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Minimizing Waste and Costs in Multi-Level Manufacturing: A Novel Integrated Lot Sizing and Cutting Stock Model Using Multiple Machines

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Abstract

Lot sizing and cutting stock problems are critical for manufacturing companies seeking to optimize resource utilization and minimize waste. This paper addresses the interconnected nature of these problems, often occurring sequentially in industries involving cut items or packaging. We propose a novel mixed integer linear programming (MILP) model that integrates the capacitated lot sizing problem with the one-dimensional cutting stock problem within a multi-level manufacturing framework. The cutting stock problem is addressed using an arc flow formulation. Our model aims to minimize setup, production, holding, and waste material costs while incorporating capacity constraints, setup requirements, inventory balance, and the use of various cutting machines. The effectiveness of our model is demonstrated through numerical experiments using a commercial optimization package. While the model efficiently generates optimal solutions for most scenarios, larger instances pose challenges within the specified time limits. Sensitivity analysis is conducted to evaluate the effect of changing essential parameters of the integrated problem on model performance and to provide managerial insights for real-life applications.

Keywords: lot sizing; cutting stock; multi-level manufacturing; mixed integer linear programming; optimization; multiple machines



Academic Editor: Wei Gao

Received: 10 May 2025

Revised: 20 June 2025

Accepted: 22 June 2025

Published: 26 June 2025

Citation: Khamis, N.; Harraz, N.; Fors, H. Minimizing Waste and Costs in Multi-Level Manufacturing: A Novel Integrated Lot Sizing and Cutting Stock Model Using Multiple Machines. *Modelling* **2025**, *6*, 56. <https://doi.org/10.3390/modelling6030056>

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1. Introduction

In today's competitive market, manufacturing firms are pressured to provide the highest quality for highly productive products at a competitive price against competitors. One effective strategy to achieve cost competitiveness involves minimizing material waste, thereby reducing overall production expenses. Moreover, firms must develop efficient production strategies, including optimal lot sizing, to prevent excessive inventory-related costs. Given the rising operational expenditure driven by inflationary pressures, it is imperative for managers to explore all feasible approaches to cost control. In this context, optimization models serve as valuable decision-support tools, enabling firms to minimize material losses and devise effective production planning methodologies.

The lot sizing problem (LSP) and cutting stock problem (CSP) are fundamental optimization challenges frequently encountered in the production planning processes of

manufacturing enterprises across various industries. LSP seeks to determine the production quantities for the products in each planning period over a limited planning horizon. The primary objective of this problem is minimizing setup, production, and holding costs. While CSP seeks to identify the optimal method for cutting large objects into smaller items to meet demand. The primary objective of this problem is to minimize the overall trim loss, thereby optimizing material utilization. Both problems are found in various industries in consecutive phases. Therefore, they should be integrated to capture the dependency existing between the decisions of these two problems. Mixed integer linear programming (MILP) is a mathematical optimization technique that extends linear programming by incorporating integer constraints, allowing for the modeling of discrete decisions. In the context of an integrated cutting stock and lot sizing model, MILP ensures efficient material utilization while optimizing production quantities. By jointly addressing cutting patterns and lot-sizing decisions, MILP minimizes waste, balances inventory levels, and enhances overall production planning in manufacturing enterprises.

Melega et al. [1] conducted a comprehensive classification and literature review of models that integrate cutting stock and lot sizing problems. Their analysis identified substantial gaps in the existing literature, particularly the lack of models that fully incorporate both problems across multiple production levels while accounting for capacity constraints and the utilization of multiple cutting machines. Moreover, according to the reviewed studies, most researchers have focused predominantly on single-level or two-level production systems, with limited attention given to capacity constraints at various stages of production. Furthermore, the inclusion of multiple cutting machines has not been extensively explored in previous research. Additionally, none of the reviewed studies have introduced the arc flow formulation for the CSP within an integrated capacitated lot sizing and one-dimensional cutting stock problem (LSCSP) framework spanning multiple production levels.

In response to these gaps, the present study aims to develop a comprehensive optimization model that concurrently addresses lot sizing and cutting stock decisions within a multi-level manufacturing environment. The objective is to minimize costs across all production levels while integrating real-world constraints, including capacity limitations at each level, setup costs and times, and the incorporation of multiple cutting machines. Thus, this study seeks to answer the following question: What are the optimal lot sizing and cutting stock decisions in a multi-level manufacturing system, considering realistic industry constraints such as capacity limitations and multiple cutting machines, to effectively minimize both material waste and overall production costs?

The main contribution of this study is twofold. First, it introduces a novel mathematical model for the integrated capacitated lot sizing and one-dimensional cutting stock problem within a multi-level production system, employing an arc flow formulation. The proposed model accounts for critical factors, including capacity constraints at each production level, setup costs and times, as well as the presence of multiple cutting machines for producing items. Second, the study conducts a comprehensive sensitivity analysis to evaluate the impact of various parameters on model performance. This analysis enhances understanding of the problem's dynamics and offers valuable insights into cost structures and the most influential factors governing the integrated decision-making process.

The structure of the paper is as follows. Section 2 gives an extensive literature review on research efforts dealing with the integrated LSCSP, along with identifying the research gaps in the reviewed articles. The proposed model for dealing with the integrated capacitated lot sizing and one-dimensional cutting stock problem in a three-level manufacturing environment is introduced in Section 3. Section 4 presents the dataset used to generate instances for computational experiments and thirty-six numerical experiments examined

to demonstrate the applicability of the proposed model. The sensitivity analysis and managerial insights related to the effect of changing essential parameters related to CSP and LSP are also illustrated in Section 4. Finally, Section 5 concludes the presented work and provides recommendations for future work.

2. Literature Review

Cutting stock and lot sizing problems appear in various industries, such as paper and furniture industries. Although there is a relationship between CSP and LSP, they are usually studied separately. However, recently, studies mentioned that better results could be achieved when handling CSP and LSP in an integrated way. This section reviews existing research on the integrated LSCSP, encompassing studies conducted across various industries as well as those that adopt a generalized modeling approach without focusing on a specific industrial context.

In the furniture industry, Gramani et al. [2] introduced a two-dimensional integrated LSCSP in a multi-level production system where final products (beds, chairs, desks, etc.) are assembled from items. These items are cut from large rectangular plates. The authors took into account the capacity of the saw-cutting machine, quantifying it as the total material area that can be cut in each period while disregarding item storage. Gramani et al. [3] extended their research and addressed the linear relaxation of the integrated problem using the column generation technique. In their extended work, they did not consider setup cost, and the model took into consideration the balance between storage of items and final products and material cost.

Santos et al. [4] presented an integrated model using rolling horizon planning. The authors assumed several object types that vary only in thickness with unbounded capacity. However, a drawback of their model is the utilization of pre-generated cutting patterns for the CSP. Alem and Morabito [5] conducted their work in a Brazilian furniture factory, where wooden plates go through different processes after being cut into smaller pieces, such as drilling, painting, and assembly. They considered stochastic demand in addition to uncertainty in setup times and drill machine capacity. A deterministic model and a two-stage stochastic model with various risk management techniques were provided by the authors. The authors in a previous work considered backlogging and overtime throughout the planning horizon [6]. Vanzela et al. [7] studied the integrated LSCSP in a small-scale furniture factory. They applied column generation to solve the linear relaxation of the proposed model. They compared the outcomes obtained from integrating the problems with the outcomes obtained from sequential solving of the problems. The integrated model performed better in reducing raw material costs and total inventory. Additional computational tests were implemented to show the impact of changing inventory and material costs on the presented approaches (sequential and integrated approaches). These approaches were presented in an earlier study [8].

In the paper industry, production lines usually include large-scale processes classified into three phases. The first phase is concerned with the production of jumbos with different types of paper. In the second phase, these jumbos are cut to produce reels with varied sizes. In the last phase, the reels are cut into paper sheets according to demand [9].

Poltroniere et al. [10] addressed the integration of a lot sizing problem with a one-dimensional cutting stock problem. They used two distinct sequences to tackle each problem independently. Initially, the LSP was solved to determine the number of jumbos to be produced. Subsequently, the CSP was tackled, taking into account the jumbos produced in the LSP phase. While this sequence provided a satisfactory solution in terms of jumbos production, it led to an increase in material waste. In the second sequence, they addressed the CSP, assuming unlimited stock of the jumbos. After that, the LSP was employed to

generate the production plan. While this approach yielded significant waste reduction, it incurred high setup costs. Computational experiments indicated that the second procedure outperforms the first one. Poltroniere et al. [11] extended their previous work by allowing the inventory of items. They also proposed using arc flow formulation for the CSP, the innovative work of Valério de Carvalho [12], which proved to achieve a better upper bound.

