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Probabilistic Retry and Threshold Multirate Loss Models for Impatient Calls

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Abstract: In this paper, a link of fixed capacity is considered that services calls from different service-classes. Calls arrive in the link according to a Poisson process, have an initial (peak) bandwidth requirement while their service time is exponentially distributed. We model this system as a multirate loss system and analyze two different multirate loss models. In the first model, named probabilistic retry loss model, if there is no available link bandwidth, a new call is blocked but retries with a lower bandwidth requirement and increased service time. To allow for the fact that a blocked call may be impatient, we assume that it retries with a probability. In the second model, named probabilistic threshold loss model, a call may reduce its bandwidth requirement (before blocking occurs) based on the occupied link bandwidth. To determine call blocking probabilities in both multirate loss models, we show that approximate but recursive formulas do exist that provide quite satisfactory results compared to simulation.

Keywords: call blocking; retry; threshold; loss; Poisson; recursive

1. Introduction

Multirate loss models (MLMs) are widely adopted in the literature for the determination of call blocking probabilities (CBP), a significant performance index in contemporary multiservice networks [1,2]. Among the MLMs, the basis is considered to be the Erlang MLM (EMLM) not only because CBP can be efficiently determined via the Kaufman-Roberts recursive formula [3,4] but also due to the fact that it has been widely adopted for the CBP computation in various types of networks (from wired [4–16], to optical [17–20], wireless [21–34] and even satellite networks [35–38]).

The EMLM is used for the analysis of a link of certain capacity (in terms of bandwidth units (b.u.)) that services calls of different service-classes with various bandwidth requirements. The call arrival process in the link is Poisson while the available b.u. are shared via the complete sharing policy. According to this policy, a new call will be accepted in the link if there exist available b.u. at the time of the call's arrival. Otherwise call blocking occurs. An accepted call remains in the link for a generally distributed service time. The fact that the steady state probabilities in the EMLM have a product form solution (PFS) is significant for the derivation of the classical Kaufman–Roberts formula, which leads to the accurate (compared to simulation) CBP computation [3,4].

In [39,40], two extensions of the EMLM are proposed. In the first extension, blocked calls may immediately retry to be accepted in the link with a reduced b.u. requirement and an increased service time. In that case, we have a retry loss model. Call blocking occurs if the reduced b.u. are not available. In the second extension, the b.u. requirement of a new call depends on the occupied link bandwidth. If the occupied link bandwidth is above a threshold (which is common for all service-classes), then the new call tries immediately to be accepted in the link with a reduced b.u. requirement and consequently an increased service time. In that case, we have a threshold loss model. Call blocking occurs if the reduced b.u. are not available. Conceptually, the main difference between the two loss models is that in a threshold loss model, a new call does not have to be blocked with its

initial bandwidth requirement in order to retry with a reduced value. It is the value of the threshold and the occupied link bandwidth that decide with which bandwidth requirement (the initial or the reduced), a new call will try to be accepted in the link. Note that in both models, calls may have elastic bandwidth requirements but fixed bandwidth allocation (the initial or the reduced bandwidth requirement) during their service time. Because of this, we classify the models of [39,40] as models that consider stream traffic. This is different from the case of elastic traffic where the bandwidth of in-service calls fluctuates during their service time [2,13].

In this paper, we focus on both loss models and study the case where a call may wish to enter a system with a reduced b.u. requirement and consequently increased service time, with a predetermined probability. Such a probability expresses the fact that the increased service time may cause impatience to calls and therefore may decide not to enter a system, although the reduced b.u. are available. The steady-state probabilities in both loss models cannot be described via a PFS due to the existence of retry (in the retry loss model), the existence of the threshold (in the threshold loss model), and the incorporation of impatience (in both models). However, we propose approximate formulas for the recursive computation of the link occupancy distribution and consequently CBP whose accuracy is validated via simulation.

Note that the characteristic of impatient calls that retry has received attention in the literature under the assumption of queueing (and not loss) systems and single-rate (and not multirate) service-class calls, which require a single b.u. for their service. The interested reader may resort to [41–44] for the analysis of such queueing models.

The organization of this paper is the following: In Section 2, we review the probabilistic retry loss model of [45]. In Section 3, we propose the probabilistic threshold model and provide recursive formulas for the computation of both the link occupancy distribution and CBP. In Section 4, simulation and analytical results are provided so as to validate the accuracy of the proposed formulas. We conclude in Section 5. Finally, in Appendix A, we present a tutorial example in order to show the intermediate calculations for the CBP computation in the probabilistic threshold loss model.
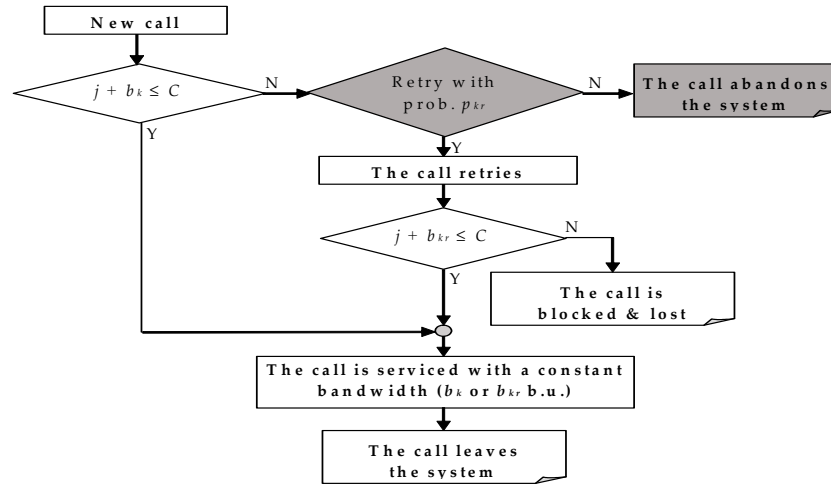
2. The Probabilistic Retry Loss Model

