

Article

Structural Integrity Evaluation of Cracked Plates with Different Types of Stiffeners: A Numerical Study [†]

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Abstract

Many structures use stiffeners to improve their strength and stability and especially to stop the growth of cracks that can appear during the manufacturing process or in service. The most used stiffeners have rectangular cross-sections, other shapes being less used to strengthen mechanical structures. A numerical study of cracked aluminum plates reinforced with different types of stiffeners is presented in this paper to study the influence of different types of stringers on the structural integrity of the plates. Continuously attached stiffeners with rectangular, L- and T-shaped cross-sections are considered in two variants: with the stiffener broken and unbroken. A numerical model is developed and validated by comparing the obtained results with those calculated using the compounding method. It is shown that an important variation in the stress intensity factor occurs though the thickness of the plate and that stiffeners with the same area yield approximately the same average values of the stress intensity factor. However, the shape of the stiffeners influences the maximum stress intensity factors, which are responsible for the crack growth. Conclusions are drawn about the shape that provides a longer lifetime and higher critical stresses at which catastrophic failure may occur.

Keywords: stress intensity factor; crack; finite element method; structural integrity; stiffener

1. Introduction

Aluminum stiffened panels are extensively used in the aeronautical, automotive, and marine industries, and in many other fields. They are light sheets reinforced by stringers in order to increase their strength and bending or buckling stiffness and are designed to withstand a variety of loading conditions. Usual cross-sections of stiffeners used in industry are rectangular, T-shaped, L-shaped, I-shaped, U-shaped, etc. They can be continuously or discretely attached to the plate by welding, bolting, riveting, bonding, etc.

Stiffeners improve the strength and stability of the structure and are also used to decrease or even stop the growth of cracks that can appear during the manufacturing process or in service. To avoid catastrophic failures, a knowledge of the crack size, stress field, material properties, and the parameters used to assess the integrity of structures containing cracks should be known or calculated. Such parameters are the stress intensity factor (SIF), the J-integral or the crack tip opening displacement. They can be obtained using analytical, numerical, or experimental methods.



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The mechanical behavior of cracked stiffened panels was studied by several researchers using different techniques. Among them, the finite element method (FEM) is one of the most employed. Huang et al. [1] performed three-dimensional finite element analyses to evaluate the influence of the out-of-plane bending and loading modes and the crack arrest effect of stiffeners in cracked stiffened plates under uniform tensile stress. They studied flat cracked plates with one or two symmetric stiffeners, and showed that the relative stiffness, location, integrity, and load sharing of the stiffeners play significant roles in crack arrest behavior. The effect of the crack position on the ultimate strength of aged marine stiffened panels was numerically studied by Bahmani et al. [2]. They undertook nonlinear finite element analyses for plates subjected to axial in-plane compression, having thin and thick stiffeners of the same area, and cracks in both longitudinal and transversal directions, and predicted that longitudinal cracks affect the behavior of the stiffened panels in the post-buckling region. Similar analyses were presented by Shi et al. [3], who also compared the finite element results with experiments made to investigate the influence of orientation, length, and location of the crack on the ultimate strength of the panels. Kim et al. [4] presented a finite element study to investigate how cracks affect the strength of plates and stiffened panels under uniaxial and biaxial compression. They analyzed a plate located between longitudinal stiffeners and transverse frames and a stiffened panel, located between longitudinal girders and transverse frames, and evaluated longitudinal, transverse, and bidirectional cracks, to reveal that stiffened panels are more resistant to cracking, whereas plates are more sensitive to crack orientation and loading direction. A rectangular stiffened plate, subjected to tensile, bending, and shear loads, and having a center crack, was studied with FEM by Palani et al. [5], using the numerically integrated modified virtual crack closure integral technique, to obtain SIFs for a large variety of crack lengths and positions. Liu et al. [6] investigated the crack propagation behavior of a stiffened plate under tensile and bending displacement loads, using the extended finite element method. They analyzed the influence of the size of the stiffened plates, various heights of rectangular stiffeners, and different loading types, namely uniaxial tensile and four-point bending on the residual ultimate strength of stiffened plates with a central crack. The same numerical technique was employed by Fei et al. [7] to analyze the ultimate strength of cracked stiffened panels under uniaxial tensile load, considering various numerical models of stiffened panels with different plate thicknesses, crack lengths, crack locations, and crack angles. They showed that thickness has little effect on the ultimate strength of stiffened plates with through-cracks and that the ultimate strength decreases with the increase of the angle between the crack and the load. A methodology based on the change in the magnitudes of elastic and plastic strain energies of the stiffened panel, before and after the cracks coalesce, was proposed by Labeas and Diamantakos [8], to calculate the residual strength of multiple cracked stiffened panels. For this purpose, they used non-linear elastic–plastic finite element analyses and showed that the residual strength values calculated with their methodology are in good agreement with the experimental results found in the literature. Srivastava and Arora [9] used finite element analyses to study cracked panels with stiffeners of different areas and shape, and calculated the values of a correction factor in an un-stiffened plate to account for the stiffener effect on the SIF at the tip of cracks. They also investigated the effect of different stiffeners on the fatigue life of panels with edge cracks. Božić et al. [10] calculated SIFs in welded stiffened panels made of mild steel using finite element analyses by a linear superposition of the SIF values due to the applied load and due to weld residual stresses to predict fatigue crack propagation, considering the effects of residual stresses due to the welding process. A meshless formulation for fracture analysis of stiffened plates was introduced by Peng et al. [11]. The authors verified the accuracy of the proposed method by calculating SIFs

for flat plates having different geometries of cracks and stiffeners, and by comparing the results with those obtained by finite element analyses. An analysis of the constraint effect of stiffeners on the displacement, stress and SIFs of stiffened plates was also undertaken in this research.

Other papers used the boundary element method to study the fracture behavior of stiffened plates. For example, Wen et al. [12] proposed a boundary element formulation for the analysis of a rectangular stiffened plate containing a single crack or double cracks subjected to uniform distributed moment on the crack surface and uniform shear load on the plate. The boundary element method was employed by Salgado and Aliabadi [13] to study cracked panels stiffened with discretely attached stringers. A determination of SIFs for different configurations with riveted stiffeners and cracks near the stiffener or crossing the stiffener, followed by fatigue crack growth simulations, were presented. Citarella et al. [14] presented a new methodology, based on the dual boundary element method and applied to reinforced cracked aeronautic panels to study the effect of through-the-thickness cracks, initiated from rivet holes, on the SIFs and crack propagation.

The equations of the stress and displacement fields near the crack tip were used to calculate the SIFs in cracked stiffened plates using different experimental techniques. Woloszyk et al. [15] studied the influence of corrosion and cracks on the compressive strength of stiffened plates used in ship structures by finite element analyses, and validated the numerical results by digital image correlation measurements. The problem of cracked plates stiffened by three-dimensional stringers was investigated using transmission photoelasticity by Nurse et al. [16]. They studied photoelastic models of the hole-in-the-plate geometry stiffened by a combination of stringers transverse and/or parallel to the applied tensile load and having cracks of different lengths emanating from one edge of the hole and approaching a stringer. Papadopoulos and Konsta-Gdoutos [17] used the optical method of caustics to experimentally obtain SIFs at the tip of an edge-crack or a V-notch reinforced with double bonded strips or with compression pre-stresses applied along the crack surfaces. The experimental determination of the fatigue crack growth rate in stiffened panels was presented in several papers. Yuen et al. [18] measured the residual stresses in welded stiffened center-cracked steel plates using the neutron diffraction method, and tested the plates further to constant amplitude fatigue loads to obtain and compare the fatigue crack growth rates of the stiffened plates to those of the corresponding unstiffened plates. Dong et al. [19] performed crack growth tests on steel stiffened plates with single-side cracks and central cracks under different loading conditions, to investigate the effects of stress amplitude, stress ratio, and the stiffener position on the crack growth behavior. Tests on the ultimate load-bearing capacity of stiffened panels under low-cycle fatigue and accumulative plastic deformation were undertaken by Xu and Dong [20], to obtain hysteresis curves of stress–strain relationships for stiffened panels under different crack locations and loading conditions. Feng et al. [21] conducted fatigue tests to assess the crack growth in a welded stiffened panel subjected to a cyclic load of a sine shape and having a central crack. Further, they employed a finite element model to estimate SIFs for different combinations of crack sizes on the plate and web of the stiffened panel and statistically analyzed the propagation rates in the stiffener web and plate on different confidence levels by using the one-sided tolerance method.

