

Article

A Fractal Topology-Based Method for Joint Roughness Coefficient Calculation and Its Application to Coal Rock Surfaces

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Abstract

The accurate evaluation of the Joint Roughness Coefficient (JRC) is crucial for rock mechanics engineering. Existing JRC prediction models based on a single fractal parameter often face limitations in physical consistency and predictive accuracy. This study proposes a novel two-parameter JRC prediction method based on fractal topology theory. The core innovation of this method lies in extracting two distinct types of information from a roughness profile: the scale-invariant characteristics of its frequency distribution, quantified by the Hurst exponent (H), and the amplitude-dependent scale effects, quantified by the coefficient (C). By integrating these two complementary aspects of roughness, a comprehensive predictive model is established: $JRC = 100.014H^{-1.5491}C^{1.2681}$. The application of this model to Atomic Force Microscopy (AFM)-scanned coal rock surfaces indicates that JRC is primarily controlled macroscopically by amplitude-related information (reflected by C), while the scale-invariant frequency characteristics (reflected by H) significantly influence local prediction accuracy. By elucidating the distinct roles of scale-invariance and amplitude attributes in controlling JRC, this research provides a new theoretical framework and a practical analytical tool for the quantitative evaluation of JRC in engineering applications.

Keywords: JRC evaluation; rough profiles; fractal topography; Hurst exponent; AFM

1. Introduction

In rock mechanics, the Joint Roughness Coefficient (JRC) serves as a fundamental parameter for evaluating the shear strength of rock joints, and its accurate determination is crucial for the stability analysis of rock mass engineering [1–3]. Natural rock and joint surfaces typically exhibit pronounced multi-scale roughness [4–7], which not only affects the mechanical properties of rocks but also constrains fluid flow within the rock mass [8–12]. Owing to the inherently complex morphology and multi-scale characteristics of structural surfaces, their quantitative characterization remains highly challenging. Among the various descriptors, the joint roughness coefficient (JRC), introduced relatively early and defined in a straightforward manner [13], has been widely recognized as a key parameter for representing surface roughness morphology [14–19]. In coal-bearing strata, the roughness of coal rock fractures directly influences fracture permeability and, consequently, the efficiency of coal-bed methane extraction. Accurate estimation of JRC is therefore of both theoretical and practical importance in reservoir evaluation.



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After the first proposal of the JRC concept, extensive research has been conducted to develop reliable methods for its quantitative determination. Early methods for obtaining JRC values primarily relied on direct or indirect measurements conducted both in the field and in laboratories [20]. These traditional approaches were often subjective, time-consuming, labor-intensive, and also faced challenges related to scale effects. Subsequently, a variety of analytical approaches have been proposed to establish the relationships between various parameters and the JRC. These parameters include the structure function [21], the root mean square of slope [22,23], and the roughness surface index and so on [24–26]. Relevant methods can be referred to in some review articles [3,27]. However, these methods often depend on the choice of scale, different sampling intervals correspond to different empirical equations, and the issue of applicability of the model requires further analysis [28–31]. Although a standardized sampling interval of 1 mm is widely recommended, it still struggles to adequately capture the rough characteristics of joint surfaces across different scales [3,32,33].

Conventional linear or single-scale parameters are often insufficient to fully characterize the roughness of rock surfaces due to their inherently complex and multi-scale properties. Researchers have introduced fractal theory, employing parameters such as fractal dimension and the Hurst exponent to achieve quantitative descriptions [34]. One important advantage of fractal theory in characterizing roughness is that it transcends the limitation of integer dimensions, by capturing correlations across multiple scales and enabling an overall description with a single fractal dimension [35,36]. However, its limitation lies in the fact that a single parameter is often insufficient to effectively distinguish scale-dependent features in different directions [37].

The fractal dimension, as a key parameter for describing scale invariance, is primarily calculated using classical approaches such as the yardstick method and the box-counting method [34,36], which quantify surface complexity by the variation of geometric features across different scales. The Yard stick method is used to calculate the fractal dimension of fractal curves while the Box-counting method is applied to calculate the fractal dimension of two-dimensional and three-dimensional discontinuous and irregular objects, both of which use a series of equal-length or equal-volume scales to measure the target object and establish the relationship between the measurement scale and the number of measurements. Compared with approaches that merely focus on the calculation of fractal dimensions, fractal topology theory provides a more systematic framework for explaining the intrinsic nature of fractal behavior and its objects [4,35,37,38]. It not only addresses the calculation of fractal dimensions but also emphasizes the definition and interpretation of what constitutes a fractal object and how fractal behavior is manifested.

Building upon fractal theory and fractal topology theory, numerous scholars have calculated the fractal characteristics of rock surfaces, for example, Lee et al. [39], and Seidel and Haberfield [40] calculated the Barton curve's fractal dimension and given the corresponding empirical equations; Xie et al. [41], Chen et al. [42], and Qin et al. [43] used the Yard stick method or Box-counting method to measure the contour lines and gave the relationship between the fractal dimension and JRC, and this method has been widely accepted because it accounts for scale effects. However, for Barton standard profiles, the fractal dimensions obtained by the yardstick (divider) method are often very small; consequently, even minor computational errors can be amplified, leading to significant inaccuracies in JRC estimates based on those fractal values. Moreover, a single parameter, such as the structure function or fractal dimension, is often insufficient to fully describe the roughness of a profile curve since a general profile contains not only information on the amplitude of its undulations but also the characteristics of frequency variations [15–19]. This is a key limitation of using a single parameter to predict the JRC.

Neural networks and machine learning are now essential for JRC prediction and rock mechanics analysis, enhancing accuracy and generalization by handling complex nonlinear relationships [19,44–47]. Key applications include: (1) CNN with SE-Net attention (SEC model) for automatic multi-parameter evaluation in JRC prediction [44]; (2) ANN predicting slope stability, confirming JRC as the most critical factor [45]; (3) SVR with factor analysis for stable, anisotropic JRC prediction [19]; (4) Shallow models (RBF, SVM, MLP) with PCA/LDA for JRC classification [46]; (5) Regression models (LR, RF, SVR) for slope vulnerability, with SVR performing best and reaffirming JRC's dominant influence [47]. These AI methods effectively integrate complex parameters, offering reliable solutions for JRC assessment, however, using high-dimensional data (e.g., multiple statistical parameters) may reduce model efficiency and prediction accuracy when the sample size is small.

Considering the significance of JRC in fluid flow and mechanical analyses, as well as the capability of fractal dimensions to integrate multi-scale information, this study proposes a novel method for JRC calculation. The method is developed by first employing the concept of directional scale independence in fractal topology theory to separate the horizontal and vertical scale features of profile lines, thereby enabling a quantitative characterization of their fractal and roughness properties. Subsequently, a two-parameter JRC model is established by linking the derived fractal parameters with the JRC values of the Barton standard profiles. Finally, the applicability and effectiveness of the proposed method is examined in characterizing the surface roughness of coal rocks.

The paper begins with an introduction reviewing the importance of JRC evaluation and the limitations of existing methods. The Theory and Models section elaborates on the method for generating profiles using the Weierstrass-Mandelbrot function, fractal topology theory, and derives a two-parameter JRC evaluation model. The Results and Discussion section verify the power-law relationship between equivalent vertical height and horizontal scale. In the fourth section, a JRC prediction model is established by fitting the standard Barton curves, and the model is applied to AFM-scanned coal rock surfaces to validate its effectiveness. The Conclusion section summarizes the main findings.

