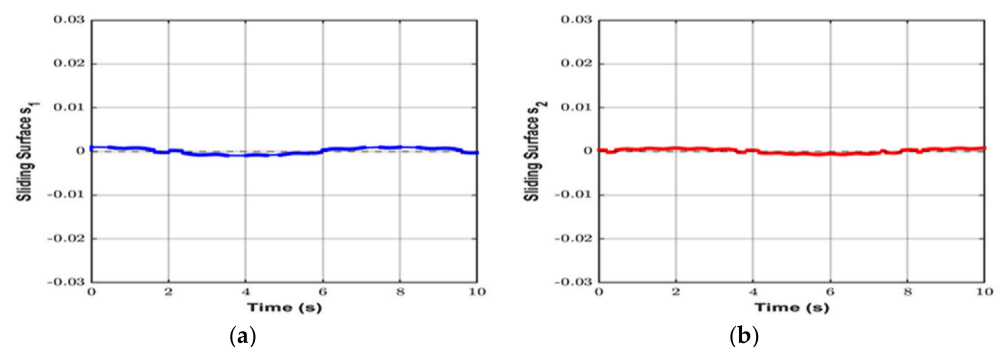
| Parameters   | Case III: $\theta_1$ | Improvement $\theta_1$<br>from Case II | Case III: $\theta_2$ | Improvement $\theta_2$<br>from Case II |
|--------------|----------------------|----------------------------------------|----------------------|----------------------------------------|
| $T_{i\_FTS}$ | 2.14 s               | 104.7%                                 | 1.66 s               | 74.4%                                  |
| ISE          | 0.0121               | 3.97%                                  | 0.0050               | 94.6%                                  |
| IAE          | 0.1139               | 165.5%                                 | 0.0611               | 22.4%                                  |
| ITAE         | 0.0754               | 50.7%                                  | 0.0298               | 16.0%                                  |
| RMSE         | 0.0348               | 95.86%                                 | 0.0223               | 12.1%                                  |

Figure 13 shows accurate tracking of the reference trajectory for both joint angles and angular velocities, while the sliding surfaces remained close to zero at all times and did not change. This reflects the robustness of the ISMC.

Figure 14a,b show that the sliding variables remain close to zero throughout the operation, confirming the effectiveness of ISMC in eliminating the reaching phase.



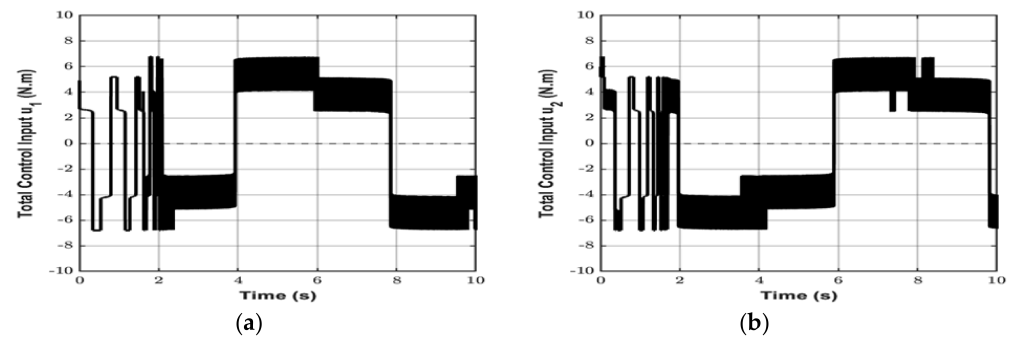
**Figure 13.** Simulation results for the PSO-tuned controller in case III. (a) Joint 1 angle tracking. (b) Joint 2 angle tracking. (c) Joint 1 velocity. (d) Joint 2 velocity.



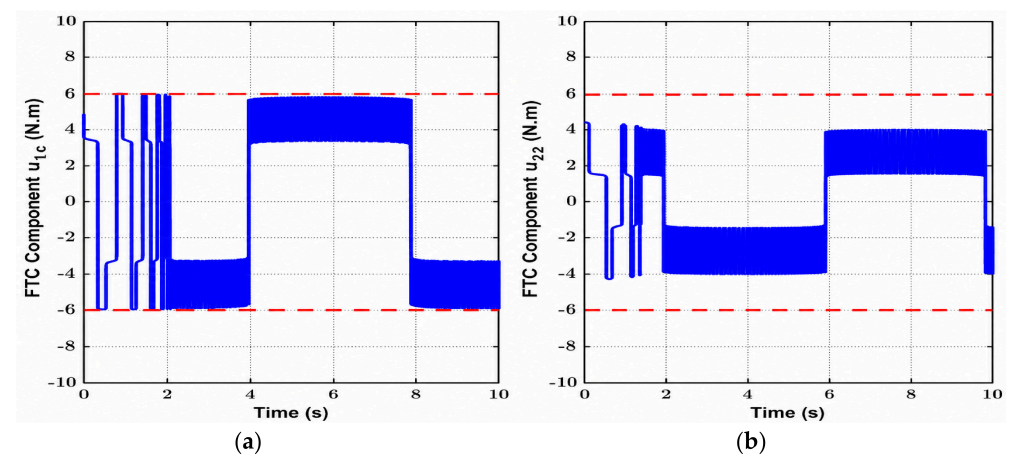
**Figure 14.** Sliding surface behavior under constrained control. (a) Sliding variable  $s_1$ . (b) Sliding variable  $s_2$ .