In the probabilistic retry loss model, a link of capacity C b.u. services calls from K service-classes. Service-class k ($k = 1, \ldots, K$) calls arrive in the link according to a Poisson process with an arrival rate λ_k and have an initial (peak) bandwidth requirement of b_k b.u. If these b_k b.u. are available, then a new service-class k call remains in the link for a service-time, which is exponentially distributed with mean μ_k^{-1}. If the available link b.u. are less than b_k, then the new call is blocked and may: (1) Abandon the link with probability $1 - p_{kr}$ or (2) immediately retry to be connected in the system with probability p_{kr}, reduced bandwidth requirement $b_{kr} < b_k$ and increased service time $\mu_{kr}^{-1} > \mu_k^{-1}$ given by $\mu_{kr}^{-1} = b_k \mu_k^{-1} / b_{kr}$ (see Figure 1, where the call admission mechanism of [39] does not include the grey parts).

The steady state probabilities in the probabilistic retry loss model cannot be determined via a PFS. However, it is proved in [45] that the computation of the link occupancy distribution, $q(j)$, with $j = 0, 1, \ldots, C$, is based on the following recursive formula:

$$q(j) = \begin{cases} 1, \text{ if } j = 0 \\ \frac{1}{j}\left(\sum\limits_{k=1}^{K} \alpha_k b_k q(j - b_k) + \sum\limits_{k=1}^{K} \alpha_{kr} p_{kr} b_{kr} \gamma_k(j) q(j - b_{kr})\right) \\ j = 1, \ldots, C \\ 0, \text{ otherwise} \end{cases} \tag{1}$$

where $\alpha_k = \lambda_k \mu_k^{-1}$ is the offered traffic-load of service-class k calls, $\alpha_{kr} = \lambda_k \mu_{kr}^{-1}$ is the retry offered traffic-load of service-class k calls accepted in the link with their reduced bandwidth

requirement (b_{kr} b.u.), p_{kr} is the retry probability of blocked calls of service-class k while $\gamma_k(j) = 1$ when $j > C - (b_k - b_{kr})$.

Figure 1. The call admission mechanism for a new call in the probabilistic retry loss model.

In order to derive (1), we assume that: (1) Local balance exists between adjacent states, which is correct in models whose steady state probabilities have a PFS and (2) retry calls do not exist in the link (i.e., the occupied link bandwidth from such calls is considered to be negligible) when the link occupancy j does not exceed the retry boundary, i.e., when $j \leq C - (b_k - b_{kr})$. The second assumption is expressed in (1) via $\gamma_k(j)$.

Having computed, via (1), the unnormalized values of $q(j)$'s, we can now calculate the CBP of service-class k calls with their reduced bandwidth requirement of b_{kr} b.u., B_{kr}, as follows:

$$B_{kr} = (1 - p_{kr}) \sum_{j=C-b_k+1}^{C-b_{kr}} G^{-1}q(j) + \sum_{j=C-b_{kr}+1}^{C} G^{-1}q(j) \tag{2}$$

where $G = \sum\limits_{j=0}^{C} q(j)$ is the normalization constant.

Similarly, we may also obtain the CBP of service-class k calls with their initial (peak) bandwidth requirement of b_k b.u., B_k, and the conditional CBP denoted as $B_{kr}^* = \text{Prob}\{B_{kr}|j > C - b_k\}$ via (3) and (4), respectively:

$$B_k = \sum_{j=C-b_k+1}^{C} G^{-1}q(j) \tag{3}$$

$$B_{kr}^* = \text{Prob}\{B_{kr}|j > C - b_k\} = \frac{B_{kr}}{B_k} \tag{4}$$

Additionally, we can determine the mean number of in-service calls of service-class k, $\bar{n}_k$, accepted in the link with b_k b.u. via the formula:

$$\bar{n}_k = \sum_{j=1}^{C} y_k(j) \frac{q(j)}{G} \tag{5}$$

where $y_k(j)$ refers to the mean number of service-class k calls with b_k b.u. in state j.

The latter can be determined via the formula:

$$y_k(j) = \frac{\alpha_k q(j - b_k)}{q(j)} \tag{6}$$

Similarly, we can compute the mean number of in-service calls of service-class k, $\overline{n}_{kr}$, accepted in the link with b_{kr} b.u. via the formula:

$$\overline{n}_{kr} = \sum_{j=1}^{C} y_{kr}(j)\frac{q(j)}{G} \tag{7}$$

where $y_{kr}(j)$ refers to the mean number of service-class k calls with b_{kr} b.u. in state j.

The latter can be determined via the formula:

$$y_{kr}(j) = \frac{\alpha_{kr}p_{kr}\gamma_k(j)q(j-b_{kr})}{q(j)} \tag{8}$$

The percentage of calls accepted in the system with b_{kr} b.u. can be obtained as $(\overline{n}_{kr}/(\overline{n}_k + \overline{n}_{kr})) \times 100\%$.