2. Theory, Models and Experiments

2.1. Weierstrass-Mandelbrot Function and Profiles Generation Method

Fractional Brownian Motion (FBM) [48], the Weierstrass-Mandelbrot (W-M) function [49], and Koch curves are commonly employed to generate fractal curves for simulating rough profiles. Among these, the W-M function is particularly advantageous due to its explicit mathematical form and its stochastic, self-affine, continuous yet nowhere-differentiable nature, which closely resembles the geometric characteristics of natural fracture profiles. These properties make it widely applied in modeling natural roughness [49,50].

The W-M function was originally introduced by Weierstrass in 1872 as a continuous but nowhere-differentiable function. Mandelbrot later recognized its fractal nature and provided a generalized expression, which is now known as the W-M function:

$$W(t) = \sum_{n=-\infty}^{\infty} \frac{(1 - e^{it\gamma^n})e^{i\phi_n}}{\gamma^{(2-D)n}} \quad (\gamma > 1, 1 \leq D < 2, \phi_n = \text{arbitrary phases}), \quad (1)$$

where γ is the frequency density factor, ϕ_n denotes the arbitrary phase, D represents the fractal dimension, and n is the summation index. In contrast, the real form of the W-M

function [51–54] is often employed for modeling the contour profile of two-dimensional rough surfaces:

$$Y(x) = G \times \sum_{i=N_s}^{N_f} \frac{\cos(\phi_i) - \cos(2\pi\gamma^i x/L_s - \phi_i)}{\gamma^{Hi}}, \quad (2)$$

where $Y(x)$ denotes the height of the two-dimensional rough surface profile at horizontal position x ; G is the fractal height factor that controls the amplitude of profile undulations; N_s and N_f represent the starting and ending terms of fractal iteration, respectively, where N_s together with the period length parameter L_s determines the minimum period length of the function, and N_f controls the minimum resolution of the fractal profile. The Hurst index H characterizes the roughness of the curve in this function and is directly related to the fractal dimension. Similarly, γ is the frequency density factor, typically set to $\gamma = 1.5$ for rough surfaces following a normal distribution. ϕ_i represents the random phase, uniformly distributed in the range $[0, 2\pi]$, which governs the stochastic properties of the fractal function. Based on this formulation, we constructed a three-component rough fracture profile, as illustrated in Figure 1.

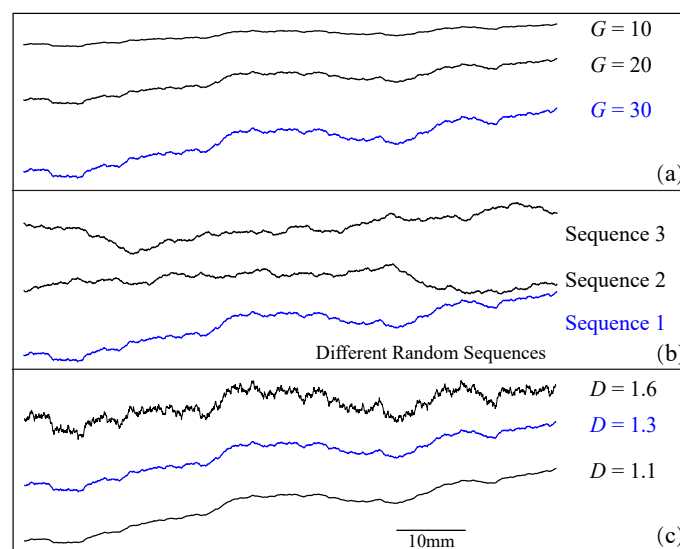


Figure 1. Effects of parameters in Weierstrass-Mandelbrot function on the morphology of simulated rough curves: (a) influence of the fractal height factor G ; (b) influence of the random phase sequence ϕ_i ; (c) influence of the fractal dimension D .

Figure 1a illustrates the influence of the fractal height factor G on the morphology of rough profiles. From top to bottom, the three curves correspond to G values of 10, 20, and 30, respectively, while keeping the fractal dimension constant at $D = 1.3$ and the same random phase sequence. It can be seen that G primarily controls the amplitude of the profiles. Figure 1b shows the influence of the random phase sequence under identical conditions of $D = 1.3$ and $G = 30$. In contrast to the effect of G shown in (a), the random sequence significantly affects the position of undulations: profiles with the same sequence exhibit similar overall shapes, whereas different sequences produce distinctly different morphologies. In Figure 1c, the profiles share the same $G = 30$ and random sequence but differ in fractal dimension ($D = 1.6, 1.3$, and 1.1 from top to bottom). As D increases, the profiles become progressively rougher, with more pronounced asperities and burr-like features.

2.2. Fractal Topology Theory

Fractal geometry describes the disordered objects using fractal dimension D . For self-similar objects, D remains constant across different scales, reflecting the scale invariance of

geometric features under magnification or reduction. This property forms a fundamental basis of fractal theory and provides the foundation for calculating D :

$$N(\delta) \sim C_0 \delta^{-D}, \quad (3)$$

where, the scale length δ represents the measuring length, which is the ruler length employed in the divider method. $N(\delta)$ is the number of the measuring with ruler δ , and C_0 is a coefficient. In this model, two parameters are the length δ and the corresponding count $N(\delta)$. Therefore, this model is known as the classical number-size relationship (R_{ns}).

Jin et al. [37] pointed out that different R_{ns} can yield the same D . This implies that D alone may not uniquely define the fractal behavior, making its characterization non-deterministic. So two scale-independent parameters are proposed to describe the fractal behavior, and they are the scaling lacunarity (P) and scaling coverage F :

$$P = \frac{\delta_i}{\delta_{i+1}}, \quad F = \frac{N(\delta_{i+1})}{N(\delta_i)}, \quad (4)$$

where, i is the i th level of the ruler. With Equations (3) and (4), D is now expressed as:

$$D = \frac{\log F}{\log P}. \quad (5)$$

For self-affine objects, a key characterizing parameter is the Hurst exponent the Hurst exponent (H). In other words, the original object $G(x, y)$ requires different scaling factors in different directions ($G(\zeta x, \zeta^H y)$) to preserve similarity. According to the definition of scaling lacunarity, it is known that:

$$P_x = \zeta^{-1}, \quad P_y = \zeta^{-H}, \quad (6)$$

where, P_x and P_y represent the scaling lacunarities in x - and y - direction, respectively. The Hurst exponent is scale-invariantly defined as:

$$H = \frac{\log P_y}{\log P_x}. \quad (7)$$

According to Equation (6), the scaling lacunarities P_x and P_y of the two adjacent cosine functions in the W-M function in the x - and y -directions can be deduced as

$$P_x = \frac{\gamma^i}{\gamma^i + 1} = \frac{1}{\gamma}, \quad P_y = \frac{\gamma^{Hi}}{\gamma^{H(i+1)}} = \frac{1}{\gamma^H}, \quad (8)$$

Since the ratio of vertical and horizontal scaling is characterized by H , it can be determined by analyzing the power-law relationship between P_y and P_x (i.e., $P_y \propto P_x^H$) derived from the function curve.

Since fractal objects exhibit different scaling ratios along the x - and y -directions, a crucial step in determining the scale-invariant parameter H of self-affine objects is to separate the scaling information along the horizontal and vertical axes. This separation provides the theoretical basis for calculating the Hurst exponent of rock profiles.