Figures 15–17 illustrate the control signals and their corresponding components. It can be observed that all control inputs remain within the imposed constraints, demonstrating

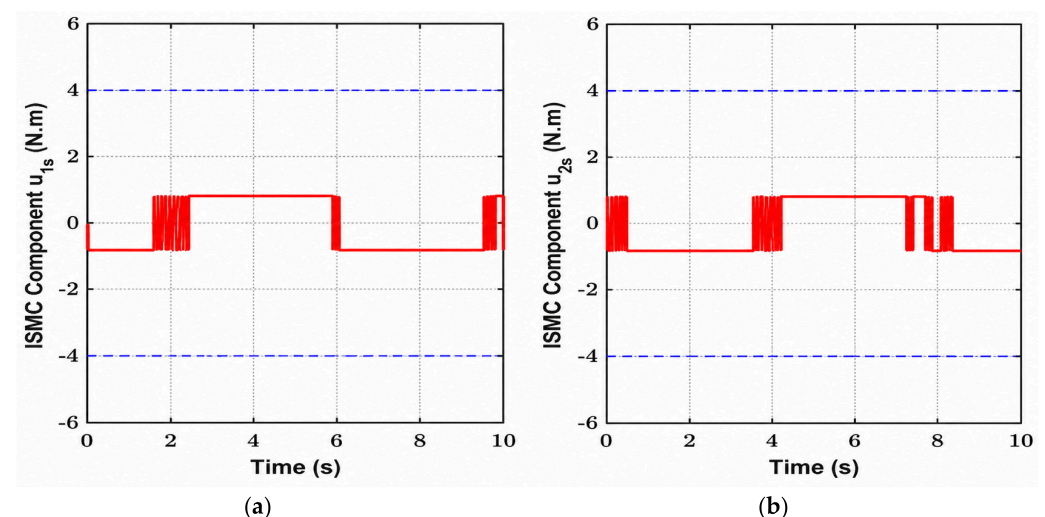
the effectiveness of the proposed control strategy in maintaining bounded control effort, with  $|u_{in}| \leq 6 \text{ Nm}$ .



**Figure 15.** Simulation results for PSO-tuned controller in case III. (a) Total control input  $u_1$ . (b) Total control input  $u_2$ .

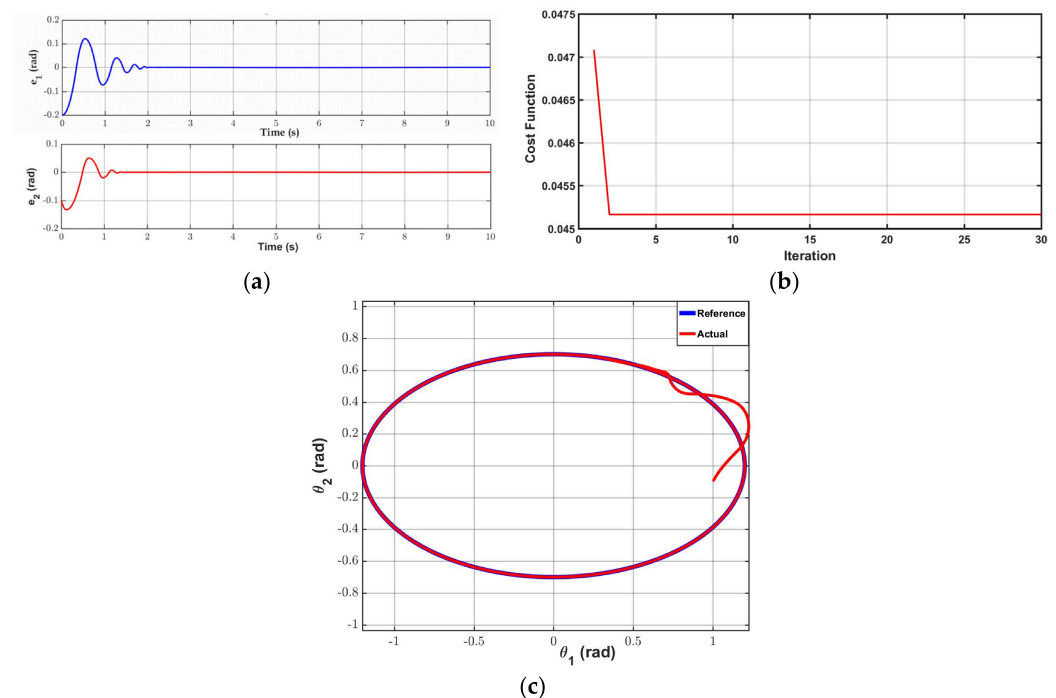


**Figure 16.** Simulation results for PSO-tuned controller in case III. (a) FTC component  $u_{1n}$ . (b) FTC component  $u_{2n}$ .



**Figure 17.** Simulation results for PSO-tuned controller in case III. (a) ISMC component  $u_{1s}$ . (b) ISMC component  $u_{2s}$ .

Figure 18a–c demonstrate that the proposed controller achieves fast and stable velocity tracking, efficient optimization through PSO, and high-precision trajectory tracking in the joint space. The combined results confirm the effectiveness of the control strategy in ensuring both dynamic performance and optimization efficiency.