3. The Proposed Probabilistic Threshold Loss Model

In the proposed probabilistic threshold loss model, a link of capacity C b.u. services calls from K service-classes. Service-class k ($k = 1, \ldots, K$) calls arrive in the link according to a Poisson process with an arrival rate λ_k and have an initial (peak) bandwidth requirement of b_k b.u. and a reduced bandwidth requirement of b_{kc} b.u. Contrary to the probabilistic retry loss model, in this model, it is the value of j (the occupied bandwidth of the link) together with the value of the threshold $J_0 \leq C - b_k$ that are closely related to the bandwidth requirement (initial or reduced) a new call will use in order to enter the system. More specifically, the call admission mechanism applied in the case of a new service-class k call is the following: (a) Assuming that $j \leq J_0$ then the call will be accepted in the link with b_k b.u. and will be serviced for an exponentially distributed service time with mean μ_k^{-1}, and (b) assuming that $j > J_0$ then the call will: (b1) abandon the link with probability 1-p_{kc} or (b2) try to be connected in the system with probability p_{kc}, reduced bandwidth requirement $b_{kc}<b_k$ and increased service time $\mu_{kc}^{-1} > \mu_k^{-1}$ computed as $\mu_{kc}^{-1} = b_k\mu_k^{-1}/b_{kc}$. Needless to say, if the b_{kc} b.u. are not available then the call will be blocked (see Figure 2, where the call admission mechanism of [38] does not include the grey parts).

To determine the unnormalized values of $q(j)$'s, we initially consider the case of three service-classes whose calls are serviced by a link of capacity C and have the following call arrival parameters $(\lambda_1, \lambda_2, \lambda_3)$ and service parameters $(b_1, b_2, b_3), (\mu_1, \mu_2, \mu_3)$. Assuming that $j > J_0$, then calls of the third service-class have the choice to enter the system with probability p_{3c}, reduced bandwidth requirement $b_{3c} < b_3$, and increased service time $\mu_{3c}^{-1} > \mu_3^{-1}$ computed as $\mu_{3c}^{-1} = b_3\mu_3^{-1}/b_{3c}$. Contrary to the calls of the third service-class, calls of the first and second service-classes do not have the option to enter the system with a reduced bandwidth requirement.
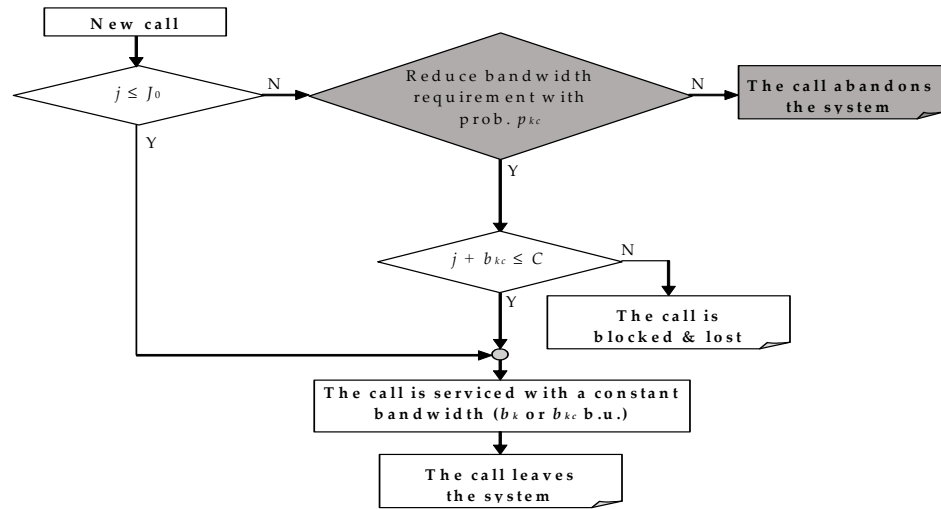
Figure 2. The call admission mechanism for a new call in the probabilistic threshold loss model.

The derivation of a recursive formula for the computation of $q(j)$'s, requires the assumption (approximation) that local balance exists between the adjacent states: (a) $j - b_k$ and j for $k = 1, 2, 3$ and (b) $j - b_{kc}$ and j for $k = 3$. More precisely, we consider the following local balance equations for calls of the first two service-classes and $j = 1, \ldots, C$:

$$\lambda_1 b_1 q(j - b_1) = y_1(j) b_1 \mu_1 q(j) \tag{9}$$

$$\lambda_2 b_2 q(j - b_2) = y_2(j) b_2 \mu_2 q(j) \tag{10}$$

where $y_1(j)$ and $y_2(j)$ refer to the average number of first and second service-class calls, respectively, that exist in the system in state j.

Regarding the local balance equations for calls of the third service-class, we have the following two equations for calls accepted with their initial bandwidth requirement and calls accepted with their reduced bandwidth requirement:

$$\lambda_3 b_3 q(j - b_3) = y_3(j) b_3 \mu_3 q(j), \text{ for } j - b_3 \le J_0 \tag{11}$$

$$\lambda_3 p_{3c} b_{3c} q(j - b_{3c}) = y_{3c}(j) b_{3c} \mu_{3c} q(j), \text{ for } j - b_{3c} > J_0, \tag{12}$$

where $y_3(j)$ refers to the average number of the third service-class calls with b_3 b.u. that exist in state j while $y_{3c}(j)$ refers to the average number of the third service-class calls with b_{3c} b.u. that exist in state j.