Leao et al. [13] addressed the integrated problem by considering multiple machines of different sizes and limited capacities. They proposed three mathematical formulations with different decomposition orientations for items, patterns, and machines. Ayres et al. [9] presented a model for integrating lot sizing with one-dimensional and two-dimensional cutting stock problems. Their model covers the three previously mentioned production phases. The authors mentioned that 16.45% gains were observed when compared to the case of no integration among the three phases. Pierini and Poldi [14] presented the integrated problem within a multi-plant environment in the paper industry. The study focuses on optimizing production, cutting, transportation, and storage decisions across multiple factories in the paper industry, considering the specific complexities of plant-to-plant transfers and cost variations among plants. They solved the model by combining column generation, a relax-and-fix heuristic, and a feasibility heuristic to efficiently solve the complex multi-plant integration with LSCSP.

The integration between LSP and CSP has been introduced in the aluminum, automotive, concrete, and aircraft industries. Suliman [15] addressed the integrated problem in the aluminum fabrication industry. The authors developed the problem as a non-linear integer programming model. They introduced an algorithm to solve LSP first, then CSP at each planning period. The algorithm showed good results when compared with what is exercised in the aluminum fabrication industry. Andrade et al. [16] presented the integrated problem in automotive spring factory. The authors formulated a mathematical model to reduce the inventory costs and losses in the cutting process of steel bars. The model considers the presence of parallel cutting machines with limitations on the number of different items that can be cut per cutting pattern and limitations on the thickness and lengths that can be used over each machine. They solved the model with real data, and the results showed far better solutions than the company's current practice. Andrade et al. [17] investigated the integrated problem within a truck suspension manufacturing context. They proposed a mixed-integer programming model that simultaneously addresses three hierarchical production stages: raw materials (steel bars), intermediate items (springs), and final products (spring bundles), with the objective of minimizing overall production and inventory holding costs. The authors applied a solution methodology based on column generation and the simplex method for solving the linear relaxation, followed by an integer solution procedure to obtain feasible solutions. The effectiveness of the proposed model was demonstrated through both real-world and randomly generated instances.

Signorini et al. [18] addressed the production of hollow-core slabs at a factory. The production planning of these structures is studied as one-dimensional CSP with multiple periods. They proposed two models based on the formulations of Kantorovich [19] and Gilmore and Gomory [20]. The computational results were achieved using real data, and both formulations delivered the same optimal solution. Sanan and Azizoglu [21] developed an integrated two-dimensional LSCSP tailored to an aircraft manufacturing environment. Their study addresses two inherently conflicting objectives: minimizing the number of steel panels utilized in cutting operations and reducing the total inventory holding cost of the resulting items. The authors formulated a mixed-integer linear programming model to systematically generate the complete set of non-dominated vectors for the stated objectives. Computational experiments demonstrated that problem instances involving fewer items

could be efficiently solved over planning horizons of up to 14 periods, whereas those with a larger number of items were solved for up to seven periods.

Regarding the generalized models, Silva et al. [22] studied the case of integrated two-dimensional LSCSP, considering allowing large leftovers to be stored and cut in the next period. The authors proposed two integer programming models. From the models' results, they concluded that neither of the models is superior to the other. They also proposed two heuristics and compared their solutions with optimum ones obtained from the models. Melega et al. [23] presented different formulations for single-level one-dimensional cutting stock and lot sizing problems. In the single-level problem, after cutting the object, the final products (items) are produced immediately without requiring additional processing. Alternative mixed integer programming (MIP) models were proposed for the integrated problem. The authors utilized the CPLEX optimization package and column generation technique to solve the models. The experimental results revealed the challenge of obtaining feasible solutions in capacitated models. Furthermore, the arc flow model yielded the highest number of feasible solutions across various experiments compared to the other proposed models. Poldi and de Araujo [24] presented the generalization of models of Gilmore and Gomory [20] and Valério de Carvalho [12] to handle the multi-period one-dimensional cutting stock problem with several object types. They mentioned that the arc flow model has advantages over the Gilmore and Gomory model. Based on the computational experiments conducted, it was concluded that the arc flow model could solve some instances with the CPLEX optimization package, and the proposed heuristic could solve hard instances. Ma et al. [25] addressed the capacitated multi-period one-dimensional CSP with pattern setup cost. The problem was to define the patterns and their occurrence in each period to minimize pattern setup, inventory, and material consumed costs. The authors proposed two models based on the formulation of the arc flow model and Gilmore and Gomory [20]. Also, two heuristics were introduced to solve the models. Computational results were achieved using randomly generated instances. The Gilmore and Gomory model worked better in solving the addressed problem due to the massive rise in the number of arcs in the other model. The two heuristics showed comparable results with good solutions in a reasonable computational time. do Nascimento et al. [26] tackled the integrated lot sizing and one-dimensional CSP with usable leftovers. A column generation method was used to solve the model's linear relaxation, and a heuristic procedure was employed to find integer solutions. Curcio et al. [27] considered uncertainty in demand and presented stochastic programming and robust optimization models to address the integrated LSCSP. The authors developed a heuristic procedure based on the column generation technique to generate the cutting patterns for solving the proposed models, which has proven to achieve high-quality solutions.

Table 1 provides a comprehensive summary of the reviewed studies addressing the integrated LSCSP. The initial three rows detail the characteristics associated with the CSP, including the problem's dimensionality, the inclusion of single or multiple object types, and the foundational approach to formulating the CSP. Subsequent rows focus on features related to the LSP and the structure of the proposed mathematical models. These characteristics encompass the type of inventory considered, the representation of setup conditions, the inclusion of capacity constraints, the model's decision variables, production structure (single- or multi-level), additional LSP-specific considerations, elements of the objective function, the proposed solution methodology, classification of the mathematical model, and whether the model is tailored to a specific industry or represents a generalized framework. The studies in this table are organized chronologically, from the earliest to the most recent.

Table 1. Summary of integrated LSCSP literature.

Characteristics	Reference	Poltroniere et al. [10]	Gramani et al. [2]	Gramani et al. [3]	Santos et al. [5]	Alem and Morabito [6]	Suliman [15]
Dimensionality	One-dimensional	•					•
	Two-dimensional		•	•	•	•	
Object Type	One type		•	•		•	•
	Several types	•			• (t)		
CSP Formulation	Gilmore and Gomory						
	Valerio de Carvalho						
	Other	•	•	•	•	•	•
Type of Inventory	Objects	•					•
	Items	•		•	•	•	•
	Residual pieces						
	Final products		•	•	•		•
Setup	Cost	•	•		•		
	Time				•		
Capacity	Capacity level	• (o)	• (m)	• (m)			
Decision Variables	Production quantity	• (o)	• (f)	• (f)	• (f)	• (i)	• (f)
	Inventory	• (o&i)		• (f)	• (i&f)		• (f)
	Setup				• (m)	• (i)	
	No. of residual pieces						
Number of Production Levels	Single level	•					•
	Multi level		•	•	•	•	
Other LSP Considerations	Safety stock						
	Multiple machines	•			•		
Objective Function Elements	Production cost	• (o)		• (f)	• (f)	• (o)	• (o)
	Trim loss cost	•	•	•	•	•	
	Setup cost	•	•		• (m)	•	•
	Storage cost	• (o&i)	• (i)	• (i&f)	• (i&f)	• (i)	
	Overtime					•	
	Non-fulfillment						•
	Purchasing cost						•
	Backlogging					•	
Solution Approach	Exact			•	•	•	•
	Heuristic	•	•				•
Mathematical Model Type	ILP						
	MILP	•	•	•	•	•	
	INLP						•
Application	Paper industry	•					
	Furniture		•	•	•	•	
	General						
	Others						•

Table 1. Cont.

Characteristics	Reference	Vanzela et al. [8]	Alem and Morabito [5]	Silva et al. [22]	Poldi and de Araujo [24]	Poltroniere et al. [11]	Melega et al. [23]
Dimensionality	One-dimensional				•	•	•
	Two-dimensional	•	•	•			
Object Type	One type	•	•				•
	Several types			•	•	•	•
CSP Formulation	Gilmore and Gomory				•	•	•
	Valerio de Carvalho				•	•	•
	Other	•	•	•			•
Type of Inventory	Objects				•	•	
	Items	•	•	•	•	•	•
	Residual pieces			•			
	Final products	•					
Setup	Cost				•		•
	Time		•				•
Capacity	Capacity level	• (m)	• (m)		• (o)	• (m)	• (i)
Decision Variables	Production quantity	• (f)	• (i)	• (i)	• (o)	• (o)	• (i)
	Inventory	• (i&f)		• (i)	• (f)		• (i)
	Setup		• (i)				• (i)
	No. of residual pieces			•			
Number of Production Levels	Single level			•			•
	Multi level	•	•		•	•	
Other LSP Considerations	Safety stock	•					
	Multiple machines	•				•	
Objective Function Elements	Production cost	• (f)	• (o)		• (o)		
	Trim loss cost	•	•	•	•	•	•
	Setup cost				•		•
	Storage cost	• (i&f)	• (i)	• (i)	• (o&f)	• (o&i)	• (i)
	Overtime		•				
	Non-fulfillment						
	Purchasing cost						
	Backlogging		•				
Solution Approach	Exact		•				•
	Heuristic	•		•	•	•	
Mathematical Model Type	ILP			•			
	MILP	•	•		•	•	•
	INLP						
Application	Paper industry				•		
	Furniture	•	•				
	General			•		•	•
	Others						

Table 1. Cont.

