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# A Second-Order Dynamic Friction Model Compared to Commercial Stick–Slip Models

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**Abstract:** Friction has long been an important issue in multibody dynamics. Static friction models apply appropriate regularization techniques to convert the stick inequality and the non-smooth stick–slip transition of Coulomb's approach into a continuous and smooth function of the sliding velocity. However, a regularized friction force is not able to maintain long-term stick. That is why dynamic friction models were developed in recent decades. The friction force depends herein not only on the sliding velocity but also on internal states. The probably best-known representative, the LuGre friction model, is based on a fictitious bristle but realizes a too-simple approximation. The recently published second-order dynamic friction model describes the dynamics of a fictitious bristle more accurately. It is based on a regularized friction force characteristic, which is continuous and smooth but can maintain long-term stick due to an appropriate shift in the regularization. Its performance is compared here to stick–slip friction models, developed and launched not long ago by commercial multibody software packages. The results obtained by a virtual friction test-bench and by a more practical festoon cable system are very promising. Thus, the second-order dynamic friction model may serve not only as an alternative to the LuGre model but also to commercial stick–slip models.

**Keywords:** dynamic friction model; commercial stick–slip friction models; long-term stick; multibody dynamics

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## 1. Introduction

Friction has long been an important issue in multibody dynamics. A detailed survey and comparison of several friction force models for dynamic analysis of multibody mechanical systems is provided in [1]. Just as in the present work, the use of internal states defines the difference between dynamic and static friction models. It starts with different variations of the static Coulomb model, including viscous parts and the Stribeck effect, discusses several regularization approaches with finite slopes at zero velocity, and ends up with dynamic friction force models, including the probably best-known representative, the LuGre friction model.

Static friction models, where an appropriate regularization technique converts the stick inequality and the non-smooth stick–slip transition of Coulomb's approach into a continuous and smooth function of the sliding velocity, are not able to maintain long-term stick. That is why commercial multibody software packages, like Adams, RecurDyn, and Simpack, also offer dynamic friction models. To reproduce stick–slip effects and maintain long-term stick, they rely on their own developments and do not simply use models known from the literature, e.g., [1]. Just the LuGre model, as published in [2], is implemented in the commercial software package Adams [3]. However, as demonstrated in [4], the LuGre approach does not appear to be an engineer's first choice because it has too many drawbacks and is not able to reproduce a pre-defined friction characteristic in dynamic applications.

A second-order dynamic friction model (FrDyn2) was introduced in [4] as a reference to the LuGre and to a standard static friction model. The FrDyn2 model produced accurate and reliable results in standard stick–slip examples as well as in a more practical model of a festoon cable system.

The present paper compares this model to commercial stick–slip models as as presented and analyzed in [5]. It turns out that the concepts of Adams and Recurdyn are rather similar but different from the Simpack stick–slip model. That is why just the Adams and the Simpack stick–slip models are used here for the comparison with the FrDyn2 model.

Section 2 illustrates a typical standard and the shifted regularization technique and provides a short description of the FrDyn2 model, as defined in [4]. Section 3 demonstrates the long-term stick potential of the FrDyn2 model, achieved by an appropriate shift in the standard regularization. It also illustrates the poor performance of a static friction model which approximates stick by slow sliding. A virtual friction test-bench was set up in Matlab for that purpose. Section 4 compares the break-away and stick–slip transition of the FrDyn2 model with the stick–slip models of Adams and Simpack. For that purpose, the virtual friction test-bench was also set up in Adams and Simpack. The focus of Section 5 is on the dynamic response to step-like force excitations with different durations and a force amplitude close to the adhesion limit. Section 6 analyzes the performance of the friction models under consideration by the more practical model of a festoon cable system, with a setup in Matlab, Adams, and Simpack. Section 7 summarizes and discusses the results and and provides an outlook for future research.

Finally, Appendix A contains a Matlab script and the corresponding Matlab functions describing the virtual friction test-bench operated with the FrDyn2 model.

## 2. Static and Dynamic Friction Models

The idealized friction model of Coulomb simply distinguishes between sticking and sliding, as seen in Figure 1a. The friction force $F_R$ depends on the sliding velocity $v$ and is realized by combining an inequality with a simple relation

$$|F_R| \leq \mu_s F_N \quad \text{if} \quad v = 0 \quad \text{and} \quad F_R = \frac{v}{|v|}\, \mu_d F_N \quad \text{if} \quad |v| > 0 \tag{1}$$

The friction force is proportional to the normal force $F_N$ and characterized by the parameters $\mu_s$ and $\mu_d$, which specify the static and the dynamic coefficients of friction.

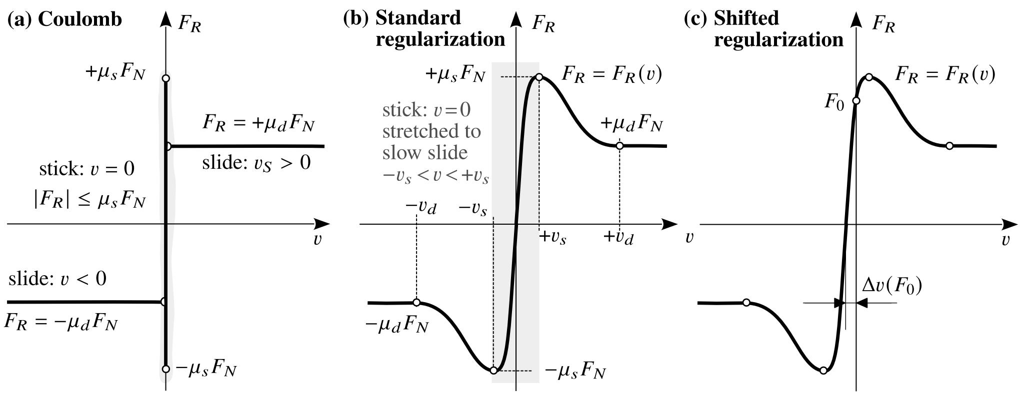


**Figure 1.** Dry friction: (**a**) Coulomb's approach, (**b**) regularized approximation, (**c**) shifted regularization.

Coulomb's dry friction approach, as defined in Equation (1), is practically unusable in general multibody dynamics due to the inequality representing stick. That is why dry friction is usually approximated by an unambiguous function, as seen in Figure 1b. The regularized characteristic $F_R = F_R(v)$ introduces the fictitious velocity $v_s$, which defines the width of the regularization interval $-v_s \leq v \leq +v_s$ and the dynamic velocity $v_d$ that characterizes at $|v| \geq v_d$ the range of full sliding, where the simple relation

$F_R = \pm\mu_d F_N$ applies. The transitions from the value pairs $(-v_d, -\mu_d F_N) \to (-v_s, -\mu_s F_N)$ and $(-v_s, -\mu_s F_N) \to (+v_s, +\mu_s F_N)$ as well as $(+v_s, +\mu_s F_N) \to (+v_s, +\mu_s F_N)$, which are usually modeled by sufficiently smooth functions, like polynomials or trigonometric functions.

Static friction models are widely used in multibody dynamics and control theory. They are able to reproduce stick–slip effects in standard and in a more practical application [4]. Even much simpler regularizations, assuming by $\mu = \mu_s = \mu_d$ just one unique friction value, are in good conformity to dynamic measurements [6]. However, static friction models cannot maintain long-term stick. They describe the friction force just as a function of the sliding velocity $F_R = F_R(v)$. Figure 1b defines a commonly used regularized friction characteristics $F_R = F_R(v)$ by a set of "static" $(\mu_s, v_s)$ and "dynamic" $(\mu_d, v_d)$ friction parameters. However, the use of dynamic friction parameters is no essential criteria for a dynamic friction model, as erroneously assumed in [7]. Here, a static friction force defined by $F_f = F_f(\mu_s, v_s, \mu_d, F_n)$ making use of the dynamic friction value $\mu_d$ is already supposed to define a dynamic friction force.