**Figure 18.** Velocity tracking, PSO convergence, and elliptical trajectory tracking in the  $(\theta_1, \theta_2)$  plane. (a) Tracking error. (b) PSO fitness function convergence. (c) Elliptical trajectory tracking with r circle.

#### 5.4. Case IV: Design of a Hybrid ISMC-FTC Controller with Constraints on Angles and Velocities and Safe Region Using PSO

A simulation was conducted for a robotic control system incorporating a built-in mechanism to impose constraints on state variables. This mechanism ensures the system operates within safe and reliable ranges for angles and velocities. The maximum allowable angle for each joint was defined as  $|\theta_i| \leq \pm 1.0$  radians; this means the joint movement must remain confined between  $-1.0$  and  $+1.0$  radians, preventing exceedance of permitted mechanical limits. The maximum allowable rotational speed for each joint was defined as  $|\dot{\theta}_i| \leq \pm 1.0$  radians/second; this ensures that the joints' rotation rates do not surpass safe values that could cause vibrations or mechanical stress. The safe region represents the set of states  $\theta_i, \dot{\theta}_i$  that satisfy the aforementioned constraints. It is described by a circle defined by the following equations.

The safe region is defined based on both kinematic constraints and workspace limitations. The end-effector position is computed as follows:

- For the first joint (base):  $x_1 = l_1 \cdot \cos(\theta_1)$ ,  $y_1 = l_1 \cdot \sin(\theta_1)$
- For the second joint (second arm):  $x_2 = l_2 \cdot \cos(\theta_1 + \theta_2)$ ,  $y_2 = l_2 \cdot \sin(\theta_1 + \theta_2)$

The total position of the end-effector is as follows:

$$x_{\text{end}} = x_1 + x_2 = l_1 \cdot \cos(\theta_1) + l_2 \cdot \cos(\theta_1 + \theta_2) \quad (29)$$

$$y_{\text{end}} = y_1 + y_2 = l_1 \cdot \sin(\theta_1) + l_2 \cdot \sin(\theta_1 + \theta_2) \quad (30)$$

where  $l_1$  = length of the first arm,  $l_2$  = length of the second arm,  $\theta_1$  = angle of the first joint and  $\theta_2$  = angle of the second joint

The equation that calculates the actual distance of the end-effector from the origin point (the base) is as follows:

$$r = \sqrt{x_{\text{end}}^2 + y_{\text{end}}^2} \quad (31)$$

For checking workspace violation,

$$\text{If } r > R_{\max} \quad (32)$$

where  $r$  = calculated actual distance and  $R_{\max}$  = maximum allowed distance (radius of the safe workspace), the theoretical workspace of the arm is given by the equation

$$R_{\min} \leq r \leq R_{\max} \quad (33)$$

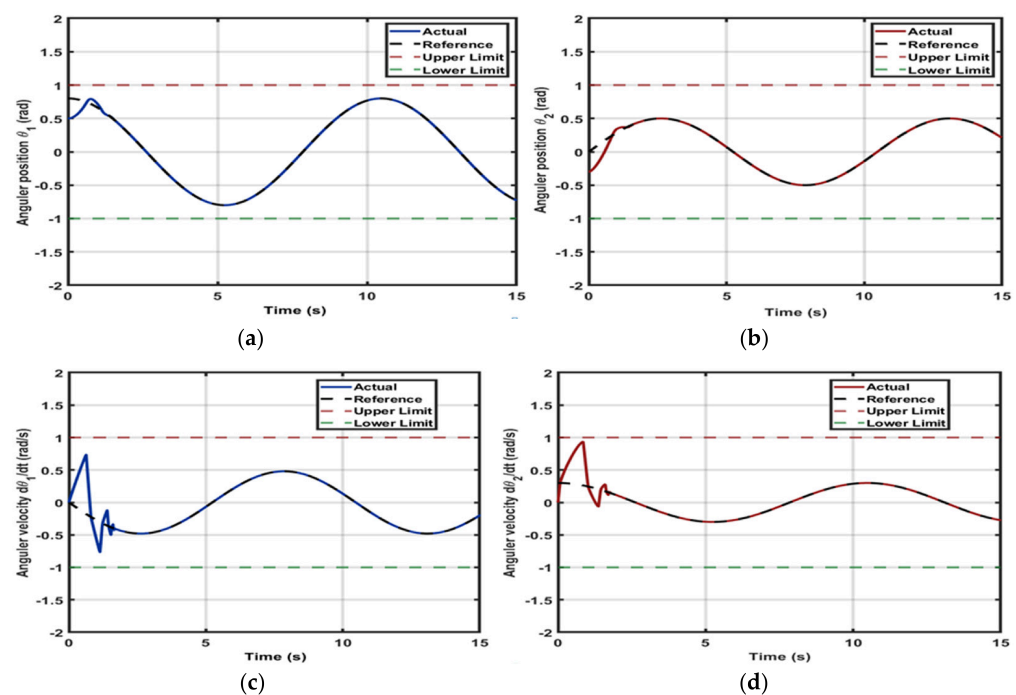
where

$$R_{\min} = |l_1 - l_2| \text{ (minimum distance, closest point);}$$

$$R_{\max} = l_1 + l_2 \text{ (maximum distance, farthest point).}$$

If the system's dynamic trajectory remains within this circle in state space, it indicates that the constraints on angles and velocities have not been exceeded. The system ensures that the final solution is not only fast but also safe, stable, and physically realizable.