Analytical methods were also employed for fracture mechanics studies on cracked stiffened panels. The problems of an edge crack in a stiffened panel and of two panels riveted together, one of which contained an edge crack, were solved analytically by Shkarayev and Moyer Jr. [22] using the analytic solution for a semi-infinite plate containing an edge crack. Nishimura [23] obtained SIFs for a cracked sheet and cracked stiffeners attached with rivets using an analytical approach based on the compatibility of displacements among the sheet,

fasteners, and stiffeners. The same approach was used by Chen [24] to solve the tension problem of a finite cracked plate with stiffened edges. Deng et al. [25] adopted the crack line field method to analyze finite-width stiffened plates with a central through-crack under uniaxial tensile loading. By matching the exact solutions for the elastic and plastic stress fields along the elastic–plastic boundary, they determined the plastic stress–strain field around crack tips, and the plastic zone length for a finite-width, center-cracked stiffened plate.

Even though stiffeners are solutions used since the design phase of a structure to enhance the stiffness and to stop the growth of prospective cracks, some repair techniques are often used to extend the fatigue life of damaged elements: drilling a hole to blunt the crack tip; or bridging the cracked section by welding, bolting or bonding the steel plates, etc. [26]. In recent years, the application of composite patches became a useful tool for repairing metallic structures. Similar to stiffeners, patches are used to slow down or to arrest the cracks that could grow due to service-variable loads and are used both for aluminum and steel plates. The effect of repairs on such structures using composite patches was investigated using FEM [27–29], experimental methods [30–32] or combined experimental and numerical techniques [33].

Although many papers in the scientific literature are dedicated to the analysis of the mechanical behavior of stiffened plates, in most papers only the case of rectangular stringers is considered; there is very little research tackling the behavior of plates reinforced with other types of stiffeners. This article is a revised and expanded version of [34], preliminary research involving only some results for rectangular stiffeners, presented at a conference, but not published in any journal. This research presents the influence of different types of stringers on the structural integrity of thin aluminum plates. Continuously attached stiffeners with rectangular, L- and T-shaped cross-sections are considered. The results obtained using the FEM are presented in two variants: with the stiffener broken and unbroken. The proposed numerical model is also validated by comparing the obtained results with those calculated using the compounding method [35]. Further, the values of the critical stress, at which failure of the plate occurs, are calculated for different crack lengths.

2. The Studied Structure and the Model

A thin aluminum plate continuously stiffened with a central stringer made of the same material was studied using the FEM for a constant remote stress $\sigma = 100$ MPa. The chosen value of the remote stress is not relevant, since all results are presented in the non-dimensional form shown in Equation (1), in the hypothesis of elastic loading. The elastic constants of the material were chosen as $E = 70$ GPa, and $\nu = 0.333$. A crack symmetric with respect to the stiffener and the plate, and having a length $2a$ was considered in two cases: broken stiffener and unbroken stiffener. Continuously attached stiffeners with rectangular, L- and T-shaped cross-sections were considered. All stiffeners had the same cross-section area, namely $A = 60$ mm². The volume of the stiffener is only 2.5% from the volume of the plate, and so is the weight, since the material is the same. In this way, the addition of a stiffener does not alter the weight of the plate too much, which is important in many practical applications. The geometry of the cracked plate is shown in Figure 1, together with the system of axes used in the analyses. The geometry of the stiffeners and their orientation with respect to the surface of the plate is depicted in Figure 2.

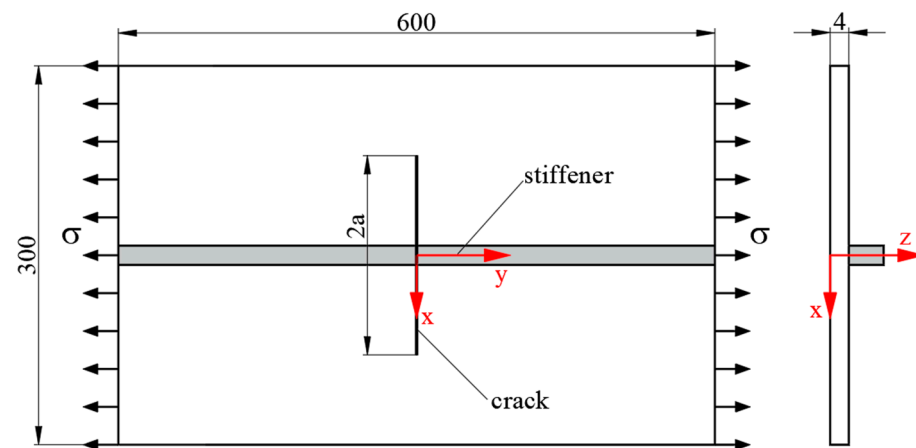


Figure 1. The studied structure.

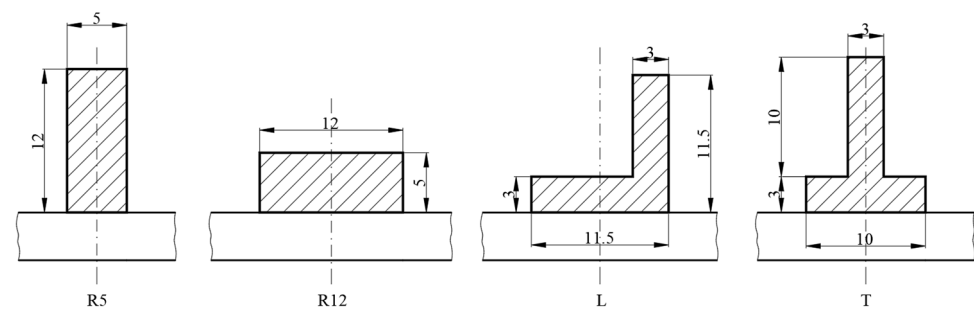


Figure 2. Geometry of the studied stiffeners and their orientation with respect to the plate (the vertical dashed line shows the center of the plate).

For each stiffener, seven different crack lengths were considered in the numerical analyses, namely $2a = 30, 60, 90, 120, 150, 180$, and 210 mm, corresponding to the ratios $2a/W = 0.1, 0.2, \dots, 0.7$, where $W = 300$ mm is the width of the plate. For each crack and stiffener, the non-dimensional mode-I stress intensity factor:

$$F = \frac{K_I}{\sigma\sqrt{\pi a}} \quad (1)$$

was determined to study the influence of the shape and position of the stiffener on the structural integrity of the plate in the case of elastic loading. In Equation (1), K_I is the mode-I stress intensity factor. It should be underlined that SIFs are expressed in non-dimensional values and they are plotted as a function of non-dimensional lengths obtained by dividing the relevant lengths by the width or thickness of the plate. In this way, the results can be extended to other lengths and applied stresses in the linear elastic range.