Dynamic friction models are characterized by the use of internal states. The friction force is then defined by a more complex function $F_R = F_R(v, s)$, where $s$ collects the internal states. Dynamic friction models are able to reproduce stick–slip effects and maintain long-term stick [8].

A software-capable formulation of a friction force model also takes the friction model parameters into account. Then, $F_R = F_R(v, p)$ characterizes a static and $F_R = F_R(v, s, p)$ a dynamic friction model, where $p$ collects the friction model parameters.

The well-known LuGre friction model uses the displacement $z$ of a fictitious bristle as an internal state. However, as demonstrated in [4], the LuGre approach represents just a first step approximation to the dynamics of a massless bristle, which results in several drawbacks of this dynamic friction model. If the mass of the fictitious bristle is also taken into account, this results in a second-order dynamic friction model (FrDyn2), which was introduced in [4] as a reference. The FrDyn2 model exhibits none of the LuGre drawbacks and performed well in standard stick–slip examples and in a more practical model of a festoon cable system.

The second-order bristle dynamics is defined by

$$m_b\,\ddot{z} \;=\; F_R - F_B \;=\; F_R(v_C - \dot{z}) - (\sigma_0\, z + \sigma_1\, \dot{z}) \tag{2}$$

where $m_b\,\ddot{z}$ approximates the inertia force of the fictitious bristle, $F_R = F_R(v_C - \dot{z})$ describes the friction force, $F_B = \sigma_0\, z + \sigma_1\, \dot{z}$ models the bristle as a visco-elastic element, and $v_C$ represents the component of the contact point velocity that is perpendicular to the contact normal. Just as with the LuGre model, $\sigma_0$ and $\sigma_1$ describe the stiffness and the damping of the fictitious bristle. The fictitious mass of the bristle is defined by

$$m_b = \frac{\sigma_1^2}{4\,\sigma_0} \tag{3}$$

which represents the aperiodic case of the homogenous second-order differential Equation (2), thus avoiding unwanted oscillations in the fictitious bristle. The FrDyn2 model implies with $s = [z,\, \dot{z}]$ two internal states. It is provided as a Matlab function in Appendix A by Listing A4.

The stick–slip models of Adams and RecurDyn describe the dynamic friction force just as a function of the contact point velocity $F_{Rd} = F_N\, \mu_d(v_C)$ and approximate the static friction force by a two-dimensional function $F_{Rs} = F_N\, \mu_s(v_C, x)$, where $x$ is a fictitious displacement [5]. A smooth function of the contact point velocity $v_C$, which is not explained in detail in the user manual, describes the transition from $\mu_s$ to $\mu_d$. The fictitious displacement serves as an internal state and generates the static friction force $F_{Rs}$ or the static friction coefficient $\mu_s$ as a nonlinear function of $x$ by adding a viscous damping term.

Simpack provides a stick–slip model which realizes, like Coulomb's approach in Figure 1a, a sudden drop from the static $\mu_s$ to the dynamic friction coefficient $\mu_d$ [5]. In the adhesion region, the friction force is approximated by a visco-elastic element whose deflection again represents an internal state of this stick–slip model.

## 3. Long-Term Stick of the Second-Order Dynamic Friction Model

By applying an appropriate horizontal shift to the regularized friction characteristics, as indicated in Figure 1c, the second-order dynamic friction model can maintain long-term stick. The steady state solution ($\ddot{z} = 0$ and $\dot{z} = v_C$) of the fictitious bristle dynamics (2) provides the required sticking force as

$$F_0 = F_R(0) = \sigma_0\, z + \sigma_1\, v_C \tag{4}$$

The FrDyn2 model describes here the transition from the static to the dynamic friction force $(\mu_s, F_s) \rightarrow (\mu_d, F_d)$ by a cubic polynomial and defines the friction force in the regularization range $-v_s \le v \le +v_s$ by a parabolic function. Then,

$$\Delta v \;=\; \frac{F_0}{|F_0|}\, v_s\, w_F \quad \text{with} \quad w_F = \begin{cases} 1 - \sqrt{1 - |F_0|/F_s} & \text{if } |F_0| \le F_s \\ 1 & \text{elsewhere} \end{cases} \tag{5}$$

delivers the corresponding horizontal shift. The static friction force is defined by $F_s = F_N\, \mu_s$ and $F_0/|F_0|$ adjusts the horizontal shift to the sign of the required sticking force value.

Figure 2 provides a virtual test-bench which is used here to demonstrate the long-term sticking quality of the FrDyn2 model and in the following for a comparison to the stick–slip models of Adams and Simpack.

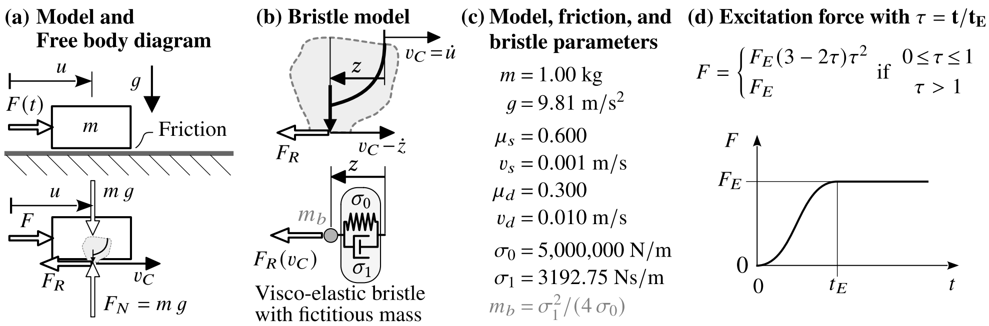


**Figure 2.** Virtual friction test-bench.

A body of unit mass $m = 1$ kg is in contact to a horizontal rough plate, as seen in Figure 2a. It is exposed to a horizontal force $F = F(t)$, which is continously increased in the interval $0 \le t \le t_E$ from $F = 0$ to the final value $F = F_E$, as defined by Figure 2d. The friction parameters $\mu_s$, $v_s$ and $\mu_d$, $v_d$, provided by Figure 2c, model a regularized friction characteristic as defined in Figure 1b. The FrDyn2 model uses the states $z$ and $\dot{z}$ of a fictitious bristle, where $m_b$ denotes its fictitious mass and $\sigma_0$ and $\sigma_1$ characterize the visco-elastic properties of the bristle, as seen in Figure 2b. The bristle parameters, as defined by Figure 2c, are adjusted to the body and the friction parameters by estimating a reference friction force of $F_{Rref} \approx 5$ N and defining a reference bristle deflection of $z_{ref} = 1 \cdot 10^{-6}$ m. Then, $\sigma_0 = F_{Rref}/z_{ref} = 5 \cdot 10^6$ N/m provides the bristle stiffness. The reference friction force corresponds to a reference mass of $m_{ref} = F_{Rref}/g = 0.51$ kg. It provides in a first guess the damping of the fictitious bristle as $\sigma_1 = 2\sqrt{m_{ref}\,\sigma_0} = 3192.75$ N/(m/s), when the aperiodic case of a fictitious oscillator consisting of $m_{ref}$, $\sigma_1$, and $\sigma_0$ is assumed hereby.

The Matlab script, provided in Appendix A by Listing A1, performs simulations with different step durations. It applies the standard implicit solver `ode15s` where the default tolerances are changed to `RelTol` = $1{\cdot}10^{-6}$ and `AbsTol` = $1{\cdot}10^{-9}$. The reference bristle deflection was chosen very small in this example. The Matlab function `dyn_fr_test_bench`, provided in Appendix A by Listing A2, computes the dynamics of the virtual friction test-bench including the FrDyn2 model as a set of first-order differential equations. The Matlab functions `Step3` and `FrDyn2`, defined in Appendix A by Listings A3 and A4, provide the step input and the dynamics of the FrDyn2 model. The simulation results, plotted in Figure 3, demonstrate that the FrDyn2 model perfectly maintains long-term stick.