The results demonstrated that the system achieved rapid stabilization, accurate tracking of the elliptical trajectory and strict adherence to the imposed constraints on angles and velocities. Figure 19a–d illustrate the tracking of joint angles and angular velocities, as well as the imposed constraints on these variables. It can be observed that all signals adhered strictly to their respective limits while maintaining precise trajectory tracking.



**Figure 19.** Joint angles and velocities tracking under PSO-tuned controller (case IV). (a) Joint 1 angle tracking. (b) Joint 2 angle tracking. (c) Joint 1 velocity. (d) Joint 2 velocity.

The workspace boundary represents the theoretical safe limits of the arm's operating area, preventing overextension. This boundary is defined by a radius  $R = 1.8$  m, calculated using Equation (31). The max radius = 1.8 m shows the farthest point reached by the end-effector during motion, as calculated in  $L_1 + L_2 = 1.0 + 0.8 = 1.8$  m. The base represents the fixed origin of the arm, serving as the reference point for all movements.

The end-effector path illustrates the complete trajectory followed by the end-effector during execution, demonstrating high-quality tracking of the desired (reference) path. Overall, these results confirm that the robot arm operated safely within all limits (angles, velocities, and workspace), without exceeding any constraints, thus performing reliably within the designated safe boundaries.

By observing the stabilization time for both joints, it is evident that the system stabilized within a reasonable time frame. Although the imposed constraints slow down the

system and reduce tracking accuracy compared with the ideal (unconstrained) case, a good balance between performance and safety was achieved. It was verified that the end-effector remained within the designated safe workspace, and all kinematic variables stayed within their permitted bounds throughout the simulation.

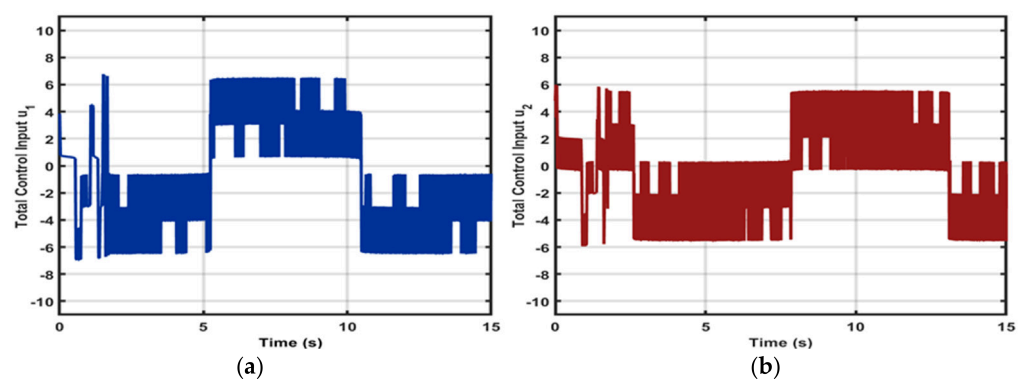
When comparing case IV with case II, Table 10 shows that the stabilization time increased by 36.4% for joint 1 and 51.7% for joint 2. While this increase is noticeable, it remains within acceptable limits given the enforced physical constraints. Regarding tracking accuracy, as reflected by the ISE and IAE metrics, a significant degradation is observed, indicating that the system takes longer to correct tracking errors under kinematic limitations. For ITAE, joint 1 shows improvement (56.4%), suggesting effective early-stage error reduction. However, joint 2 exhibits deterioration (153.8%), indicating a slower response in the later stages of convergence.

**Table 10.** Performance metrics in case IV and comparison with cases II and III.

| Parameters   | $\theta_1$ | $\theta_2$ | $\theta_1$ : IV vs. II | $\theta_2$ : IV vs. III |
|--------------|------------|------------|------------------------|-------------------------|
| $T_{i\_FTS}$ | 1.43 s     | 1.44       | ↑ 36.4%                | ↓ 33.4%                 |
| ISE          | 0.0308     | 0.0462     | ↑ 51.7%                | ↓ 13.0%                 |
| IAE          | 0.1539     | 0.2072     | ↑ 144.4%               | ↑ 154.5%                |
| ITAE         | 0.0667     | 0.0901     | ↓ 50.2%                | ↑ 824.0%                |
| RMSE         | 0.0449     | 0.0565     | ↑ 258.7%               | ↑ 35.1%                 |

In contrast, when case IV is compared with case III, the stabilization time is notably shorter, making case 4 faster in terms of convergence. This is because the kinematic constraints in case IV do not limit the control effort, unlike case III, where the FTC component is capped at 6 N.m. In terms of accuracy (ISE, IAE), case III outperforms case IV significantly, due to its ability to utilize higher control authority, which enables faster and more precise error correction. Regarding stability metrics (ITAE and RMSE), case III demonstrates superior performance across all indicators, showing smoother and more efficient convergence.

Figure 20a,b present the workspace that the robot arm's end-effector can reach and move within during task execution. The control signals remain bounded within the imposed limits throughout the simulation. This demonstrates that the controller successfully enforces actuator constraints while maintaining effective control performance and system stability.



**Figure 20.** Total control inputs demonstrating constraint compliance. (a) Total control input  $u_1$ . (b) Total control input  $u_2$ .