Characteristics	Reference	Vanzela et al. [7]	Leao et al. [13]	Ma et al. [25]	do Nascimento et al. [26]	Ayres et al. [9]	Andrade et al. [16]
Dimensionality	One-dimensional		•	•	•	•	•
	Two-dimensional	•				•	
Object Type	One type			•			
	Several types	• (t)	•	•	•	•	•
CSP Formulation	Gilmore and Gomory			•	•	•	•
	Valerio de Carvalho			•			
	Other	•	•			•	
Type of Inventory	Objects		•			•	
	Items	•	•	•		•	•
	Residual pieces				•		
	Final products	•				•	•
Setup	Cost		•	•			
	Time		•				
Capacity	Capacity level	• (m)	• (m)	• (o)		• (o&m)	
Decision Variables	Production quantity		• (o)		• (o)&(i)	• (o)	• (i)&(f)
	Inventory	• (f)	• (o)&(i)	• (i)	• (i)&(l)	• (o&i&f)	• (i)&(f)
	Setup		• (o)	• (p)			
	No. of residual pieces				•		
Number of Production Levels	Single level		•		•		•
	Multi level	•		•		•	
Other LSP Considerations	Safety stock	•					
	Multiple machines		•			•	•
Objective Function Elements	Production cost	• (o&f)	• (o)			• (o)	
	Trim loss cost		•	•	•	•	•
	Setup cost		• (o)	•			
	Storage cost	• (i&f)	• (o)&(i)	• (i)	• (i)&(l)	• (o&i&f)	• (i)&(f)
	Overtime						
	Non-fulfillment						
	Purchasing cost						
	Backlogging						
Solution Approach	Exact						
	Heuristic	•	•	•	•	•	•
Mathematical Model Type	ILP						
	MILP	•	•	•	•	•	•
	INLP						
Application	Paper industry		•			•	
	Furniture	•					
	General			•			
	Others						•

Table 1. Cont.

Characteristics	Reference	Signorini et al. [18]	Curcio et al. [27]	Pierini and Poldi [14]	Sanan & Azizoglu [21]	Andrade et al. [17]	This Study
Dimensionality	One-dimensional	•		•		•	•
	Two-dimensional		•		•		
Object Type	One type		•		•		
	Several types	•		•		•	•
CSP Formulation	Gilmore and Gomory	•					
	Valerio de Carvalho						•
	Other	•	•	•	•	•	
Type of Inventory	Objects			•		•	•
	Items	•	•	•	•	•	•
	Residual pieces						
	Final products					•	•
Setup	Cost			•			•
	Time						•
Capacity	Capacity level	• (o)	• (i)	• (m)		• (m)	• (o&i&f)
Decision Variables	Production quantity	• (i)	• (i)	• (o&i)	• (o&i)	• (i&f)	• (o&i&f)
	Inventory	• (i)	• (i)	• (o&i)	• (i)	• (o&i&f)	• (o&i&f)
	Setup	• (o)		• (m)			• (o&m&f)
	No. of residual pieces						
Number of Production Levels	Single level	•	•		•		
	Multi level			•		•	•
Other LSP Considerations	Safety stock						
	Multiple machines			•			•
Objective Function Elements	Production cost	• (o)		• (o)	• (o)		• (o&i&f)
	Trim loss cost	•	•	•		•	•
	Setup cost			• (o)			• (o&m&f)
	Storage cost	• (i)	• (i)	• (o&i)	• (i)	• (o&i&f)	• (o&i&f)
	Overtime						
	Non-fulfillment						
	Purchasing cost						
	Backlogging		•				
Solution Approach	Exact				•		•
	Heuristic	•	•	•		•	
Mathematical Model Type	ILP			•			
	MILP	•	•		•	•	•
	INLP						
Application	Paper industry			•			
	Furniture						
	General		•				•
	Others	•			•	•	

Notes: o: objects, i: items, f: final products, m: cutting machines, l: leftovers, t: thickness, p: pattern, ILP: integer linear programming, MILP: mixed integer linear programming, and INLP: integer non-linear programming.

Based on the literature explored, the following findings are concluded:

- **Cutting stock problem:** The CSP is almost divided evenly between one and two-dimensional CSP, and the most recent papers are studying the one-dimensional problem. The one-dimensional case appears widely in the paper industry, while the two-dimensional case appears in the furniture industry. Most of the recent papers consider the presence of multiple types of objects. The most used formulations for CSP are Gilmore and Gomory and Valerio de Carvalho, and some authors compared them.
- **Lot sizing problem:** Most papers considered inventory of items, and less than half of the papers considered inventory of objects and final products. Regarding including setup in LSP, setup cost was included in approximately 40% of the papers, and few of them incorporated setup time. The authors concentrated on handling the integrated problem across either a single or two levels, where two operations are sequentially executed. There is a scarcity in integrating both problems over three production levels. Few research works considered the capacity constraints in the different production levels, and the capacity of cutting machines was the focus of the researchers. Multiple cutting machines were not considerably included in the reviewed papers.
- **Mathematical model:** The primary objective in most of the models was to minimize material waste and storage costs. In addition to these primary objectives, some authors also considered production or setup costs. The integrated problem is mostly formulated as a mixed integer linear programming model.
- **Application area:** The integrated problem appeared widely in the paper and furniture industries.

A thorough classification and literature review of the models combining the cutting stock and lot sizing problems was carried out by Melega et al. [1]. Significant gaps in the literature were noted by the authors, especially the absence of models that fully integrate both problems at various production levels, consider capacity constraints into account, and take into account multiple machines for cutting stock problems. Hence, the primary contribution of this study is the development of a novel mathematical model for the integrated one-dimensional LSCSP, incorporating three production levels and multiple cutting machines. The model is constructed based on arc flow formulation. To the best of our knowledge, the arc flow CSP formulation for the integrated three-level LSCSP with capacity limits, setup times and costs, and multiple cutting machines were not presented together in any of the reviewed literature.

3. Proposed Model for the Integrated Problem

The developed model in this research work is an extension of our previous work [28], where capacity constraints for the production of final products are introduced, leading to having capacity constraints over the different production levels. Also, more extensive computational experiments are deployed to evaluate model performance against changes in the number of periods and the number of different types of objects, items, and final products. Lastly, a sensitivity analysis is performed to assess the effect of changing model parameters on model performance, giving a better understanding of the problem and providing insights regarding different costs and the most effective parameters of the integrated problem.

3.1. Model Description

Consider a three-level manufacturing framework, where objects are cut into items, and these items are assembled into final products. Initially, quantities of objects produced/purchased and stored are decided. Then, in the second level, the cutting patterns for these objects are specified over the available cutting machines, and the amounts of pro-

duced and stored quantities for each item are determined. Subsequently, the acquired items in the second level are assembled into final products at the third level, where production and stored quantities of these final products are determined.

A mixed integer linear programming model (MILP) is developed to solve the integrated capacitated lot sizing problem (LSP) and cutting stock problem (CSP). The model minimizes different production-related costs at all production levels and minimizes waste material costs. The integration between CSP and LSP in a three-level manufacturing environment with multiple time periods is illustrated in Figure 1.