Based on (9)–(12) and since $\alpha_k = \lambda_k \mu_k^{-1}$ for $k = 1, 2, 3$ and $\alpha_{kc} = \lambda_k \mu_{kc}^{-1}$ for $k = 3$ are the corresponding offered traffic-loads, we can write the following equations:

$$\alpha_1 b_1 q(j - b_1) + \alpha_2 b_2 q(j - b_2) + \alpha_3 b_3 q(j - b_3) = (y_1(j) b_1 + y_2(j) b_2 + y_3(j) b_3) q(j), \text{for } j \le J_0 + b_{3c} \tag{13}$$

$$\alpha_1 b_1 q(j - b_1) + \alpha_2 b_2 q(j - b_2) + \alpha_3 b_3 q(j - b_3) + \alpha_{3c} p_{3c} b_{3c} q(j - b_{3c}) = jq(j), \text{for } J_0 + b_{3c} < j \le J_0 + b_3 \tag{14}$$

$$\alpha_1 b_1 q(j - b_1) + \alpha_2 b_2 q(j - b_2) + \alpha_{3c} p_{3c} b_{3c} q(j - b_{3c}) = (y_1(j) b_1 + y_2(j) b_2 + y_{3c}(j) b_{3c}) q(j), \text{for } J_0 + b_3 < j \le C. \tag{15}$$

In order to combine (13)–(15) into a single recursive formula for the computation of $q(j)$'s, two necessary approximations are adopted: (1) in (13), the value of $y_{3c}(j) = 0$ when $j \le J_0 + b_{3c}$ and (2) in (15), the value of $y_3(j) = 0$ when $J_0 + b_3 < j \le C$. Based on

these approximations we can write the following recursive formula, assuming that $z_k(j)$ expresses the second approximation and $z_{3c}(j)$ expresses the first approximation:

$$q(j) = \begin{cases} 1, \text{ if } j = 0 \\ \frac{1}{j}\left(\sum\limits_{k=1}^{3} \alpha_k b_k z_k(j)q(j - b_k) + \alpha_{3c}p_{3c}b_{3c}z_{3c}(j)q(j - b_{3c})\right), \\ j = 1, \ldots, C \\ 0, \text{ otherwise} \end{cases} \quad (16)$$

where $z_k(j) = 1$, if $b_{kc} > 0$ and $j \leq J_0 + b_k$, or if $b_{kc} = 0$ and $j \leq C$ while $z_{3c}(j) = 1$ when $J_0 + b_{3c} < j$.

Assuming the general case where a link accommodates calls of K service-classes, Equation (16) takes the following form:

$$q(j) = \begin{cases} 1, \text{ if } j = 0 \\ \frac{1}{j}\left(\sum\limits_{k=1}^{K} \alpha_k b_k z_k(j)q(j - b_k) + \sum\limits_{k=1}^{K} \alpha_{kc}p_{kc}b_{kc}z_{kc}(j)q(j - b_{kc})\right), \\ j = 1, \ldots, C \\ 0, \text{ otherwise} \end{cases} \quad (17)$$

where $z_k(j) = 1$, if $b_{kc} > 0$ and $j \leq J_0 + b_k$, or if $b_{kc} = 0$ and $j \leq C$ while $z_{kc}(j) = 1$ when $J_0 + b_{kc} < j$.

Having computed, via (17), the unnormalized values of $q(j)$'s in the probabilistic threshold loss model, we can now calculate the CBP of service-class k calls with their reduced bandwidth requirement of b_{kc} b.u., B_{kc}, as follows:

$$B_{kc} = (1 - p_{kc})\sum\limits_{j=J_0+1}^{C-b_{kc}} G^{-1}q(j) + \sum\limits_{j=C-b_{kc}+1}^{C} G^{-1}q(j), \quad (18)$$

where $G = \sum\limits_{j=0}^{C} q(j)$ is the normalization constant.

Similarly, we may also obtain the conditional CBP denoted as $B_{kc}^* = \text{Prob}\{B_{kc}|j > J_0\}$ via:

$$B_{kc}^* = \text{Prob}\{B_{kc}|j > J_0\} = \frac{B_{kc}}{\sum\limits_{j=J_0+1}^{C} G^{-1}q(j)} \quad (19)$$

The probabilistic threshold loss model provides the same CBP with those obtained via the probabilistic retry loss model if we set $J_0 = C - b_k$ and $p_{kc} = p_{kr}$.

Apart from the CBP, we can also determine the mean number of in-service calls of service-class k, $\overline{n}_k$, accepted in the link with b_k b.u. via (5) where $y_k(j)$ can be computed via:

$$y_k(j) = \frac{\alpha_k z_k(j)q(j - b_k)}{q(j)}. \quad (20)$$

Similarly, we can compute the mean number of in-service calls of service-class k, $\overline{n}_{kc}$, accepted in the link with b_{kc} b.u. via the formula:

$$\overline{n}_{kc} = \sum\limits_{j=1}^{C} y_{kc}(j)\frac{q(j)}{G}, \quad (21)$$

where $y_{kc}(j)$ refers to the mean number of service-class k calls with b_{kc} b.u. in state j.

The latter can be determined via the formula:

$$y_{kc}(j) = \frac{\alpha_{kc}p_{kc}z_{kc}(j)q(j - b_{kc})}{q(j)}. \quad (22)$$

The percentage of calls accepted in the system with b_{kc} b.u. can be obtained as $(\bar{n}_{kc}/(\bar{n}_k + \bar{n}_{kc})) \cdot 100\%$.