Figure 21a,b show the tracking errors for both joints. The errors decrease rapidly and settle near zero within a short time interval, indicating high tracking accuracy and excellent steady-state performance.

Figure 22a illustrates the elliptical trajectory tracking, where the actual path closely follows the reference trajectory with minimal deviation. This reflects accurate coordination



between the joints and high-quality trajectory tracking. Figure 22b shows the convergence of the PSO fitness function, which decreases monotonically, confirming stable and effective optimization. Figure 23 presents the workspace of the robotic manipulator.

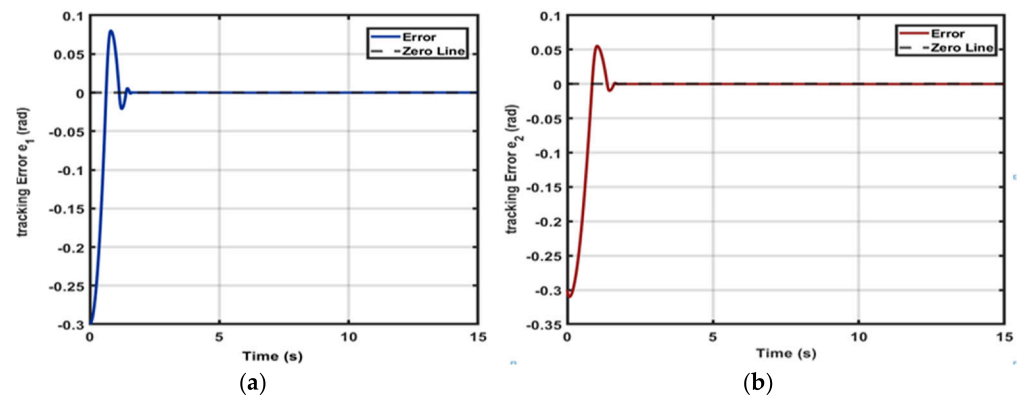


Figure 21. Tracking error convergence for both joints. (a) Tracking error  $e_1$ . (b) Tracking error  $e_2$ .

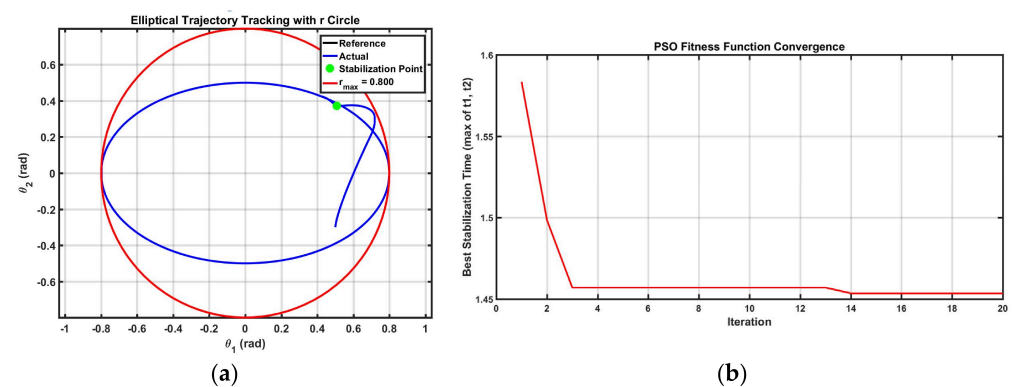


Figure 22. Elliptical trajectory tracking and PSO convergence behavior. (a) Elliptical trajectory tracking with r circle. (b) PSO fitness function convergence.

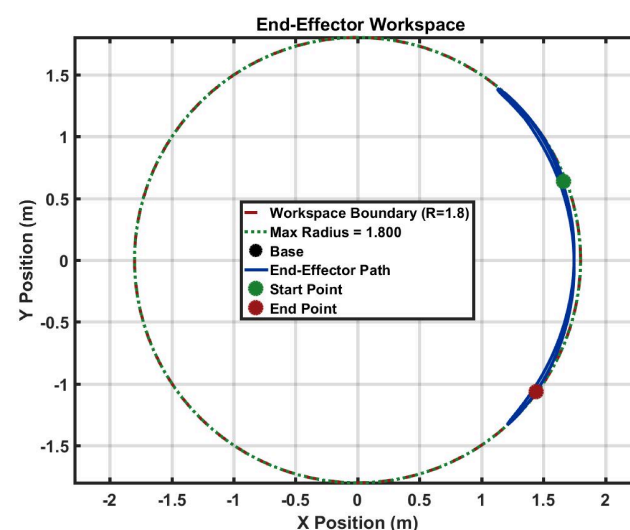


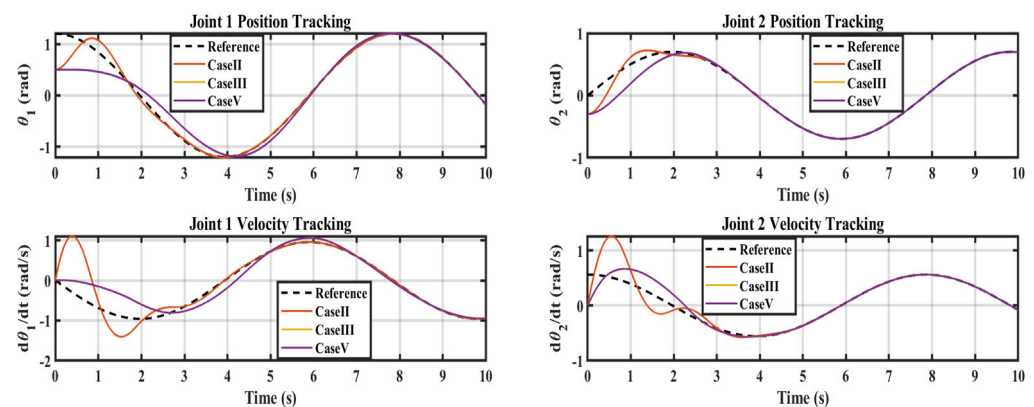
Figure 23. End-effector workspace and boundary constraints.