The critical stress, defined as the stress at which the crack reaches a critical length a_{cr} where unstable crack propagation occurs, was further calculated for each type of stiffener and crack length. The critical crack length was obtained from the condition that the SIF becomes equal to the fracture toughness of the material K_{Ic} (the SIF at which sudden crack propagation occurs, leading to the failure of the plate). In the aerospace industry, stiffened panels are mainly made of aluminum from series 2000 or 7000. The 2000 series is characterized by high strength and heat resistance, while the 7000 family of aluminum has a high mechanical strength [36]. Values of fracture toughness between 20 and $29 \text{ MPa}\cdot\text{m}^{0.5}$ are reported for the aluminum 7000 series [37]. In this paper, an average value $K_{Ic} = 25 \text{ MPa}\cdot\text{m}^{0.5}$ was considered for the calculation of the critical stress.

The three-dimensional numerical analyses were undertaken using the software Ansys [38]. Each model was meshed with tetrahedral elements, suitable for fracture me-

chanics analyses in Ansys, to obtain the SIF. All meshes contained up to 125,000 nodes and 90,000 elements. To see if the size of the crack front elements and number of nodes/elements would influence the values of the obtained stress intensity factors, a mesh convergence study was performed first for the case of the R5 broken stiffener with $2a/W = 0.2$. In this study, the mesh was progressively refined in the crack area, starting from 47,894 nodes and a size of the crack front elements of 0.203 mm, to a more refined mesh with 121,976 nodes and a size of the crack front elements of 0.0707 mm. Figure 3 depicts the results for the maximum stress intensity factor in non-dimensional form F given by Equation (1) as a function of the number of nodes. It can be seen that the variation in the results is insignificant, showing thus that the results are not sensitive to the size of the crack front elements in the above-mentioned range. A further reduction in the size of the elements does not improve the results.

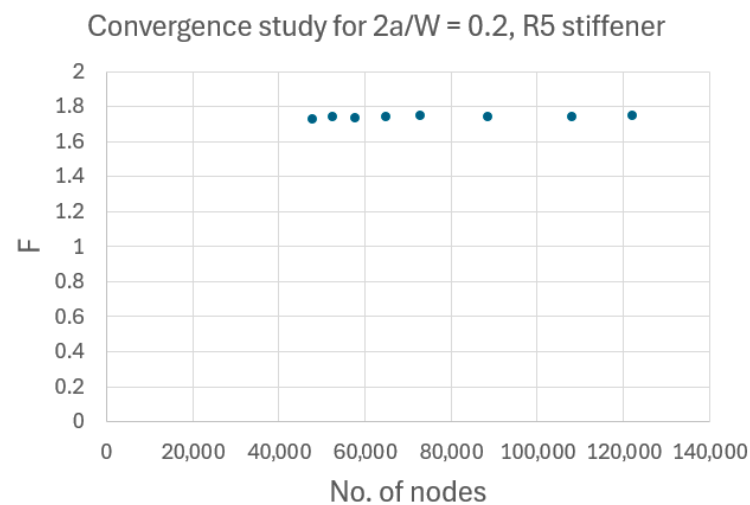


Figure 3. Results of the convergence study for R5 stiffener, $2a/W = 0.2$.

Details of the FE models for $a/W = 0.2$ are shown in Figures 4 and 5 for the T-shaped stiffener.

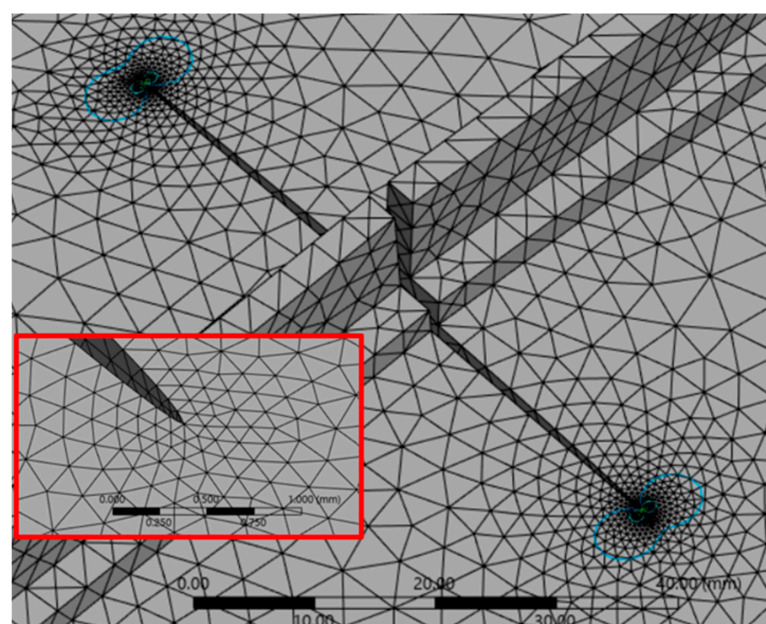


Figure 4. Mesh in the crack tips area for $2a/W = 0.2$, broken T-shaped stiffener with detail at the crack tip: 123,461 nodes, 83,319 tetrahedral elements, crack front element size 0.0776 mm.

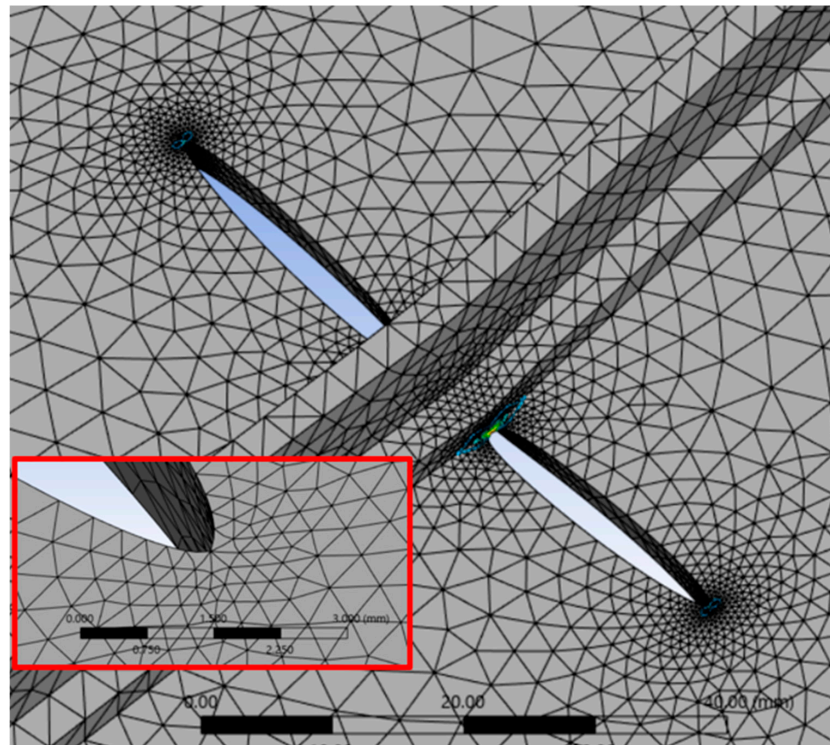


Figure 5. Mesh in the crack tips area for $2a/W = 0.2$, unbroken T-shaped stiffener with detail at the crack tip: 123,711 nodes, 83,268 elements, crack front element size 0.161 mm.

3. Validation of the Numerical Model

To validate the numerical model, SIFs were also calculated using the compounding method [35]. In this method, a complex structure containing n boundaries (sheet edges, holes, other cracks, etc.) is separated into a number of simpler ancillary configurations, each of these containing only one boundary that interacts with the crack and having known solutions.

The SIF for the complex structure is calculated as a function of the SIFs for each ancillary configuration with the equation:

$$K_{1N} = K_0 + \sum_{n=1}^N (K_n - K_0) \quad (2)$$

where K_n is the stress intensity factor for the cracked structure having only the n^{th} boundary and $K_0 = \sigma\sqrt{\pi a}$ is the SIF for the cracked structure in the absence of all boundaries (crack in an infinite plate). It is more convenient to express Equation (2) in a normalized form by dividing it by K_0 :

$$F_{1N} = 1 + \sum_{n=1}^N (F_n - 1) \quad (3)$$

where $F_{1N} = K_{1N}/K_0$ and $F_n = K_n/K_0$.