4. Performance Evaluation

In this section, we consider an application example of a single link and present both analytical and simulation CBP results of the proposed probabilistic threshold loss model as well as the corresponding analytical and simulation results of the probabilistic retry loss model. For reference we also present the corresponding analytical CBP results of the models of [39,40] together with those obtained via the EMLM. The simulation results presented in this section are mean values of seven runs and are based on Simscript III [46]. In each run, 1 million calls are generated while the initial 2% of these generated calls is not considered in the blocking probability results, so as to account for a warm-up period. Regarding reliability ranges, they are less than two orders of magnitude and therefore are not presented in the following figures.

In our application example, two service-classes are considered whose calls are accommodated in a link of 32 b.u. Calls of the first service-class have an initial bandwidth requirement of 6 b.u. and they do not have the option to retry with a reduced bandwidth requirement. On the other hand, calls of the second service-class have an initial (peak) bandwidth requirement of 24 b.u. In the case of the probabilistic retry loss model, blocked calls of this service-class retry with probability p_{2r} and a reduced bandwidth requirement of 8 b.u. In the case of the probabilistic threshold model, calls of this service-class try to be connected in the link with probability $p_{2c} = p_{2r}$ and a reduced bandwidth requirement of 8 b.u., when $j > J_0$. The value of the threshold J_0 is set to 4 b.u. Regarding the values of p we consider the cases of: (a) $p_{2r} = p_{2c} = 0.2$ and (b) $p_{2r} = p_{2c} = 0.6$. Calls of both service-classes arrive in the link according to a Poisson process with arrival rates $\lambda_1 = 0.8$ and $\lambda_2 = 0.1$ while their service time is exponentially distributed with $\mu_1^{-1} = \mu_2^{-1} = 1.0$, and $\mu_{2r}^{-1} = \mu_{2c}^{-1} = 3.0$. Based on these values, the initial offered traffic-load values (in erl) are the following: $\alpha_1 = 0.8$, $\alpha_2 = 0.1$ and $\alpha_{2r} = \alpha_{2c} = 0.3$.

In the x-axis of Figures 3–6, we consider an increase in the value of α_1 in steps of 0.2 erl. Thus, point 1 (in the x-axis) is: $\alpha_1, \alpha_2, \alpha_{2r} = (0.8, 0.1, 0.3)$ and point 7 is: $\alpha_1, \alpha_2, \alpha_{2r} = (2.0, 0.1, 0.3)$.

Figure 3 presents the analytical together with the simulation CBP of the first service-class calls assuming the probabilistic retry loss model (p-RM in Figure 3) for $p_{2r} = 0.2$ and $p_{2r} = 0.6$ and the corresponding analytical CBP for the retry loss model of [39] (RM in Figure 3) and the EMLM. On the same hand, in Figure 4, we present the corresponding CBP results of B_{2r}. Both figures show that the analytical results obtained via the probabilistic retry loss model are quite close to the corresponding CBP simulation results. Regarding Figure 3, we observe that decreasing p_{2r} results in a CBP decrease for the first service-class calls compared to the values of [39] or even the values of the EMLM (where retries are not permitted). As far as Figure 4 is concerned, we observe that a decrease of p_{2r} significantly increases the CBP of calls of the second service-class in the case of the probabilistic retry loss model, compared to the case of the model of [39]. Regarding the EMLM, it is obvious that it cannot capture the behavior of the proposed model or the model of [39].
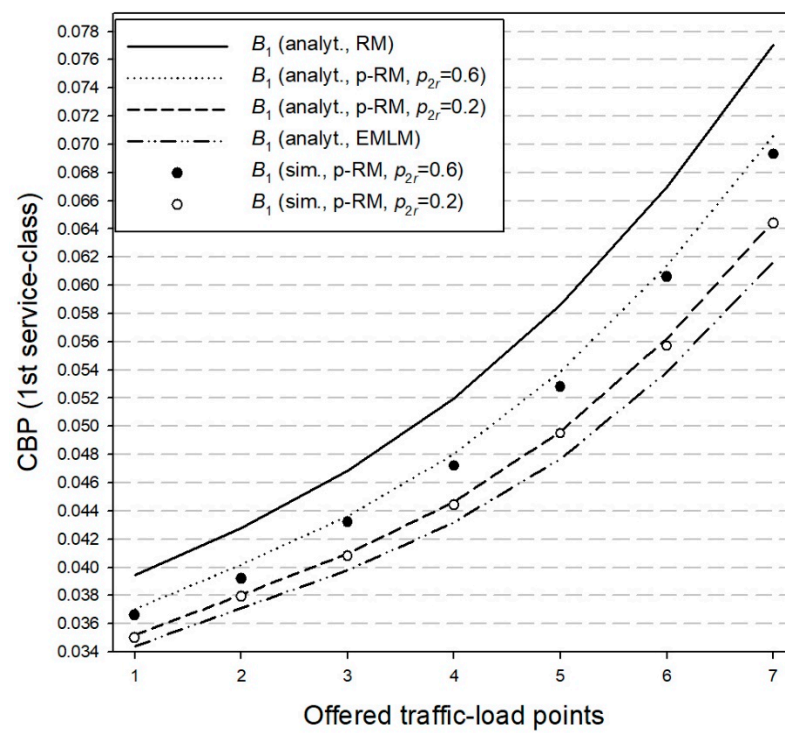
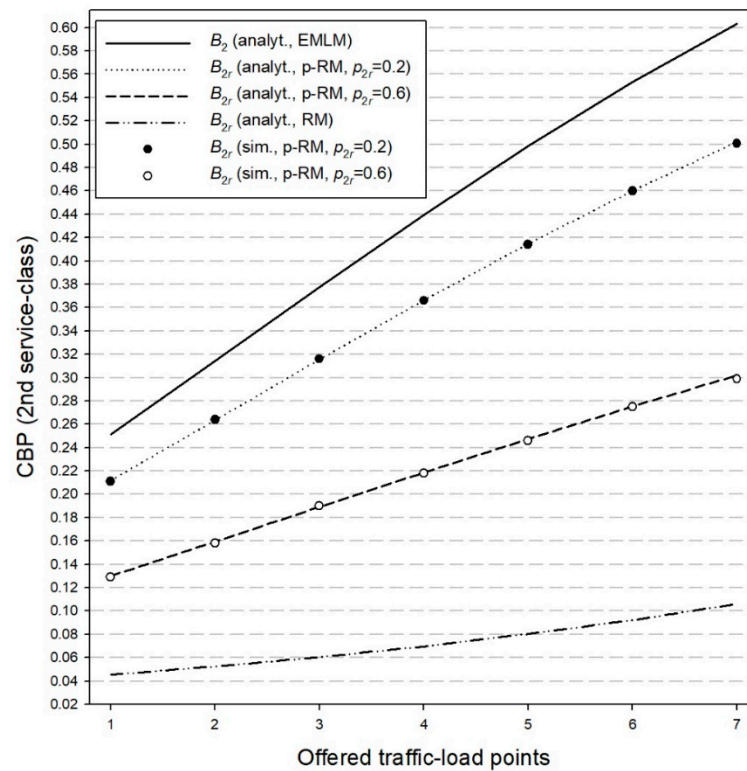**Figure 3.** CBP—First service-class (probabilistic retry loss model).