2.3. Calculation of the Fractal Dimension and JRC for Fracture Profiles

As that in Equations (7) and (8), the expression of the W-M function allows for the theoretical determination of a Hurst exponent. To develop a universal method for calculating the Hurst exponent of profile lines (such as the profiles generated using W-M function), it

is essential to propose a scale-separation approach that can effectively distinguish between horizontal and vertical scaling information in rough curves.

The commonly used method for calculating D of a simple curve is the yardstick method as shown in Equation (3). This approach evaluates the scaling properties of a curve by varying the length of the measuring yardstick, thereby obtaining the corresponding number and fractal dimension. The number and the length are not the same dimension, thereby, Brown [55] modified it, unlike measuring along the profile trajectory with a measuring tape δ , he selected a measuring tape δ_h in the horizontal direction. This leads to a relationship between the profile length $L(\delta_h)$ and the measuring length δ_h :

$$L(\delta_h) \sim C_0 \delta_h^{1-D}, \quad (9)$$

where C_0 is a constant and D is the fractal dimension, which can be determined through the slope value of the $\ln N(\delta) \sim \ln \delta$ or $\ln L(\delta_h) \sim \ln \delta_h$ plots.

Dou et al. [56] derived that the expectation $(E((Y(x + \delta_h) - Y(x))^2))$ of the squared height difference for a self-affine profile at a length interval δ_h is expressed as:

$$E((Y(x + \delta_h) - Y(x))^2) = \sigma^2 \delta_h^{2H}, \quad (10)$$

where σ represents the volatility of the profile, and its magnitude reflects the vertical height fluctuations. By applying the Pythagorean theorem, this is further used to derive the explicit expression for the length of a self-affine profile line [57]:

$$L(\delta_h) = \lambda_0 * \sqrt{1 + \sigma^2 \delta_h^{2H-2}}, \quad (11)$$

where λ_0 is the horizontal length of the profile. By comparing Equations (9) and (11), it is known that these two equations become equivalent when σ is large enough (meaning 1 is relevant small compared to $\sigma^2 \delta_h^{2H-2}$). By decomposing the total length into individual measurement segments, the equivalent vertical height ($V(\delta_h)$) can be derived through Equations (12) and (13), and the detailed derivation can be found in Ref. [57]:

$$\frac{L(\delta_h)^2 - \lambda_0^2}{\lambda_0^2} = \frac{V(\delta_h)^2}{(N * \delta_h)^2} = \sigma^2 \delta_h^{2H-2} \quad (12)$$

$$V(\delta_h) = C \delta_h^H. \quad (13)$$

This concise equation provides the theoretical basis for calculating H for self-affine profiles. In this equation, $V(\delta_h)$ and δ_h represent the scale information in the vertical and horizontal direction respectively. This equation is a power-law model representing a statistical scaling relationship.

However, Equation (13) presents the following deficiencies:

- **Dimensional Inconsistency:** The left side, $V(\delta_h)$, has dimensions of length. The right side has δ_h also with dimensions of length, H is a dimensionless parameter ($0 < H \leq 1$), thus, the fitting parameter C should carry the necessary dimension $\delta_h^{(1-H)}$ to ensure the dimensional consistency.
- **Scale Dependence:** Because this equation represents a nonlinear relationship, the constant C becomes dependent on δ_h . Its value will vary depending on the study area, data sampling interval, and total profile length. This means that if the same profile proportionally, the fitted value of C will change. For example, suppose an original profile with a horizontal scale $1 \leq \delta_h \leq 5$ satisfies the relationship $V(\delta_h) = 1 \times \delta_h^{0.7}$,

resulting in $1 \leq V(\delta_h) \leq 3.0852$. If we scale both $V(\delta_h)$ and δ_h by a factor of 5 and refit the data, the resulting C would be 1.6207, different from the original value of 1.

Therefore, using a parameter non-dimensionalization method like $V(\delta_{h,1})/V(\delta_{h,2}) = (\delta_{h,1}/\delta_{h,2})^H$ (where $V(\delta_{h,1})$ and $V(\delta_{h,2})$ represent the equivalent vertical height corresponding to the adjacent measurement scale of $\delta_{h,1}$ and $\delta_{h,2}$) to avoid the scale effects. And another approach is to replace the denominator on the left with the equivalent vertical height $V(\delta_{h,s})$ corresponding to the measurement starting value $\delta_{h,s}$, and similarly replace the denominator on the right with $\delta_{h,s}$.

Now, consider two profiles: profile A and profile B, where, profile B is obtained by scaling profile A by a factor of $k(k \geq 1)$. Based on fundamental geometric relations, we have:

$$V_A(\delta_{h,A}) = C_A \times \delta_{h,A}^H \quad \text{and} \quad V_B(\delta_{h,B}) = C_B \times \delta_{h,B}^H \quad (14)$$

Given $V_B(\delta_{h,B}) = k \times V_A(\delta_{h,A})$ and $\delta_{h,B} = k \times \delta_{h,A}$, substituting yields:

$$k \times \delta_{h,A} = C_B \times (k \times \delta_{h,A})^H \quad (15)$$

Solving for C_B :

$$C_B = C_A \times k^{1-H} \geq C_A \quad (\text{since } k \geq 1 \text{ and } H \leq 1) \quad (16)$$

Therefore, the fitted value C increases with larger profile scales and decreases with smaller scales. While, H is a scale-invariant parameter.

Here we provide an example to illustrate the role of the coefficient C . When the profile is a straight line: if the line is horizontal, the equivalent vertical height $V(\delta_h)$ is zero, with the Hurst exponent H being 1 and the C being 0; if the line is inclined, $V(\delta_h)$ exhibits a linear relationship with δ_h , H remains 1, and C is positively correlated with the slope of the line. This demonstrates that the coefficient C captures the amplitude (height-related) information of the profile.

When the profile is a rough curve, the vertical fluctuations do not increase linearly with the horizontal measurement scale. Consequently, the growth of $V(\delta_h)$ is slower than that of δ_h , resulting in a Hurst exponent H less than 1. Furthermore, the more intricate or high-frequency the local fluctuations of the curve, the smaller the H exponent tends to be.

Re-examining the relationship $V(\delta_h) = C\delta_h^H$, it is clear that C is a coefficient related to the scale of a profile. A profile possesses not only scale information but also indicators of frequency variation. Therefore, when estimating the JRC of a profile, it is necessary to consider both the scale-invariant information and the scale relationship. On this basis, the relationship between JRC, the Hurst exponent, and the coefficient C is further analyzed, leading to the establishment of a two-parameter JRC prediction model $f(C, H)$.

$$\text{JRC} = f(C, H). \quad (17)$$

3. Results and Discussion

3.1. Fractal Properties of W-M Profiles

To systematically investigate the scaling effects on fracture surface morphology, a series of self-affine fractal profiles were generated using (2). The fractal dimension was varied across ten values (1.001, 1.1, 1.2, 1.25, 1.3, 1.4, 1.5, 1.55, 1.6, and 1.7), while the height scaling factor G was assigned seven values (1, 50, 100, 500, 1000, 5000, and 10,000) for each fractal dimensional value, covering a wide range of amplitude scaling. The fractal characteristics of these profiles were quantified by analyzing the relationship between measured profile lengths and corresponding measurement scales via a fitting procedure.