Equations (2) and (3) can be used with good accuracy if the interaction between the boundaries is negligible, which is a correct assumption in most cases, especially when straight boundaries (as sheet edges or other cracks) are present. If this interaction is not negligible, a supplementary term that takes it into account must be added in Equations (2) and (3). This term is known as the boundary-to-boundary interaction term. Rooke et al. [39] showed that this term becomes significant only in the case of very short cracks and that its evaluation is difficult, involving complex calculations in which it is expressed in terms of the residual stress fields at the boundaries by using the Schwarz alter-

nating technique. An approximate method to obtain the boundary-to-boundary interaction term was developed by Rooke [40], but only for the case of a hole with two equal-length cracks located symmetrically in a finite-width strip. In the papers where the compounding method is applied, this term is usually neglected. It will be further shown (see Table 1) that, except for the smallest crack, in all the other studied cases in this research, this term is less than 4% from the total value of the stress intensity factor.

Table 1. Comparison between FE and compounding results for R5 stiffener.

λ	Non-Dimensional SIF F					Difference [%]
	Finite Element			Compounding		
	$F_{\max}$	$F_{\min}$	F_{average}	F_{2D}	F_{3D}	
2	2.036	0.376	1.206	1.011	1.073	11.07
4	1.747	0.524	1.136	1.032	1.095	3.60
6	1.703	0.575	1.139	1.063	1.127	1.02
8	1.711	0.649	1.180	1.111	1.178	0.19
10	1.761	0.742	1.252	1.176	1.248	0.31
12	1.881	0.855	1.368	1.272	1.348	1.43
14	2.109	0.996	1.553	1.407	1.492	3.87

The ancillary configurations for the studied structure are presented in Figure 6. As is shown in [40], in the case when a crack crosses a boundary (hole or stiffener), a modification is needed in the compounding procedure because the stiffener has a significant effect on the displacements of the crack faces. It means that these displacements are different from the ones of a crack of the same length $2a$, in the absence of the stiffener. Rooke [40] showed that the effect of adding a stiffener to a sheet containing a crack of length $2a$ is assumed to be the same as the effect of adding the stiffeners to an infinite sheet containing a crack of an equivalent length $2a'$. The value of the equivalent length is obtained from the condition that such a crack in an infinite plate should produce the same stress intensity factor as the crack of length $2a$ crossing a stiffener:

$$a' = F_0^2 a \quad (4)$$

where a is the half crack length and F_0 is the non-dimensional SIF for a crack crossing the stiffener in an infinite plate. Consequently, a crack of equivalent length $2a'$ has to be introduced in the ancillary configurations instead of the crack crossing a stiffener. Another issue highlighted in [40] is that, in order to obtain the same effect for the SIF, the distance from the tip of the equivalent crack to any boundary should be equal to the one in the original problem (length b in Figure 6).

The upper and lower boundaries of the plate are far enough from the crack, and they do not influence the SIFs.

Values of F_0 were taken from [41] while SIFs at the crack tip in the ancillary configurations were obtained from [42], for all the seven considered crack lengths, in both cases (cracked and uncracked stiffener).

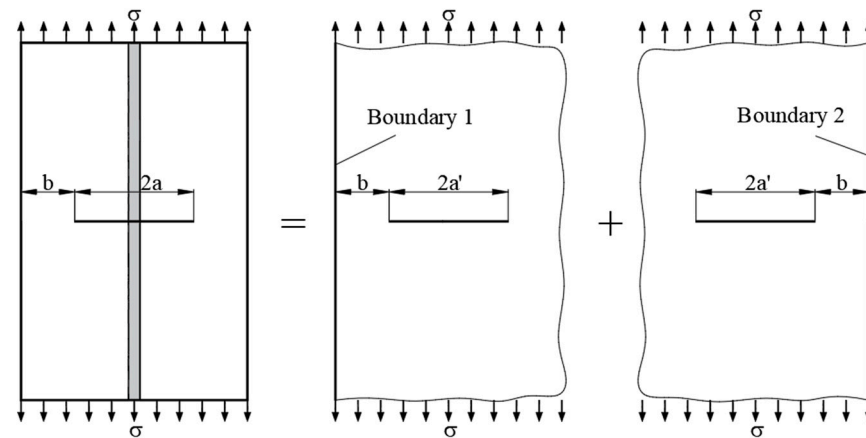


Figure 6. Ancillary configurations for the compounding method.

4. Results and Discussion

For all studied cases, the main goal of the numerical analyses was to obtain the stress intensity factor K_I as the fundamental parameter in linear elastic fracture mechanics, used to assess the structural integrity of a cracked structure. Results for the SIF are presented in the dimensionless form F shown in Equation (1) as a function of the non-dimensional parameter

$$\lambda = \frac{2E_1at}{AE_2} \quad (5)$$

where E_1 and E_2 are the Young's moduli of the plate and stiffener, $A = 60 \text{ mm}^2$ is the cross-section area of the stiffener, and $t = 4 \text{ mm}$ is the thickness of the plate.

4.1. Variation in SIF Along the Thickness

In a three-dimensional analysis, the SIF is not constant along the thickness. The presence of a stringer increases the difference between the maximum and minimum SIFs, compared to the variation for a simple cracked plate. The variation in the SIF along the thickness obtained by finite element analysis for the case $2a/W = 0.2$ ($\lambda = 4$) in the case of a broken and unbroken stiffener is presented in Figures 7 and 8, as a function of the non-dimensional parameter d/t where d is the depth, measured from the side of the plate where the stiffener is attached.

When the stiffener is broken, the maximum SIF occurs near the stiffener side, it decreases along the thickness and there is a significant variation in the SIF along the thickness, due to the hogging of the plate (out-of-plane bending with tension on the stiffener side), visible in Figure 4. Similar conclusions were drawn by Moreira et al. [43] for the case of a plate with two stiffeners. If the stiffener remains intact, the maximum SIF occurs on the face of the plate opposed to the one where the stiffener is attached (back side) due to the sagging of the plate, see Figure 5. In this case, the variation in the SIF is smaller. Also, as is expected, smaller SIFs are noticed for the unbroken stiffener, which shows that an intact stiffener increases the structural integrity of the structure. The SIFs for broken stiffeners in all studied cases become equal at the middle of the thickness. A similar trend can be noticed in the case of the unbroken stiffener, except for the R5 stiffener which shows a higher SIF along the thickness, except for the values close to the back side which are approximately equal to the ones for the R12 stiffener, but still greater than those for L and T stiffeners. For all crack lengths, a similar behavior was noticed, both for the broken and unbroken stiffener.

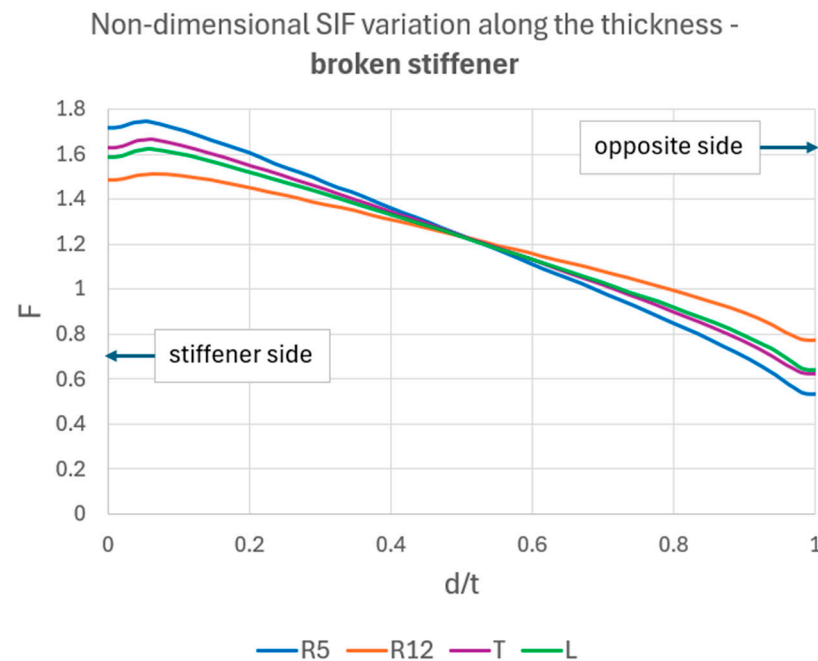


Figure 7. SIF variation for $2a/W = 0.2$, broken stiffener.