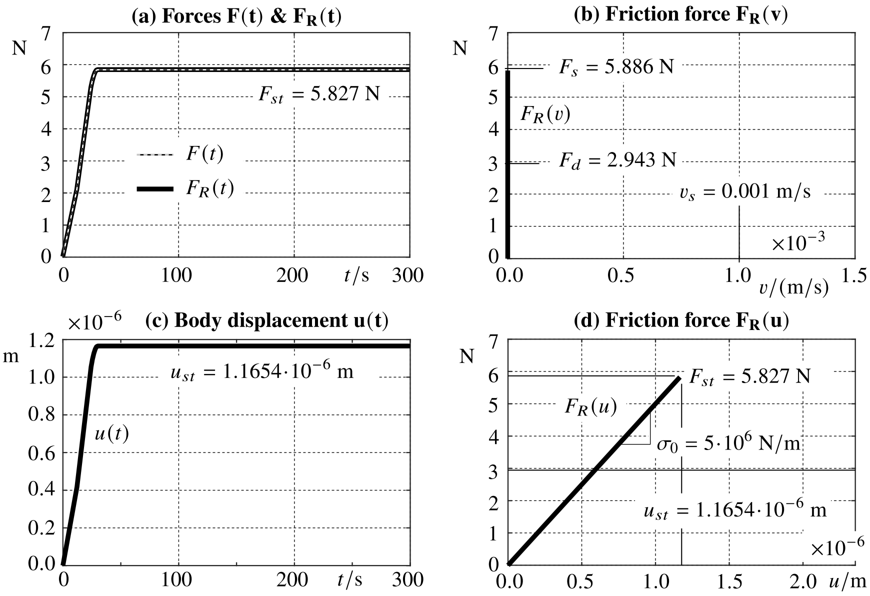


**Figure 3.** Long-term stick potential of second-order dynamic friction model.

The excitation force is slowly increased in the time interval $0\ \text{s} \leq t \leq 30\ \text{s}$ from $F = 0$ to 99% of the static friction force $F_s$, which is given here by $F_s = \mu_s F_N = 0.6 \cdot 1\ \text{kg} \cdot 9.81\ \text{m/s}^2 = 5.886\ \text{N}$. In the subsequent time interval $30\ \text{s} \leq t \leq 300\ \text{s}$, the force $F_E = 0.99 \cdot 5.886\ \text{N} = 5.827\ \text{N}$ is kept constant. The friction force $F_R(t)$, generated by the FrDyn2 model, counteracts the excitation force $F_R = F$, as seen in the upper-left plot in Figure 3. It is worth noting that the friction force, as defined by the free-body diagram in Figure 3b, points in the opposite direction of the contact point velocity. The tip of the bristle sticks to the ground because the excitation force does not exceed the static friction force. Then, the bristle deflection $z$ coincides with the body displacement $u$ and provides the friction force as a function of the bristle deflection $z = u$, as demonstrated by the lower-right plot in Figure 3. As a consequence, the body shifts slightly and comes to a stand-still at the steady state value of $z_{st} = u_{st} = F_E/\sigma_0 = 5.827\ \text{N}/5.0 \cdot 10^6\ \text{N/m} = 1.1654 \cdot 10^{-6}\ \text{m}$. The friction force diagram $F_R(v)$, in the upper-right plot of Figure 3, shows that the FrDyn2 model can reproduce the ambiguous part of Coulomb's friction law at vanishing contact point velocities $v = 0$.

A static friction model describes the friction force just as a function of the sliding velocity, $F_R = F_R(v)$. A typical regularization without any horizontal shift is illustrated in Figure 1. A static friction model can reproduce the required friction force $F_R = F$ but definitely cannot maintain stick for longer time intervals, as seen in Figure 4.

In the regularization range, the friction force characteristic $F_R = F_R(v)$ is described by a parabolic function. This pre-defined function, plotted in the right graph of Figure 4 by a thin dashed line, perfectly coincides with the computed friction force $F_R$ plotted by a solid thick line. The parabola delivers the required steady state friction force of $F_R = F_{st} = 0.99 \cdot F_s$ at the velocity of $v_{st} = 0.9 \cdot v_s = 0.9 \cdot 10^{-3}\ \text{m/s}$. As a consequence, the body does not come to rest but continues to move inexorably at this velocity.

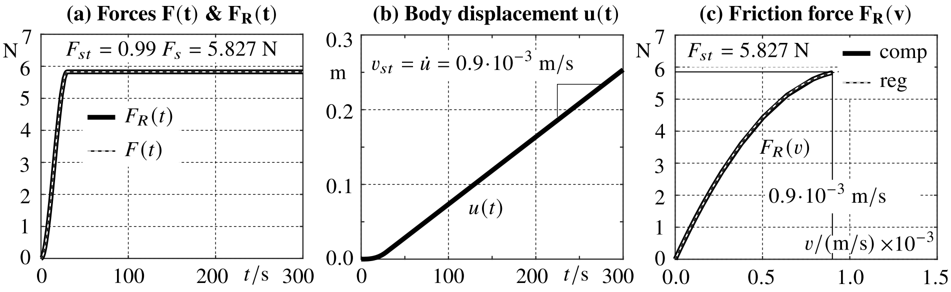


**Figure 4.** Slow sliding approximates sticking at a standard static friction model.

## 4. Break-Away and Stick–Slip Transition

The virtual friction test-bench, defined in Figure 2, consists of a body in contact to a horizontal rough plate. The excitation force $F = F(t)$ is now slowly increased within 10 s from $F = 0$ to the final value $F = F_E = 1.05\, F_s = 6.180$ N, which exceeds the adhesion limit of $F_s$ by 5%. The excitation force $F = F(t)$, modeled by a third-order polynomial, reaches the adhesion limit $F = F_s = 5.886$ N at time $t = t_B = 8.6808$ s.

The simulation results are plotted in Figure 5. The dashed blue line, the solid black line, and the dotted red line represent the results computed with the FrDyn2 model, the Adams stick–slip model, and the Simpack stick–slip model.