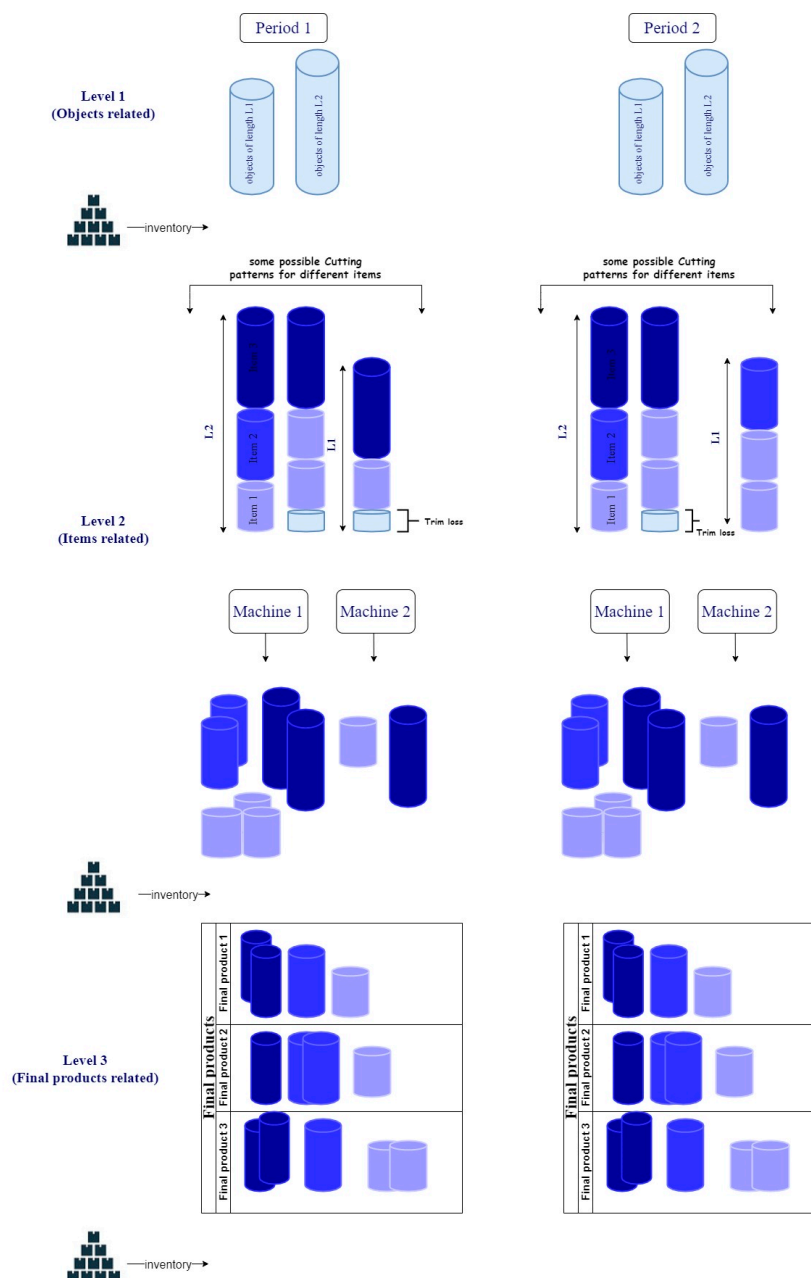


Figure 1. The three production levels for the integrated lot sizing and cutting stock problem over two periods.

Although alternative modeling strategies—such as decomposition techniques and heuristic frameworks—may enhance scalability for large-scale problem instances, this study adopts a monolithic MILP formulation to achieve an integrated and exact representation of the lot sizing and cutting stock problem. This modeling choice facilitates the explicit

capture of interdependence between the decisions of these two problems, thereby enabling the generation of exact solutions and the execution of in-depth sensitivity analyses. Furthermore, the incorporation of an arc flow formulation for the cutting stock problem enhances the strength and compactness of the overall MILP model while minimizing reliance on heuristic pattern generation.

The proposed model is based on the WWVCMO (Wagner and Whitin, Valério De Carvalho, and Multiple object types) model presented by Melega et al. [23]. They dealt with one-to-one systems, where final products are produced directly from raw material after performing a single process with no further subassemblies. However, the proposed model considers three production levels in addition to considering different cutting machines in the problem. The authors also neglected production cost and capacity constraints.

All feasible cutting patterns are generated according to the arc-flow model proposed by Valério de Carvalho [12] without limitations on the number of used cutting patterns in each period or the maximum number of items in each cutting pattern. The arc-flow model is a well-known method for solving CSP. The model determines a valid solution for different objects of length L_k by considering a path in a directed acyclic graph $G = (V, A)$, with vertices $V = \{0, 1, \dots, \max(L_k)\}$ and set of arcs in the graph $A = \{(g, l); 0 \leq g < l_i < L_k \text{ and } l - g = l_i \text{ for all } i \leq I\}$ where l_i is the item length. Furthermore, the losses generated from cutting objects are represented as additional arcs between $(g, g + 1)$ vertices.

The proposed model is formulated under the assumption of deterministic demand within a fixed planning horizon; however, its formulation permits extension to dynamic planning environments. Specifically, it can be incorporated into a rolling horizon framework in which updated demand information is periodically introduced and the model re-optimized accordingly. Such adaptability enhances the model's relevance to industrial applications characterized by evolving production requirements.

3.2. Model Assumptions

- No initial inventory is considered over the three production levels;
- No limitations on stored quantities over the three production levels;
- Any item produced at a certain time period can be used in producing the final products within the same time period;
- Cutting machines have different capacities and costs;
- Safety stock levels are included in the final products' demand.

3.3. Model Elements

3.3.1. Indexes

$t = 1, \dots, T$: number of time periods;

$k = 1, \dots, K$: number of objects;

$i = 1, \dots, I$: number of items;

$f = 1, \dots, F$: number of final products;

$m = 1, \dots, M$: number of cutting machines.

3.3.2. Parameters

L_k	Object length of type k ;
sck_{kt}	setup/ordering cost of object type k in time period t ;
stk_{kt}	setup time of object type k in period t ;
vck_{kt}	production/purchasing cost of object type k in time period t ;
vtk_{kt}	production time of object type k in time period t ;
$Capk_t$	production time capacity to produce objects in time period t ;
hck_{kt}	holding cost of object type k in period t ;

l_i	item length of type i ;
cw	unit length cost of raw material waste;
scm_{mt}	setup cost of machine type m in time period t ;
stm_{mt}	setup time of machine type m in time period t ;
vc_{kmt}	production cost of cutting object type k on machine m in time period t ;
vtc_{kmt}	production time of cutting object type k on machine m in time period t ;
$Capi_{mt}$	cutting time capacity of machine m in time period t ;
hci_{it}	holding cost of item type i in time period t ;
d_{ft}	demand of final product type f in time period t ;
r_{if}	number of required items of type i for assembling final product f ;
scf_{ft}	setup cost of final product type f in time period t ;
stf_{ft}	setup time of final product type f in time period t ;
vcf_{ft}	production cost of final product type f in time period t ;
vtf_{ft}	production time of final product type f in time period t ;
$Capf_t$	production time capacity to produce final products in the time period t ;
hcf_{ft}	holding cost of final product type f in time period t ;
M	big positive number.

3.3.3. Decision Variables

Xk_{kt}	Quantity produced/purchased of object type k in the time period t ;
Yk_{kt}	binary variable that indicates whether there is a production of object type k or not in the time period t ;
Sk_{kt}	quantity stored of object type k at end of the time period t ;
fl_{kmt}	flow through the network of object type k cut on machine type m in the time period t ;
Xi_{imt}	quantity produced of item type i on machine type m in the time period t ;
Ym_{mt}	binary variable that indicates whether machine type m is utilized or not in the time period t ;
Si_{it}	quantity stored of item type i at the end of the time period t ;
z_{glmt}	number of items cut from an object on machine type m in the time period t , corresponding to an item of length (l-g) allocated at a distance g from the beginning of the object (considering all cutting patterns);
Xf_{ft}	quantity produced of final product type f in the time period t ;
Yf_{ft}	binary variable that indicates whether there is a production of final product type f or not in the time period t ;
Sf_{ft}	quantity stored of final product type f at the end of time period t .

3.3.4. Objective Function

The objective function seeks to reduce the various costs related to the three production levels (objects, items, and final products). The object-related costs are setup/ordering, production/purchasing, and holding costs. The item-related costs involve machine setup and production, holding, and waste material costs. Lastly, final product costs include setup, production, and holding costs.

$$\begin{aligned}
& \text{Min} \sum_{t=1}^T \sum_{k=1}^K sck_{kt} Yk_{kt} + vck_{kt} Xk_{kt} + hck_{kt} Sk_{kt} \\
& + \sum_{t=1}^T \sum_{m=1}^M scm_{mt} Ym_{mt} + \sum_{t=1}^T \sum_{m=1}^M \sum_{k=1}^K vc_{kmt} fl_{kmt} + \sum_{t=1}^T \sum_{i=1}^I hci_{it} Si_{it} \\
& + cw \left(\sum_{t=1}^T \sum_{m=1}^M \sum_{k=1}^K L_k fl_{kmt} - \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^I \sum_{(l,l+l_i) \in A} l_i z_{l,l+l_i,mt} \right) \\
& + \sum_{t=1}^T \sum_{f=1}^F scf_{ft} Yf_{ft} + vcf_{ft} Xf_{ft} + hcf_{ft} Sf_{ft}
\end{aligned} \tag{1}$$

3.3.5. Constraints

$$Xk_{kt} + Sk_{k,t-1} = \sum_{m=1}^M fl_{kmt} + Sk_{kt}, \forall k, t \tag{2}$$

$$\sum_{m=1}^M Xi_{imt} + Si_{i,t-1} = \sum_{f=1}^F r_{if} Xf_{ft} + Si_{it}, \forall i, t \tag{3}$$

$$Xf_{ft} + Sf_{f,t-1} = d_{ft} + Sf_{ft}, \forall f, t \tag{4}$$

$$Xk_{kt} \leq M Yk_{kt}, \forall k, t \tag{5}$$

$$\sum_{i=1}^I Xi_{imt} \leq M Ym_{mt}, \forall m, t \tag{6}$$

$$Xf_{ft} \leq M Yf_{ft}, \forall f, t \tag{7}$$

$$-\sum_{(0,l) \in A} z_{0lmt} = -\sum_{k=1}^K fl_{kmt}, \forall m, t \tag{8}$$

$$\sum_{(g,l) \in A} z_{glmt} - \sum_{(l,h) \in A} z_{lhmt} = 0, l = \{1, \dots, L_{max} - 1\} (l \neq L_k, \forall k), \forall m, t \tag{9}$$