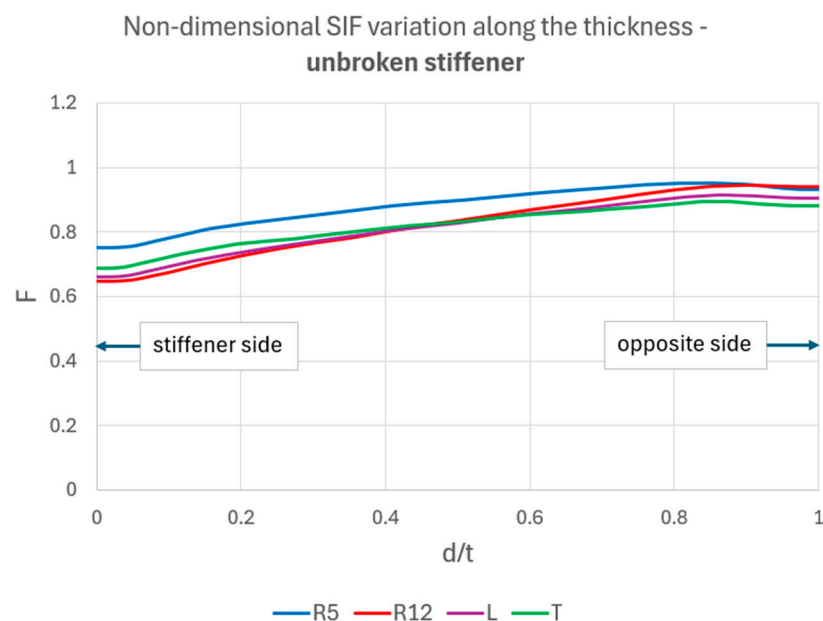


Figure 8. SIF variation for $2a/W = 0.2$, unbroken stiffener.

4.2. Validation of the Finite Element Model

The models were validated by comparing the results with those obtained with the compounding method. As was already emphasized, if a three-dimensional FE analysis is performed, the SIF varies along the thickness of the plate [43]. On the other hand, the compounding results, obtained using equations available in the literature, are two-dimensional. Kwon and Sun [44] showed that a 2D analysis underestimates the SIFs of thin plates, and introduced a simple equation to obtain approximate 3D non-dimensional SIFs from 2D solutions for thin flat plates, using the equation:

$$F_{3D} = F_{2D} \sqrt{\frac{1}{1 - \nu^2}} \quad (6)$$

where F_{3D} is the non-dimensional 3D SIF at the mid-plane of a flat plate, F_{2D} is the non-dimensional 2D SIF result, and ν is Poisson's ratio. The correction factor from Equation (6) was established presuming that, although the stress field near the crack front is three-dimensional, the deformation field very close to the crack front near the mid-plane of the plate is close to a state of plane strain. To prove this hypothesis, Kwon and Sun [44] calculated a parameter which measures the variation in the constraint factor with respect to the thickness direction, a parameter known as degree of plane strain [45]:

$$D_{pe} = \frac{\sigma_z}{\nu(\sigma_x + \sigma_y)} \quad (7)$$

where σ_z is the normal stress along the axis parallel to the thickness of the plate. The degree of plane strain is zero for a state of plane stress, and is unity where plane strain occurs. To show that Equation (6) can be used for the stiffened plates studied in this paper, the degree of plane strain was calculated close to the crack front for $2a/W = 0.3$ for the R5 stiffener used in the comparison between finite element results and the compounding values (Table 1), using the stresses obtained from the finite element analysis on a path near the crack front. Figure 9 shows the variation of D_{pe} along the thickness as a function of the non-dimensional parameter d/t . It can be seen that at the middle of the plate, a degree of around 0.82 (very close to unity) was obtained, showing thus that the deformation field is close to a state of plane strain and thus the use of this correction factor is appropriate.

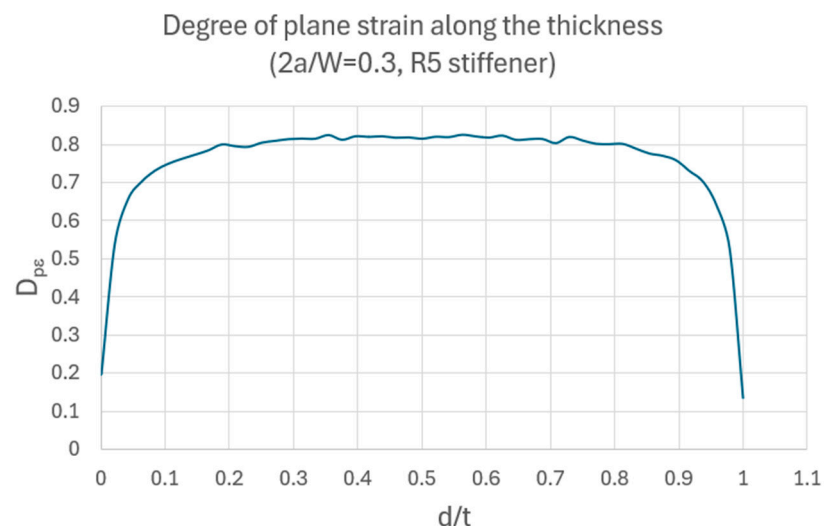


Figure 9. Degree of plane strain for R5 stiffener, $2a/W = 0.3$.

One can notice from Figure 7 that the average value of the SIF along the thickness occurs approximatively in the mid-plane of the thickness. That is why the maximum and minimum finite element results for each crack length were averaged and further compared to the compounded ones, corrected with the factor calculated with Equation (6). The comparison $F_{average}$ (finite element) versus F_{3D} (compounding) was made for the case of the R5 broken stiffener, and the results are listed in Table 1.

One can notice that, except for the shortest crack ($\lambda = 2$), the differences between the FE and compounding results are less than 4%, thus validating the proposed FE model. It should be underlined that all compounding results are smaller than the FE ones, due to the boundary-to-boundary interaction term which was neglected. The small difference between the FE and compounding results proves that indeed this term is not important, excepting the case $\lambda = 2$, where the tips of the crack are close to the stiffener, and thus the boundary-to-boundary influence term becomes significant.

4.3. Influence of the Position of the Stiffener

The SIFs for a crack crossing a stiffener in an infinite plate are presented in [41] as a function of the parameter λ , calculated with Equation (5). In this equation, the area A of the cross-section of the stiffener appears, but there is no mention about the shape or the position of the stiffener. In this paragraph, a comparison between the results obtained for the same rectangular stiffener but placed in two different positions on the surface of the plate (R5 and R12, see Figure 2) is undertaken.

The variation in the non-dimensional SIF F for both cases of rectangular stiffeners is presented in Figures 10 and 11, as a function of the parameter λ . One can notice that the position in which the stiffener is attached to the plate has an important influence both on the maximum and minimum values of the SIF. If the stiffener is broken, the maximum SIF is higher and the minimum SIF is smaller in the case R5. These results can be explained by the fact that the connection of the stiffener with the plate on a smaller length decreases the stiffness of the structure and thus increases the magnitude of the stresses near the stiffener side, compared to R12. Nevertheless, it is interesting to notice that the average SIF is practically the same and it is close to the two-dimensional results obtained by the compounding method. In the case when the stiffener is unbroken, the maximum SIF is approximately the same in both cases while the minimum SIF is smaller in the case R12. The average SIF is not the same, but the differences are not significant.