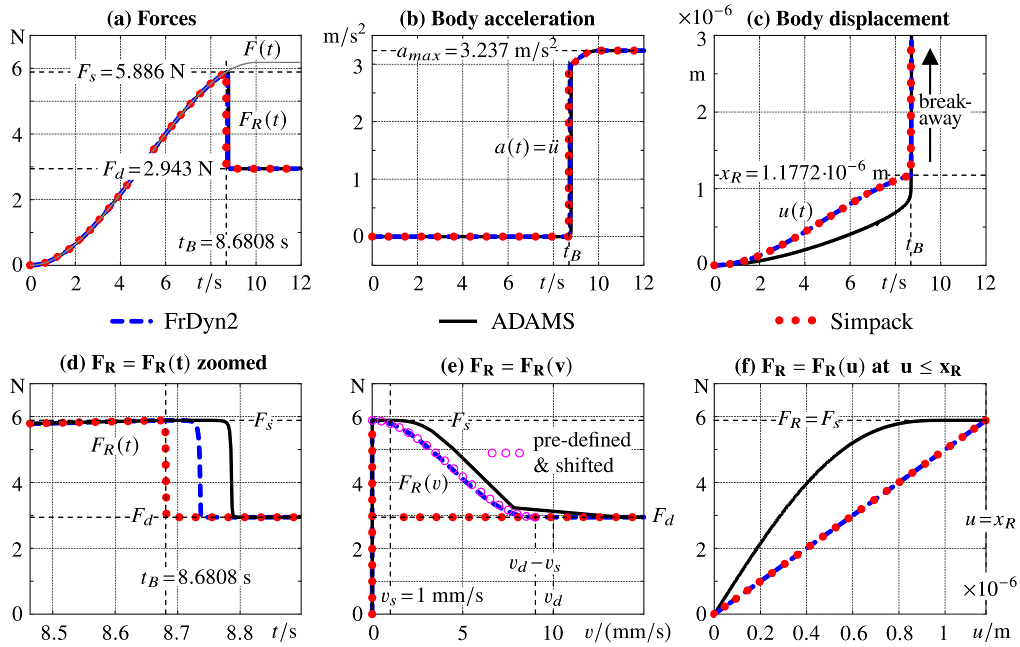


**Figure 5.** Virtual friction test-bench simulation results with a force that exceeds the adhesion limit.

At first ($0 \le t \le t_B$), the body remains in a quasi-static equilibrium, where the slowly increasing excitation force $F(t)$ is perfectly counteracted by a friction force $F_R(t) = F(t)$ generated by each of the friction models, as demonstrated in Figure 5a, where the lines for $F(t)$ and $F_R(t)$ perfectly coincide in the time interval $0 \le t \le t_B$. In general, the friction models under consideration generate friction forces depending not only on the contact velocity but also on internal states. The FrDyn2 model uses the displacement $z$ of a fictitious bristle and its time derivative $\dot{z}$ as internal states $s = [z,\, \dot{z}]$. In the quasi-static equilibrium mode, the velocity of the body and the time derivative of the bristle deflection are negligibly small $v \approx 0$ and $\dot{z} \approx 0$. In this mode, the tip of the fictitious bristle sticks to the ground, which results in a bristle deflection that equals the body displacement $z = u$. The compliance of the fictitious bristle is modeled by a viscous force element, $F_B = \sigma_0\, z + \sigma_1\, \dot{z}$. In the quasi-static equilibrium mode, the body acceleration $a = \ddot{u}$ is negligibly small too, as indicated in Figure 5b by the time history $a = a(t)$ in the time interval $0 \le t \le t_B$.

According to Equation (2), the friction force generated by the FrDyn2 model corresponds then to the elastic part of the bristle force, $F_R \rightarrow \sigma_0\, z = \sigma_0\, u$. The quasi-static force $F_R = \sigma_0\, u$ equals the static friction force $F_s$ at the reference displacement of

$$u = x_R = \frac{F_s}{\sigma_0} = \frac{5.886\ \mathrm{N}}{5 \cdot 10^6\ \mathrm{N/m}} = 1.1772 \cdot 10^{-6}\ \mathrm{m} \tag{6}$$

The adhesion range is characterized by a vanishing sliding velocity ($v \approx 0$) and extends here to displacements in the range of $0 \leq u \leq x_R$. In this range, the friction force is generated as a function of the displacement, where the FrDyn2 model and the stick–slip model of Simpack apply a linear and the stick–slip model of Adams a nonlinear digressive function, as seen in Figure 5f. The Adams manual does not specify the type of nonlinearity but, as indicated by the solid black line in Figure 5f, it approaches the limit value $F_R = F_s$ at the reference displacement $x_R$ defined in (6) with a vanishing inclination.

The stick–slip model of Simpack is based on Coulomb's approach, where the friction force drops in an instant from the static to the dynamic value as soon as the excitation force exceeds the static friction force at $t = t_B$, as seen in Figure 5a,d in particular. The transition from the static to the dynamic friction force $F_s \rightarrow F_d$ are modeled in the FrDyn2 and the Adams stick–slip models as functions of the velocity $v$ controlled by the parameters $v_s$ and $v_d$. The FrDyn2 model applies a cubic polynomial which is shifted in the horizontal direction to maintain stick at $v = 0$, as indicated in Figure 1. As can be seen by inspecting Figure 5e, the FrDyn2 model generates a friction force characteristic (dashed blue line) which reproduces the pre-defined and horizontally shifted one (magenta colored circles) nearly perfectly. The friction characteristics produced by the Adams stick–slip model is rather similar (solid black line). Most likely, Adams models the transition $F_s \rightarrow F_d$ by a fifth-order polynomial. As a consequence, the FrDyn2 and the Adams stick–slip models produce slightly more delayed drops in the time histories of the computed friction forces, as seen in Figure 5d.

Figure 5b,c illustrate the break-away effect at $t = t_B$ by the time histories of the body acceleration $\ddot{u} = a = a(t)$ and the body displacement $u = u(t)$. All friction models under consideration approximate sliding at $v \geq v_d$ by a constant friction force $F_R(v \geq v_d) = F_d = 2.943$ N. Viscous components in the friction force are not considered here. The free body diagram in Figure 2b delivers the linear momentum

$$m\, \ddot{u} = F - F_R \tag{7}$$

for the body of mass $m = 1$ kg. At $t > 10$ s which includes $t > t_B$, the applied force is defined by $F = F_E = 1.05 F_s = 6.180$ N and the friction force is represented by its dynamic value $F_R = F_d = 2.943$ N. Then, the maximum acceleration of the body is defined by

$$a_{max} = \ddot{u}(t > 10\ \mathrm{s}) = (F_E - F_d)/m = (6.180\ \mathrm{N} - 2.943\ \mathrm{N})/1\ \mathrm{kg} = 3.237\ \mathrm{m/s^2} \tag{8}$$

which is exactly reproduced by the friction models, as seen in Figure 5b.

## 5. Dynamic Response

A pulse load excitation, performed in [4], revealed the tendency of dynamic friction models to produce dynamic overshoots in the friction force time histories. That is why the virtual test-bench defined in Figure 2 is now exposed to excitation forces where the amplitude $F_E = 0.95\, F_s = 5.592$ N is 5% less than the static friction force $F_s = 5.886$ N and the step duration is varied from $t_E = 0.1$ s to $t_E = 0.0001$ s. The corresponding simulation results are shown in Figure 6. The solid thin gray line represents the excitation force $F = F(t)$, the dashed blue line, the solid black line, and the dotted red line mark the friction forces $F_R = F_R(t)$ computed with the FrDyn2, the Adams stick–slip, and the Simpack stick–slip models.

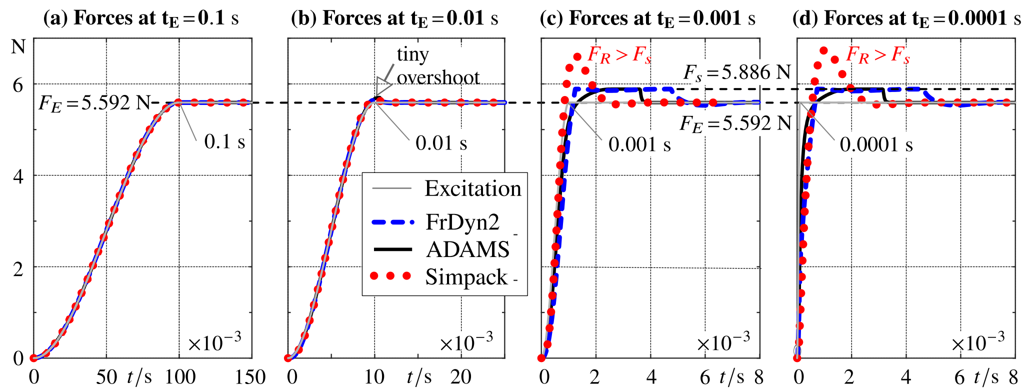


**Figure 6.** Dynamic friction forces resulting from step force excitation with different durations.

The time histories of the friction forces perfectly coincide with the excitation force $F_R(t) = F(t)$ at a step duration of $t_E = 0.1$ s, as seen in Figure 6a. All friction models operate here in a quasi-static sticking mode where the friction forces are practically generated as a function of the body displacement, as already illustrated in Figure 5e, and in this specific case by Figure 7.