$$\sum_{(g,L_k) \in A} z_{gL_kmt} = fl_{kmt}, \forall k, m, t \tag{10}$$

$$\sum_{(h,h+l_i) \in A} z_{h,h+l_i,mt} = Xi_{imt}, \forall i, m, t \tag{11}$$

$$\sum_{k=1}^K stk_{kt} Yk_{kt} + vtk_{kt} Xk_{kt} \leq Capk_t, \forall t \tag{12}$$

$$\sum_{k=1}^K vtc_{kmt} fl_{kmt} + Stm_{mt} Ym_{mt} \leq Capi_{mt}, \forall m, t \tag{13}$$

$$\sum_{f=1}^F stf_{ft} Yf_{ft} + vtf_{ft} Xf_{ft} \leq Capf_t, \forall t \tag{14}$$

$$Xk_{kt}, Sk_{kt} \geq 0, \forall k, t \tag{15}$$

$$Xi_{imt} \geq 0, \forall i, m, t \tag{16}$$

$$Si_{it} \geq 0, \forall i, t \quad (17)$$

$$Xf_{ft}, Sf_{ft} \geq 0, \forall f, \forall t \quad (18)$$

$$Yk_{kt} \in \{0, 1\}, \forall k, t \quad (19)$$

$$Ym_{mt} \in \{0, 1\}, \forall m, t \quad (20)$$

$$Yf_{ft} \in \{0, 1\}, \forall f, t \quad (21)$$

$$zgl_{mt} \in \mathbb{Z}_+, \forall (g, l) \in A, m, t \quad (22)$$

$$fl_{kmt} \in \mathbb{Z}_+, \forall k, m, t \quad (23)$$

Constraints (2) and (3) are inventory balance constraints for the objects and items production levels. For any production level and during each period, the quantities produced in this period and the stored quantities from the previous period should be equal to the required and stored quantities for that period. Constraint (4) is the demand balance constraint for final products. Constraint (5) assures that the setup variable of objects Yk_{kt} is assigned a value of one if production takes place at that period. Constraint (6) forces the machine setup variable Ym_{mt} to have a value of one if the production of items takes place on that machine. Constraint (7) guarantees that the setup variable of the final products Yf_{ft} is assigned a value of one if production takes place at that period. The flow conservation constraints (8)–(10) are related to the arc-flow model. Constraint (11) combines the decision of cutting stock and lot sizing problems. It ensures that there are enough items cut to meet the anticipated amounts of parts that need to be assembled into finished products. Capacity constraints (12)–(14) ensure that the available production time for producing objects, items, and final products is not exceeded in each period. Constraints (15)–(23) determine the type of decision variable.

4. Numerical Experiments, Sensitivity Analysis, Managerial Insights

4.1. Numerical Experiments

The performance of the proposed model to solve the integrated LSCSP is assessed through numerical experiments. It is evaluated using thirty-six instances. The instances vary in the number of objects, items, and final products and in the number of periods. The experimental datasets, instances' classes, and computational results of these instances are presented in the following sub-sections. The computational experiments were performed on a computer configured with a 2.4 GHz Intel Core i5 processor and 6 GB of RAM. GUROBI optimizer software (version 9.1) was used to solve the developed mathematical model.

4.2. Data Generation

The dataset used to generate instances for computational tests is based on data presented in previous literature addressing lot sizing and cutting stock problems. The CUT-GEN1 generator, developed by Gau and Wäscher [29] for the one-dimensional Cutting Stock Problem (CSP), is utilized in this study. Although CUTGEN1 is designed to handle instances involving a single object length, the proposed model incorporates multiple object lengths. Accordingly, the generation of varying object lengths was adapted based

on the original CUTGEN1 framework. The lot sizing problem (LSP) data are based on Trigeiro et al. [30], other literature, or randomly generated. Table 2 illustrates the value/interval of the model parameters.

Table 2. Value/interval of the model parameters.

Parameter	Value/Interval
Object length (L_k)	[300, 1000]
Item length (l_i)	[0.1 $\bar{L}$, 0.4 $\bar{L}$]
Average object length ($\bar{L}$)	$\frac{\sum_{k=1}^K L_k}{K}$
Setup cost of object, machine, final product ($sck_{kt}, scm_{mt}, scf_{ft}$)	[100, 500]
Setup time of object, machine, final product ($stk_{kt}, stm_{mt}, stf_{ft}$)	[10, 50]
Production/purchasing cost of object (vck_{kt})	0.05 L_k
Production time of object (vtk_{kt})	0.01 L_k
Production cost of cutting an object on machine (vc_{kmt})	0.1 L_k
Production time of cutting an object on machine (vtc_{kmt})	0.02 L_k
Production cost of final product (vcf_{ft})	2
Production time of final product (vtf_{ft})	5
Holding cost of object (hck_{kt})	βL_k
Holding cost of item (hci_{it})	αl_i
Holding cost of final product (hcf_{ft})	$\gamma \max(hci_{it}) \times 10$
Inventory factors to calculate holding costs of object and item (α, β)	{0.0, 0.01, 0.1, 0.5}
Inventory factors to calculate holding cost of final product (γ)	{0.0, 0.5, 1.0, 5.0}
Unit length cost of raw material waste (cw)	1
Demand of final product (d_{ft})	[1, 50]
Number of required items for assembling final product (r_{if})	[0, 2]
Number of cutting machines (m)	2

The capacity is generated based on the average of lot-by-lot policies. The number of resources needed to produce exactly the demand in each period is determined. Then, the capacity is computed by dividing the total number of resources for all periods by the number of periods. It should be noted that the expected average number of necessary objects DK_{avg} must be computed to determine $Capk_t$ and $Capi_{mt}$.

$$DK_{avg} = \frac{\sum_{t=1}^T \sum_{f=1}^F \sum_{i=1}^I d_{ft} r_{if} l_i}{\bar{L}}, \text{ where } \bar{L} = \frac{\sum_{k=1}^K L_k}{K}$$

$$Capk_t = \frac{\sum_{t=1}^T \sum_{k=1}^K DK_{avg} * vtk_{kt} + stk_{kt}}{TK}$$

$$Capi_{mt} = \frac{\sum_{t=1}^T \sum_{m=1}^M \sum_{k=1}^K DK_{avg} * vtc_{kmt} + stm_{mt}}{TM}$$

$$Capf_t = \frac{\sum_{t=1}^T \sum_{f=1}^F d_{ft} * vtf_{ft} + stf_{ft}}{TF}$$

4.3. Instances' Classes

The test instances are categorized into 12 distinct classes, each characterized by varying numbers of items and time periods, as detailed in Table 3. This classification yields a total of thirty-six instances.

Table 3. Defining instances' classes.

Instance	No. of Items	No. of Periods	No. of Objects	No. of Final Products	Instance	No. of Items	No. of Periods	No. of Objects	No. of Final Products
1	5	2	2	2	19	15	2	2	2
2			4	3	20			4	3
3			6	4	21			6	4
4	5	4	2	2	22	15	4	2	2
5			4	3	23			4	3
6			6	4	24			6	4
7	5	6	2	2	25	15	6	2	2
8			4	3	26			4	3
9			6	4	27			6	4
10	10	2	2	2	28	20	2	2	2
11			4	3	29			4	3
12			6	4	30			6	4
13	10	4	2	2	31	20	4	2	2
14			4	3	32			4	3
15			6	4	33			6	4
16	10	6	2	2	34	20	6	2	2
17			4	3	35			4	3
18			6	4	36			6	4

4.4. Computational Results of Instances

The previously mentioned thirty-six instances are solved using GUROBI optimization software with a time limit of two hours. The input data used in instances is randomly generated. After solving each instance, different outputs were observed, as summarized in Table 4. The material waste and material waste percentage are calculated using the following formulas. The material waste percentage is calculated by dividing the material waste by the total length of used objects.

$$\text{Material Waste} = \sum_{t=1}^T \sum_{m=1}^M \sum_{k=1}^K L_k f l_{kmt} - \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^I l_i X i_{imt}$$

$$\text{Material Waste (\%)} = \frac{MW}{\sum_{t=1}^T \sum_{m=1}^M \sum_{k=1}^K L_k f l_{kmt}} * 100$$

Optimal solutions were reached in 19 instances, as depicted in Table 4. The optimization software was unable to find a solution in the specified time in six instances (30, 32, 33, 34, 35, and 36). It should be noted that if the time limit is increased to three hours to solve instance number 36 for example, a solution could be obtained with an objective function value of 464,185, 0.36% Gap (%), and material waste (%) of value 0.021%. In the remaining instances, the optimization software reached a solution within the time limit, with a Gap (%) less than 0.61%. As indicated in Table 4, an increase in the number of time periods—as observed in instances 8, 9, 10, 16, 17, 18, 24, 25, and 26—leads to a notably higher Gap (%) compared to other instances. The analysis further reveals that material

waste tends to decrease as the number of objects with varying lengths increases, thereby improving material utilization. In general, the computational complexity of the problem escalates with the number of periods and items, resulting in an exponential growth in the time required to obtain an optimal solution. Moreover, the complexity is further exacerbated by the inclusion of items with differing lengths, which increases the number of nodes. This effect is particularly pronounced when item lengths are relatively small in comparison to the length of the available objects.