We present in Figure 2 the measurement scale δ_h and corresponding measurement lengths $L(\delta_h)$ for structural profiles with a Hurst exponent of 0.3 and G values of 1, 5, 10, 50, 100, 500, and 1000. It can be observed that $L(\delta_h)$ decreases with increasing δ_h , indicating that as the measurement scale becomes larger, the profile curve captures fewer morphological details. In the double logarithmic coordinate system, when $G = 1$, the data exhibit a clear power-law relationship in the initial segment. In the latter segment corresponding to larger δ_h , however, the variation of measurement length becomes more gradual, indicating that the influence of the measurement scale has significantly diminished. As the G value increases, the range over which the power-law relationship holds gradually expands; when $G = 1000$, a well-defined power-law relationship between measurement scale and length is observed across the entire figure. The exponents of the fitting curves are $\{-0.277, -0.657, -0.709, -0.736, -0.737, -0.737, -0.737\}$, which means the calculated H are $\{0.723, 0.343, 0.291, 0.264, 0.263, 0.263, 0.263\}$.

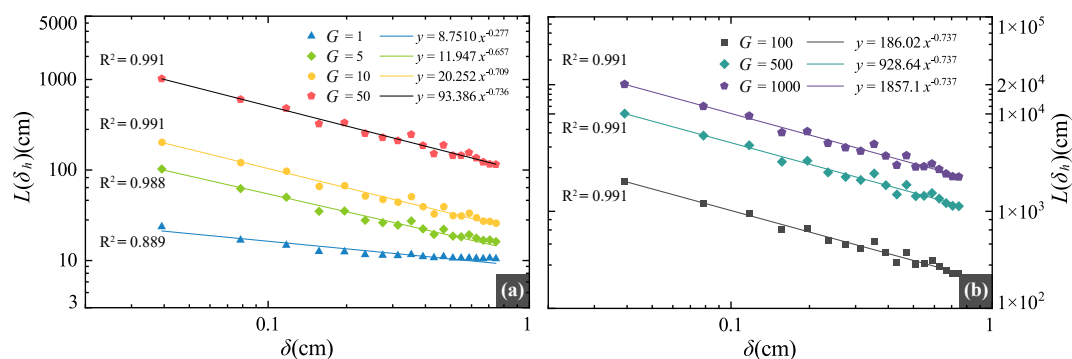


Figure 2. Power-law relationship between $L(\delta_h)$ and δ_h for profiles with $H = 0.3$: (a) data for $G = 1, 5, 10$, and 50 , (b) data for $G = 100, 500$, and 1000 .

This indicates that Brown's method of calculating the fractal dimension by scaling the height on the y -axis is reasonable. Furthermore, we computed the Hurst exponent H_e for self-affine profiles corresponding to different G values and plotted them in Figure 3. The x -axis represents H from Equation (2), and the y -axis represents H_e calculated using $L(\delta_h) \sim C\delta_h^{H-1}$. As G increases, H_e gradually decreases and stabilizes. The graph shows that when $G = 500$ and $G = 10,000$, the calculated H_e values are nearly identical. From the graph, it can also be observed that when the calculation results tend to stabilize, their values are lower than the initial set values used to generate the profiles. Moreover, the larger the value of H , the greater the deviation. For specific reasons, please refer to the relevant explanation in Dong et al. [57].

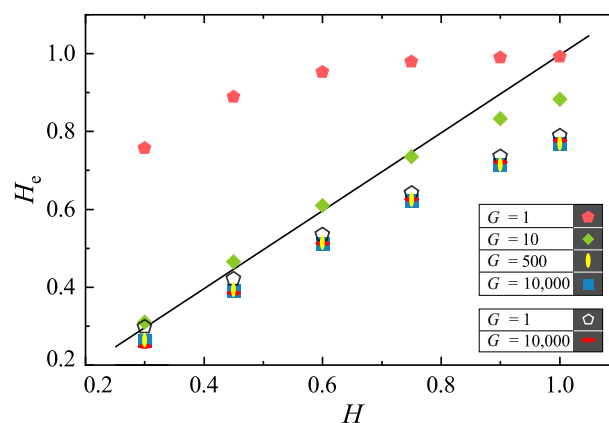


Figure 3. Influence of the height scaling factor G and estimation methods on the calculation results for the self-affine profile with $H = 0.3$.

3.2. Power Law Relationship of $V(\delta_h)$ and δ_h

Although the Hurst exponent calculated from the relationship $L(\delta_h) \sim C\delta_h^{H-1}$ provides valuable insight into the scaling properties of rough profiles, the measured Hurst exponent obtained by this method shows a considerable deviation from the theoretical value H . Accurate results could be achieved when height scaling factor G is extremely large, which limits the applicability of this method.

To characterize the scaling properties of self-affine fractures, the method proposed by Dong et al. [57] was employed. Specifically, Equations (11) and (13) were used to calculate the Hurst exponent, a key parameter describing the long-memory dependence in fracture topography. The outcomes of applying this analytical procedure are as follows:

The results obtained from Equation (11) are shown in Figure 4a,b, and the only difference is the value of G . The y -axis of these two graphs corresponds to the derived parameter $(L(\delta_h)/\lambda_0)^2 - 1$. The right side of the equation becomes $\sigma^2\delta_h^{2H-2}$, therefore the exponent of the fitting results is $2H - 2$.

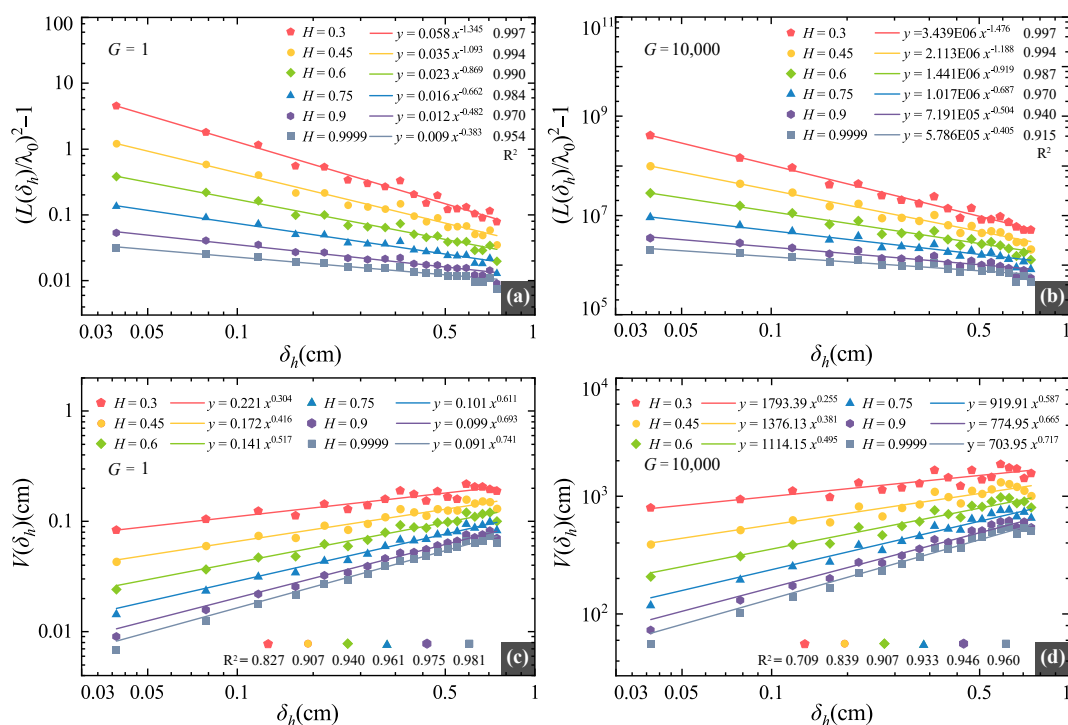


Figure 4. The data of Equations (11) and (13): (a) Equation (11), $G = 1$; (b) Equation (11), $G = 10,000$; (c) Equation (13), $G = 1$; (d) Equation (13), $G = 10,000$.