Figure 4. CBP—Second service-class (probabilistic retry loss model).
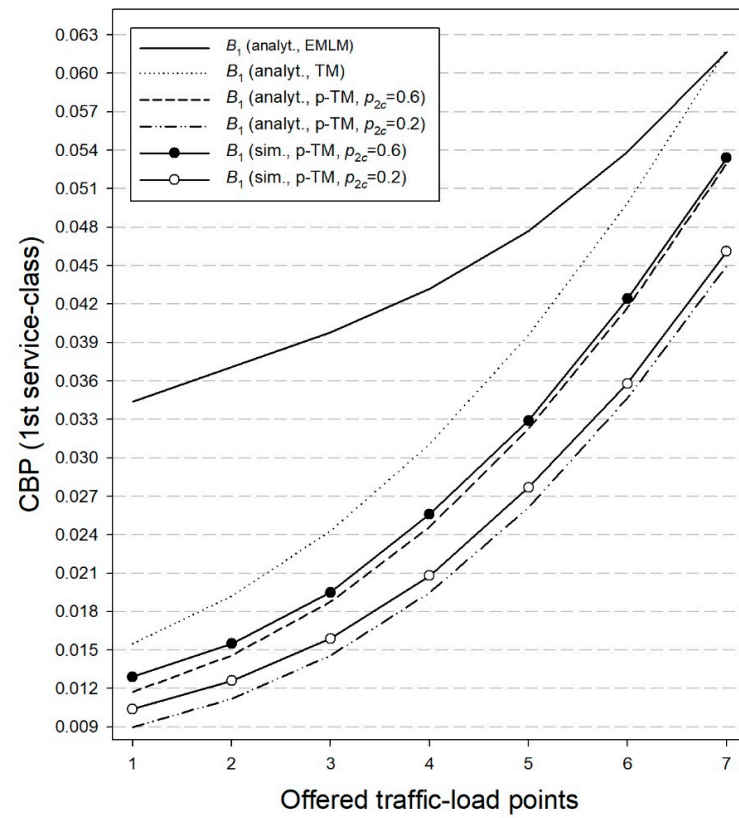
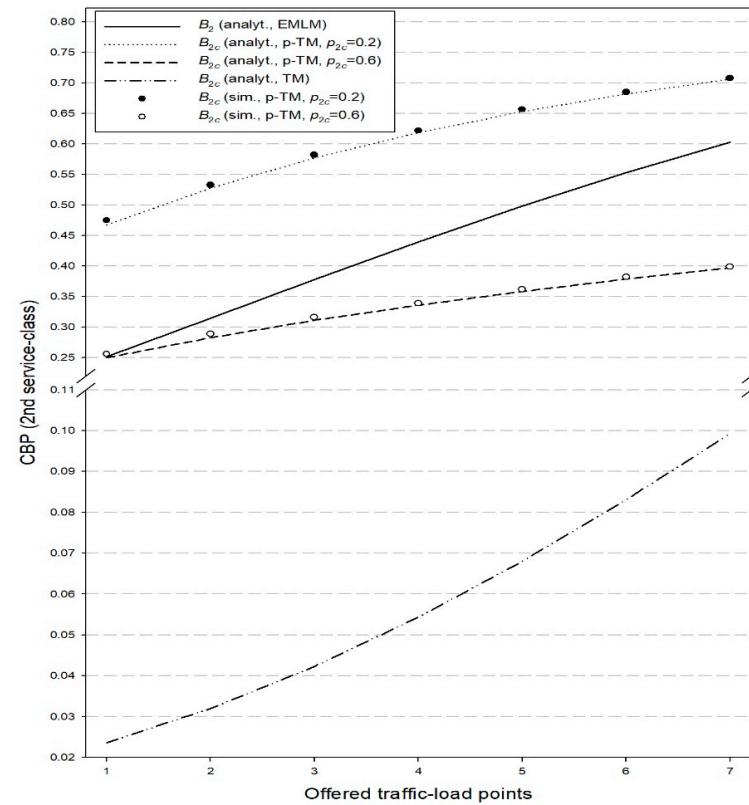**Figure 5.** CBP—First service-class (probabilistic threshold loss model).