The values of the average SIF for all the studied broken stiffeners are shown in Table 2. It can be noticed that, in all cases, the values for each crack length (or parameter λ) are almost the same. This can lead to the conclusion that indeed the average SIF depends only on the area of the stiffener and not on its shape and position, as reported in [41].

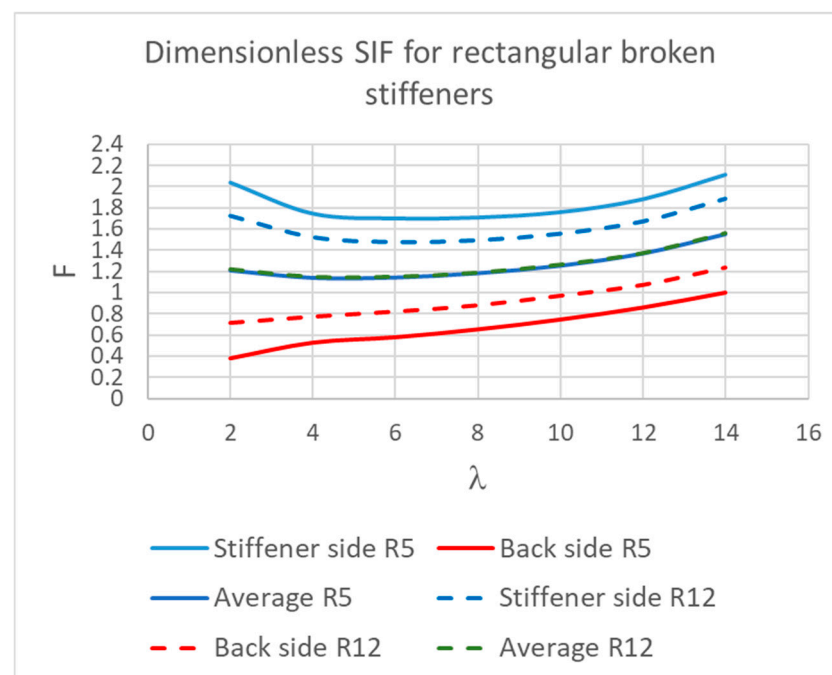


Figure 10. Variation in non-dimensional SIF for rectangular broken stiffeners.

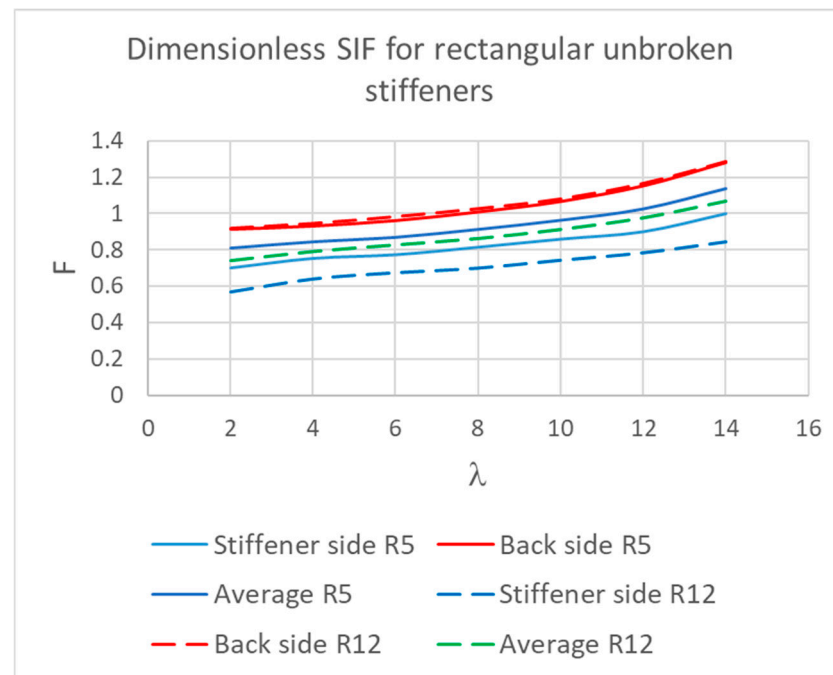


Figure 11. Variation in non-dimensional SIF for rectangular unbroken stiffeners.

Table 2. Average values of the non-dimensional SIF F for broken stiffener.

λ	F —Broken Stiffener			
	R5	R12	T	L
2	1.206	1.219	1.216	1.227
4	1.136	1.147	1.143	1.142
6	1.139	1.148	1.142	1.148
8	1.180	1.186	1.185	1.190
10	1.252	1.264	1.254	1.258
12	1.368	1.374	1.375	1.372
14	1.552	1.565	1.572	1.563

In the case of an unbroken stiffener, the variation in the SIF with the parameter λ is presented in Figure 12. One can notice here similar values, except the R5 stiffener, which exhibits values greater with about 7–8%. The conclusion about the different behavior of R5 in the case of an unbroken stiffener was also presented in the previous paragraph. This is due to the fact that the length of the contact zone of the stiffener with the plate is much smaller in the case of R5 than for all the other stiffeners, making the structure in this case less stiff.

4.4. Influence of the Stiffener Shape on the Maximum SIF

As the SIF at the crack tip is not constant along the thickness of the plate, it is of interest to study mainly the maximum values of this parameter, due to which sudden failure can occur if the crack length reaches a critical value during stable crack growth. In the case of a broken stiffener, these values occur near the stiffener side of the plate. The variation of the non-dimensional maximum SIF for all considered types of stiffeners is presented in Figure 13 for the case of a broken stiffener and in Figure 14 for an unbroken stiffener.

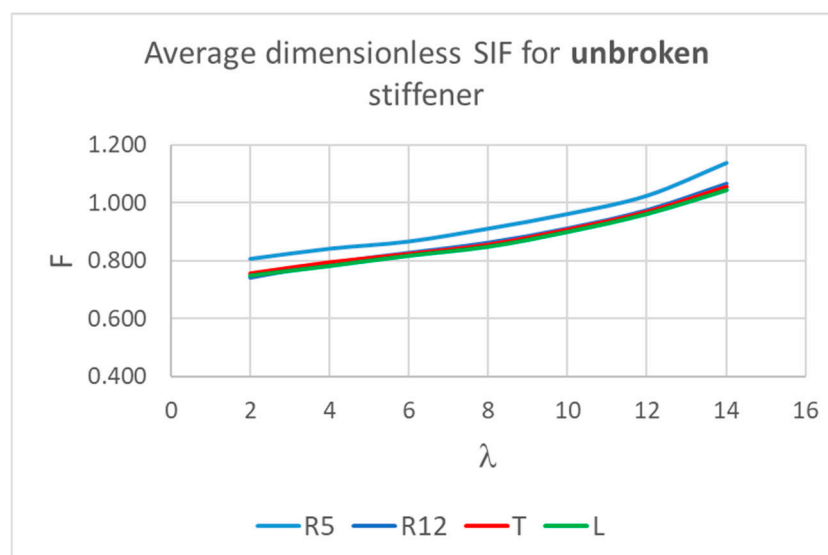


Figure 12. Average values of the non-dimensional SIF F for unbroken stiffener.