Similarly, Figure 4c,d illustrate the results of Equation (13), where the y -axis is the equivalent vertical height $V(\delta_h)$. Thus, the exponent of the fitting results is the Hurst exponent. Both δ_h and $V(\delta_h)$ have dimension of length (cm).

As shown in these four figures, they all exhibit well-defined power-law relationships. More importantly, the Hurst exponents calculated using Equations (11) and (13) show excellent agreement. This indicates that for the same profile, the two distinct methods yield consistent estimates of the Hurst exponent, regardless of the vertical scaling (amplitude along the y -axis, governed by the parameter G), demonstrating the robustness of the method.

In addition to demonstrating robustness, this study further validated the accuracy of the computational results. As shown in Figure 3, the Hurst exponents obtained via Equation (13) show remarkable consistency with those calculated after amplifying the profile heights by factors of 500 and 10,000. The hollow shapes in the figure represent the

results calculated by this method, and the excellent agreement under significant amplitude scaling convincingly verifies its effectiveness and reliability.

4. The New Method for Evaluating Rock Joint JRC and Its Application on AFM Surfaces

4.1. The Fitting Equation of JRC for Barton Curves

As indicated in the previous Section 2.3, Equation (13) allows for the calculation of the Hurst exponent without requiring significant amplification of the vertical height of the profile, while yielding more reasonable results. In conjunction with the fundamental framework for evaluating JRC (Equation (17)), this study conducted a systematic analysis of the ten standard profile curves with known JRC values as proposed by Barton. Fitting was performed, using $V(\delta_h) = C\delta_h^H$ relation, to calculate the Hurst exponent H and the coefficient C .

The physical and mechanical properties of rock masses are strongly influenced by the distribution and surface morphology of their structural surfaces, a subject extensively studied in the literature [58–61]. Consequently, the ten standard profile curves established by Barton have become a cornerstone for quantitatively evaluating structural surface roughness. These standard curves are presented in Figure 5.

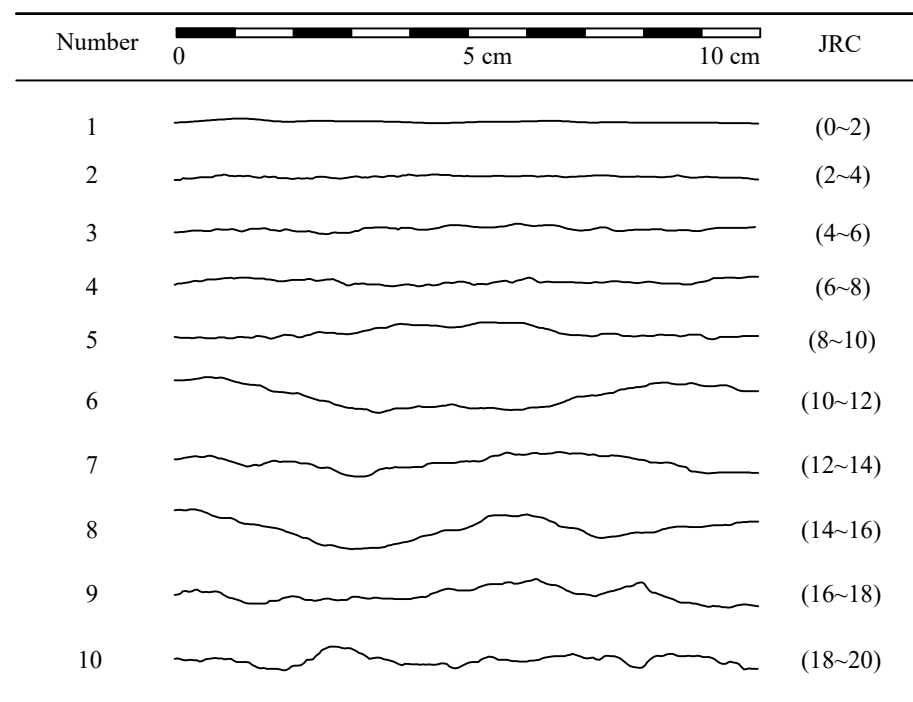


Figure 5. The morphology of the Barton curves and their JRC.

The JRC of the ten curves in Figure 5 from top to bottom increase from 0 to 20. These curves show increased volatility and higher-frequency variations. Then, we calculate the 10 standard structural surface curves using the power-law relationship between $V(\delta_h)$ and δ_h , and illustrate the fitting results in Figure 6.

As shown in the figure, the Hurst exponent for profile 1 is close to 1, indicating a relatively smooth surface with minimal fluctuations, which aligns with visual observations. However, a significant decrease is observed for profile 2, which exhibits a much smaller Hurst exponent than the others. This suggests a high proportion of high-frequency components, corresponding to rapid local ascents and descents. A magnified view of profile 2 confirms these frequent, albeit small-amplitude, fluctuations, validating its exceptionally low Hurst exponent. The low Hurst exponent observed in Profile 2 is due to the nature of

its low-amplitude fluctuations, not the rapidity of its frequency variations. Coefficient C shows a slowly increasing trend, suggesting that the the overall amplitude or scale of the profile height progressively increases.

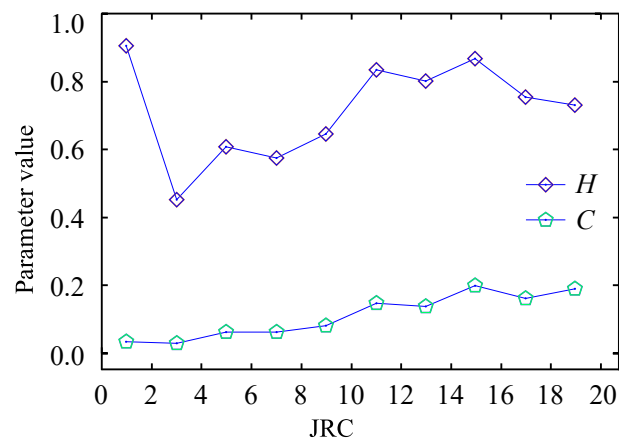


Figure 6. Fitted H and C for Barton curves with different JRC.

Given that the independence test confirmed no significant correlation between the two parameters, a power-law model was adopted with C and H as independent variables and JRC as the dependent variable. This model shows a good fit to the data, following the relationship in Equation (18).

$$\text{JRC} = 100.014 \times H^{-1.5491} \times C^{1.2681} \quad (R^2 = 0.993). \quad (18)$$

This equation indicates a positive correlation between JRC and the coefficient C , and a negative correlation with H , and the coefficient of determination (R^2) is 0.993. Consequently, a greater profile amplitude corresponds to a higher JRC value [16], while a higher proportion of high-frequency components in the height distribution-indicating rapid local variations-also leads to an increase in JRC. This pattern of correlations suggests that surfaces perceived as rougher, characterized by sharp and frequent elevation changes, exhibit reduced height correlation between adjacent points, which is reflected by a lower H value.

The fitting results for the ten standard Barton curves are presented in Figure 7a, where the white dashed lines represent contours of the fitted surface, and the ten blue points superimposed on it correspond to the Barton curve data. The three axes denote the Hurst exponent, the coefficient C , and the JRC, respectively. Using Equation (18) and the fitted values of H and C from Figure 6, the JRC values of the Barton curves are calculated. A comparison between these calculated values using Equation (18) and the given JRC values is shown in Figure 7b. All data points are clustered around the brown line ($y = x$), indicating strong agreement. The corresponding data can be found in Table 1.