### 5.5. Case V: Robustness Evaluation Under Matched Disturbances and Model Uncertainties

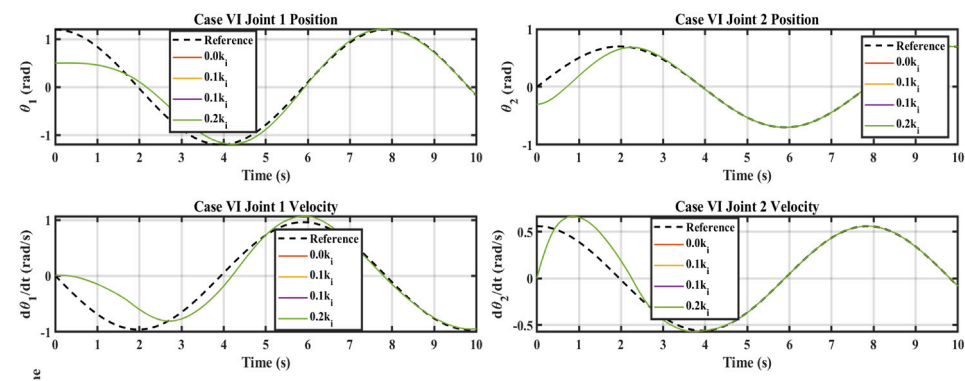
To further evaluate the robustness of the proposed hybrid ISMC-FTC controller, an additional simulation scenario was carried out under the simultaneous presence of matched bounded disturbances and model uncertainties. Unlike the previous cases, which mainly focused on improving convergence speed and satisfying physical constraints, this case

investigates the controller performance under realistic operating conditions where external disturbances and plant parameter mismatches coexist.

In this study, matched disturbances were modeled as additive torque disturbances acting through the same control channel as the control input, while model uncertainties were represented by variations in the plant dynamic gain. The disturbance magnitude was selected as  $0.2k_i > \Delta_{max}$ , which satisfies the theoretical ISMC robustness condition  $k_i > \Delta_{max}$ . Compared with the previous cases, Figure 24 demonstrates that cases III and V maintain trajectory and velocity tracking performance nearly identical to the reference trajectory. In contrast, case II exhibits more pronounced transient deviations. Although the simultaneous presence of matched disturbances and model uncertainties slightly increases the RMSE values in case V, the overall tracking accuracy remains high. Figure 25 further confirms that both position and velocity responses closely track the reference trajectories across all tested disturbance levels. These findings indicate that increasing the disturbance magnitude within the theoretical robustness bound has a negligible effect on the overall controller performance, thereby validating the robustness of the proposed approach.



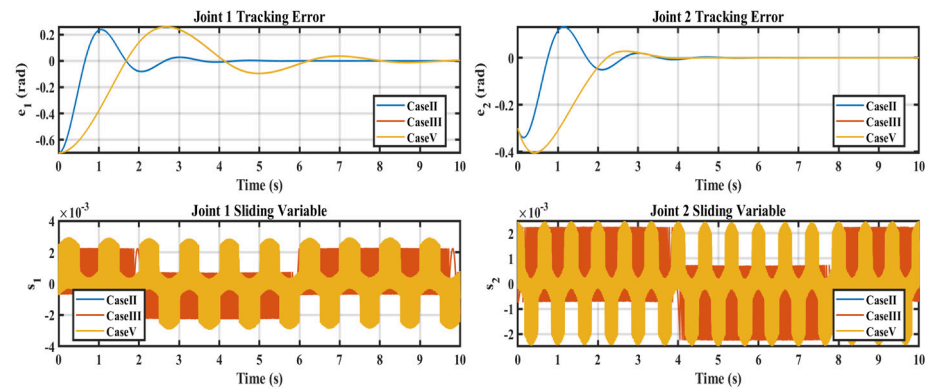
**Figure 24.** Position and velocity, under disturbances and model uncertainties.