Table 4. Results of the instances using the GUROBI optimization package.

Instance	Objective Function Value	Gap (%)	Lower Bound	Computational Time (seconds)	Material Waste	Material Waste (%)	No. of Nodes
1	19,724.7	0.00	19,722.7	371.51	219	0.199	10,192
2	17,181.6	0.00	17,181.6	89.24	81	0.089	1850
3	25,233.9	0.00	25,231.6	82.78	5	0.003	9632
4	22,800.4	0.00	22,799.4	339.38	576	0.496	7686
5	50,724.2	0.00	50,724.2	199.76	1044	0.358	4348
6	39,775.7	0.00	39,773.5	1897.12	206	0.096	27,193
7	14,253.9	0.00	14,253.9	49.68	469	0.801	524
8	52,542.7	0.17	52,452.3	7200.19	544	0.191	52,220
9	77,912.3	0.06	77,860.8	7200.24	1696	0.39	66,484
10	41,411.5	0.05	41,388.3	7201.67	623	0.249	500,964
11	18,640.9	0.00	18,640.9	178.27	362	0.358	1321
12	70,412.5	0.00	70,405.7	454.63	355	0.080	7075
13	43,840.9	0.01	43,836.5	6557.63	883	0.350	174,909
14	87,617.4	0.01	87,600.4	7202.69	759	0.141	186,336
15	116,252	0.01	116,230.7	7200.35	167	0.022	77,910
16	112,293	0.60	111,618.1	7200.13	1681	0.246	18,943
17	144,217	0.29	143,793.9	7200.48	407	0.045	16,339
18	181,211	0.24	180,767.6	7200.05	943	0.083	15,836
19	65,535.4	0.00	65,528.8	1123.81	1393	0.340	74,304
20	59,811.1	0.01	59,801.0	7200.29	12	0.003	114,555
21	97,321.1	0.00	97,312.8	660.10	31	0.004	1431
22	70,141.6	0.00	70,137.7	727.47	27	0.006	3033
23	143,671	0.00	143,657.8	4853.52	299	0.032	48,380
24	125,631	0.61	124,853.4	7200.08	61	0.007	4404
25	110,676	0.40	110,225.2	7200.12	27	0.003	11,645
26	160,181	0.29	159,710.8	7200.05	108	0.010	13,180
27	97,321.1	0.00	97,312.8	782.96	31	0.004	1431
28	30,261.6	0.01	30,257.8	7200.36	104	0.057	159,310
29	68,549.5	0.00	68,544.0	1062.46	28	0.006	1058
31	59,493	0.02	59,480.2	7200.61	857	0.239	192,541

4.5. Sensitivity Analysis

The proposed model is analyzed under different settings. The subsequent sub-sections provide a detailed analysis of the model's behavior in response to variations in key parameters associated with the cutting stock and lot sizing problems. The parameters examined include the ratio of item length to object length, material waste cost, setup costs, capacity constraints, and holding costs.

4.5.1. Studying Changing Items' Length with Respect to Object Length

A critical parameter in the CSP that influences the feasibility of cutting items of varying lengths from a single object with minimal material waste is the relationship between item length and object length. To investigate this relationship and its impact on material waste, item lengths are systematically adjusted relative to object length by varying two coefficients, denoted as a and b , which define the item length specification. The item's length is calculated using the following equation $l_i \in [a\bar{L}, b\bar{L}]$, where $\bar{L} = \frac{\sum_{k=1}^K L_k}{K}$. A different dataset is used in each instance while changing the two factors (a and b). Table 5 illustrates the results obtained from the different instances when changing items' length with respect to object length. The results indicate that smaller item lengths are associated with lower material waste percentages—including scenarios with zero waste—whereas waste percentages tend to increase as item lengths grow.

Table 5. Results obtained when changing items' length with respect to object length.

Instance Size	a	b	Average Object Length	Average Item Length	Objective Function Value	Gap (%)	Computational Time (seconds)	Material Waste	Material Waste (%)
Small	0.02	0.1	479.0	38.6	11,034.8	0.00	1813.60	205	0.576
	0.04	0.2	858.0	63.0	13,257.6	0.00	860.62	71	0.130
	0.08	0.4	640.0	156.6	47,820.1	0.00	158.38	678	0.245
	0.1	0.6	682.5	230.4	24,239	0.00	45.89	3817	3.769
Medium	0.02	0.1	699.5	40	16,430.5	0.00	4032.60	0	0
	0.04	0.2	571.0	84.3	22,657	3.82	7200.07	20	0.018
	0.08	0.4	756.0	185.8	62,717.8	0.07	7200.46	1605	0.441
	0.1	0.6	717.0	260.9	93,539.1	0.00	79.60	10,857	2.135

Notes: a and b are factors used to change the item's length with respect to object length.

To get a clear visualization of the effect of changing items' length with respect to object length on the material waste, Figure 2 illustrates this effect for small and medium-sized instances. It can be observed that the material waste percentage increases as items' length increases with respect to object length. This is because smaller items' length allows for better cutting pattern configurations, leading to lower material waste. However, there is an increase in computational time when dealing with smaller items' length, as illustrated in Figure 3, due to the massive increase in the number of arcs in the network. Also, it can be noticed that computational time increases at higher rates in medium-sized instances than in small-sized instances. Finally, all small instances were solved before reaching the time limit, as shown in Figure 3.

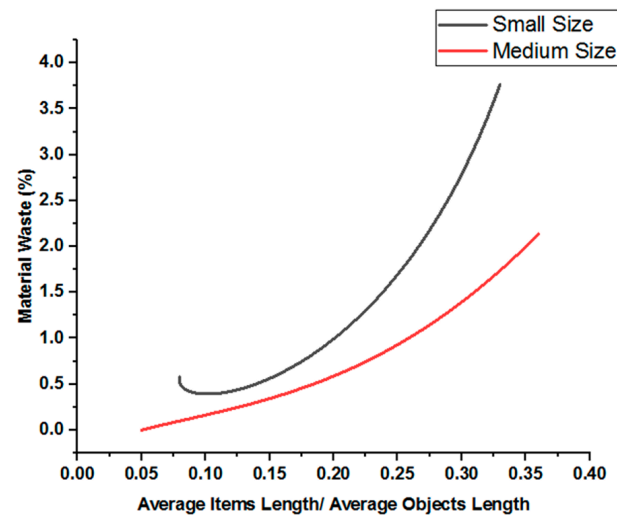


Figure 2. Effect of changing items' length with respect to object length on material waste percentage.

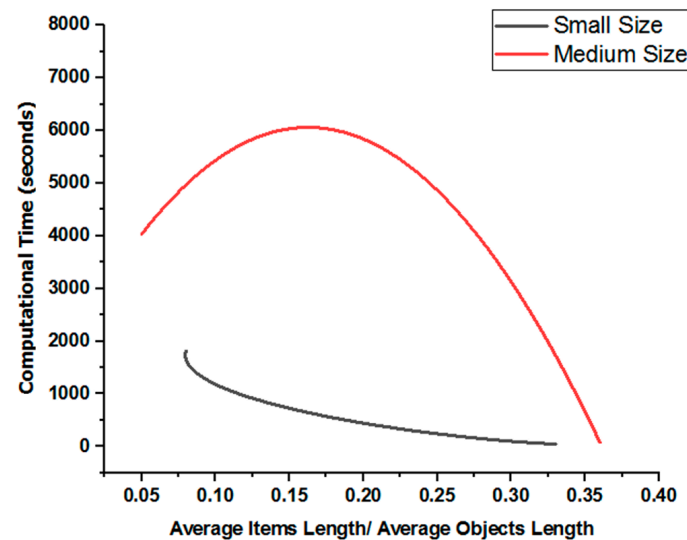


Figure 3. The computational time of small and medium instances in case of changing items' length with respect to object length.

4.5.2. Studying Different Material Waste Cost

The primary objective of CSP is to reduce material waste or achieve the maximum possible utilization of the available raw material. Recently, manufacturers have given much attention to minimizing material waste due to the high cost of raw materials. The thirty-six instances previously mentioned were optimized while considering low material waste cost ($c_w = 1$). In this sub-section, the effect of changing the material waste cost from 1 to 100 per unit length of raw material is illustrated. To evaluate the material waste cost effect on the model behavior correctly, the capacity levels were increased by 30% for the three production levels, thus allowing storing of objects, items, and final products. The study was applied in two instances, small and medium-sized. The results obtained when changing the material waste cost are presented in Table 6.

Table 6. Obtained results at different material waste costs.

Instance Size	cw	Objective Function Value	Gap (%)	Computational Time (seconds)	Material Waste (%)	Increased Total Cost (%)
Small	1	55,430.7	0.00	899.27	0.629	-
	10	72,493.4	0.00	32.63	0.576	30.7
	50	142,744.0	0.00	19.06	0.543	157.5
	100	230,293	0.00	53.58	0.543	315.4
Medium	1	41,981.2	0.01	7201.28	0.238	-
	10	44,623.2	0.02	7200.16	0.082	4.7
	50	49,726.3	0.00	274.02	0.028	13.9
	100	51,400.2	0.00	84.06	0	16.99

Notes: cw is the unit length cost of raw material waste.