Figure 6. CBP—Second service-class (probabilistic threshold loss model).

In Figure 5, we present the analytical together with the simulation CBP of the first service-class calls assuming the probabilistic threshold loss model (p-TM in Figure 5) for $p_{2c} = 0.2$ and $p_{2c} = 0.6$ and the corresponding analytical CBP for the threshold loss model of [40] (TM in Figure 5) and the EMLM. On the same hand, in Figure 6, we present the corresponding CBP results of B_{2c}. In both figures, we see that the simulation results and the analytical results obtained via the probabilistic threshold loss model are quite close. Regarding Figure 5, we observe that decreasing p_{2c} results in a CBP decrease for the first service-class calls compared to the values of [40] or those obtained via the EMLM. In addition, the CBP of the first service-class are lower to the corresponding CBP results obtained via the probabilistic retry loss model (compare Figure 3 with Figure 5). This can be explained by the fact that in the probabilistic threshold loss model, calls from the second service-class may decide earlier (due to the value of $J_0 = 4$) not to enter the system, a fact that increases the available b.u. for calls of the first service-class. As far as Figure 6 is concerned, we observe that a decrease of p_{2c} significantly increases the CBP of calls of the second service-class in the case of the probabilistic threshold loss model, compared to the model of [40] or the probabilistic retry loss model (compare Figure 4 with Figure 6). Regarding the EMLM, it is obvious that it cannot capture the behavior of the proposed model or the model of [40]. As a general comment for both the probabilistic retry and the probabilistic threshold loss models, note that particular attention is necessary when choosing the values of p_{kc} and p_{kr} for calls of service-class k.

5. Conclusions

We propose multirate loss models for a single link that services different service-classes whose calls follow a Poisson process. New calls have an initial and a reduced bandwidth requirement. In the probabilistic retry loss model, calls that cannot be accepted in the link with their initial bandwidth requirement are allowed to retry with their reduced bandwidth requirement at the cost of an increased service time. The latter may lead some impatient calls to decide not entering the system. To this end, it is assumed that blocked calls retry with a predetermined probability. In the probabilistic threshold loss model, new calls may reduce their bandwidth requirement (before blocking takes place) based on the occupied link bandwidth. The proposed multirate loss models do not have a PFS. However, we show that recursive formulas do exist for the computation of CBP. The accuracy of the proposed formulas is quite satisfactory compared to simulation. As a future work, we will extend these models in order to incorporate the interesting case of elastic and adaptive multirate traffic, where the bandwidth given to in-service calls is not constant during their service time [47–54].

Conflicts of Interest: The author declares no conflict of interest.

Appendix A

Calls of two service-classes are accommodated in a link whose capacity is 5 b.u. First service-class calls have a bandwidth requirement of 1 b.u. while calls of the second service-class require 3 b.u. and may use their reduced bandwidth requirement of 2 b.u., with probability $p_{2c} = 0.4$, when the occupied link bandwidth is above the threshold $J_0 = 1$. In that case, the increased service time will be $\mu_{2c}^{-1} = \frac{b_2}{b_{2c}}\mu_2^{-1} = \frac{3}{2}1.0 = 1.5$. In addition, let $\lambda_1 = \lambda_2 = \mu_1 = 1.0$.

Assuming that the in-service calls of both service-classes are denoted via n_1, n_2 and n_{2c} (the latter expresses calls of the second service-class accepted in the system with b_{2c}), then the state space of this multirate loss system consists of the following 16 states $\boldsymbol{n} = (n_1, n_2, n_{2c})$:

$$\boldsymbol{n} = \begin{pmatrix} (0,0,0), (0,0,1), (0,0,2), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,0,2), \\ (1,1,0), (2,0,0), \ (2,0,1), (2,1,0), (3,0,0), (3,0,1), (4,0,0), (5,0,0) \end{pmatrix}$$

The computation of CBP for each service-class requires the solution of a system of 16 global balance equations, where the notation $P(\boldsymbol{n})$ refers to the steady state probability of $\boldsymbol{n}$:

$$P(1,0,0) + P(0,1,0) + 2/3\,P(0,0,1) - 2P(0,0,0) = 0$$
$$P(0,1,1) + 4/3\,P(0,0,2) + P(1,0,1) - (1.4 + 2/3)\,P(0,0,1) = 0$$
$$P(1,0,2) + 0.4\,P(0,0,1) - 7/3\,P(0,0,2) = 0$$
$$P(0,0,0) + P(1,1,0) + 2/3\,P(0,1,1) - 2.4\,P(0,1,0) = 0$$
$$0.4P(0,1,0) - 5/3\,P(0,1,1) = 0$$
$$2/3\,P(1,0,1) + P(0,0,0) + 2P(2,0,0) + P(1,1,0) - 3\,P(1,0,0) = 0$$
$$4/3\,P(1,0,2) + P(0,0,1) + 2P(2,0,1) - (8/3 + 0.4)\,P(1,0,1) = 0$$
$$0.4P(1,0,1) + P(0,0,2) - 7/3\,P(1,0,2) = 0$$
$$P(1,0,0) + P(0,1,0) + 2P(2,1,0) - 3P(1,1,0) = 0$$
$$P(2,1,0) + P(1,0,0) + 3P(3,0,0) + 2/3P(2,0,1) - 3.4P(2,0,0) = 0$$
$$0.4P(2,0,0) + P(1,0,1) + 3P(3,0,1) - 11/3\,P(2,0,1) = 0$$
$$P(1,1,0) - 3P(2,1,0) = 0$$
$$2/3\,P(3,0,1) + P(2,0,0) + 4P(4,0,0) - 4.4P(3,0,0) = 0$$
$$P(2,0,1) + 0.4P(3,0,0) - 11/3\,P(3,0,1) = 0$$
$$5P(5,0,0) + P(3,0,0) - 5P(4,0,0) = 0$$
$$P(4,0,0) - 5P(5,0,0) = 0$$