Table 1. Comparison between calculated JRC and theoretical JRC.

Theoretical JRC	Calculated JRC	Relative Error
1	1.45235	45.2%
3	3.78783	26.3%
5	6.22889	24.6%
7	7.07520	1.07%
9	8.09291	10.1%
11	11.3585	3.26%
13	11.4609	11.8%
15	16.1090	7.39%
17	15.4732	8.98%
19	19.7513	3.95%

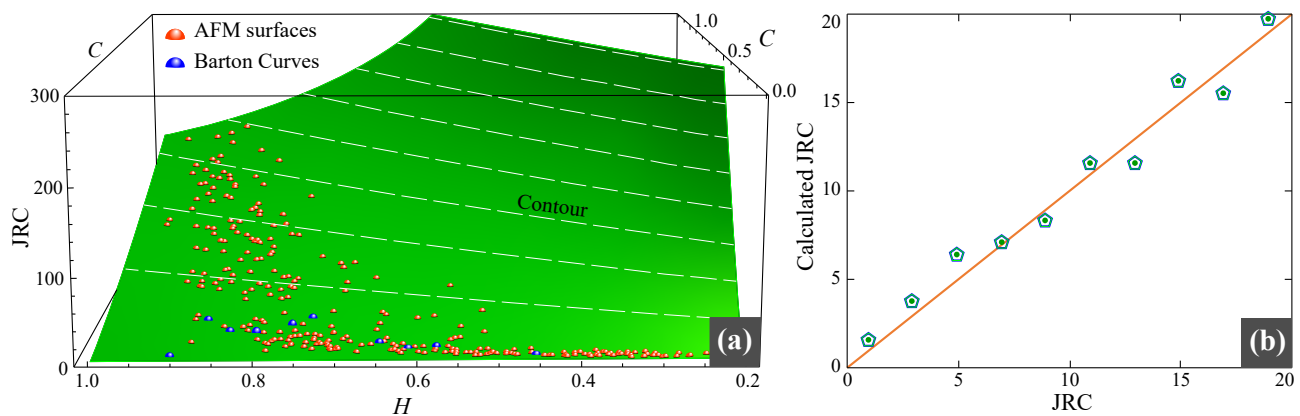


Figure 7. The verification of JRC evaluation method: (a) the fitting surface of Barton data (represented by the 10 blue points); (b) Comparison between given JRC and calculated JRC using Equation (18).

4.2. Validation and Comparison of the New JRC Model with Other Methods

The root mean square of slope Z_2 and the fractal dimension D are commonly used as parameters for estimating the roughness of rock joints, and researchers have established a quantitative relationship between JRC and Z_2 or D , some of them are listed in Table 2.

Table 2. Empirical equations for estimating JRC values.

Proposer	Equation	Interval
Yu and Vayssade [21]	$JRC = 60.32Z_2 - 4.51$	0.25 mm
	$JRC = 61.79Z_2 - 3.47$	0.5 mm
	$JRC = 60.32Z_2 - 4.51$	1.0 mm
Yang et al. [23]	$JRC = 32.69 + 32.98 \log_{10}(Z_2)$	0.5 mm
Jang et al. [22]	$JRC = 51.16(Z_2)^{0.531} - 11.44$	0.5 mm
	$JRC = 53.15(Z_2)^{0.632} - 6.32$	1.0 mm
	$JRC = 51.16(Z_2)^{0.65} - 6.40$	2.0 mm
Turk et al. [62]	$JRC = -1138.6 + 1141.6D$	—
Xie et al. [41]	$JRC = 85.2671(D - 1)^{0.5679}$	—
Chen et al. [42]	$JRC = 15179W_d^{0.79}(D - 1)^{1.46}$	—
You et al. [63]	$JRC = 287.76(D - 1)^2 + 126.9(D - 1)$	—

As applied by Li and Xiao [64], the computed JRC values of the Barton curves were normalized by forming the ratio of the calculated value to the given theoretical value, resulting in the normalized parameter N_{JRC} . The reliability of the method is assessed by evaluating its deviation from 1.

$$N_{JRC} = JRC_{cv} / JRC_{th}, \quad (19)$$

where JRC_{cv} is the JRC value calculated by the empirical model, and JRC_{th} is the theoretical mean JRC value. N_{JRC} , the normalized parameter, closer to 1 indicates a better agreement between the empirically calculated JRC and the theoretical value. The validation results are presented in Figure 8, where Figure 8a₁–a₄ show the N_{JRC} results computed using the JRC- Z_2 empirical formula at different sampling intervals (Figure 8a₁ interval is 0.25 mm, Figure 8a₂ interval is 0.5 mm, Figure 8a₃ interval is 1 mm, Figure 8a₄ interval is 2 mm), and Figure 8b presents the corresponding results for the JRC- D empirical models.