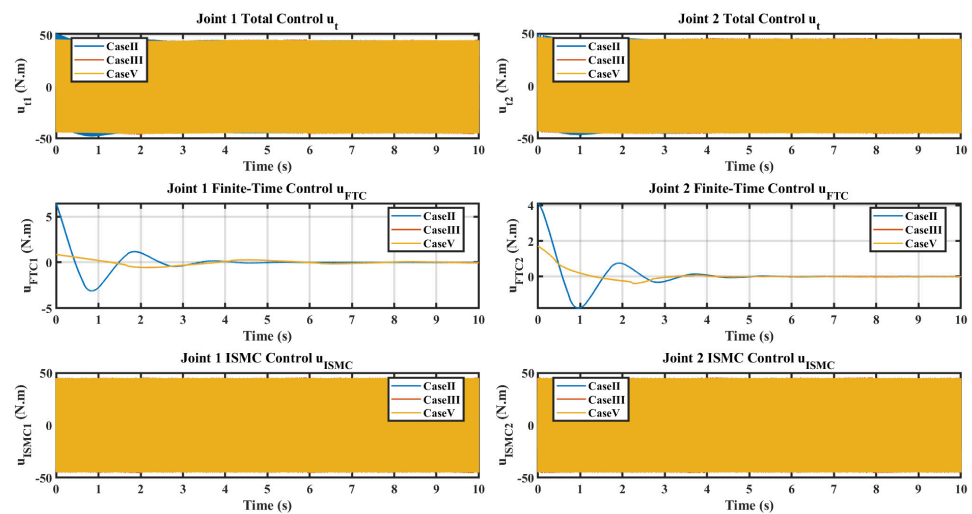
**Figure 25.** Tracking performance and stability versus disturbance level.

Figure 26 indicate that sliding variables remain bounded within a narrow region around the sliding manifold. Furthermore, the phase portraits exhibit smooth and stable trajectories converging toward the equilibrium point, confirming the robust stability of the proposed hybrid ISMC–FTC controller even in the presence of disturbances and model uncertainties.

Figure 27 shows that the total control input remains bounded for all cases despite the presence of matched disturbances and model uncertainties. Moreover, the finite-time control component rapidly decays to zero after the transient response, while the ISMC component continues compensating for disturbances, demonstrating efficient control effort and preserving robust closed-loop stability.

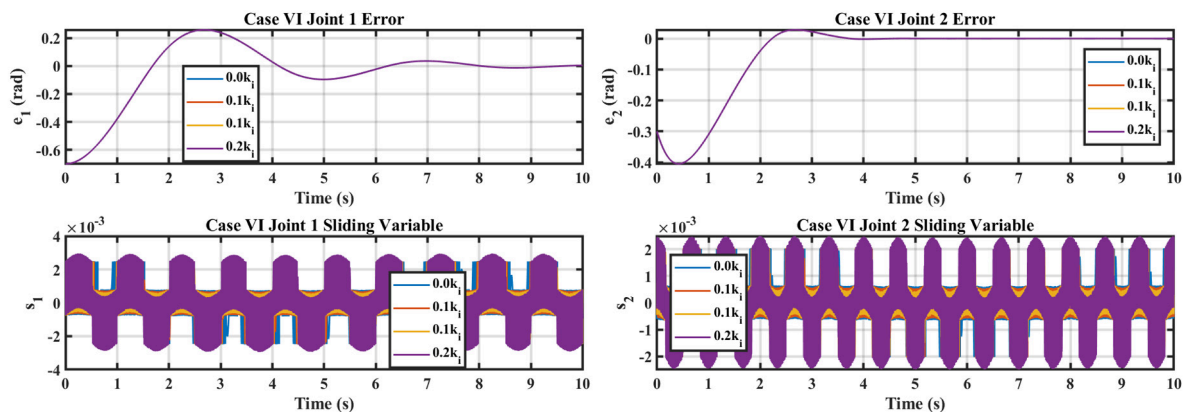


**Figure 26.** Tracking errors, sliding variables and phase portraits under disturbances and model uncertainties.



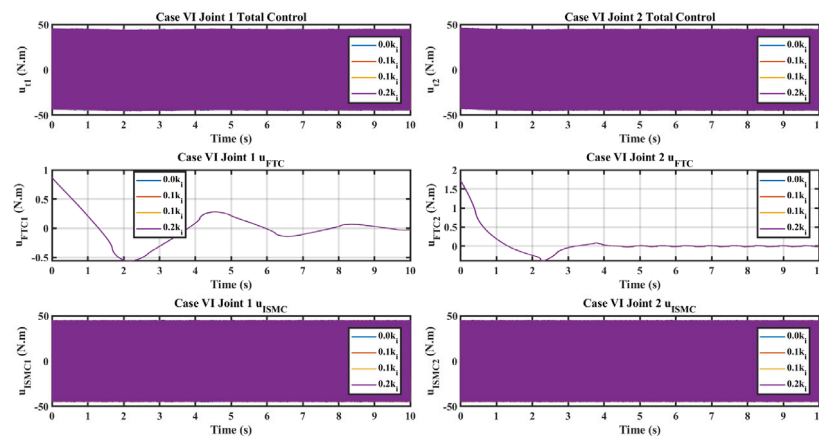
**Figure 27.** Total control input under disturbances and model uncertainties.

Increasing the disturbance level up to  $0.2 k_i$  produces only minor variations in the tracking error and sliding variables, as can be seen in Figure 28. These results verify that the sliding motion is preserved as long as the ISMC robustness condition  $k_i > \Delta_{max}$  is satisfied.



**Figure 28.** Effect of disturbance magnitude on sliding motion stability.

The total control input remains bounded for all disturbance levels, as can be seen in Figure 29, whereas the finite-time control component rapidly diminishes after the transient stage. This demonstrates that the ISMC component effectively compensates for increasing disturbances while preserving stable closed-loop performance.