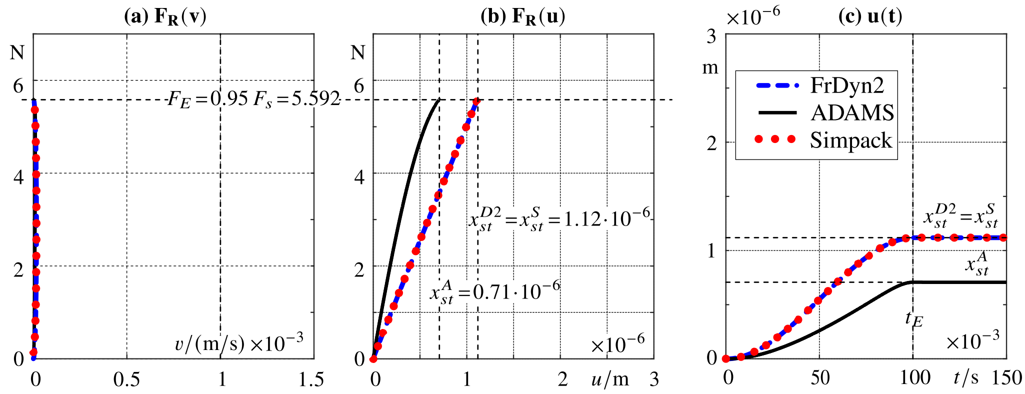


**Figure 7.** Friction force characteristics and body displacements at a step duration of $t_E = 0.1$ s.

The forces $F_R$ generated by the friction models depend here practically not on the velocity $v = \dot{u}$ but only on the displacement of the body $u$, as seen in Figure 7a,b. In case of the FrDyn2 and the Simpack stick–slip models $F_R = \sigma_0\, u$ holds, which provides the friction force $F_R = F_E = 5.592$ N at the steady state displacement $u = x_{st}^{D2} = x_{st}^{S} = 5.592\,\mathrm{N}/5{\cdot}10^6\,\mathrm{N/m} = 1.12{\cdot}10^{-6}$ m, as indicated in Figure 7b,c by thin dashed black lines. Adams models the friction force a quasi-static sticking mode by a strongly nonlinear and degressive function of the displacement. The Adams manual does not specify this function but the simulation results provide the friction force $F_R = F_E = 5.592$ N at the steady state displacement $u = x_{st}^{A} = 0.71{\cdot}10^{-6}$ m, as seen in Figure 7b,c. As expected from the time histories $F_R = F_R(t)$ plotted in Figure 6a, the time histories of the body displacement reach their steady state values $u = x_{st}^{D2} = x_{st}^{S}$ and $u = x_{st}^{A}$ at $t > t_E$ without any overshoots, as seen in Figure 7c.

In a quasi-static mode, the tip of the fictitious bristle, which forms the basis of the FrDyn2 model, sticks to the ground. Then, the linear momentum (7) of the body in the virtual friction test-bench simplifies to

$$m\,\ddot{u} = F(t) - \sigma_0\, u \quad \text{or} \quad m\,\ddot{u} + \sigma_0\, u = F(t) \tag{9}$$

where the quasi-static friction force is generated by the bristle compliance $F_R = \sigma_0\, z$ and $z = u$ holds in addition. The simplified equation of motion (9) is characterized by the eigen-frequency $\omega_0 = \sqrt{\sigma_0/m}$ and delivers the value and its corresponding oscillation period as

$$\omega_0 = \sqrt{(5{\cdot}10^6\ \mathrm{N/m})/(1\ \mathrm{kg})} = 2236\ \mathrm{s}^{-1} \quad \text{and} \quad T = \frac{2\pi}{\omega_0} = 0.003\ \mathrm{s} \tag{10}$$

As a consequence, even a rather short step duration of $t_E = 0.01$ s will still represent a subcritical excitation of the virtual friction test-bench. The time histories of the friction forces $F_R(t)$ exhibit just a small overshoot at $t > t_E = 0.01$ s, as seen in Figure 6b.

The situation becomes complicated for step durations $t_E < T$, as seen in Figure 6c,d. The Simpack stick–slip model (dotted red line) generates now significant overshoots, which amount to

$$F_{Rmax}\Big|_{Simpack}^{t_E=0.001\ \mathrm{s}} = 6.61\ \mathrm{N} \quad \text{and} \quad F_{Rmax}\Big|_{Simpack}^{t_E=0.0001\ \mathrm{s}} = 6.75\ \mathrm{N} \tag{11}$$

The values exceed the steady state value $F_{Rst} = F_E = 5.592$ N by 18.2% and 20.7% and even the static friction value $F_s = 5.886$ N by 12.3% and 14.5%, which calls into question the physical basis of this stick–slip model. The time histories of the friction forces generated by the FrDyn2 and the Adams stick–slip models (dashed blue and solid black lines) differ somehow. But both models limit the friction force to the static value $|F_R| \le F_s$, as expected from friction models in general.

The friction models now generate friction forces which strongly depend on the body velocity $v = \dot{u}$ and the body displacement $u$, as seen in Figure 8a,b.

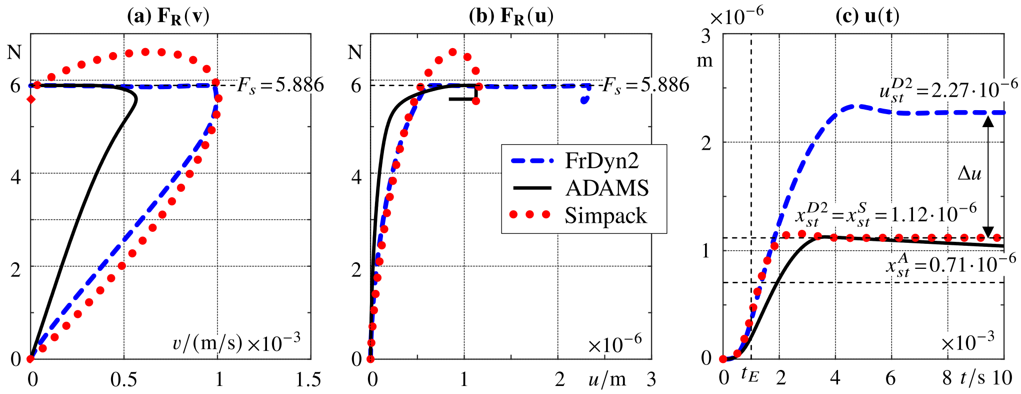


**Figure 8.** Friction force characteristics and body displacements at a step duration of $t_E = 0.001$ s.

The Simpack stick–slip model (dotted red line) generates a time history of the body displacement which approaches the quasi steady state value $u = x_{st} = 1.12{\cdot}10^{-6}$ m with a small overshoot shortly after the step duration of $t_E = 0.001$ s, as seen in Figure 8c. The FrDyn2 model overshoots and partly slides, resulting in displacements at $t > t_E$ which exceed with $u_{st}^{D2} = 2.27{\cdot}10^{-6}$ m the quasi steady state value of $x_{st} = 1.12{\cdot}10^{-6}$ m significantly. Hence, the FrDyn2 model generates a dynamic break-away effect at high frequent excitation loads, which are close (here, 95%) to the static friction force. The time history of the body displacement $u = u(t)$ shows a strange behavior for the Adams stick–slip model, as exemplified by the solid black line in Figure 8c. At first ($t < 3.5{\cdot}10^{-3}$ s), it approaches the quasi steady state value $u = x_{st}^{D2} = x_{st}^{S} = 1.12{\cdot}10^{-6}$ m of the FrDyn2 and Simpack solution and then ($4{\cdot}10^{-3}$ s $< t \le 10{\cdot}10^{-3}$ s) it starts to decrease very slowly, but the simulated time interval $0 \le t \le 10{\cdot}10^{-3}$ s is too short to indicate a limit value.

A simulation with the Adams stick–slip model over a longer time period results in the time history of the body displacement $u = u(t)$, as plotted in Figure 9.

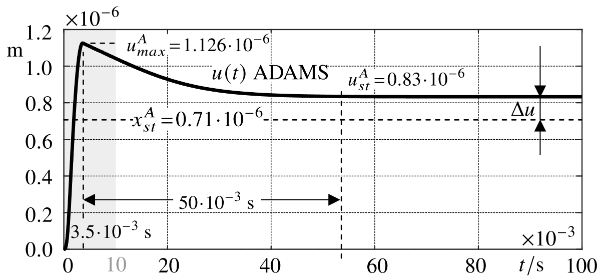


**Figure 9.** Body displacements generated with Adams at a step duration of $t_E = 0.001$ s.

The section shown in Figure 8c has a light gray background. It seems that Adams applies in its stick–slip model different time constants for the increase and decrease in the body displacement $u = u(t)$. The force excitation with a step duration of $t_E = 0.001$ s is much faster than the dynamics of the virtual friction test-bench. The body displacement reaches its maximum value at $t \approx 0.0035$ s, which corresponds to the oscillation period computed in (10). The decay from the maximum displacement to the steady state value takes about 0.05 s, which is fourteen times as much. This strange behavior was also reported in [5], wherein pulse loads are applied to a single mass resting on a horizontal plate. At the end of a series of impulse loads each of magnitude $0.8F_s$, the body is returned to its initial position. However, in the present example, a small but permanent deviation of $\delta u = u^A_{st} - x^A_{st} = 0.12 \cdot 10^{-6}$ m remains, as seen in Figure 9. This indicates that the Adams stick–slip model also tends to partly slip, when high frequent excitation loads close to the static friction force are applied.