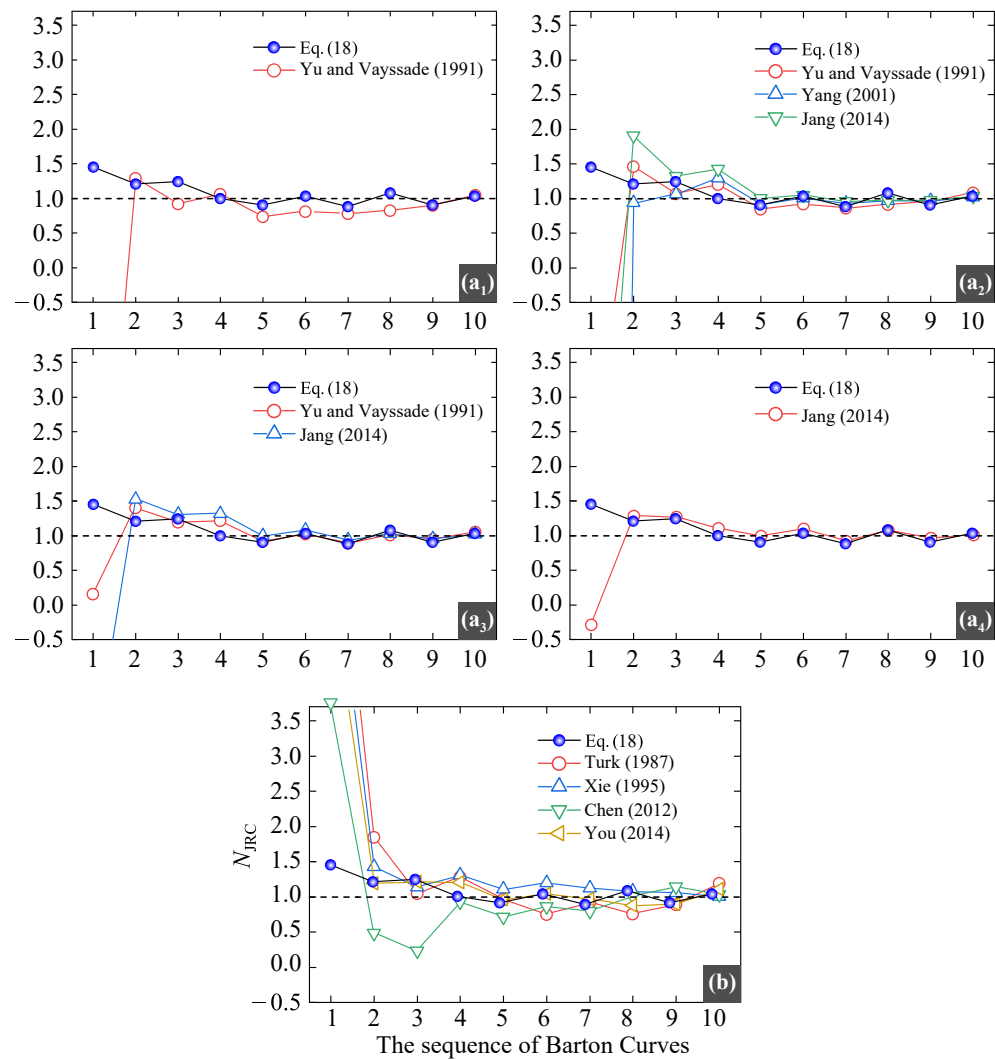


Figure 8. Evaluated results of different JRC models: (a₁–a₄) Models based on Z_2 ((a₁) interval is 0.25 mm, (a₂) interval is 0.5 mm, (a₃) interval is 1 mm, (a₄) interval is 2 mm); (b) Results of models based on fractal dimension

As shown in Figure 8a₁–a₄, the JRC values calculated using the proposed Equation (18) align closely with the theoretical values across all sampling intervals. While notable deviations are observed in the results for Curve 1 across different equations since the theoretical value is 1. The Hurst value calculated in this research is 1.4589, more accurate than other models. Besides, for profiles 2 to 10, various calculation methods yielded reasonably accurate values; however, the model proposed in this paper achieved smaller errors. This outcome not only validates the integrity of the data but also confirms the enhanced accuracy of our model.

The results in Figure 8b are the comparison of N_{JRC} obtained using different relationships between JRC– D . The situation is similar, for profile 1, the results from the present study are markedly more accurate than the previous ones, while for the other profiles, the proposed model demonstrates superior stability in its computed results.

4.3. The Application and Analysis of JRC Evaluation Method on AFM Surfaces

Six coal samples were prepared for this study, and high-resolution scanning of the coal surface microstructure was conducted using a Bruker Dimension Icon atomic force microscope (AFM). The experiments were performed with a Multimode AFM equipped with a silicon probe (Si Probe) operating in Tapping Mode to minimize potential surface

damage and enhance measurement accuracy. The scanning area was set to $2\ \mu\text{m} \times 2\ \mu\text{m}$ with a resolution of 256×256 pixels, resulting in a data point spacing of approximately 7.81 nm, which effectively captures nanoscale surface topography variations. The acquired data were subsequently analyzed to investigate the surface roughness characteristics and fractal properties and JRC of the coal samples. The processed 3D rough surfaces of the coal samples are listed in Figure 9.

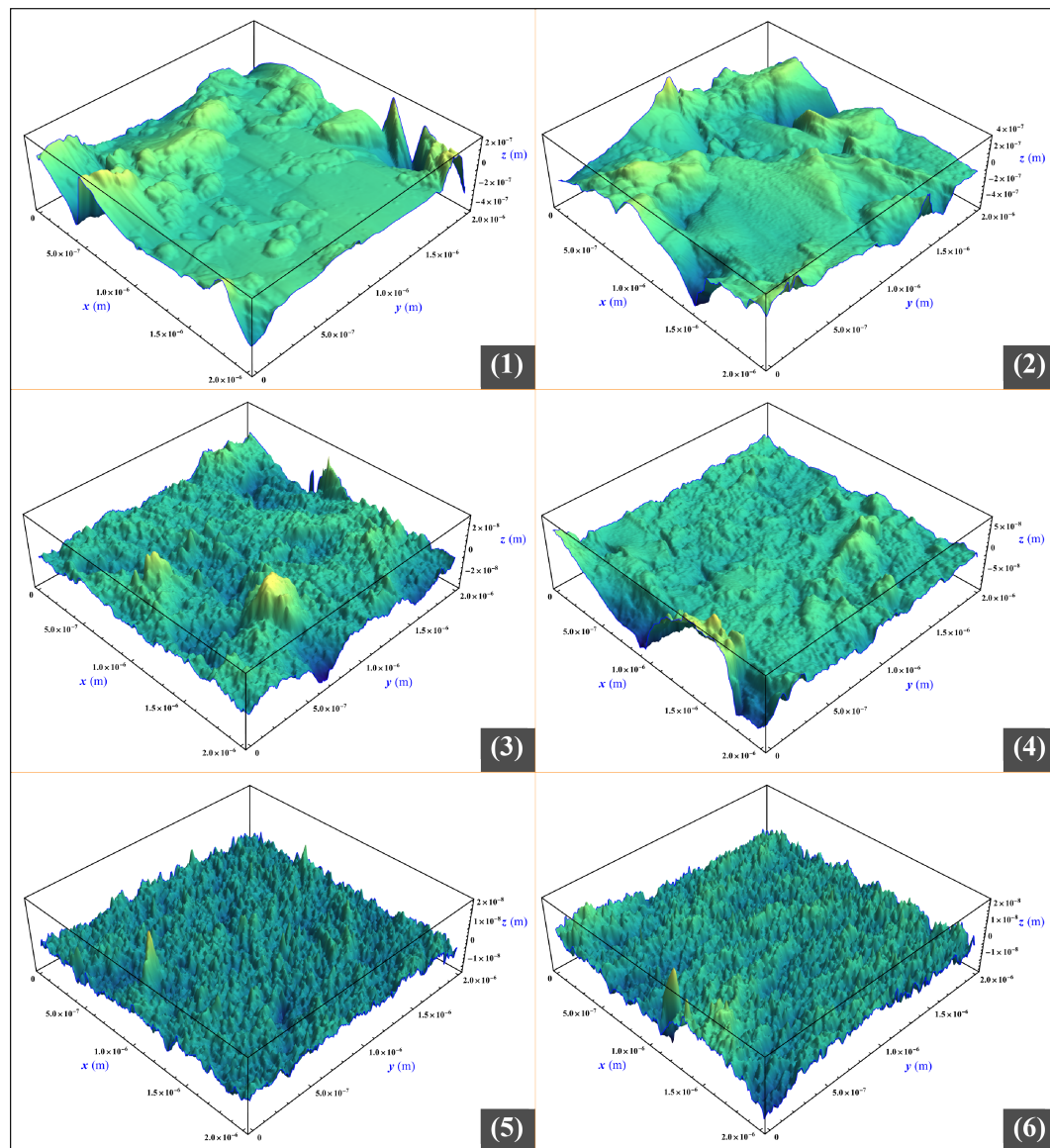


Figure 9. The surface morphologies of the six rock samples scanned using AFM all have dimensions of $2\ \mu\text{m}$ along both x -axis and y -axis. (1)–(6) represent the surface morphologies of the six samples after scanning. In the figure, colors ranging from dark (dark blue) to light (yellow) indicate the relative elevation along the z -axis from low to high.

For the six surfaces depicted in Figure 9, the $x:y:z$ aspect ratio is uniformly set to 1:1:0.3 in order to provide adequate visual detail in the z -axis direction. The surface morphology of all samples exhibit significant roughness characteristics. Samples 1 and 2 demonstrate greater height variations along the z -axis, yet their macroscopic surfaces appear relatively stable without rapid transitions between high and low elevations. In contrast, Samples 3 through 6 display topographies characterized by rapid and frequent fluctuations. These distinct morphological features make them highly suitable for investigating surface roughness properties.