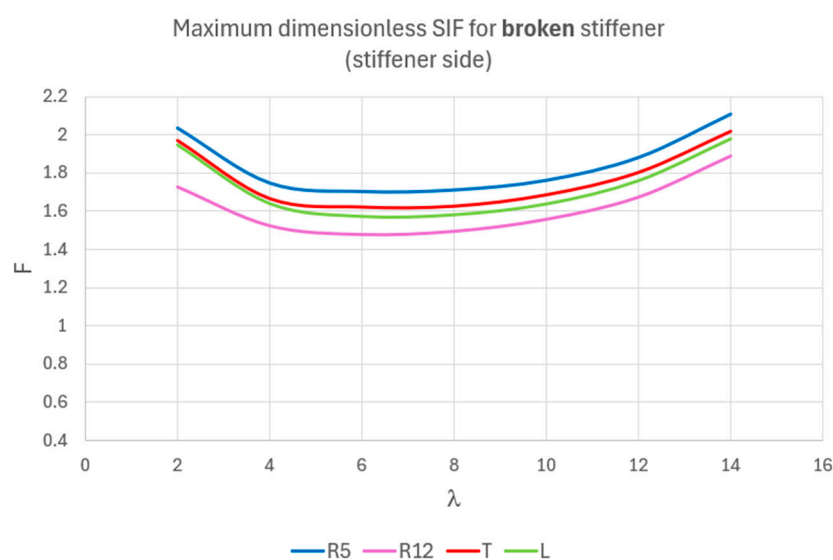


Figure 13. Maximum dimensionless SIF F for broken stiffener.

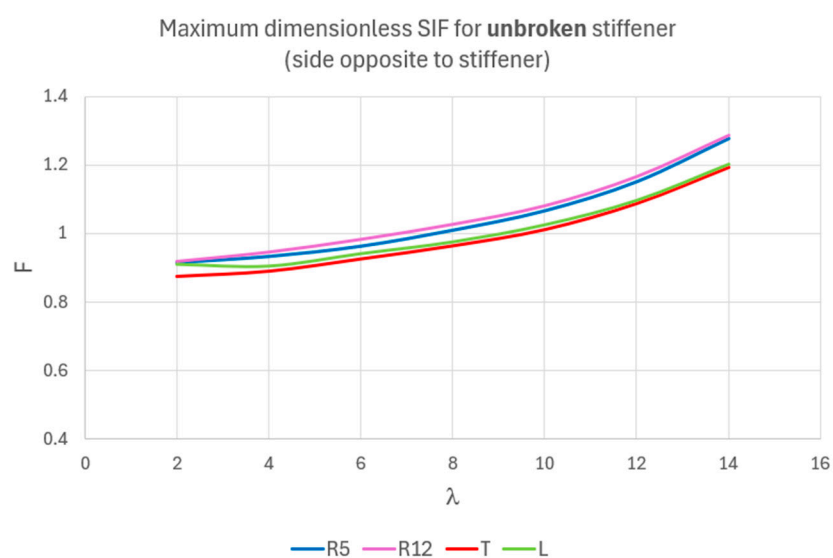


Figure 14. Maximum dimensionless SIF F for unbroken stiffener.

For a broken stiffener, one can notice similar trends of variation for all stiffeners, with values close to one another, but with the R12 stiffener showing the smallest SIFs. Consequently, for this stiffener, the safety of the structure is the highest and it can be used with greater confidence to stop or to slow the growth of cracks that could appear during the manufacturing process or in service.

In the case of an unbroken stiffener, the trend is also similar and the values for each type of stiffener are even closer, one to the other, than in the case of a broken stiffener. In this case, the T- and L-shaped stiffeners offer the best safety, especially for longer cracks, the values of the SIF for the T-shaped stiffener being slightly smaller.

4.5. Influence of the Stiffener Shape on the Crack Face Displacements

The increase in the plate stiffness due to the addition of stringers can be quantified by an analysis of the crack face displacements for each type of stiffener. Due to the bending effect seen in Figures 4 and 5, the mode-I displacements on the crack faces on a line parallel to the crack are different through the thickness. In the case of a broken stiffener, displacements near the stiffener side will be bigger than on the other side, due to the hogging effect, while for unbroken stiffeners, the sagging effect produces greater displacement on the side of the plate opposite to the stiffener. That is why the displacement plot is made for the values on the side with the maximum stress intensity factor which is on the tension side of the bent plate. In Figures 15 and 16, the crack face displacements v in the case $2a/W = 0.2$ are plotted as a function of the distance L from the left crack tip for broken and unbroken stiffeners, respectively.

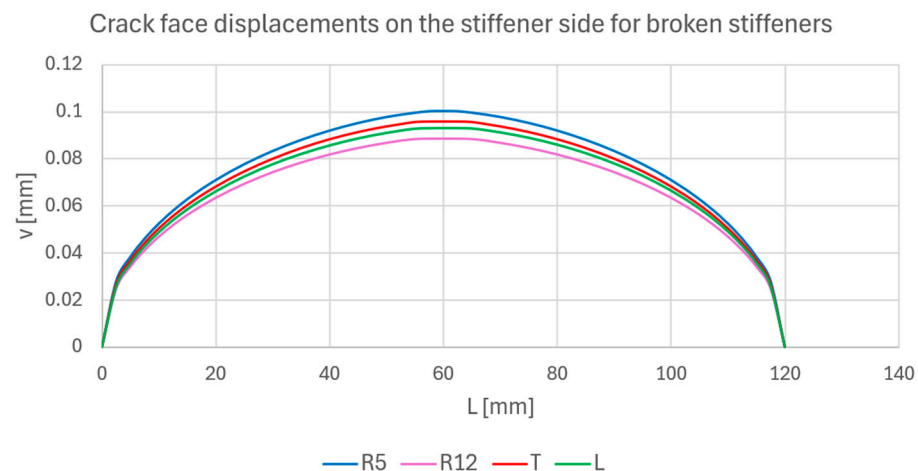


Figure 15. Crack face displacements for broken stiffeners, stiffener side, $2a/W = 0.2$.

It can be seen that crack face displacements follow the same trend and the symmetry of displacements is present for all types of broken stiffeners. Although the L-shaped stringer itself is not symmetric, crack face displacements maintain symmetry since the stringer is broken and thus its lack of symmetry has no influence on the displacements. Also, it can be noticed that the displacements follow the same trend as the maximum values of the SIF: the biggest displacements occur for the stiffener where the maximum stress intensity factor is the biggest, namely the R5 stringer, as can be seen in Figure 13. The smaller the maximum SIFs, the smaller the crack face displacements for the broken stringers. A similar trend can be observed for the unbroken stiffeners, where the maximum SIFs occur for the rectangular stiffeners, as also happens with the displacements, while the L and T stiffeners exhibit smaller values of maximum SIFs and smaller displacements too.

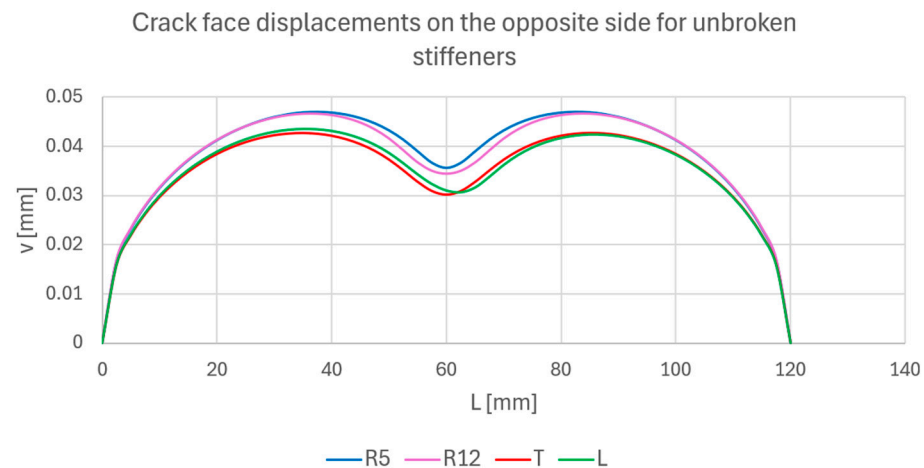


Figure 16. Crack face displacements for unbroken stiffeners, opposite to stiffener side, $2a/W = 0.2$.