The solution of this system of equations is the following:
$P(0,0,0) = 0.173022$, $P(0,0,1) = 0.044023$, $P(0,0,2) = 0.012907$, $P(0,1,0) = 0.137834$, $P(0,1,1) = 0.03308$, $P(1,0,0) = 0.178861$, $P(1,0,1) = 0.040692$, $P(1,0,2) = 0.012507$, $P(1,1,0) = 0.135727$, $P(2,0,0) = 0.100354$, $P(2,0,1) = 0.032044$, $P(2,1,0) = 0.045242$, $P(3,0,0) = 0.031912$, $P(3,0,1) = 0.012221$, $P(4,0,0) = 0.007978$, $P(5,0,0) = 0.001596$.

The normalized values of $q(j)$'s, denoted as $Q(j)$'s, are:

$$Q(0) = P(0,0,0) = 0.173022$$
$$Q(1) = P(1,0,0) = 0.178861$$
$$Q(2) = P(0,0,1) + P(2,0,0) = 0.144377$$
$$Q(3) = P(1,0,1) + P(0,1,0) + P(3,0,0) = 0.210438$$
$$Q(4) = P(0,0,2) + P(2,0,1) + P(1,1,0) + P(4,0,0) = 0.188656$$
$$Q(5) = P(0,1,1) + P(2,1,0) + P(1,0,2) + P(3,0,1) + P(5,0,0) = 0.104646$$

Based on the above, we compute B_1 and B_{2c} as follows (in the parenthesis we provide the corresponding values in the case of the probabilistic retry loss model):

$$B_1 = \sum_{j=C-b_1+1}^{C} Q(j) = Q(5) = 0.104646\ (0.148955)$$

$$B_{2c} = (1 - p_{2c})\sum_{j=J_0+1}^{C-b_{2c}} Q(j) + \sum_{j=C-b_{2c}+1}^{C} Q(j) = 0.6(Q(2) + Q(3)) + Q(4) + Q(5) = 0.50619\ (0.45411).$$

For reference only, the CBP results in the EMLM are: $B_1 = 0.09744$, $B_2 = 0.52077$.

The corresponding recursive calculations in the case of the probabilistic threshold loss model are based on (17) and are the following (assuming that $q(0) = 1$, $\alpha_1 = \alpha_2 = 1.0$ erl, $\alpha_{2c} = 1.5$ erl while $z_2(j) = 1$, for $1 \le j \le 4$ and $z_{2c}(j) = 1$, for $j > 3$).

$j = 1: \ q(1) = \alpha_1 b_1 q(1 - b_1) + 0 = 1.0 \ \Rightarrow q(1) = 1.0$
$j = 2: \ 2\,q(2) = \alpha_1 b_1 q(2 - b_1) + 0 = 1.0 \ \Rightarrow q(2) = 0.5$
$j = 3: \ 3\,q(3) = \alpha_1 b_1 q(3 - b_1) + \alpha_2 b_2 q(3 - b_2) = 3.5 \ \Rightarrow q(3) = 1.16667$

$j = 4: \ 4\,q(4) = \alpha_1 b_1 q(4 - b_1) + \alpha_2 b_2 q(4 - b_2) + \alpha_{2c} p_{2c} b_{2c} q(4 - b_{2c}) = 4.76666 \ \Rightarrow q(4) = 1.19166$
$j = 5: \ 5\,q(5) = \alpha_1 b_1 q(5 - b_1) + \alpha_{2c} p_{2c} b_{2c} q(5 - b_{2c}) = 2.59166 \ \Rightarrow q(5) = 0.51833$

and $G = 5.37666$ while:

$$Q(0) = \frac{q(0)}{G} = 0.185989, \; Q(1) = \frac{q(1)}{G} = 0.185989, \; Q(2) = \frac{q(2)}{G} = 0.092944, \; Q(3) = \frac{q(3)}{G} = 0.216987,$$
$$Q(4) = \frac{q(4)}{G} = 0.2216357, \; Q(5) = \frac{q(5)}{G} = 0.0964037.$$

Based on the previous values, we can compute the CBP of the two service-classes, as follows (in the parenthesis, we repeat the exact values obtained via the global balance equations):

$$B_1 = \sum_{j=C-b_1+1}^{C} Q(j) = Q(5) = 0.096404 \; (0.104646)$$

$$B_{2c} = (1 - p_{2c}) \sum_{j=J_0+1}^{C-b_{2c}} Q(j) + \sum_{j=C-b_{2c}+1}^{C} Q(j) = 0.6(Q(2) + Q(3)) + Q(4) + Q(5) = 0.50403 \; (0.50619).$$

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