Each surface is composed of a grid of 256×256 data points. Sixteen profiles were extracted along the x -axis for each sample. H and C were subsequently calculated for these profiles using Equation (13), followed by the determination of JRC employing Equation (18). The computational results are presented in Figure 10. It should be noted that when applying Equation (18), we performed a uniform scaling transformation on the AFM-scanned data, stretching the original $2 \mu\text{m}$ length to the standard 10 cm. Without this scaling, the calculated JRC values would be extremely small (mostly less than 1), because the fitting process in Equation (13) would yield a very small coefficient C .

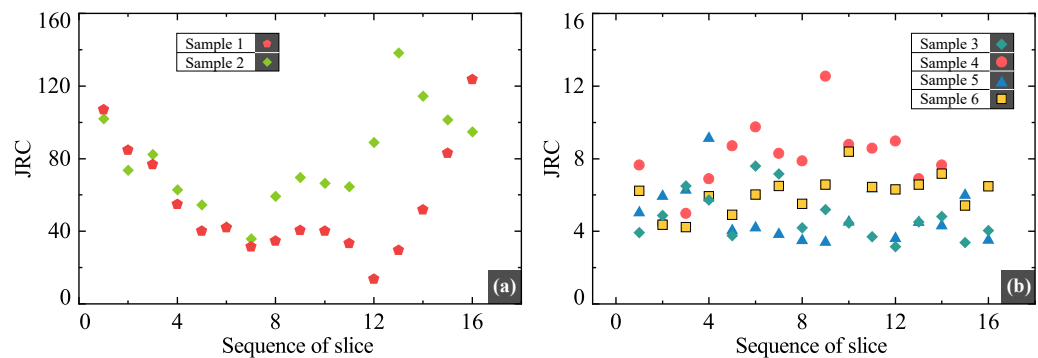


Figure 10. Evaluated Hurst exponents of the six coal samples. (a) represents the calculated Hurst exponent for Sample 1 and Sample 2, while (b) denotes the Hurst exponent for Samples 3, 4, 5, and 6.

As illustrated in Figure 10, the JRC values of Sample 1 and Sample 2 (Figure 10a) are significantly higher than those of the other four samples (Figure 10b). This discrepancy primarily stems from the markedly greater height values of Sample 1 and Sample 2, which exceed those of the other samples by approximately one to two orders of magnitude (data comparisons are listed in Table 3). This substantial difference in surface topography is likely the main reason for the notably elevated JRC values observed in these two samples.

After magnification, the microstructure of rock samples may differ from the actual macroscopic profile. For instance, it may exhibit greater relative height in the vertical direction, or the selected measurement profile might lie on a locally steeper inclined surface with more pronounced undulations. These factors could lead to an overestimation of the calculated JRC value. Nevertheless, such high-resolution data at the micro-scale also offer richer information, facilitating a deeper analysis of the controlling parameters and underlying mechanisms influencing JRC.

Table 3. The range and the standard deviation of rock surface heights.

Sample Number	Range (m)	Standard Deviation (m)
1	7.15806×10^{-7}	6.80622×10^{-8}
2	8.61101×10^{-7}	9.68752×10^{-8}
3	6.46883×10^{-8}	6.46630×10^{-9}
4	1.67089×10^{-7}	1.39056×10^{-8}
5	3.71305×10^{-8}	2.41970×10^{-9}
6	4.28529×10^{-8}	3.65181×10^{-9}

Based on the intuitive observation that greater surface height variations correspond to higher JRC, we further computed the range and standard deviation (as in Table 3) of the rock surface heights to quantitatively analyze the relationship between surface fluctuation characteristics and the calculated JRC. The specific analysis is presented below.

The relationship between the standard deviation of heights and JRC is illustrated in Figure 11a, showing an approximately proportional trend. This indicates that the JRC is macroscopically governed by the height information of the profile. Similarly, the rela-

relationship between the height range and JRC also exhibits a direct proportionality, further verifying that the JRC is significantly controlled by the height information of the structural surface. Subsequently, data points with JRC less than 30 were selected to re-plot the correlation between JRC and the standard deviation in Figure 11b. Although the data points display a general proportional trend macroscopically, their distribution is highly scattered. This suggests that relying solely on the standard deviation of height is insufficient for accurately determining the actual JRC value. Finally, C and H were incorporated alongside JRC, and all three parameters were projected onto the fitted 3D surface, as shown in Figure 7a by the red points.

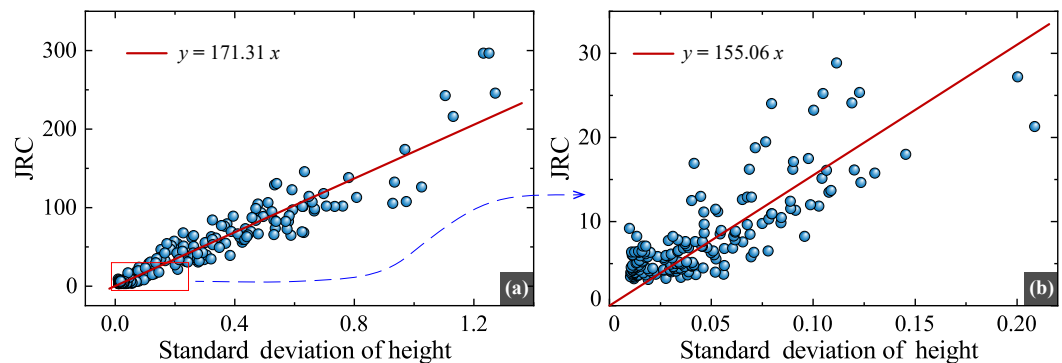


Figure 11. Relationship between Hurst and the standard deviation of height: (a) The whole JRC data in this study, (b) Data with JRC < 30.

The analysis reveals a general positive correlation between JRC and height-derived parameters such as standard deviation or range. However, significant local scatter exists because JRC is inherently a comprehensive empirical parameter. Height parameters only reflect vertical deviations and cannot capture frequency-related morphological differences, which significantly influence mechanical behaviors such as friction, wear, and shear strength of joints. Moreover, Barton's JRC was originally determined through subjective visual comparison with ten standard profiles, a process that inherently integrates empirical judgments on amplitude, wavelength, slope, and mechanical performance. Consequently, using a single height statistic to fit such a composite parameter inevitably leads to localized errors.

5. Conclusions

This research focuses on the quantitative characterization of joint roughness profiles and proposes a new JRC evaluation method based on fractal topology theory. Through systematic analysis of the standard Barton profiles, a two-parameter JRC prediction model was established. The main conclusions are as follows:

- (1) In the model $V(\delta_h) = C\delta_h^H$, the Hurst exponent H is a scale-invariant parameter reflecting the scale-invariant characteristics of the profile's frequency distribution, while the coefficient C is a scale-dependent parameter capturing amplitude-related scale effects. These two parameters describe different aspects of rock profile morphology and are not contradictory.
- (2) As an empirical parameter comprehensively reflecting the roughness of a joint surface, JRC incorporates both scale information and undulation characteristics. This study suggests that the combined effect of both parameter C (reflecting height information) and H (reflecting frequency information) should be considered to fully characterize the rough features of structural surfaces.
- (3) Analysis based on AFM scanning of coal rock samples confirms that JRC is primarily controlled macroscopically by height-related parameters but is also influenced by frequency distribution characteristics. The proposed two-parameter model

$JRC = 100.014H^{-1.5491}C^{1.2681}$, by integrating both the height and frequency attributes of profiles, achieves higher predictive accuracy.

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