**Figure 29.** Control components under different disturbance levels.

Tables 11 and 12 collectively demonstrate that increasing the matched disturbance level from  $0.02 k_i$  to  $0.2 k_i$  has only a negligible effect on the overall performance of the proposed hybrid ISMC–FTC controller. Although the robustness margin gradually decreases with increasing disturbance magnitude, it remains positive for both joints, ensuring that the theoretical robustness condition  $k_i > \Delta_{\max}$  is satisfied throughout all tested cases. Furthermore, the practical stabilization time, tracking accuracy, error indices, sliding variables, maximum control input, and control energy exhibit only minor variations, confirming that the proposed controller maintains stable, accurate, and robust performance even under increasing bounded matched disturbances.

**Table 11.** Stabilization time and tracking accuracy.

| $\Delta_{\max}/k_i$ | $T_s^{J1}$ (s) | $T_s^{J2}$ (s) | RMSE <sup>J1</sup> (rad) | RMSE <sup>J2</sup> (rad) |
|---------------------|----------------|----------------|--------------------------|--------------------------|
| 0.02                | 7.31           | 2.54           | 0.217545                 | 0.131942                 |
| 0.05                | 7.32           | 2.54           | 0.217574                 | 0.131942                 |
| 0.10                | 7.32           | 2.54           | 0.217579                 | 0.131940                 |
| 0.20                | 7.31           | 2.55           | 0.217577                 | 0.131942                 |

**Table 12.** Error performance for both joints under different disturbance levels.

| $\Delta_{\max}/k_i$ | ISE <sup>J1</sup> | ISE <sup>J2</sup> | IAE <sup>J1</sup> | IAE <sup>J2</sup> | ITAE <sup>J1</sup> | ITAE <sup>J2</sup> |
|---------------------|-------------------|-------------------|-------------------|-------------------|--------------------|--------------------|
| 0.02                | 0.473237          | 0.174084          | 1.290930          | 0.569992          | 2.596168           | 0.498623           |
| 0.05                | 0.473364          | 0.174084          | 1.291129          | 0.570019          | 2.596114           | 0.498813           |
| 0.10                | 0.473387          | 0.174080          | 1.291216          | 0.570036          | 2.596538           | 0.498974           |
| 0.20                | 0.473379          | 0.174084          | 1.291123          | 0.570083          | 2.595835           | 0.499290           |

One can investigate other control techniques like adaptive control [66], back-stepping control [67,68], fuzzy logic control [69–72], artificial neural network-based control [73,74], observer-based control [75] and PID-based control [76] for comparing the tracking performances when applied for robotic manipulator. This will be the future extension of this study.

## 6. Conclusions

This study presented a hybrid ISMC–FTC framework optimized by PSO for a 2-DOF robotic manipulator to achieve finite-time stabilization and accurate elliptical trajectory tracking under disturbances and uncertainties. The proposed approach combines the robustness of ISMC with the fast convergence capability of FTC, while PSO effectively optimizes the controller parameters to improve dynamic performance. Robustness validation

under simultaneous matched disturbances and model uncertainties (case V) confirms that the controller maintains stable performance and satisfies the condition  $k_i > \Delta_{\max}$ . The integration of PSO with the ISMC–FTC structure has proven to be highly effective, demonstrating that optimization is not merely a supplementary tool but a critical enabler for unlocking the full potential of advanced nonlinear control strategies. In the absence of PSO, the system stabilizes within the predefined time bounds (e.g.,  $T_1 \leq 5.08$  s,  $T_2 \leq 4.43$  s), but the response is suboptimal in terms of speed and tracking accuracy. In contrast, the PSO-optimized controller (case 2) achieves a fivefold reduction in stabilization time ( $T_1 = 1.05$  s,  $T_2 = 0.95$  s) and improves tracking accuracy by an order of magnitude, as evidenced by significant reductions in ISE, IAE, and ITAE.

The study further investigated the impact of practical constraints on system performance through three additional cases. Case III, which imposes limits on the FTC control effort ( $|u_{\text{in}}| \leq 6$  N·m), represents a practical and safe solution. While a trade-off in convergence speed is observed (stabilization times increase to 2.1430 s and 1.6550 s), the system maintains excellent steady-state accuracy and robustness, with all kinematic and control variables remaining within feasible bounds. Case IV extends this safety-oriented approach by enforcing strict constraints on joint angles ( $|\theta_i| \leq 1.0$  rad), joint velocities ( $|\dot{\theta}_i| \leq 1.0$  rad/s), and the end-effector workspace ( $r \leq 1.8$  m). While this case sacrifices some speed and tracking precision compared with the unconstrained optimal case (case II), it ensures that the robot operates strictly within its physical and mechanical limits, achieving a smooth and stable final response. This work highlights the powerful synergy between intelligent optimization algorithms and robust nonlinear control, paving the way for autonomous, adaptive control systems capable of meeting stringent performance and safety requirements under complex dynamic conditions.

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## Abbreviations

The following abbreviations are used in this manuscript:

|      |                                          |
|------|------------------------------------------|
| ISMC | Integral sliding mode control            |
| FTC  | Finite-time controller                   |
| FTS  | Finite-time stability                    |
| PSO  | Particle swarm optimization              |
| SMC  | Sliding mode control                     |
| 2DOF | Two-degree-of-freedom                    |
| PID  | Proportional–integral–derivative         |
| ITAE | Integral of time-weighted absolute error |
| IAE  | Integral of absolute error               |
| ISE  | Integral of squared error                |
| RMSE | Root mean square error                   |



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