In the case of the unbroken L-shaped stiffener, the deformed shape of the crack shifts from the symmetry line to the right, on the stiffener side, which is expected since the stiffener is not broken and thus its lack of symmetry is expected to influence the symmetry of the displacement field. Nevertheless, the deviation from the symmetry in the case of the non-symmetric L-shaped unbroken stiffener is very small. Consequently, the lack of symmetry of the stress field and the stress intensity factor in the case of the L-shaped stringer should also be insignificant, a conclusion that is supported by the results for the left and right crack tip in this case: for example, for the crack length given by $2a/W = 0.2$, the maximum non-dimensional stress intensity factors are $F = 1.6364$ and $F = 1.6269$ respectively (difference of 0.5%) while for the unbroken L stiffeners the values are $F = 0.9145$ and $F = 0.8881$. In this case, the difference is 2.41%, slightly bigger than in the case of the broken stiffener, which is to be expected, but again not significant, although the distance from the crack tips to the stiffener in this case is small. For longer cracks, this difference decreases.

4.6. Calculation of the Critical Stress for the Most Favorable Stiffener

It is of interest to find the values of the stress at which the length of a crack becomes critical, leading to the failure of the plate. This happens when the SIF becomes equal to the fracture toughness of the material, namely

$$F\sigma\sqrt{\pi a} = K_{Ic} \quad (8)$$

For each crack length, the value of the stress can be calculated from Equation (8), provided that the variation in F with the crack length is known. This can be achieved by plotting the non-dimensional maximum SIF values obtained by FEM as a function of λ , a parameter which includes the crack length, and finding a fit for this variation. In this paragraph, the stiffeners which provide the greatest level of safety, namely the R12 cracked stiffener, and the T-shaped uncracked stiffeners are considered to illustrate the determination of the critical stress.

The following obtained polynomial fits were obtained by curve fitting and considered in the calculations:

$$F = 5 \cdot 10^{-6}\lambda^5 + 0.0003\lambda^4 - 0.0069\lambda^3 + 0.074\lambda^2 - 0.3893\lambda + 2.2601 \quad (9)$$

with a coefficient of determination $R^2 = 1$ for the R12 broken stiffener, and

$$F = 2 \cdot 10^{-5}\lambda^4 - 0.0005\lambda^3 + 0.0057\lambda^2 - 0.0141\lambda + 0.884 \quad (10)$$

with a coefficient of determination $R^2 = 0.9997$ for the T-shaped unbroken stiffener.

The obtained results for both cases are shown in Figure 17, where the variation in the critical stress with the non-dimensional crack length $2a/W$ is plotted, for the values considered in this research.

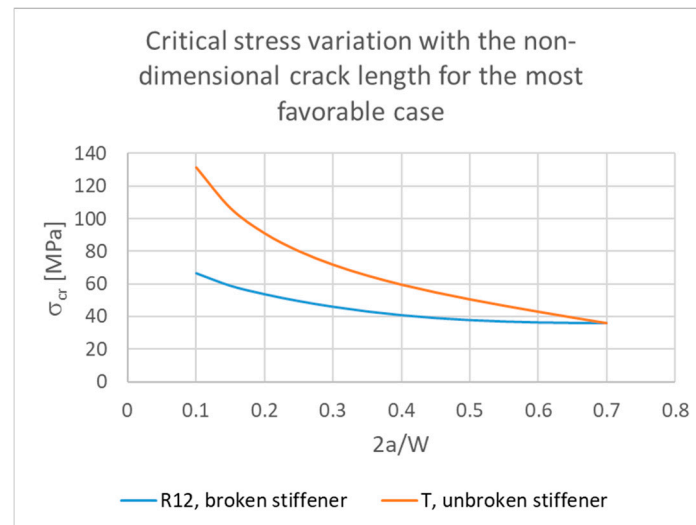


Figure 17. Variation in the critical stress with the non-dimensional crack length.

As is expected, the unbroken stiffener provides greater safety, namely higher values of the critical stress, but it can also be underlined that the difference decreases as the crack length increases and the crack tips are farther from the stiffener, ending in almost the same value of the critical stress for $2a/W = 0.7$ (35.8 MPa for the broken stiffener versus 36.3 MPa for the unbroken stiffener). For the smallest crack length ($2a/W = 0.1$), where the influence of the stiffener at the crack tip is important, the plate with the unbroken stiffener can withstand stresses almost twice as large as the same stiffener but broken (131.6 MPa compared to 66.7 MPa).

5. Conclusions

Extensive finite element analyses were conducted to study the structural integrity of thin aluminum stiffened plates with cracks crossing the stiffener and having different lengths. Four types of cross-sections were considered for the stringers to find the influence of the shape and position of the broken and unbroken stiffener on the SIF values at the crack tips.

An important variation in the SIF along the thickness was emphasized in both cases, but mainly in the case of the broken stiffener, where the maximum values of the SIF, obtained close to the stiffener side, are between 1.3 and 5.4 times bigger than the minimum values, which occur near the back side of the plate. The ratio depends on the crack length, being much bigger for small cracks. In the case of the unbroken stiffener, the difference between the maximum and minimum SIF is smaller, ranging between 1.3 and 1.7, with higher values also on the stiffener side of the plate, but with maximum values of the SIF on the back side of the plate.

The finite element model was validated by calculating the SIF using the compounding method. The comparison was made between the average values of the SIFs obtained numerically and the compounded ones. It was shown that the differences between the results are less than 4%, except for the case of the shortest crack, where the influence of the supplementary so-called “boundary-to-boundary interaction term” that was not considered in the compounding calculations becomes important.

The average value of the SIF obtained as half of the sum between the maximum and minimum SIF was found to be almost the same for all cases of broken stiffeners. In this case, it was proved that all stiffeners with the same area yield approximately the same result, regardless of the shape. For unbroken stiffeners, the same trend was noticed, except for the R5 stiffener, which showed slightly higher SIFs compared to all other three due to the very small contact length with the plate, thus decreasing the overall stiffness of the structure. However, the maximum SIF values are influenced by the shape of the stringer. For broken stiffeners, the R12 stringer provides the smallest values of the SIF, and, consequently, the best level of safety. For unbroken stiffeners, the highest safety level was obtained for the T-shaped stiffener.

Crack face displacements were also calculated for all types of broken and unbroken stiffeners, and it was shown that the stiffeners that exhibit the biggest maximum SIFs also have the biggest displacements, which is an expected result. Also, it can be noticed that although the L-shaped stringer produces a geometric lack of symmetry for the structure, the deviation from symmetry of the results is not significant, even in the case of small cracks where the crack tips are close to the stiffener.

The determination of the stress at which catastrophic failure of the cracked plate occurs shows that the safety of the structure can be ensured at almost twice as high stresses if the stiffener is unbroken, but the difference decreases as the crack length increases. For the longest cracks considered in this study ($2a/W = 0.7$), the critical stresses were found to be almost equal.

Although other harder materials may be used as stiffeners, the numerical analyses shown in this study can be used to predict the mechanical behavior of aluminum plates reinforced with continuously attached stiffeners of the same material and different cross-section shapes. This is due to the fact that such structures can be realized either by integral machining or attached later to the plate by friction stir welding, a technique specific to aluminum. Stiffeners made of other materials are difficult to be continuously attached, due to the limited welding properties of aluminum, but can be attached by bonding with adhesives or discretely attached with rivets or bolts, resulting in a different stiffness of the structure.

This research can be a useful tool in choosing a stiffener shape that could ensure a greater level of safety for stiffened plates used in industry.

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Abbreviations

The following abbreviations are used in this manuscript:

SIF Stress Intensity Factor
FEM Finite Element Method

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