## 6. The Festoon Cable System Model

A planar model of a festoon cable system is used in [4] to asses different friction models in a more practical example. The model consists of three cable $m_C$ and two trolley $m_T$ masses, as seen in Figure 10.

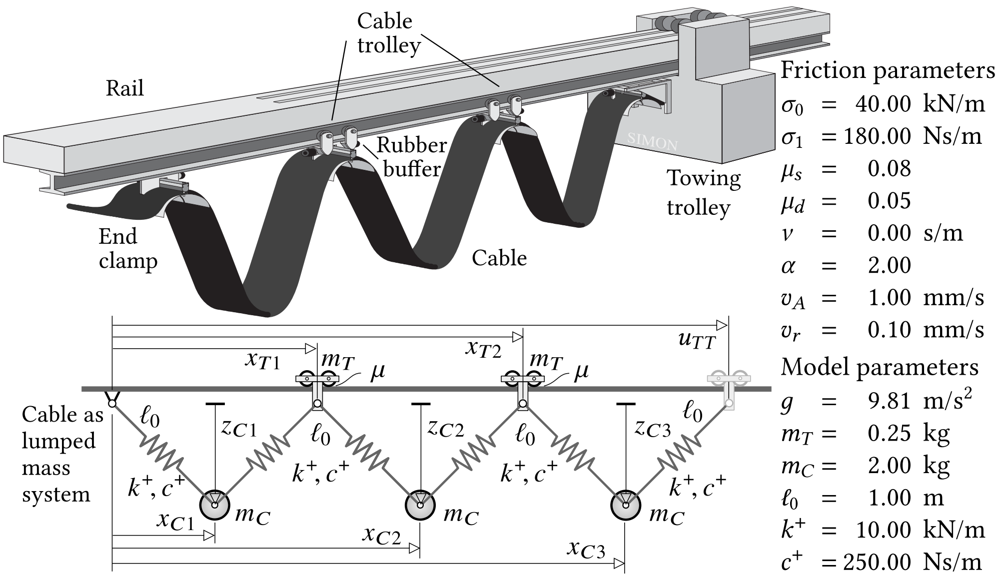


**Figure 10.** Multibody model of a crane festoon system as defined in [4].

At the beginning $(t = 0)$, the towing trolley is fixed at $u_{TT}(0) = 1.5529$ m. The equilibrium position of the cable system places the trolleys at $x_{T1}(0) = u_{TT}(0)/3$, $x_{T2}(0) = 2\,u_{TT}(0)/3$, and locates the cable masses at $x_{C1}(0) = u_{TT}(0)/6$, $x_{C2}(0) = u_{TT}(0)/2$, $x_{C3}(0) = 5\,u_{TT}(0)/6$, as well as $z_{C1}(0) = z_{C2}(0) = z_{C3}(0) = 0.96698$ m. The non-holonomic constraint $\dot{u}_{TT} = v_{TT}$ relates the movements of the towing trolley to a pre-defined velocity profile $v_{TT} = v_{TT}(t)$. The velocity profile, defined by the solid gray lines in Figure 11a, models an extension maneuver, which moves the towing trolley from the initial position $u(t=0) = u_{TT}(0) = 1.5529$ m to a final position of $u_{TT}(t > 5\text{ s}) = 5.553$ m.

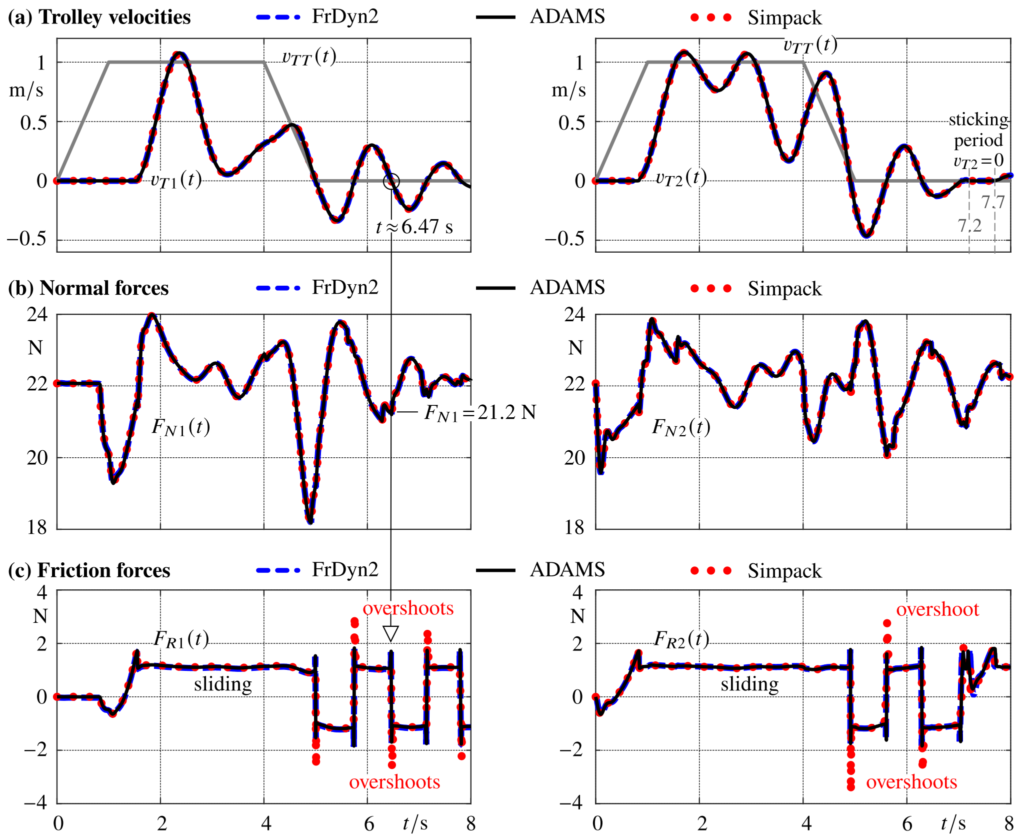


**Figure 11.** Results of a simulated festoon extension maneuver.

The Matlab simulation with the FrDyn2 model generates the output at every simulation step. It applies the Matlab standard solver for stiff differential equations `ode15s` with error tolerances of `RelTol = ` $1{\cdot}10^{-6}$ and `AbsTol = ` $1{\cdot}10^{-9}$.

The Adams and Simpack simulations were performed with an output step size of $\Delta t = 1{\cdot}10^{-4}$ s. The dashed blue, the solid black, and the dotted red lines mark the results obtained by the FrDyn2, the Adams, and the Simpack stick–slip models.

The movement of the towing trolley ends at $t = 5$ s. After that, the trolleys perform to and fro motions which at $t \geq 5$ s are indicated by sign changes in the time histories of trolley velocities $v_{T1}(t)$ and $v_{T2}(t)$. An arrow pointing from $v_{T1}(t \approx 6.47\text{ s}) = 0$ over $F_{N1}$ down to $F_{R1}$ highlights such an event, in particular. The dynamic motions of the cable masses induce variations in the normal forces $F_{N1}$ and $F_{N2}$ acting between the trolleys and the rail, as seen in Figure 11b. The time histories of the velocities $v_{T1}(t)$, $v_{T2}(t)$ and the normal forces $F_{N1}(t)$, $F_{N2}(t)$ generated with FrDyn2 and the stick–slip models of Adams and Simpack match nearly perfectly. However, the time histories of the friction forces $F_{R1}(t)$ and $F_{R2}(t)$ exhibit some discrepancies, as seen in Figure 11c. In particular, when the trolley velocities change their signs or during a sticking period of trolley 2.