Figure 4 demonstrates the impact of increasing material waste costs on the objective function value, material waste percentage, and computational time for small- and medium-sized instances. As material waste cost rises, the objective function value correspondingly increases. Notably, higher waste costs lead to a substantial reduction in computational time, primarily due to the elimination of a massive number of possible arcs. Predictably, the material waste percentage declines as material waste cost increases. In Table 6, the increase in total cost confirms the importance of CSP, especially if material costs are high.

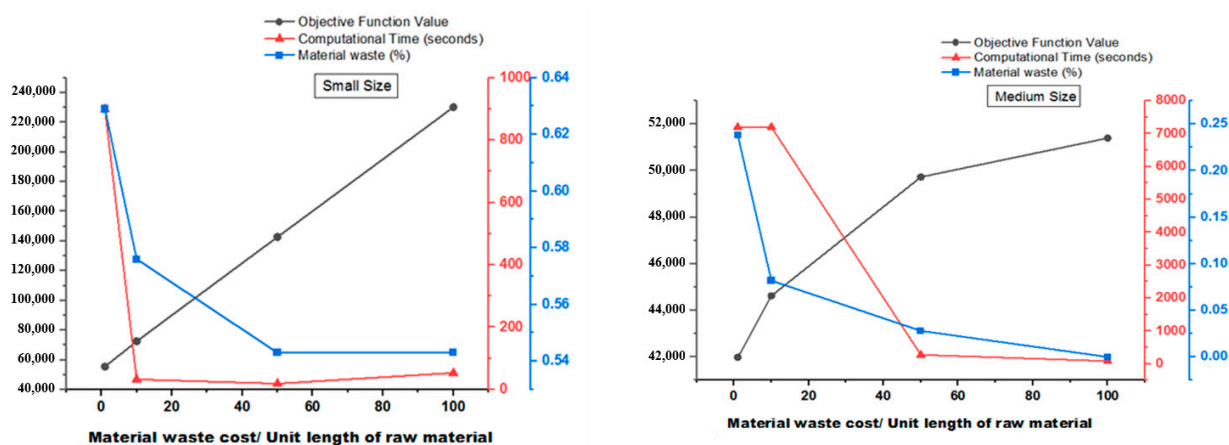


Figure 4. Effect of increasing material waste cost on objective function value, material waste percentage, and computational time for the small-sized and medium-sized instances.

4.5.3. Studying Neglecting Setup Costs of Each Production Level

The mathematical model includes setup costs for producing objects, cutting objects into items, and producing final products. The effect of neglecting setup costs of each production level on model performance is studied in small and medium-sized instances. The results obtained from these studies are presented in Table 7. The results indicate that final product setup costs have a higher effect on minimizing the objective function value over the others. While the setup costs of cutting objects into items have the lowest effect on minimizing the objective function value. Regarding the computational time, it increased when neglecting setup costs of objects and cutting objects into items and decreased in case of neglecting the setup costs of final products.

Table 7. Results obtained in case of neglecting setup costs of each production level.

Instance Size	Neglecting Setup Cost of	Objective Function Value	Gap (%)	Computational Time (seconds)	Total Objects Setup Cost	Total Machines Setup Cost	Total Final Products Setup Cost	Total Cost Reduction (%)
Small	-	22,279.1	0.00	790.48	1014.0	856	1701	-
	Objects	20,700.4	0.00	1556.71	0.0	856	1701	7.08
	Cutting objects into items	21,447.6	0.19	1579.90	1235	0.0	1701	3.73
	Final products	20,542.9	0.00	638.75	1356	856	0.0	7.79
Medium	-	43,840.9	0.01	6557.63	650	580	1568	-
	Objects	40,727.2	0.00	6904.24	0.0	580	1568	7.10
	Cutting objects into items	41,403.4	0.03	7200.18	650	0.0	1568	5.55
	Final products	40,413.2	0.01	4196.93	650	580	0.0	7.81

4.5.4. Studying Changing Capacity Levels

One of the constraints that increase the complexity of the proposed model to achieve a feasible solution is capacity constraints. The tighter the limits, the harder it is to reach a solution. To assess the influence of capacity levels, different capacity levels are applied to small and medium-sized instances. For each instance, the capacity level is increased for all production levels by 10, 20, 30, and 40 percent, as presented in Table 8. The increase in capacity levels allowed the production of objects, items, and final products earlier than needed, which led to a slight decrease in total costs, as seen in Figure 5. However, increasing capacity levels has raised the computational time needed to reach an optimal solution in the small-sized instance as shown in Figure 5.

Table 8. Results of changing capacity levels of all production levels.

Instance Size	Capacity Level	Objective Function Value	Gap (%)	Computational Time (seconds)	Material Waste (%)	Inventory Value of		
						Objects	Items	Final Products
Small	Tightened	34,885.3	0.00	25.21	1.96	0	18	0
	Increased by 10%	34,722.7	0.00	74.52	1.92	0	28	5
	Increased by 20%	34,722.7	0.00	102.96	1.92	0	28	5
	Increased by 30%	34,722.7	0.00	216.56	1.92	0	28	5
	Increased by 40%	34,665.3	0.00	5014.24	2.08	0	12	13
Medium	Tightened	40,279.3	0.20	7200.36	0.112	0	21	0
	Increased by 10%	39,794	0.36	7200.08	0.147	1	4	7
	Increased by 20%	39,455.1	0.17	7200.12	0.112	2	12	18
	Increased by 30%	39,095.6	0.02	7200.35	0.112	2	8	20
	Increased by 40%	38,712.6	0.03	7200.28	0.215	3	23	20

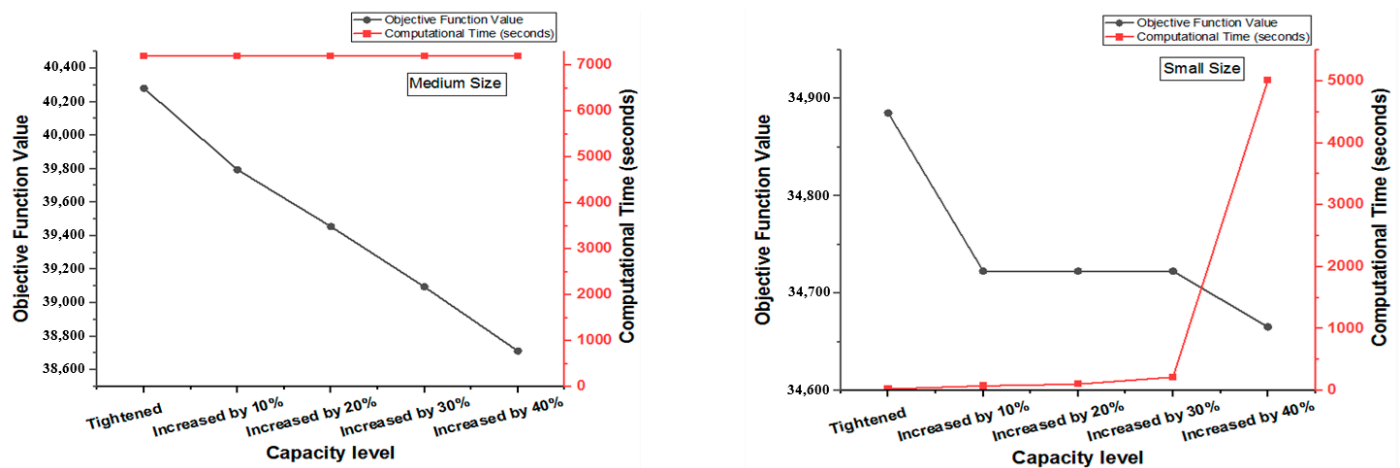


Figure 5. Objective function value and computational time of small-sized and medium-sized instances in case of changing capacity levels.

4.5.5. Studying Varying Inventory Holding Costs

The main objective of the LSP is to achieve an optimal trade-off between setup costs and inventory holding costs. This sub-section examines how variations in inventory holding costs for objects, items, and final products influence the model's behavior. The object, item, and final product holding costs are calculated using the following equations $hck_{kt} = \beta L_k$, $hci_{it} = \alpha l_i$, $hcf_{ft} = \gamma \max(hci_{it}) \times 10$, respectively. Different values for the inventory factors (α , β , γ) are studied. The capacity levels are increased by 30% to allow storing of objects, items, and final products. The study was applied to a small and medium-sized instance, and only one inventory cost was changed at a time while keeping the other costs constant. Tables 9 and 10 show the outcomes of changing objects, items, and final products holding costs in small and medium-sized instances.

Table 9. Results of varying inventory costs in the small-sized instance.