The plots in Figure 12 focus on a sign change in the trolley velocity $v_{T1}$ at $t \approx 6.47$ s and a sticking period of trolley 2 in the time interval $7.2\text{ s} \leq t \leq 7.8$ s.

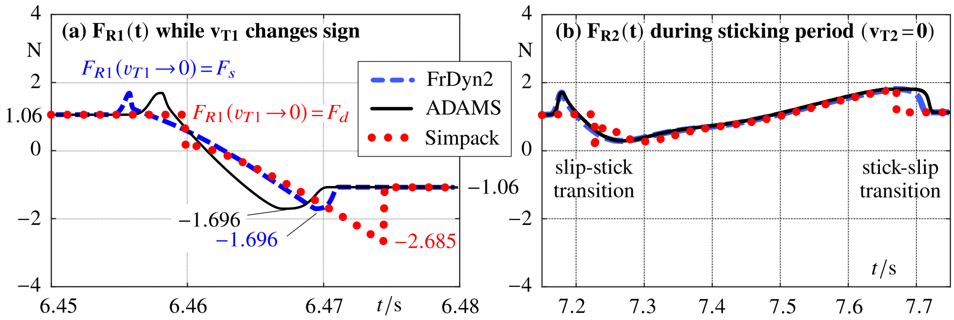


**Figure 12.** Friction forces in specific time intervals.

In the very short time interval 6.45 s $\leq t \leq$ 6.48 s, the normal force between trolley 1 and the rail amounts to $F_{N1} = 21.2$ N, as indicated in Figure 11b. The friction values defined in Figure 10 provide in this case a static friction force of $F_s = \mu_s F_{N1} = 0.08{\cdot}21.2 = 1.696$ N and a dynamic friction force of $F_d = \mu_d F_{N1} = 0.05{\cdot}21.2 = 1.06$ N. At times $t = 6.45$ s and $t = 6.48$ s, the first trolley is in a full sliding mode, as indicated in Figure 12a by the friction forces $F_{R1}(t = 6.45\text{ s}) = +F_d = 1.06$ N and $F_{R1}(t = 6.48\text{ s}) = -F_d = -1.06$ N. These sliding modes are perfectly reproduced by the friction models under consideration. Shortly before the sign change in the trolley velocity, the friction forces computed by the FrDyn2 and the Adams stick–slip model make use of the Stribeck effect, which models a velocity-dependent transition from the static to the dynamic friction force and vice versa. The Simpack stick–slip model approaches $v_{T1} \to 0$ with the dynamic force value and does not reproduce a potential velocity-dependent increase in the friction force. The FrDyn2 and the Simpack stick–slip models describe the friction force at $v = 0$ by a linear spring, whereas the Adams stick–slip model uses a nonlinear approach. That is why, the FrDyn2 model (dashed blue line) corresponds in the time interval 6.46 s $\leq t \leq$ 6.47 s more to the Simpack (dotted red line) than to the Adams stick–slip model (solid black line). The friction forces of the FrDyn2 and the Adams stick–slip models are limited to the static value $|F_{R1}| \leq F_s$, which results in $F_{R1}^{D2} = F_{R1}^{A} = -1.696$ N at $t \approx 6.47$ s. However, The Simpack stick–slip model overshoots and produces the peak value of $F_{R1}^{S} = -2.685$ N, which exceeds by nearly 60% the static friction force $F_s$ or $-F_S$, respectively.

The sticking period 7.2 s $\leq t \leq$ 7.8 s is represented rather similarly by the friction models under consideration, as seen in Figure 12b. Again, the FrDyn2 and the Adams stick–slip models increase the friction forces from the dynamic to the static value when approaching stand-still at $t \approx 7.2$ s. However, the small time delay of the peak values visible in Figure 12a is not noticeable due to the large time interval applied in this plot. The Simpack stick–slip model is based on Coulomb's approach, which results in the discontinuities at the slip–stick and stick–slip transitions in the dotted red line.

## 7. Discussion

The present manuscript compares a second-order dynamic friction (FrDyn2) model with the commercial stick–slip models of Adams and Simpack. The comparison is performed here with a virtual friction test-bench and a more practical model of a festoon cable system.

All models can maintain long-term stick. The FrDyn2 model corresponds partly to the Adams and partly to the Simpack stick–slip models. The FrDyn2 and the Adams stick–slip models show dynamic break-away effects at high frequent excitation loads, which are close (here 95%) to the static friction force. The Simpack stick–slip model avoids dynamic break-away effects by overshoots in the friction force that far exceed the static friction force. Adams models the decay of a friction force overshoot much slower than the increase, whereas the FrDyn2 model considers in both cases the dynamics of the fictitious bristle.

The FrDyn2 model is based on a fictitious bristle characterized by its mass, stiffness, and damping. The fictitious mass of the bristle is automatically adjusted to the stiffness and damping parameters. The pre-defined friction force characteristic $F_R = F_R(v)$ is described here by piecewise-defined polynomials but not limited to this. The bristle parameters can easily be derived from estimated reference friction forces and estimated bristle deflections.

The results obtained by the FrDyn2 model are reliable and based on the physical nature of the friction model approach, which makes the second order dynamic friction model a suitable alternative to commercial stick–slip models.

The dynamics of the FrDyn2 model are governed by the friction characteristics. A rapid transition from the static to the dynamic force, modeling the Stribeck effect, results in a stiff performance of the FrDyn2 model. But the shifted regularized friction characteristic is completely continuous and smooth, which makes it possible to apply any standard stiff ode solver for a multibody system which incorporates the FrDyn model.

Future works will implement the FrDyn2 model as an external force element in Adams and Simpack. Then, the run-time performance of this model can also be compared to the corresponding commercial stick–slip models. The influence of the velocities $v_s$ and $v_d$, which model the regularization and the Stribeck effect on the results and on the run-time performance, will be studied in addition.

**Author Contributions:** Both authors validated the results, wrote, and reviewed the manuscript. G.R. developed the second-order dynamic friction model, performed the Matlab Simulations, and produced the figures. M.S. performed the simulations with the Adams and the Simpack stick–slip models. All authors have read and agreed to the published version of the manuscript.

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**Data Availability Statement:** Appendix A provides a Matlab script and functions which realize the simple friction test-bench including the second-order dynamic friction model.

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**Conflicts of Interest:** The authors declare no conflict of interest.