	Objective Function Value	Gap (%)	Computational Time (seconds)	Material Waste	Objects Inventory Value	Avg. Object Inventory Cost/Unit	Items Inventory Value	Avg. Item Inventory Cost/Unit	Final Product Inventory Value	Avg. Final Products Inventory Cost/Unit
β										
Objects' Inventory										
0	21,617.7	0	378.87	346	30	0	37	0.67	62	6.56
0.01	21,652.7	0.00	281.61	576	0	5.6	19	0.67	63	6.56
0.1	21,652.7	0.00	1741.11	576	0	153.1	19	0.67	63	6.56
0.5	21,652.7	0.00	388.55	576	0	382.75	19	0.67	63	6.56
α										
Items' Inventory										
0	21,554.1	0.00	129.17	346	7	5.6	366	0	30	6.56
0.01	21,652.7	0.00	164.12	576	0	5.6	19	0.67	63	6.56
0.1	21,801.5	0.00	2411.6	576	0	5.6	4	6.7	68	6.56
0.5	21,818.2	0.00	219.2	474	8	5.6	0	98.25	71	6.56
γ										
Final products' Inventory										
0	21,216	0.00	275.85	576	0	5.6	4	0.67	68	0
0.5	21,441.6	0.00	160.47	576	0	5.6	4	0.67	68	3.27
1	21,652.7	0.00	153.1	576	0	5.6	19	0.67	63	6.56
5	22,462.8	0.03	7200.64	346	58	5.6	42	0.67	7	32.8

Notes: α , β , and γ are inventory factors used to calculate the holding costs of objects, items, and final products.

Table 10. Results of varying inventory costs in the medium-sized instance.

	Objective Function Value	Gap (%)	Computational Time (seconds)	Material Waste	Objects Inventory Value	Avg. Object Inventory Cost/Unit	Items Inventory Value	Avg. Item Inventory Cost/Unit	Final Product Inventory Value	Avg. Final Products Inventory Cost/Unit
β					Objects' Inventory					
0	39,075.9	0.09	7200.43	257	2	0	8	1.96	20	22.28
0.01	39,095.6	0.02	7200.42	257	2	7.65	8	1.96	20	22.28
0.1	39,134.7	0.05	7201.05	257	0	76.5	16	1.96	20	22.28
0.5	39,136.2	0.15	7200.56	257	0	382.75	18	1.96	21	22.28
α					Items' Inventory					
0	40,191.8	0.13	7200.1	567	2	7.65	82	0	0	22.28
0.01	40,279.3	0.20	7200.45	257	0	7.65	21	1.96	0	22.28
0.1	40,507.6	0.36	7200.28	335	0	7.65	6	19.65	0	22.28
0.5	40,740.8	0.25	7200.05	339	5	7.65	3	98.25	0	22.28
γ					Final products' Inventory					
0	38,600.9	0.11	7200.16	257	0	7.65	11	0.67	22	0
0.5	38,878.6	0.16	7200.05	285	2	7.65	6	0.67	20	11.14
1	39,095.6	0.02	7200.24	257	2	7.65	8	0.67	20	22.28
5	39,634	0.14	7200.32	490	4	7.65	78	0.67	1	111.4

Notes: α , β , and γ are inventory factors used to calculate the holding costs of objects, items, and final products.

From Tables 9 and 10, by comparing the objective function value at $\alpha = 0$, $\beta = 0$, and $\gamma = 0$ in small-sized and medium-sized instances, the least objective function value is observed at $\gamma = 0$ as the final product holding costs have the highest value over the other costs. Also, it is more likely to have items in inventory than objects because of the possibility of cutting the objects during the same production period. Hence, there is no need to keep objects in inventory.

Increasing objects' inventory costs did not change the objective function value significantly as the model prefers to produce objects and cut them into items in the same period because items' inventory costs are lower than objects' inventory costs, as reported in Tables 9 and 10. It was also observed that increasing the holding costs of items led to a reduction in their inventory value, accompanied by a corresponding increase in the inventory value of final products. This shift reflects the model's strategy to produce items and immediately assemble them into final products within the same period to avoid incurring high item holding costs. Conversely, when the holding costs of final products were increased, the model responded by raising the inventory value of items as their relative holding costs became more favorable. In such cases, items were produced and stored for later assembly into final products when needed.

4.6. Managerial Insights

Firstly, the proposed general model could help managers and decision-makers in solving the capacitated lot sizing and one-dimensional cutting stock problems within a multi-level manufacturing framework. It could be modified to fit the conditions of a certain industry, including cutting solid raw materials such as wood, paper, steel, glass, aluminum, leather, or plastic film into items with different lengths. These items are assembled to form final products. The developed model helps minimize the setup, production, holding, and waste material costs while incorporating capacity constraints, setup requirements, inventory balance, and the use of various cutting machines.

Secondly, the different sensitivity analyses conducted can give complete insight to managers regarding different costs and the most effective parameters of the integrated

problem, enabling them to achieve better practical results. According to these analyses, the following conclusions were reached:

- As changing items' length with respect to object length showed an increase in material waste for large items' length and resulted in more computational time for smaller items' length. Then, managers should consider a leftover strategy in case of large items' length to counter the increase in material waste. They should also balance material utilization with computational efficiency.
- Increasing material waste costs lead to decreased material waste percentage. Therefore, managers willing to achieve sustainability can increase material waste costs to reduce material waste.
- Neglecting setup costs for final products lowered the time consumed to solve the model. This gives insights to managers to neglect setup costs in case of having low setup costs, and in case of higher setup costs, they should focus on minimizing frequent changeovers and increasing batch sizes.
- Increasing capacity levels allowed for storing objects, items, and final products, which resulted in a decrease in total cost. Accordingly, managers should assess if expanding production capacity can provide long-term cost savings.
- Ignoring inventory cost for final products resulted in the lowest objective function value. Hence, managers should analyze the storage costs of their objects, items, and final products to decide on the optimal stocking strategies. In case of high storage costs for final products, they should prioritize storing items instead and assemble final products when needed.

5. Conclusions

Manufacturing businesses compete hard in today's market to cut total costs and minimize material waste. Two key issues that frequently arise in the production planning of manufacturing businesses across various industries are the cutting stock problem (CSP) and the lot sizing problem (LSP). To meet the demand, the CSP chooses the best configuration to cut large objects into smaller items. While the LSP seeks to determine the production volumes for each product over a planning horizon. According to recent research, integrating the cutting stock problem with the lot sizing problem is gaining much attention in several industries and new industries has been tackled.

In this paper, the integration between the capacitated LSP and one-dimensional CSP within a three-level manufacturing framework is addressed. A mixed integer linear programming mathematical model based on arc flow formulation is proposed to tackle the integrated problem. The model minimizes inventory costs, setup costs, production costs, and material waste. In the first level, the quantities of objects to be produced or procured, as well as those to be stored, are determined. In the second level, optimal cutting patterns for these objects are established across the available cutting machines, and the corresponding production and storage quantities for each item are specified. The resulting items are then assembled into final products. In the third level, the production and inventory levels of the final products are defined. The model considers three production levels, capacity limits at each production level, setup costs and times at all levels, and availability of different cutting machines.

GUROBI optimizer was used to solve the developed model. The model was evaluated using thirty-six randomly generated instances that vary in number of objects, items, final products, and time periods. Based on the results of computational experiments, optimal solutions were reached in most instances. In some instances, solving the problem within the allotted time was challenging, especially as the number of time periods rose. For

small-sized instances, the optimal solutions were found, while for medium-sized instances, a suitable gap percentage was found.

Small and medium-sized instances were used to analyze the model in various settings, improving comprehension of the problem and offering insights into various costs and the most effective parameters of the integrated problem. The analysis included studying the effect of changing essential parameters related to CSP and LSP, such as items' length with respect to object length, material waste cost, material waste cost at different items' length, capacity levels, setup costs, and inventory holding costs. Changing items' length with respect to object length showed an increase in material waste for large items' length, and on the other hand, smaller items' length resulted in increasing computational time. Increasing material waste cost increased the objective function value, and the computational time decreased significantly. Increasing capacity levels allowed the storing of objects, items, and final products, resulting in a decrease in the total cost.

The limitations encountered in this study highlight opportunities for future research and model development. Firstly, numerical experiments validated the effectiveness of our model; however, its applicability could be further examined through a real-world case study. Secondly, the proposed model addresses one-dimensional CSPs; however, its capabilities could be enhanced by adapting it to handle two- or three-dimensional CSPs. Thirdly, the proposed model can be further developed to accommodate multiple objectives and integrate uncertainty into its framework. In addition, sustainability aspects in CSP, such as including usable leftovers while generating the cutting patterns, could be considered. Finally, other formulations for cutting stock and lot sizing problems or a heuristic/metaheuristics procedure to solve large instances could be tested.

Author Contributions: Conceptualization and methodology: N.H., H.F. and N.K.; funding acquisition: N.H.; investigation, software, visualization, and writing—original draft: N.K.; supervision and writing—review and editing: N.H. and H.F. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Academy of Scientific Research and Technology (ASRT)—Egypt, PRIMA program (Call 2020, Section 2, Project number 26), project entitled “Innovation in the By-Product Supply chain of citrus in Mediterranean area (ImPUiSe)”.

Data Availability Statement: The data supporting the findings of this study will be made available upon reasonable request.

Conflicts of Interest: The authors declare that they have no competing interests.

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