## Appendix A. Virtual Friction Test-Bench Realized in Matlab

**Listing A1.** Virtual friction test-bench exposed to step-like force inputs.

```
% This Matlab script is part of the paper
% Rill, G.; Schuderer, M. A second order dynamic friction model.
% Modelling 2023, 4(3), 366-381; https://doi.org/10.3390/modelling4030021
% It performs simulations with the virtual friction test-bench operated with
% the second order dynamic friction model as defined by Figure 2, described
% in Section 5, and provided by the Matlab function dyn_fr_test_bench.
% It displays the time histories of the excitation force f and the dynamic
% friction force fr generated by the second order dynamics friction model
% as displayed in Figure 6 by the dashed blue lines. The displacements u,
% the velocity v, and the acceleration a of the body are plotted in addition.

clear, close all

% define options for the Matlab ode solver ode15s
opts = odeset(RelTol= 1.e-6, AbsTol=1.e-9);

% define force amplitude relative to the static friction force
amp = 0.95;

% define 4 different durations of the step force excitation
te  = [ 0.1, 0.01, 0.001, 0.0001 ]; % in s

% define body parameters as specified in Figure 2c
bp.g = 9.81;           % gravity in m/s^2
bp.m = 1;              % mass in kg

% define friction model parameters as specified in Figure 2c
fp.sigma0 = 5.0e6;    % stiffness of fictitious bristle in N/m
fp.sigma1 = 3192.75;  % bristle damping in N/(m/s)
fp.mus    = 0.6;      % static friction
fp.vs     = 1e-3;     % velocity where mu(vs)=mus
fp.mud    = 0.3;      % dynamic friction
```

```
fp.vd     = 10e-3;    % velocity where mu(vd)=mud

% compute static friction force
fs = fp.mus*bp.m*bp.g;

% compute steady state bristle deflection at f=fs
zb = fs/fp.sigma0;

% define step force excitation with different durations
sp.ts = 0.0;          % time in s where step starts
sp.te = [];           % time(s) in s where step ends to be defined
sp.fa = amp*fs;       % compute amplitude of excertation force in N

% solve the dynamics of the virtual friction test-bench with the
% second order dynamic friction model for different step durations

for k = 1:4  % loop according to the number of step durations

  % assign actual time where step ends to element of structure
  sp.te = te(k);

  % set trivial initial conditions
  t0 = 0;  x0 = [ 0; 0; 0; 0 ];

  % define an appropriate time interval and perform simulation
  tspan = [ 0,  (sp.te + 0.75*sp.te-sp.ts + 15*fp.sigma1/fp.sigma0) ];
  [t,x] = ode15s( @(t,x) dyn_fr_test_bench(t,x,bp,fp,sp),tspan,x0,opts );

  % get excitation force f, friction force fr, and body acceleration a
  f=zeros(size(t)); fr=f; a=f;
  for i=1:length(t)
    [ dxdt, out ] = dyn_fr_test_bench(t(i),x(i,:).',bp,fp,sp);
    f(i) = out.f;  fr(i) = out.fr;  a(i) = dxdt(2);
  end

  % generate plots
  subplot(4,4,k) % f(t) and fr(t) as part of Figure 6
    plot(t,f,'k',LineWidth=1),hold on,grid on
    plot(t,fr,':b',LineWidth=3),xlabel('t/s'),ylabel('N')
    title(['Forces @ tE=',num2str(sp.te),' s']),ylim([0,1.05*fs])
    legend('F(t)','Fr(t)','Location','southeast')
  subplot(4,4,k+4)  % body displacement u(t) as additional information
    plot(t,x(:,1),'r',LineWidth=2),grid on,xlabel('t/s'),ylabel('u/m')
    title(['Body position @ tE=',num2str(sp.te),' s']),ylim([0,zb])
  subplot(4,4,k+8)  % body velocity v(t) as additional information
    plot(t,x(:,2),'g',LineWidth=2),grid on,xlabel('t/s'),ylabel('v/(m/s)')
    title(['Body velocity @ tE=',num2str(sp.te),' s'])
  subplot(4,4,k+12) % body acceleration a(t) as additional information
    plot(t,a,'m',LineWidth=2),grid on,xlabel('t/s'),ylabel('a/(m/s^2)')
    title(['Body acceleration @ tE=',num2str(sp.te),' s'])

end
```

**Listing A2.** Dynamics of a virtual friction test-bench operated with the FrDyn2 model.

```
function ...                  % === out (<--) and in (-->) in SI-Units
[ xdot ...                    % <-- state derivatives
, out ...                     % <-- additional output structure
] = dyn_fr_test_bench ...     % === dynamics of a body on a fixed plate
( t ...                       % --> time
, x ...                       % --> states of virtual friction test-bench
, bp ...                      % --> body parameter structure
, fp ...                      % --> friction model parameter structure
, sp ...                      % --> simulation control parameter structure
)
% This Matlab function is part of the paper
% Rill, G.; Schuderer, M. A second order dynamic friction model.
% Modelling 2023, 4(3), 366-381; https://doi.org/10.3390/modelling4030021
% It computes the dynamics of the virtual friction test-bench and
% the dynamics of the second order friction model exposed to a step-like
% excitation force as defined by Figure 2
% The vector x = [ xb; vb; s ] collects the states of the body (xb,vb)
% and the states s = [ zb, zbdot ] of the fictitios bristle.
% The Matlab functions Step3 and FrDyn2 provide the step-like
% force excitation and the dynamic friction force
```

```
% extract body velocity and bristle states
  udot = x(2);    s = x(3:4);

% compute external force applied step-like to body
  f = Step3( t, sp.ts , 0, sp.te, sp.fa );

% normal force
  fn = bp.m*bp.g;

% contact velocity (body versus plate)
  vc = udot;

% dynamic friction force generated by the FrDyn2 model
  [sdot,fr] = FrDyn2( fn, vc, s, fp );

% compute acceleration of body according to
% free body diagram in Figure 2a and Equation (7)
  a = ( f - fr  ) / bp.m;

% derivatives of body and bristle states
  xdot = [ udot; a;  sdot ];

% excitation and friction forces as elements of the output structure
  out.f=f;  out.fr=fr;

end
```

**Listing A3.** Smoothed step by a third-order polynomial.

```
function y = Step3(x,x0,y0,xS,yS)
% This Matlab function provides a cubic polynomial y=y(x), which
% realizes a smooth step starting at y(x0)=y0 and ending at y(xS)=yS
% where dydx(x0)=0 and dydx(xS)=0 hold in addition.

% defaults
  if x < x0, y = y0; return, end
  if x > xS, y = yS; return, end

% cubic polynomial as smooth step approximation
  if xS > x0
    xi = (x-x0)/(xS-x0);
    y  = y0 + (yS-y0)*(3-2*xi)*xi^2;
  end

end
```

**Listing A4.** Second-order dynamic friction model including shifted regularization.

```
function ...      % === out (<--) and in (-->) in SI-Units
[ sdot ...        % <-- derivatives of bristle states
, fr ...          % <-- friction force
] = FrDyn2 ...    % === second order dynamic friction model
( fn ...          % --> normal force
, vc ...          % --> contact velocity (body versus body)
, s ...           % --> bristle states [ z; zdot ]
, fp ...          % --> friction model parameter structure
)
% This Matlab function is part of the paper
% Rill, G.; Schuderer, M. A second order dynamic friction model.
% Modelling 2023, 4(3), 366-381; https://doi.org/10.3390/modelling4030021
% It computes the dynamics of the second order friction model

% defaults for the derivatives of the bristle states and the friction force
  sdot  = [ 0; 0 ];  fr = 0;

% extract bristle states (displacement z and velocity zdot)
  z = s(1);   zdot = s(2);

% friction characteristics with shifted regularization

  if fn > 0  % perform comptation only if contact is detected

  % shift characteristics according to parabolic regularisation
```

```
    f0 = fp.sigma0*z + fp.sigma1*vc;     % Equation (4)
    dis = 1.0 - abs(f0)/(fp.mus*fn);
    if dis < 0;  dis=0;  end
    vshift = fp.vs * ( 1.0 - sqrt(dis) ); % Equation (5)
    if f0 < 0;  vshift=-vshift;  end

  % slidung velocity including appropriate velocity shift
    vs  = ( vc - zdot )  +  vshift;       % according to Figures 1c and 2b
    vsa = abs(vs);

  % force characteristic according to Figs. 1b and 1c
    if vsa <= fp.vs                       % parabola
      xi = vsa/fp.vs;
      fr = xi*(2.0-xi)*fp.mus*fn;
    elseif vsa > fp.vs  &&  vsa < fp.vd   % cubic transition
      xi = ( vsa-fp.vs ) / (fp.vd-fp.vs);
      mu = fp.mud + (fp.mus-fp.mud)*(1-xi^2*(3-2*xi));
      fr = mu*fn;
    else
      fr = fp.mud*fn;                     % straight line for full sliding
    end

    % adjust sign
    if vs < 0 ;  fr = -fr;  end

  end

% visco-elastic bristle force as defined by Equation (4)
  fb = fp.sigma0*z + fp.sigma1*zdot;

% bristle velocity
  sdot(1) = zdot;

% inverse of fictitious mass and bristle acceleration from Equations (2) and (3)
  mbi = 4*fp.sigma0 / fp.sigma1^2;
  sdot(2) = ( fr - fb ) * mbi;

end
```

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